Direct reactions with weakly-bound systems: a one-dimensional model

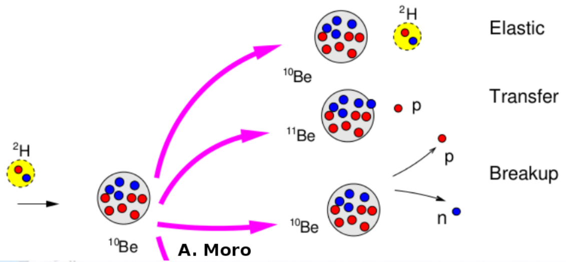
Laura Moschini
Andrea Vitturi and Antonio Moro

Outline

DREB2016
Direct Reactions with Exotic Beams
July 11-15, 2016
Halifax, Canada

Introduction

Direct reactions involve different channels simultaneously



Study of **direct reactions** applied to **weakly-bound** halo systems on neutron drip-line: inert core + **one** or two **valence neutrons**

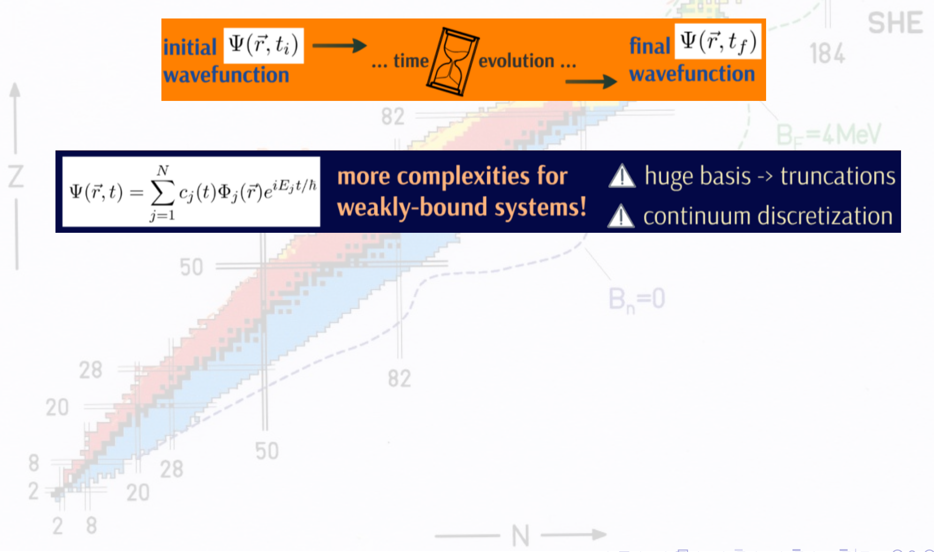
Introduction

initial $\Psi(\vec{r}, t_i)$ wavefunction → ... time evolution ... → final $\Psi(\vec{r}, t_f)$ wavefunction

$$\Psi(\vec{r}, t) = \sum_{j=1}^{N} c_j(t)\Phi_j(\vec{r})e^{iE_jt/\hbar}$$

more complexities for weakly-bound systems!

- ⚠ huge basis -> truncations
- ⚠ continuum discretization



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How to simplify the problem?

Let's move to one dimension!

$$\Psi(x, t) = \sum_{j=1}^{N} c_j(t)\Phi_j(x)e^{iE_jt/\hbar}$$

we can follow both time evolutions

using time dependent or coupled-channels methods,

understand the limitations of approximations,

and in particular study the **role of continuum**

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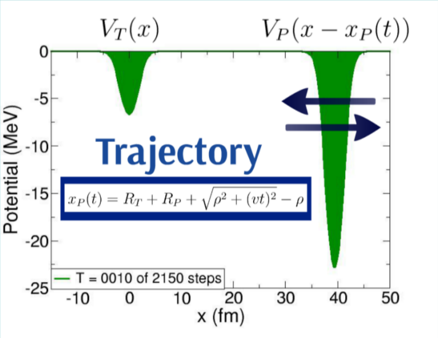
Hamiltonian

Potentials and trajectory

$$\mathcal{H}(x,t) = -\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + V_T(x) + V_P(x - x_P(t))$$



