Probing nuclear sizes of unstable nuclei with total reaction cross sections

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Systematic analysis of total reaction cross sections

- Total reaction cross section: σ_R ⇔ Size of nucleus
- Recent advances in RI beam facilities
  - …, C, O → Ne, Mg, … isotopes
  - Special features in unstable nuclei
    - Deformation, halo, skin, etc.

- Systematic study of σ_R for unstable nuclei
  - Reaction: Glauber theory
    - Microscopic high-energy scattering theory
    - Optical Limit Approximation, Input: nuclear densities
  - Structure: Skyrme-Hartree-Fock method on 3D mesh
    - Nuclear deformation
    - Cover a wide mass range

How to probe neutron-skin thickness?

- Neutron skin-thickness $\delta(N,Z) = r_n(N,Z) - r_p(N,Z),$
  - Nuclear isovector size properties
  - Close connection to the equation of state (EOS) of asymmetric nuclear matter and mass of a neutron star
  - Poorly known, difficult to probe
    - PREX: S. Abrahamyan et al., PRL108, 112502 (2012)
    - (p,p'), polarizability of ^{208}Pb

      A. Tamii et al., PRL107, 062402 (2011)
    - Proton elastic scatterings

      S. Terashima et al., Phys. Rev. C 77, 024312 (2008)

      J. Zenihiro et al., Phys. Rev. Lett. 107, 062502 (2011)

- Electron-scattering of unstable nuclei (plans: SCRIT, ELISe)

  T. Suda, M. Wakasugi, PPNP, 55 417 (2005), A.N. Antonov et al., NIMA637, 60 (2011)
- **Total reaction cross section** ⇔ Size of nucleus
  - Easy to measure, applicable to almost all nuclei

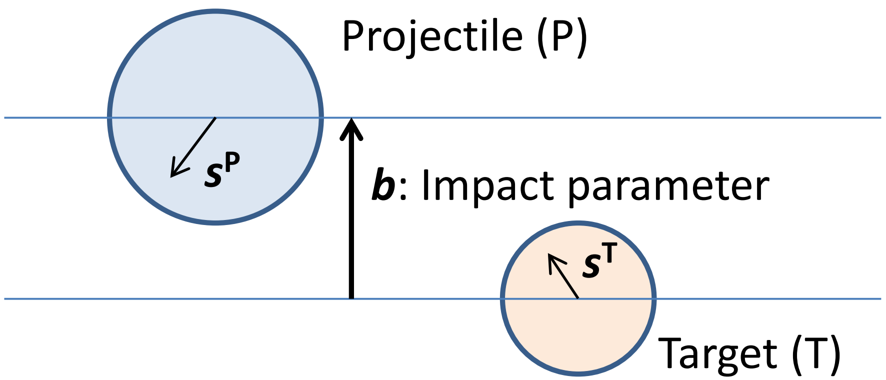
    → Possible measure of skin-thickness?
  - Charge changing cross sections
    - ^{12}C target is promising

      Y. Suzuki, WH et al., Phys. Rev. C94, 011692(R) (2016)

Reaction cross section in the Glauber model

Final state wave function

$$\Psi_f = e^{i\chi(\boldsymbol{b})}\Psi_i$$



Total reaction cross section

$$\sigma_R = \int d\boldsymbol{b}\,(1 - |e^{i\chi(\boldsymbol{b})}|^2)$$

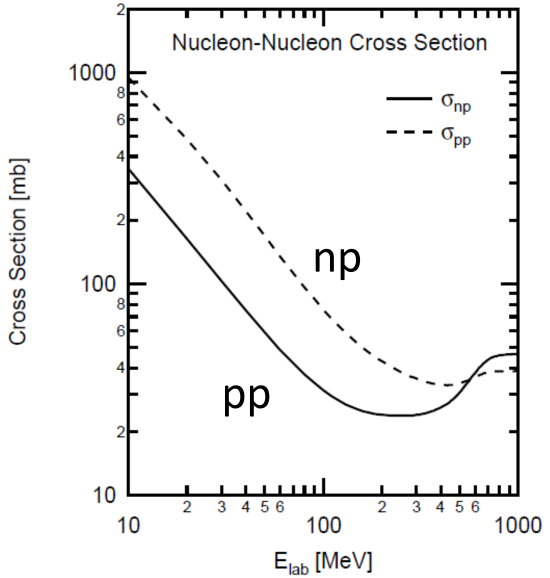
Phase shift function

$$e^{i\chi(\boldsymbol{b})} = \langle \Phi_0^P \Phi_0^T | \prod^{A_P} \prod^{A_T} [1 - \Gamma_{NN}(\boldsymbol{s}_i^P - \boldsymbol{s}_j^T + \boldsymbol{b})] | \Phi_0^P \Phi_0^T \rangle$$

Profile function $\Gamma_{NN}(\boldsymbol{b}) = \frac{1 - i\alpha}{4\pi\beta}\sigma_{NN}^{\mathrm{tot}}\exp\left(-\frac{b^2}{2\beta}\right)$

α, β: determined so as to reproduce the NN scattering

B. Abu-Ibrahim et al., PRC77, 034607 (2008)



Phase-shift function: Many-body operator

Approximate using a cumulant expansion

→ Input: Nuclear density and Profile function

no adjustable parameters

OLA vs. NTG: ^{12}C-^{12}C collision

Optical Limit Approximation (OLA)

$$e^{i\chi_{\rm OLA}(\boldsymbol{b})} = \exp\left[-\iint d\boldsymbol{r}^P d\boldsymbol{r}^T \rho_P(\boldsymbol{r}^P)\rho_T(\boldsymbol{r}^T)\Gamma_{NN}(\boldsymbol{s}^P - \boldsymbol{s}^T + \boldsymbol{b})\right]$$

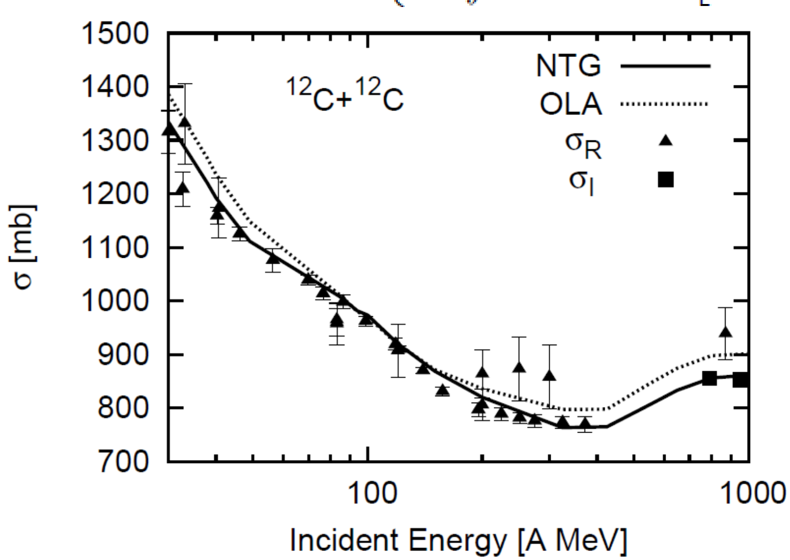
Nucleon-Target profile function in the Glauber model (NTG)

