Towards ab initio description of nuclear radiative captures

Jérémy Dohet-Eraly

TRIUMF, 4004 Wesbrook Mall, Vancouver BC V6T 2A3, Canada

Direct Reactions wih Exoic Beams, Halifax, Canada, July 12th, 2016.

Motivations

Nuclear reaction in stars

- Radiative captures play an important role in the stellar nucleosynthesis
- Reactions rates are essential for describing quantitatively the evolution of the stars
- Radiative capture processes take place mostly at low energies (Gamow peaks $\sim$10 keV) $\Rightarrow$ Coulomb barrier strongly suppresses the capture cross sections $\Rightarrow$ out of reach of the experiments
- $\Rightarrow$ NUCLEAR THEORY IS NEEDED to predict the capture cross sections at low energies

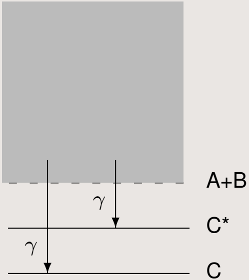
Outline

- An *ab initio* theoretical approach:
  the no-core shell model with continuum (NCSMC)
- Application to the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and $^3\mathrm{H}(\alpha,\gamma)^7\mathrm{Li}$ reactions

  in collaboration with
  - P. Navrátil (TRIUMF)
  - S. Quaglioni (LLNL)
  - W. Horiuchi (Hokkaido U.)
  - G. Hupin (IN2P3→CEA)
  - F. Raimondi (TRIUMF→U. Surrey)
- Application to the $^{11}\mathrm{C}(p,\gamma)^{12}\mathrm{N}$ reaction

  in collaboration with
  - A. Calci (TRIUMF)
  - P. Navrátil (TRIUMF)
  - R. Roth (TUD)
  - E. Gebrerufael (TUD)
  - S. Quaglioni (LLNL)
  - G. Hupin (CEA)

Theoretical description



Radiative capture

We NEED

- Unified approach to describe bound and continuum states

$$\Rightarrow \Psi_{ini} \text{ and } \Psi_{fin}$$

We use the No-Core Shell Model with Continuum (NCSMC) approach

- Efficient way to calculate photoemission/photoabsorption matrix elements between bound and continuum states

$$\Rightarrow \langle \Psi_{fin} | \mathcal{M}^{E}_{\lambda\mu} | \Psi_{ini} \rangle$$

Starting point

Microscopic Schrödinger equation

$$\Big(\sum_{i=1}^{A}\frac{p_i^2}{2m_N}+\sum_{i>j=1}^{A}v_{ij}+\sum_{i>j>k=1}^{A}v_{ijk}-T_{\mathrm{c.m.}}\Big)|\Psi_A^{J^\pi T}\rangle=E|\Psi_A^{J^\pi T}\rangle$$

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kinetic energy
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kinetic energy
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chiral NN (+3N) interactions
typically softened by the
similarity renormalization group method
to facilitate convergence

D. R. Entem and R. Machleidt, Phys. Rev. C 68, 041001 (2003)
P. Navrátil, Few-Body Syst. 41, 117 (2007)
S. K. Bogner, R. J. Furnstahl, and R. J. Perry, Phys. Rev. C 75, 061001 (2007)

TRIUMF

Starting point

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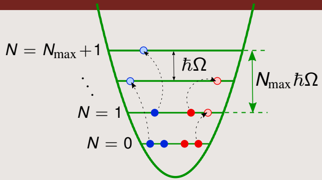
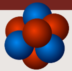
No-core shell model
with continuum w.f.

No-core shell model with continuum



No-core shell model (NCSM)



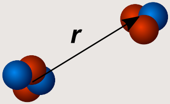
- Slater determinants of harmonic oscillator functions
- Exact c.m. factorization
- Short- and medium-range correlations
- Bound-state method

$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda c_\lambda^{J\pi T} \underbrace{|\quad\rangle}_{NCSM}$$

No-core shell model with continuum

+NCSM/resonating group method (RGM)

- NCSM cluster wave functions
- Long-range correlations
- Bound and scattering states; reactions



$$|\Psi_A^{J^\pi T}\rangle = \sum_\nu \int dr\, r^2 \frac{\gamma_\nu^{J\pi T}(r)}{r} \mathcal{A}_\nu \underbrace{|\quad\rangle}_{NCSM\,/\,RGM}$$

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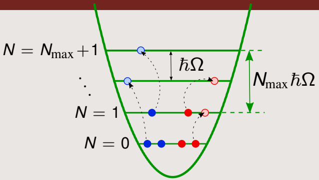

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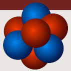

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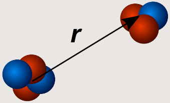

= No-core shell model with continuum (NCSMC)

[S. Baroni, P. Navratil, and S. Quaglioni, PRL 110, 022505 (2013); PRC 87, 034326 (2013).]