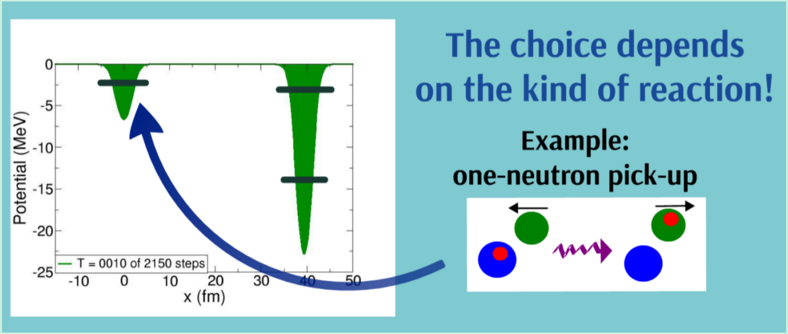
Esbensen H, Broglia RA and Winther A 1983 *Ann. Phys.* **146** 149–173

Initial wavefunction

obtained by solving the time independent Schroedinger equation

LM, Pérez-Bernal and Vitturi, *J. Phys. G: Nucl. Part. Phys.* **43** 045112 (2016)

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dx^2} + V_j(x)\right]\Phi_n^{(j)}(x) = E_n^{(j)}\Phi_n^{(j)}(x)$$



By definition we can expand the wavefunction $\Psi(x,t)$ as a combination of target and projectile basis states depending on a set of coefficients $c_j(t)$

$$\Psi(x,t) = \sum_{j=1}^{N_T} c_j^T(t)\Phi_j^T(x)e^{iE_j^T t/\hbar} + \sum_{j=1}^{N_P} c_j^P(t)\Phi_j^P(x)e^{iE_j^P t/\hbar}$$

We can follow the time evolution of:

$\Psi(x,t)$ $\rightarrow$ exact model
$c_j(t)$ $\rightarrow$ coupled-channels formalism

Exact model

Solution of time dependent Schroedinger equation

Vitturi and LM, *J. Phys.: Conf. Ser.* **590,** 012007 (2015)
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Same usual initial parameters of a reaction:

- distance of closest approach
- incident energy
- Q-value

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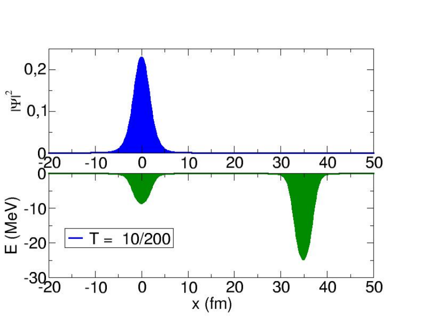

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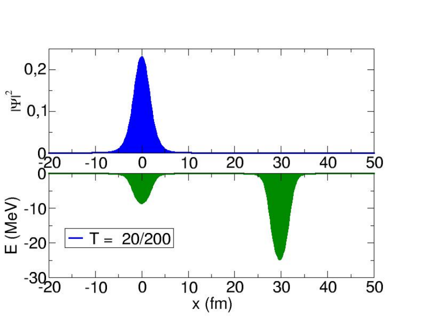

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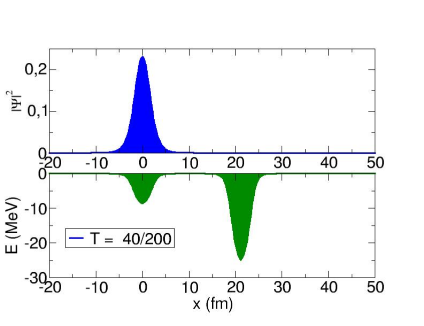

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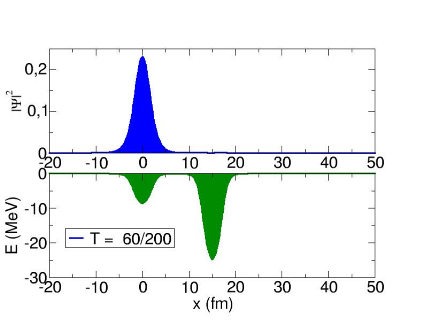

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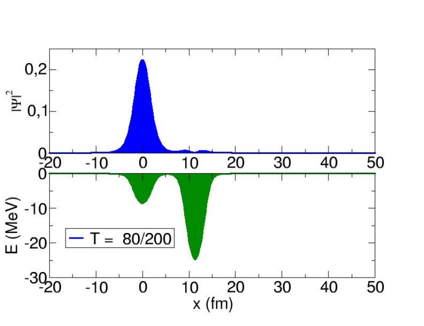