Multiple scattering effect is included

B. Abu-Ibrahim and Y. Suzuki, PRC62, 034608 (2000)

$$\Gamma_{\rm NT}(\boldsymbol{b}) = 1 - \langle\Phi_0^{\rm T}|\prod_{j\in{\rm T}}[1-\Gamma(\boldsymbol{b}-\boldsymbol{s}_j^{\rm T})]|\Phi_0^{\rm T}\rangle \approx 1 - \exp\left[-\int d\boldsymbol{r}^{\rm T}\rho_{\rm T}(\boldsymbol{r}^{\rm T})\Gamma(\boldsymbol{b}-\boldsymbol{s}^{\rm T})\right]$$

$$e^{i\chi_{\rm NTG}(\boldsymbol{b})} = \exp\left\{-\frac{1}{2}\int d\boldsymbol{r}\rho_P(\boldsymbol{r})\exp\left[1-\exp\left(-\int d\boldsymbol{r}'\rho_T(\boldsymbol{r}')\Gamma_{NN}(\boldsymbol{s}^P-\boldsymbol{s}^T+\boldsymbol{b})\right)\right]\right\}$$
$$\times\exp\left\{-\frac{1}{2}\int d\boldsymbol{r}\rho_T(\boldsymbol{r})\exp\left[1-\exp\left(-\int d\boldsymbol{r}'\rho_P(\boldsymbol{r}')\Gamma_{NN}(\boldsymbol{s}^T-\boldsymbol{s}^P+\boldsymbol{b})\right)\right]\right\}$$



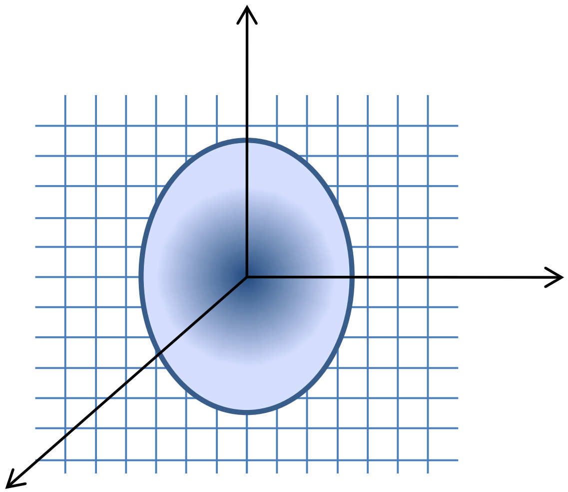
Reproduce the measured values for a wide energy range

W.H., Y. Suzuki, B. Abu-Ibrahim, A. Kohama, PRC75, 044607 (2007).

Systematic analysis of unstable nuclei

- Skyrme-Hartree-Fock on 3D mesh
  - Skyrme interactions
    - SkM*: well account for deformations
    - SLy4: total binding energies
  - One-body density ρ(r)
    - O, Ne, Mg, Si, S isotopes