$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda c_\lambda^{J\pi T} \underbrace{|\;\;\rangle}_{NCSM} + \sum_\nu \int dr\, r^2 \frac{\gamma_\nu^{J\pi T}(r)}{r} \mathcal{A}_\nu \underbrace{|\;\; r \;\;\rangle}_{NCSM\,/\,RGM}$$

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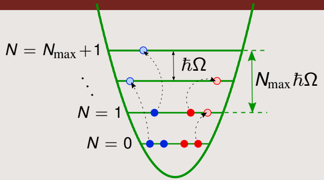

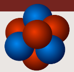

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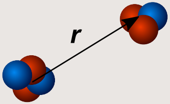

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unknowns

NCSMC equations

$$|\Psi_A^{J^\pi T}\rangle = \sum_\lambda \underbrace{c_\lambda^{J\pi T}}_{\text{unknown}} \; |$$

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- Variational amplitudes ($c_\lambda^{J\pi T}$ and $\gamma_\nu^{J\pi T}$) obtained by solving the NCSMC equations

$$\begin{pmatrix} E_\lambda \delta_{\lambda\lambda'} & \langle\ |H\mathcal{A}_\nu|\ \xrightarrow{r}\ \rangle \\ \langle\ \xrightarrow{r}\ |\mathcal{A}_{\nu'} H|\ \rangle & \langle\ \xrightarrow{r}\ |\mathcal{A}_{\nu'} H \mathcal{A}_\nu|\ \xrightarrow{r}\ \rangle \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix} = E \begin{pmatrix} \delta_{\lambda\lambda'} & \langle\ |\mathcal{A}_\nu|\ \xrightarrow{r}\ \rangle \\ \langle\ \xrightarrow{r}\ |\mathcal{A}_{\nu'}|\ \rangle & \langle\ \xrightarrow{r}\ |\mathcal{A}_{\nu'} \mathcal{A}_\nu|\ \xrightarrow{r}\ \rangle \end{pmatrix} \begin{pmatrix} c \\ \gamma \end{pmatrix}$$

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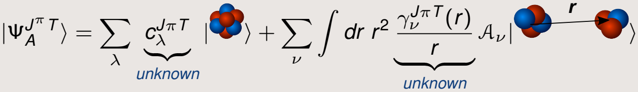
NCSMC equations



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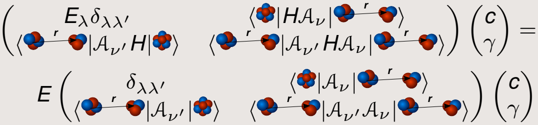

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- Scattering matrix and asymptotic normalization coefficients from matching solutions to known asymptotic with coupled-channel microscopic *R*-matrix method (MRM) on Lagrange mesh [M. Hesse, J.-M. Sparenberg, F. Van Raemdonck, and D. Baye, Nucl. Phys. A 640, 37 (1998)]

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- Astrophysical *S* factor from the electromagnetic matrix elements between the initial scattering state and the final bound state.

Applications

Physics Letters B 757 (2016) 430–436

Contents lists available at ScienceDirect

Physics Letters B

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^{3}He$(\alpha, \gamma)^{7}$Be and ^{3}H$(\alpha, \gamma)^{7}$Li astrophysical S factors from the no-core shell model with continuum

Jérémy Dohet-Eraly [a,*], Petr Navrátil [a], Sofia Quaglioni [b], Wataru Horiuchi [c], Guillaume Hupin [b,d,1], Francesco Raimondi [a,2]

Motivations

Extra motivation

Applications

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Motivations

- calculate the primordial $^7\mathrm{Li}$ abundance in the universe
- Relative rates of the $^3\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$ and $^3\mathrm{He}(^3\mathrm{He},2p)^4\mathrm{He}$ determine which fraction of *pp*-chain terminations resulting in $^7\mathrm{Be}$ or $^8\mathrm{B}$ neutrinos.