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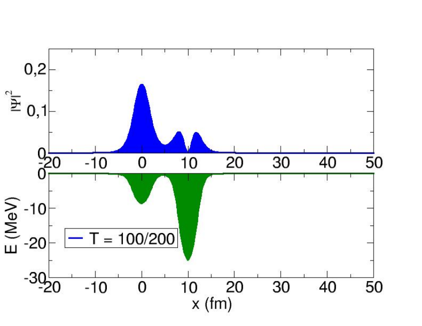

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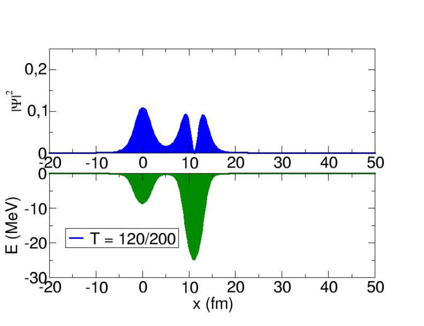

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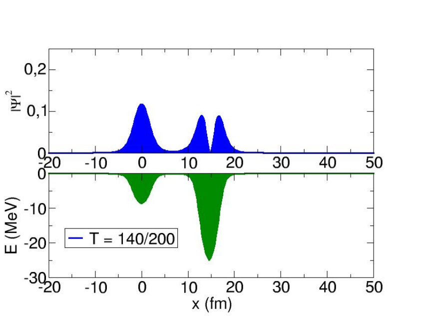

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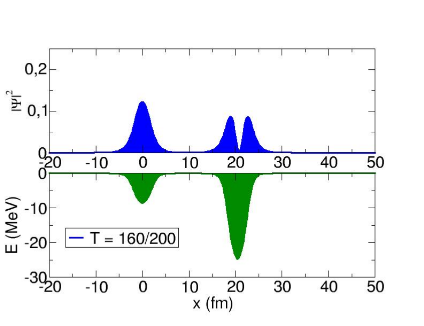

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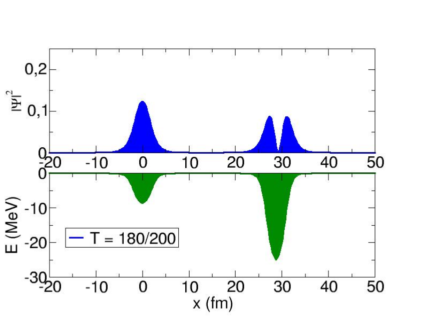

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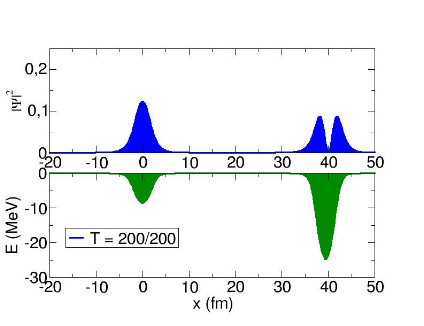

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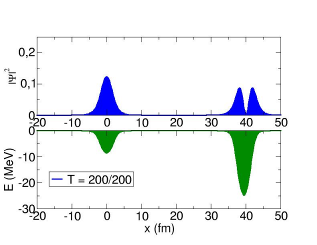
Final probabilities

$$P_{inelastic} = |<\Psi(x,t_{fin})|\Phi(x)_{target}>|^2$$

$$P_{transfer} = |<\Psi(x,t_{fin})|\Phi(x)_{proj}>|^2$$

$$P_{break-up} = 1 - P_{inelastic} - P_{transfer} = |\int <\Psi(x,t_{fin})|\Phi(k)>dk|^2$$

overlap with continuum
(pseudostates, exact continuum ...)