      $\rho(r)=\int\rho(\mathbf{r})d\Omega$
    - Center of mass motion

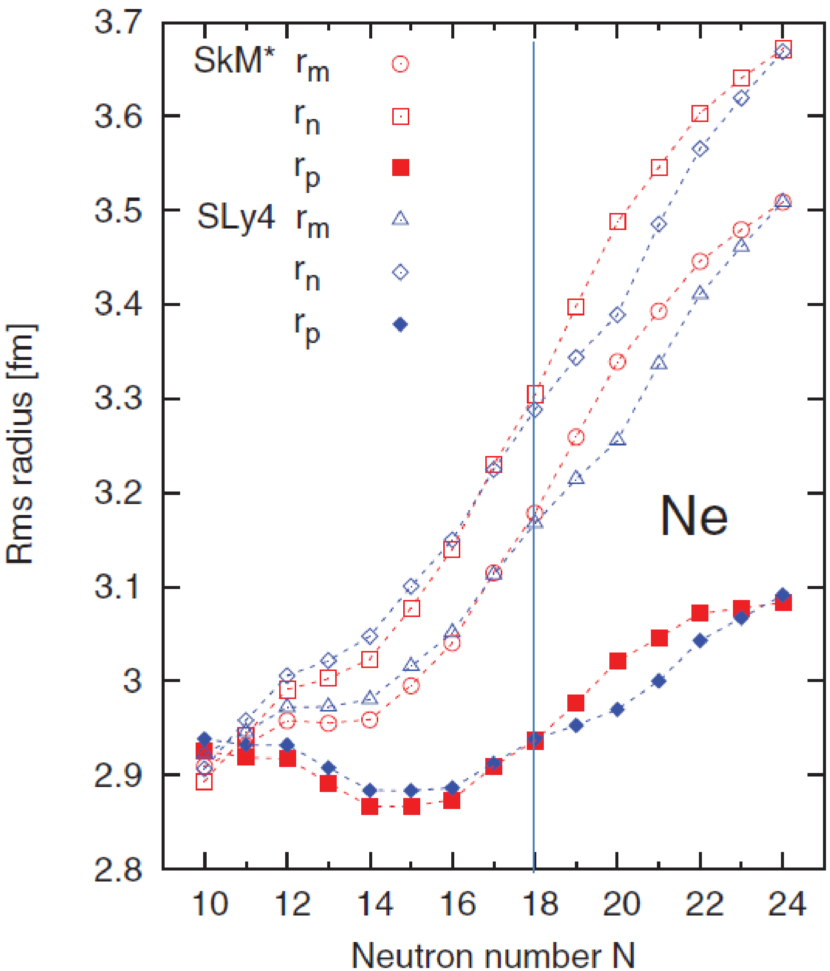


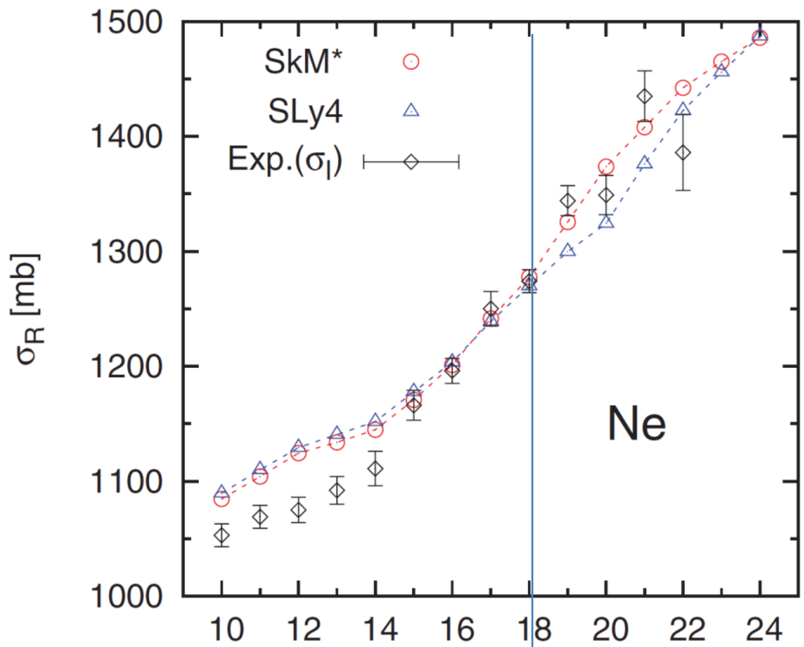
$$\int d\boldsymbol{r} x\rho(\boldsymbol{r}) = \int d\boldsymbol{r} y\rho(\boldsymbol{r}) = \int d\boldsymbol{r} z\rho(\boldsymbol{r}) = 0,$$

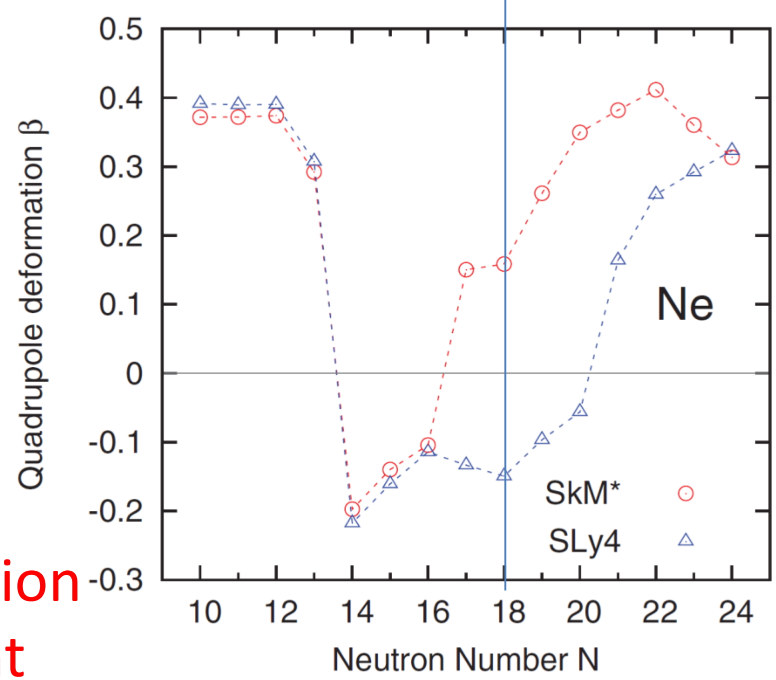
$$\int d\boldsymbol{r} xy\rho(\boldsymbol{r}) = \int d\boldsymbol{r} yz\rho(\boldsymbol{r}) = \int d\boldsymbol{r} zx\rho(\boldsymbol{r}) = 0.$$

Deformation effect in Ne isotopes

Exp. M. Takechi et al., MPLA25, 1878 (2010)