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CrossMark

Jérémy Dohet-Eraly [a,*], Petr Navrátil [a], Sofia Quaglioni [b], Wataru Horiuchi [c], Guillaume Hupin [b,d,1], Francesco Raimondi [a,2]

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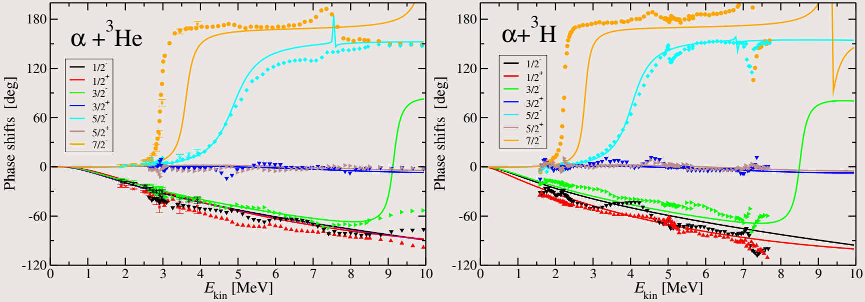
Extra motivation

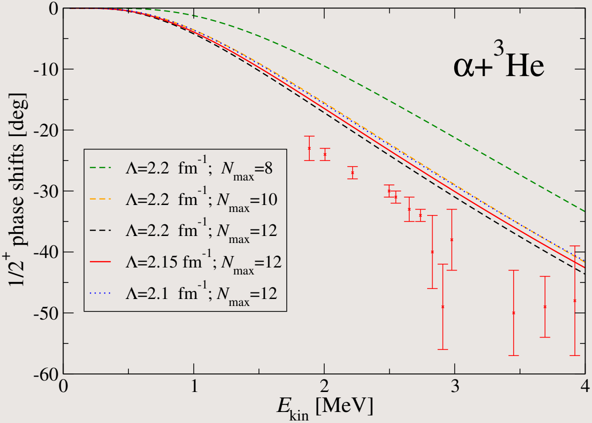
- Coulomb barrier strongly suppresses the capture cross sections $\Rightarrow$ at low energies out of reach of the experiments

$\alpha + ^3\mathrm{He}/^3\mathrm{H}$ phase shifts



- NCSMC calculations with SRG N^{3}LO *NN* potential ($\lambda = 2.15\ \mathrm{fm}^{-1}$)
- $N_{max} = 12; \hbar\Omega = 20$ MeV ; $^3\mathrm{He}/^3\mathrm{H}$, α ground state
- 8 (6) eigenstates with negative (positive) parity of $^7\mathrm{Be}/^7\mathrm{Li}$

$\alpha + {}^3$He phase shifts



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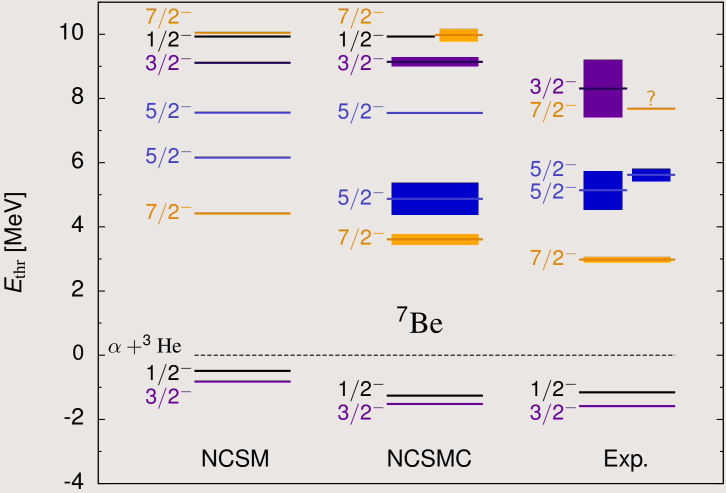
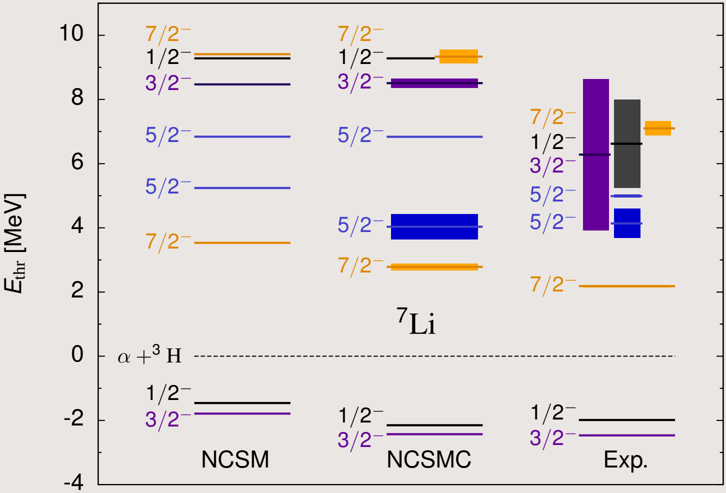
^{7}Be spectrum