Coupled-Channels

Initial condition: $c_j^P(t=-\infty)=0$ **and** $c_j^T(t=-\infty)=\delta_{i,j}$

$$i\hbar\frac{\partial c_j^T}{\partial t}=\sum c_k^T\langle\omega_j^T|V^P|\Psi_k^T\rangle+\sum c_k^P\langle\omega_j^T|V^T|\Psi_k^P\rangle$$
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Esbensen H, Broglia RA and Winther A 1983 *Ann. Phys.* **146** 149–173

- inclusion of target Ψ^T AND projectile Ψ^P bases
- dual bases: time-dependent functions
  associated with the two wells (ω^T and ω^P) based on overlaps
  between target and projectile states
  to solve non-orthogonal problem
  $\langle\Psi_m^I|\omega_n^J\rangle=\delta_{I,J}\delta_{n,m}$

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New feature: continuum included!!!

pseudostates obtained by diagonalizing the potential in different bases

(infinite square well, harmonic oscillator, transformed HO)

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Final probabilities

$$P_j^{(T,P)}(t_f)=|c_j^{(T,P)}(t_f)|^2$$

non-orthogonal basis states
=> tot probability is not
conserved during collision

To be calculated after the collision when overlaps are zero

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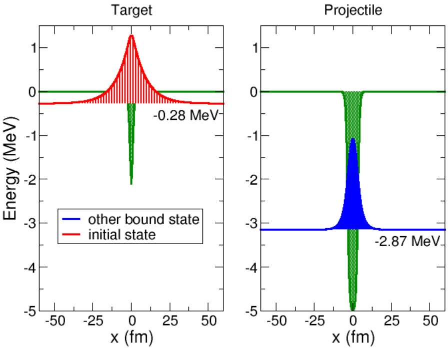

Target and projectile potentials and corresponding eigenstates

Incident energy: 5MeV

Evolution of exact wavefunction

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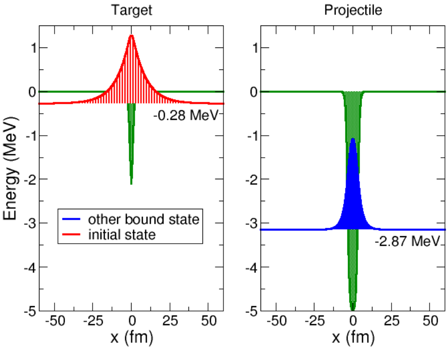

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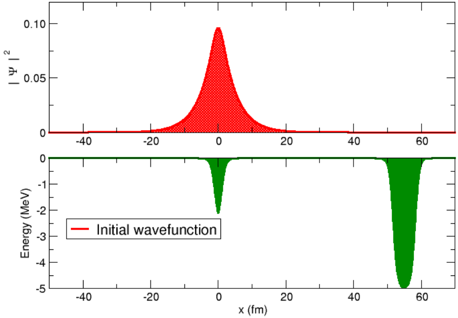

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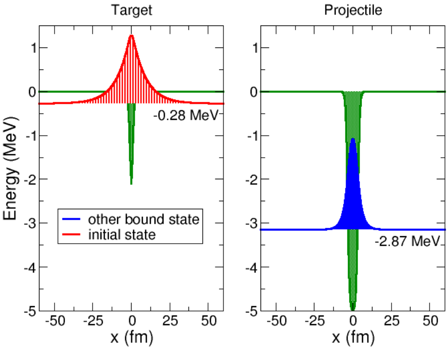

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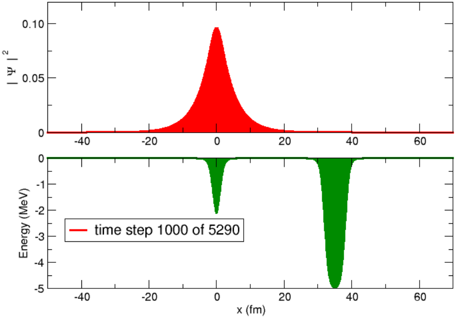

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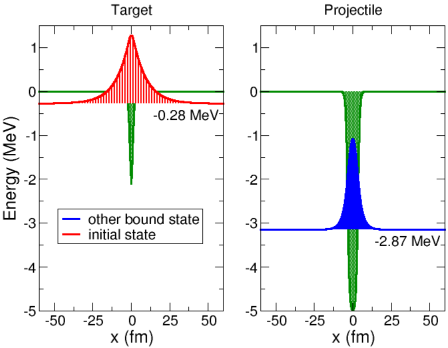

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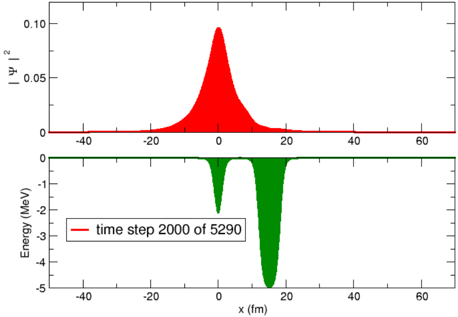