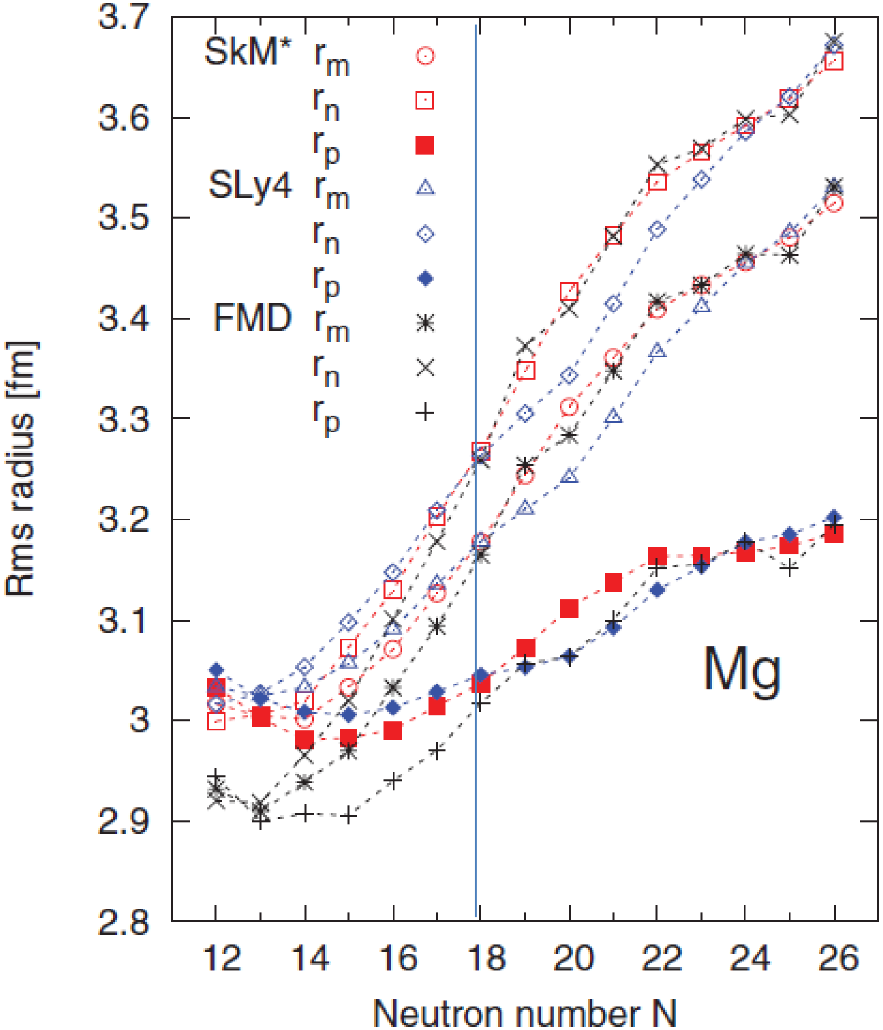


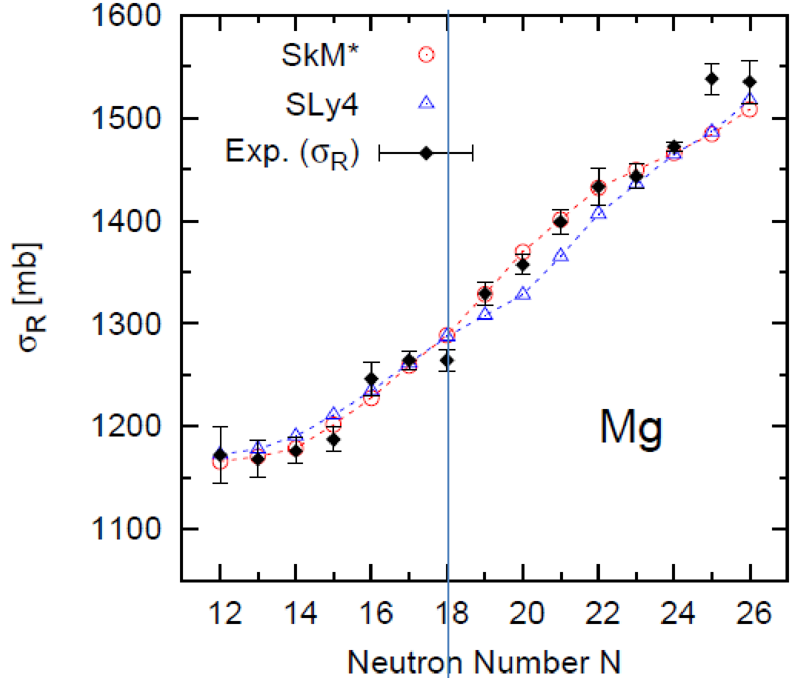
The kink behavior (N～20): deformation

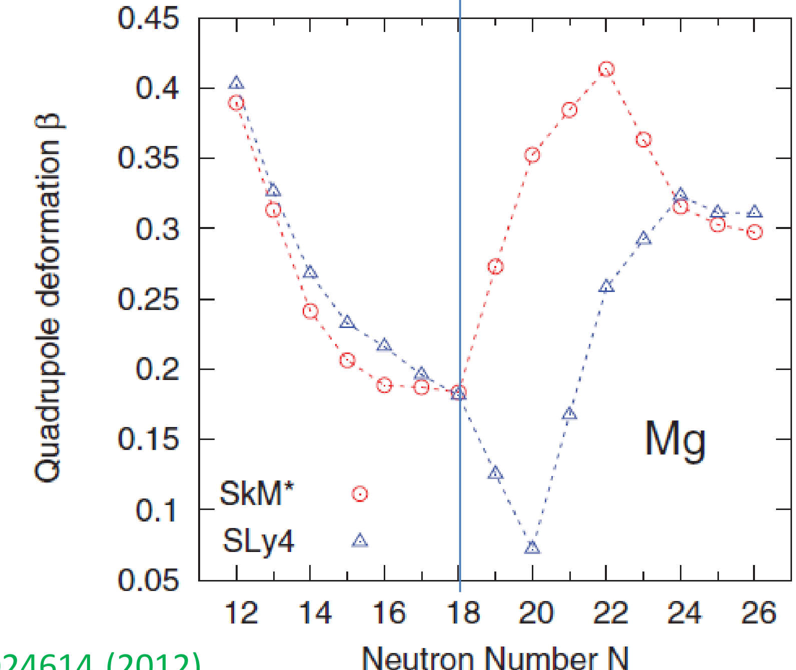
Good agreement with the experiment

New σ_R data available: PRC89, 044610 (2014)

Mg isotopes







Similar kink behavior to that of Ne

Good agreement with the recent experimental cross sections

W.H., I. Inakura, T. Nakatsukasa, Y. Suzuki, PRC86, 024614 (2012)

Probing neutron-skin thickness with σ_R

- Total reaction cross section, σ_R: Glauber model

  ^{1}H and ^{12}C target

  Reaction cross section: $\sigma_R(E)=\pi[R_P+R_T+\Delta(E)]^2$

  → "reaction radius" $a_R=(\sigma_R(E)/\pi)^{1/2}$

- Density distribution : Skyrme-Hartree-Fock on 3D coordinate space

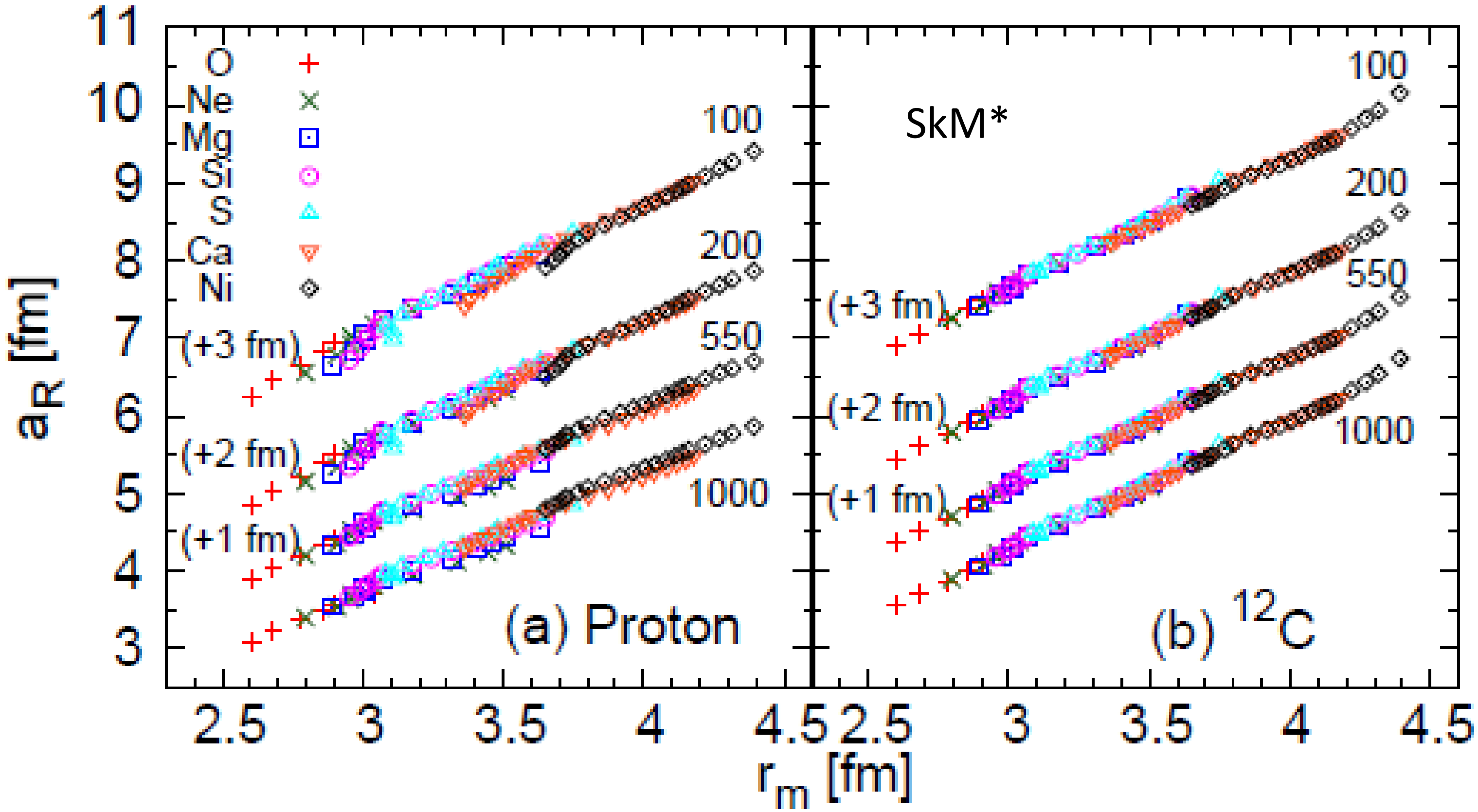
  Systematic calculation for a wide mass range