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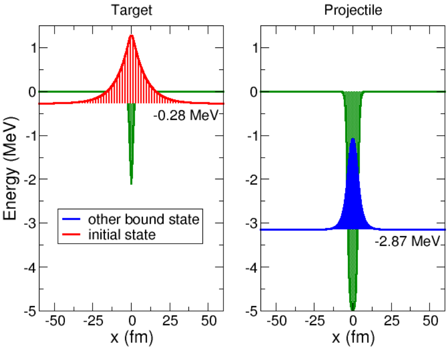

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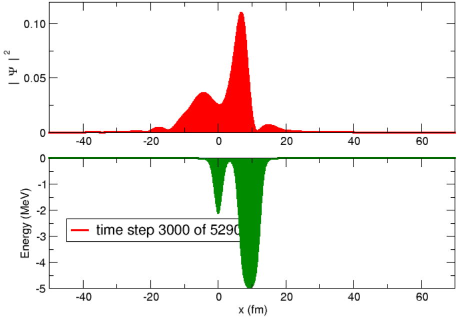

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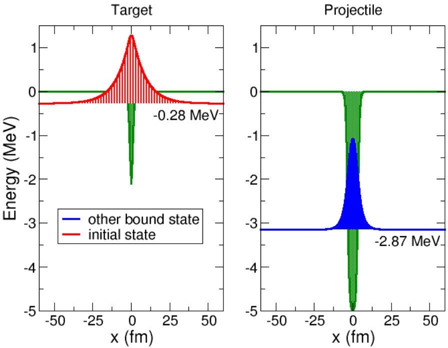

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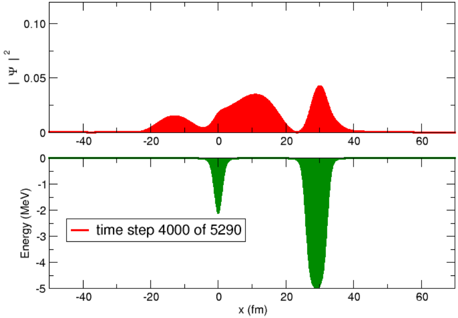

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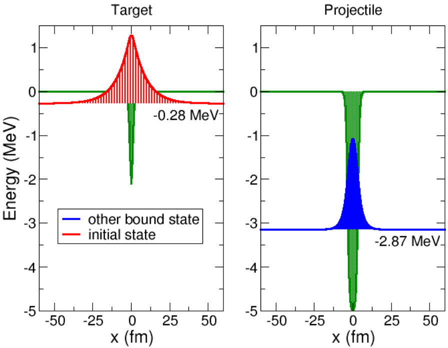

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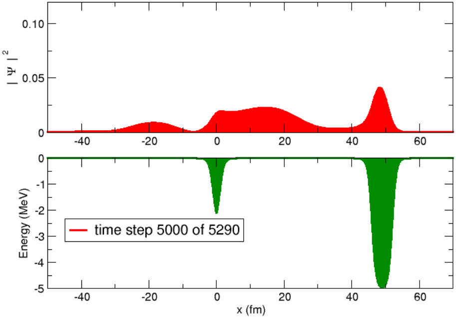
Incident energy: 5MeV

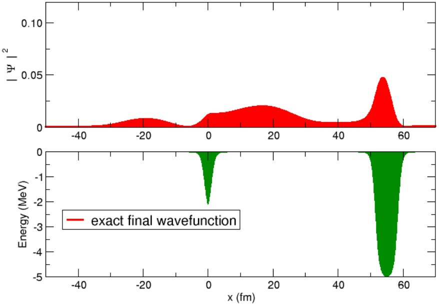
Evolution of exact wavefunction



Results

Final probability

Final exact wavefunction



	Exact	First approx	CC (a)	CC (b)	CC (c)
elastic	**21%**	**-**	**21.4%**	**95%**	**21%**
transfer	**5%**	**100%**	**-**	**5%**	**0.04%**
breakup	**74%**	**150%**	**78.6%**	**-**	**79%**

Exact: exact time evolution results

First approx: probability to excite directly the system from initial to final state

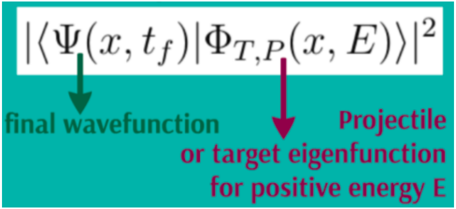
CC (a): only target basis, including continuum pseudostates

CC (b): target AND projectile bases, no continuum

CC (c): target AND projectile bases, including target's continuum pseudostates

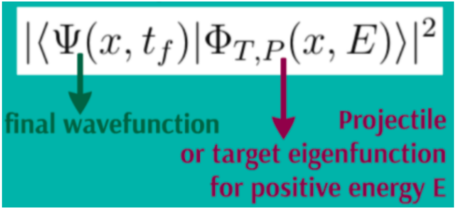
Results

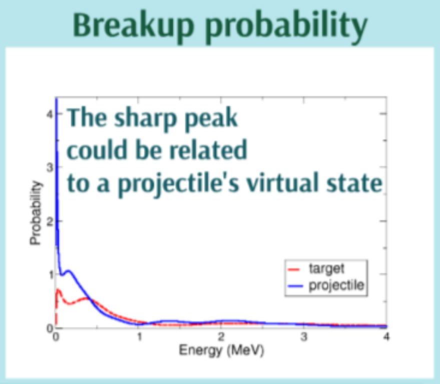
Breakup probability



Results

Breakup probability





Results

Breakup probability

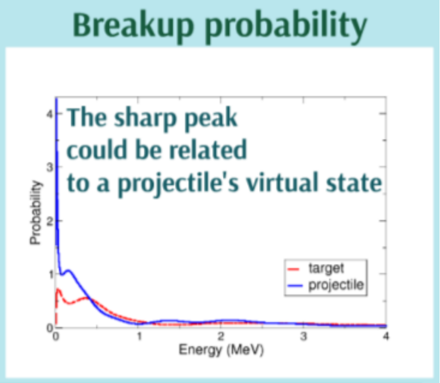
$$|\langle \Psi(x,t_f)|\Phi_{T,P}(x,E)\rangle|^2$$

final wavefunction

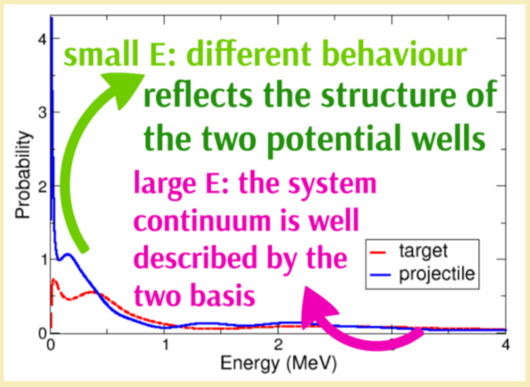
Projectile or target eigenfunction for positive energy E

→ large E: same behaviour independent to the basis

→ small E: different behaviour



Conclusions



Example: a 3D system

11Be + p

10Be + n + p

10Be + d

It is important to consider the continuum of each system because at small energy the different bases are not equivalent

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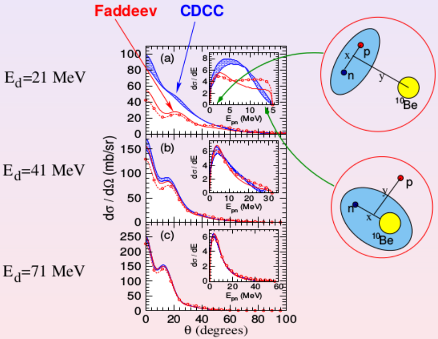
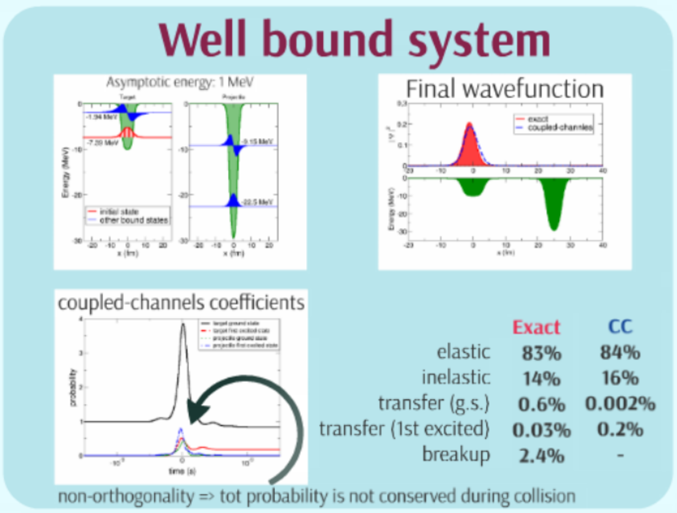
Breakdown of the CDCC ansatz: d+ ^{10}Be → p+n+ ^{10}Be