  → 91 even-even nucleus with A=14-86, Z=8-16, 20, 28 for SkM* and SLy4

Reaction radius vs. matter radius

$$a_R(N,Z,E,T) = \sqrt{\sigma_R(N,Z,E,T)/\pi},$$

$$r_m(N,Z) = \sqrt{\frac{Z}{A}r_p^2(N,Z) + \frac{N}{A}r_n^2(N,Z)},$$



- Linearity on the matter radius
- Scattered distributions for ^{1}H target

Δa_R vs. skin thickness δ

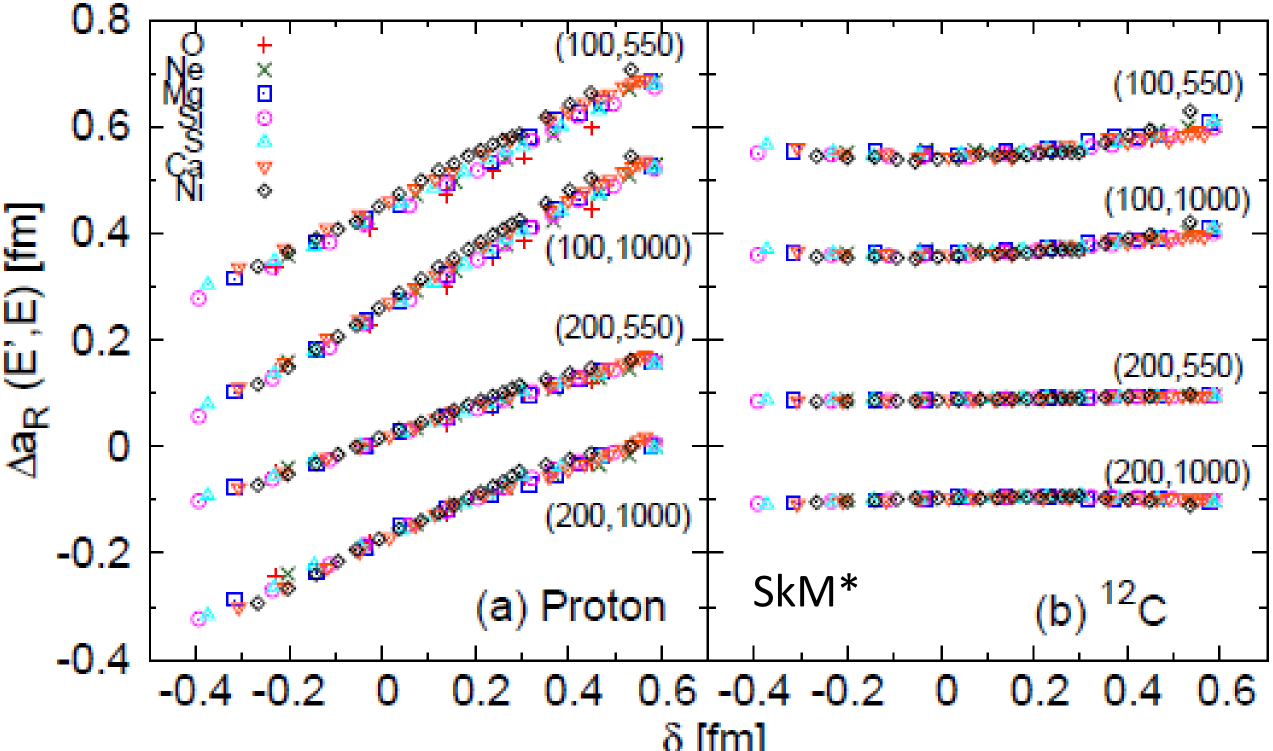
Difference between two reaction radii with different incident energies

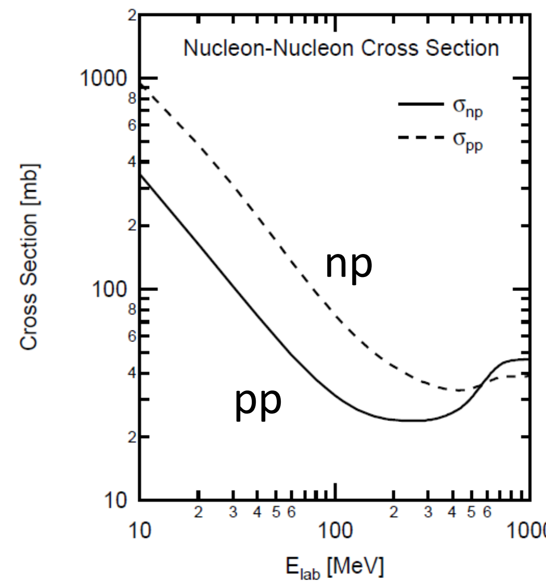
$$\Delta a_R(N, Z, E', E) = a_R(N, Z, E') - a_R(N, Z, E)$$

Four energies are chosen: 100, 200, 550, 1000 MeV/u

Δa_R (E'<E) ^{1}H: Linearities for all energy sets → sensitivity to δ

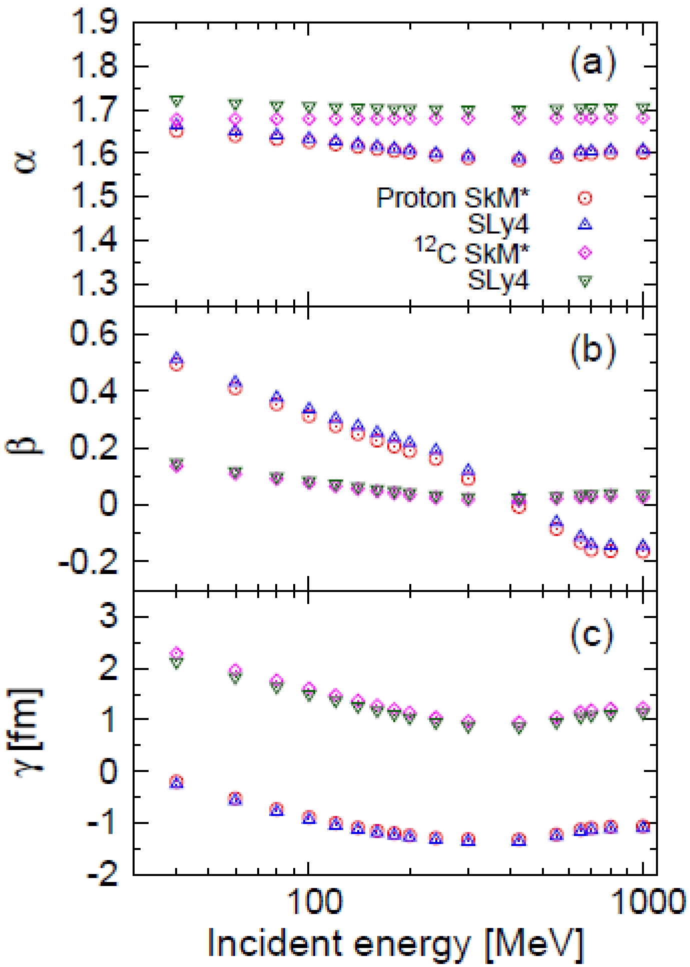
^{12}C: Constant behavior → No sensitivity to δ





Least squares fitting of reaction radius

$$a_R(N, Z, E) = \alpha(E) r_m(N, Z) + \beta(E)\delta(N, Z) + \gamma(E)$$



- Weak energy dependence of α(E)
- β(E): Proton is sensitive to δ
  No sensitivity to δ for ^{12}C
- γ(E): Similar trend
- Universal functions
  - No isotope dependence
  - Two Skyrme interactions give the virtually the same results

We can extract neutron-skin thickness and matter radius by measuring σ_R at different energies.

E< 200 and E>550 are recommended

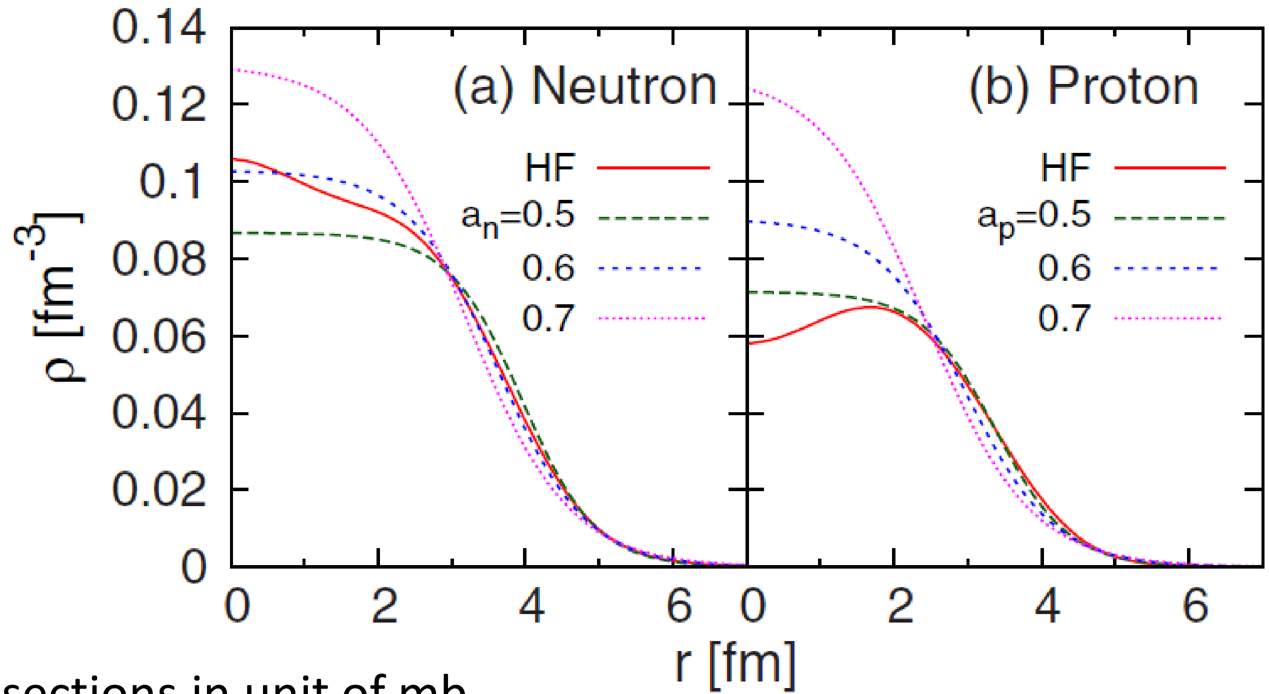
Possible ways to extract matter and skin thickness

- Measure the cross sections on ^{1}H at different E
  - Assume model density, e.g. Fermi-type, etc.
  - Do cross section calculations
  - Determine the parameters which reproduce the two cross section simultaneously
- Or measure the cross sections on ^{1}H and ^{12}C

Sensitivity to nuclear distributions

Fermi-type densities
$\rho(r)=\rho_0/[1+\exp\{(r-R)/a]$
vs. HF densities

All densities gives the same r_m and δ



p-^{40}Si total reaction cross sections in unit of mb

$(a_p, a_n)\backslash E$	100	120	140	160	200	300	425	550	800	1000
(0.5, 0.5)	742	700	667	642	607	564	550	575	615	619
(0.6, 0.6)	752	706	672	646	609	565	551	577	619	622
(0.7, 0.7)	759	711	675	648	609	564	549	576	621	623
(0.5, 0.7)	756	708	673	646	608	563	549	575	617	620
HF(SkM*)	747	703	670	644	608	565	551	576	617	620

Good agreement with HF

Error at most 1.6%

Fermi-type analysis: ^{12}C target case

p-^{40}Si total reaction cross sections in unit of b

$(a_p, a_n)\backslash E$	100	140	160	200	300	425	550	800	1000
(0.5, 0.5)	1.73	1.62	1.58	1.52	1.45	1.44	1.48	1.55	1.56
(0.6, 0.6)	1.78	1.66	1.62	1.56	1.48	1.47	1.51	1.59	1.60
(0.7, 0.7)	1.84	1.71	1.67	1.60	1.52	1.51	1.55	1.64	1.65
(0.5, 0.7)	1.80	1.68	1.64	1.57	1.49	1.48	1.53	1.61	1.62
HF(SkM*)	1.74	1.63	1.59	1.53	1.45	1.44	1.49	1.56	1.57

- ^{12}C probes more details of the size properties
- Uncertainty ～5% → the former way is better