☞ But...differences have been evidenced for breakup at small incident energies (N.J. Upadhyay, A. Deltuva, F.M. Nunes, PRC85, 054621 (2012))

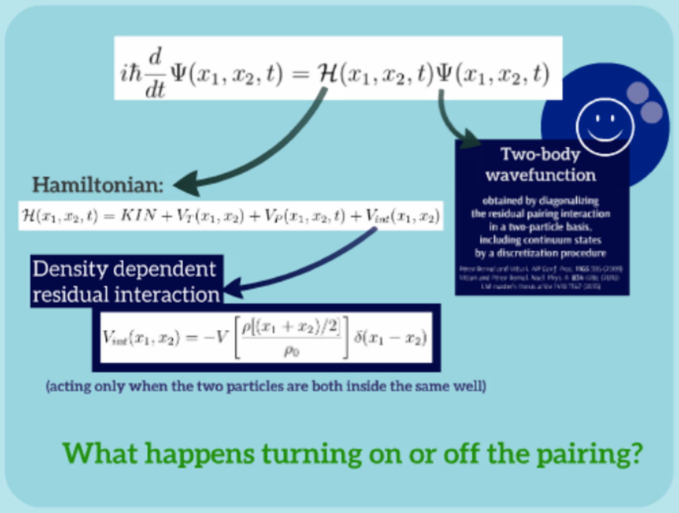


☞ CDCC can be (and must be!) improved

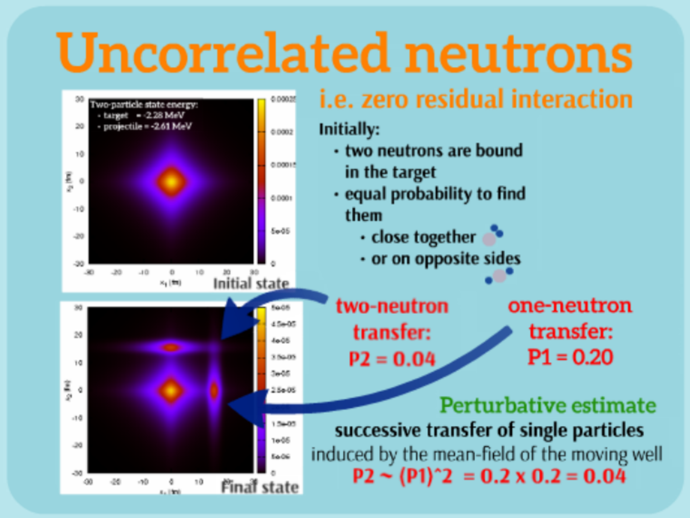
Well-bound system



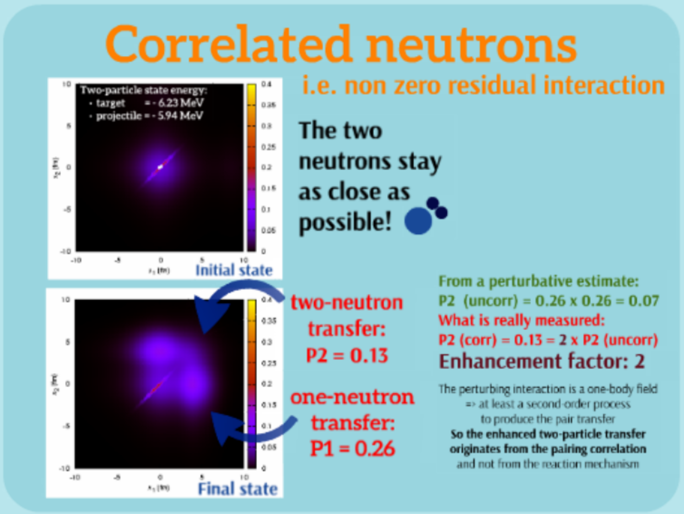
Two-neutron system



Two-neutron system



Two-neutron system



Virtual state