Nuclear sizes of heavier nuclei

- Extension to medium to heavier nuclei
  - Systematic analysis
    - Skyrme-Hartree-Fock+BCS on 3D mesh
      - 103 species $^{40\text{-}60}$Ca, $^{56\text{-}84}$Ni, $^{80\text{-}122}$Zr, $^{100\text{-}140}$Sn, $^{156\text{-}196}$Yb, $^{190\text{-}214}$Pb
      - 3 kinds of Skyrme interactions (SkM*, SLy4, SkI3)
  - Coulomb breakup effect?
  - Does "Universality" still hold?

$$a_R(N, Z, E) \simeq \alpha(E) r_m(N, Z) + \beta(E)\delta(N, Z) + \gamma(E),$$

$\sigma_R(N, Z, E)$ → "Reaction radius" $a_R(N, Z, E) = \sqrt{\sigma_R(N, Z, E)/\pi}$

Estimate of Coulomb breakup

Divergent problem of Glauber theory at large distances ← adiabatic approx.
Systematic analysis → Simple method   Equivalent Photon Method

C. A. Bertulani, G. Baur, Phys. Rep. 163 299, (1988).

- Dissociation from the Coulomb source(Target; charge $Z_T e$)
- Widely used for Coulomb breakup of halo nuclei

Reaction probability $P_C(\boldsymbol{b}) = \int_0^\infty d\omega N(\boldsymbol{b},\omega)\sigma_\gamma(\omega)$

Photon number $N(\boldsymbol{b},\omega) = \frac{Z_T^2 e^2}{\pi^2 \hbar c}\left(\frac{c}{v}\right)^2 \frac{\xi^2}{\omega b^2}\left[K_1^2(\xi) + \frac{1}{\gamma^2}K_0^2(\xi)\right]$

$\xi = b\omega/(\gamma v)$

Electric dipole (E1) responses obtained with Canonical-basis Time-Dependent Hartree-Fock-Bogoliubov (Cb-TDHFB)
S. Ebata et al., Phys. Rev. C 82, 034306(2010)

Photoabsorption cross section
$$\sigma_\gamma(\omega) = \frac{16\pi^2}{9}\frac{\omega}{c}\frac{dB(E1)}{dE}$$

Coulomb breakup cross section

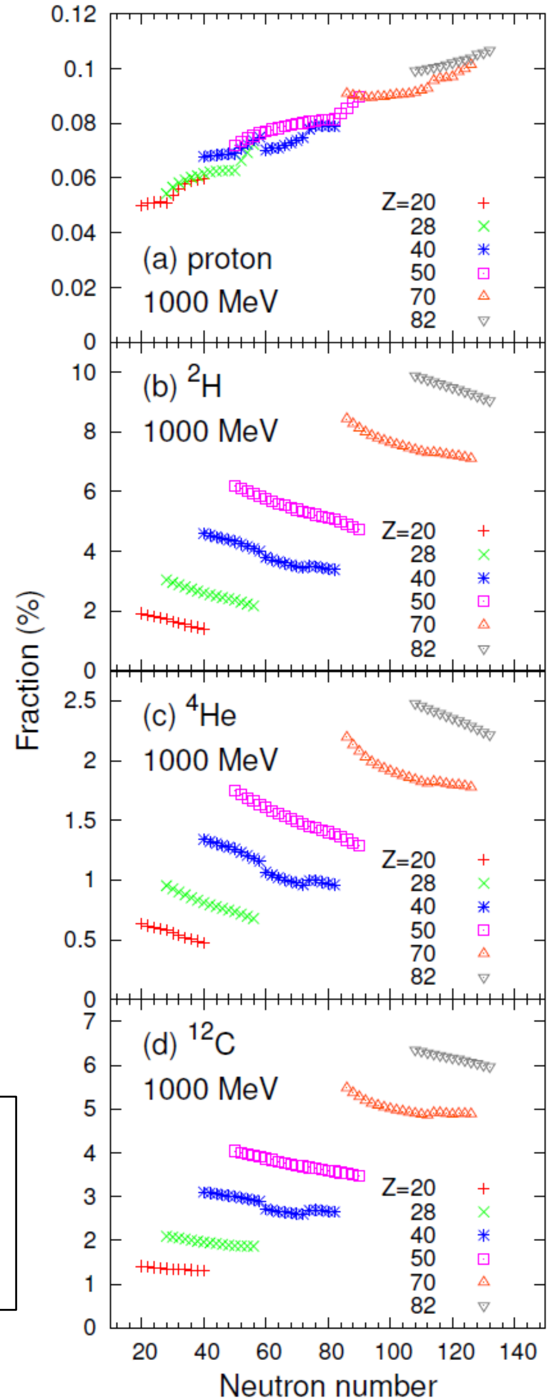
$$\sigma_C = \int_{|b|\geq b_{\min}} d\boldsymbol{b} P_C(\boldsymbol{b}) \qquad b_{\min} \sim \sqrt{5/3}(r_m^P + r_m^T)$$

Interference term → small   B. Abu-Ibrahim, Y. Suzuki, PRC62, 034608 (2000)

Nuclear breakup ＋Coulomb breakup of projectile and target

Contribution of Coulomb breakup (E1 approximation)

- Fraction of Coulomb breakup cross section to total reaction cross sections (σ_C/σ_R)
  - Proton target (less than about 0.1%)
  - ^{12}C target (at most 7%)
  - ^{2}H target (at most 10%)
    - Target excitation dominance
  - ^{4}He target (at most 2.5%)
    - Target preparation?

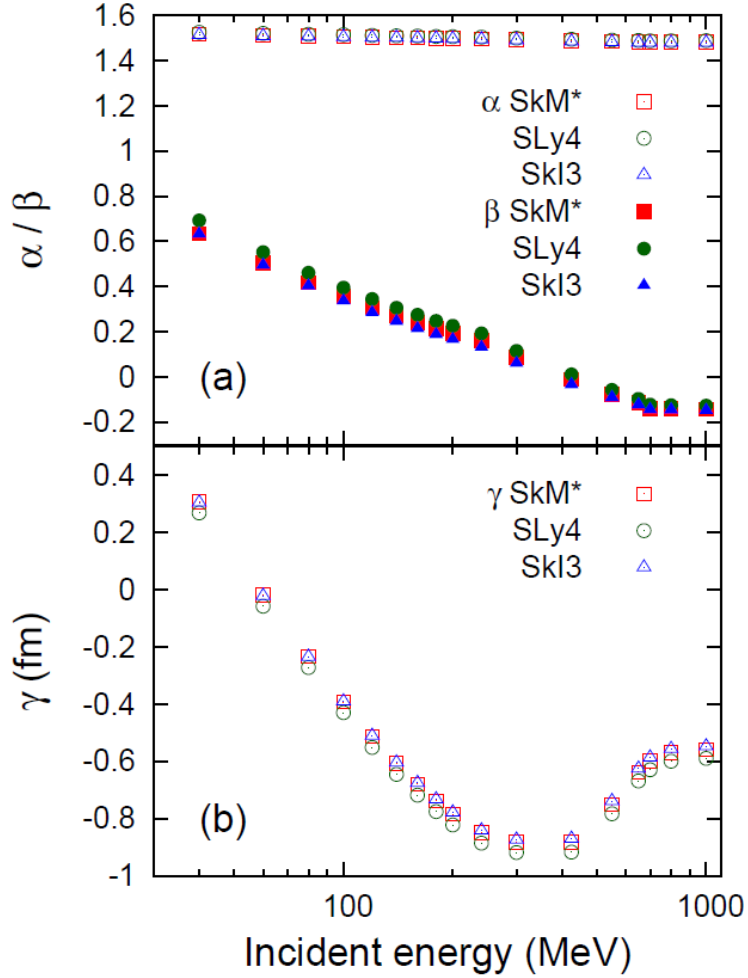


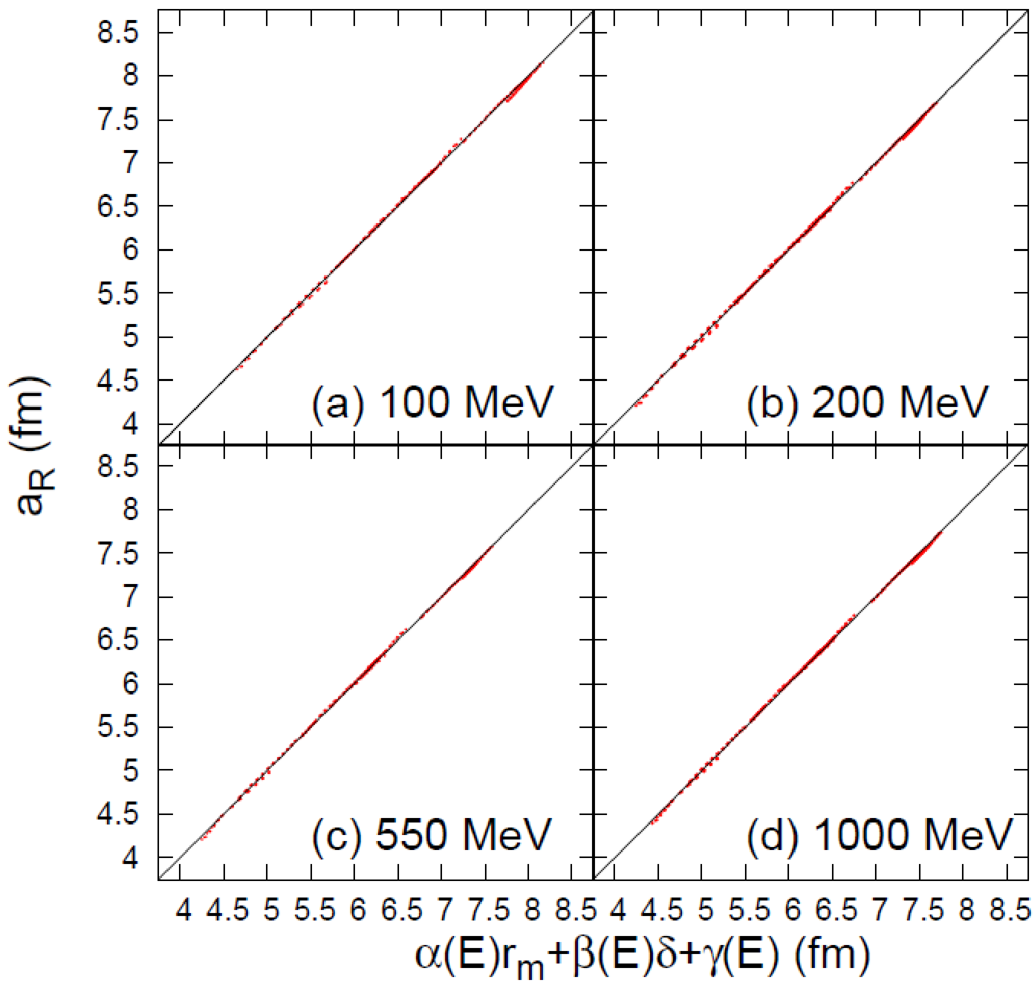
Proton target

→ Complicated Coulomb breakup can be avoided

Advantageous to extract the sizes of heavy nuclei

$$a_R(N,Z,E) \simeq \alpha(E) r_m(N,Z) + \beta(E)\delta(N,Z) + \gamma(E),$$





Least square fitting of the parameters

$$\chi^2(E) = \frac{1}{\mathcal{N}} \sum_{N,Z}^{\mathcal{N}} [a_R^{\rm HF}(N,Z,E) - a_R^{\rm Fit}(N,Z,E)]^2$$ ∼0.01-0.02 fm

Universal function
→ σ_R of r_m and δ

Summary

- Systematic analysis of total reaction cross sections (σ_R)
  - More than 100 isotopes with Skyrme-Hartree-Fock (+BCS) method
    - Three kinds of Skyrme interactions (SkM*, SLy4, SkI3)
  - **No adjustable parameters** in the structure and reaction theory

- Matter radius ⇔ Total reaction cross sections on ^{12}C
  - **Nuclear deformation**, halo, etc.
  - Almost no sensitivity to the neutron-skin thickness
- Proton and neutron radii ⇔ Energy dependence of σ_R on proton
  - Coulomb excitation negligible
  - "Reaction radius" of proton target $a_R(N, Z, E) = \sqrt{\sigma_R(N, Z, E)/\pi}$

    → two-valued linear function of matter radius and skin thickness

$$a_R(N, Z, E) \simeq \alpha(E) r_m(N, Z) + \beta(E)\delta(N, Z) + \gamma(E),$$

Universal function of incident energy E, **no dependence on density profiles**

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  - Shuichiro Ebata (Hokkaido Univ., Nuclear Data Center)
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  - Tsunenori Inakura (Niigata Univ.)
  - Takashi Nakatsukasa (Univ. of Tsukuba)
  - Yasuyuki Suzuki (Niigata Univ., RIKEN)

- References
  - Nuclear deformation

    - W.H., T.Inakura, T. Nakatsukasa, Y. Suzuki, Phys. Rev. C 86, 024614 (2012)

  - Neutron-skin thickness

    - W.H., Y. Suzuki, T.Inakura, Phys. Rev. C 89, 011601(R) (2014)
    - W.H., S. Hatakeyama, S. Ebata, Y. Suzuki, Phys. Rev. C 93, 044611 (2016)