^{7}Li spectrum



Phenomenological NCSMC

- NCSMC equations

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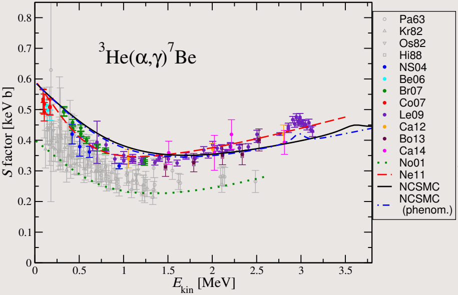
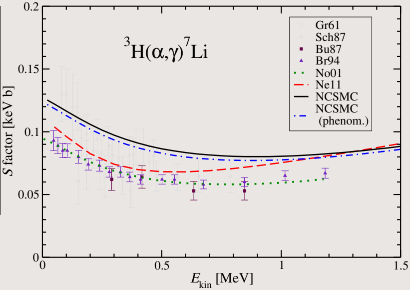
- Considering E_λ as adjustable parameters to reproduce the bound-state and resonance energies

$^3He(\alpha,\gamma)^7Be$ and $^3H(\alpha,\gamma)^7Li$





(Possible) quantitative agreement with experiments requires to include three-nucleon forces! (underway)

Applications

Reactions

Motivations

Extra motivation

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- $^{11}\mathrm{C} + p$ scattering and $^{11}\mathrm{C}(p,\gamma)^{12}\mathrm{N}$

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Extra motivation

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- Understanding the $^{12}\mathrm{N}$ spectrum (not well known experimentally)
- $^{11}\mathrm{C}(p,\gamma)^{12}\mathrm{N}$ is a part of the hot pp chain $\Rightarrow$ can bypass the triple-alpha process to produce $^{12}\mathrm{C}$

$$^{7}\mathrm{Be}(\alpha,\gamma)^{11}\mathrm{C}(p,\gamma)^{12}\mathrm{N}(e^{+}\nu_{e})^{12}\mathrm{C}$$

$$^{8}\mathrm{B}(\alpha,p)^{11}\mathrm{C}(p,\gamma)^{12}\mathrm{N}(e^{+}\nu_{e})^{12}\mathrm{C}$$

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- $^{11}\mathrm{C}(p,\gamma)^{12}\mathrm{N}$ can compete with β^{+} decay and so impacts on the $^{11}\mathrm{B}$ synthesis in novae

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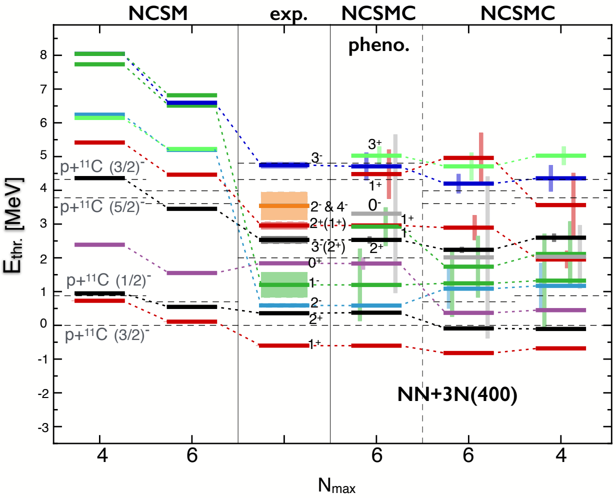
Extra motivation

- $^{11}\mathrm{C} + p$ scattering experiment planed at TUDA facility at TRIUMF
- $^{11}\mathrm{C}(p,\gamma)^{12}\mathrm{N}$ cannot be measured (at present) due to low yield of $^{11}\mathrm{C}$ beam

^{12}N spectrum

- NCSMC calculations with chiral $NN + 3N$



$^{11}C + p$ scattering

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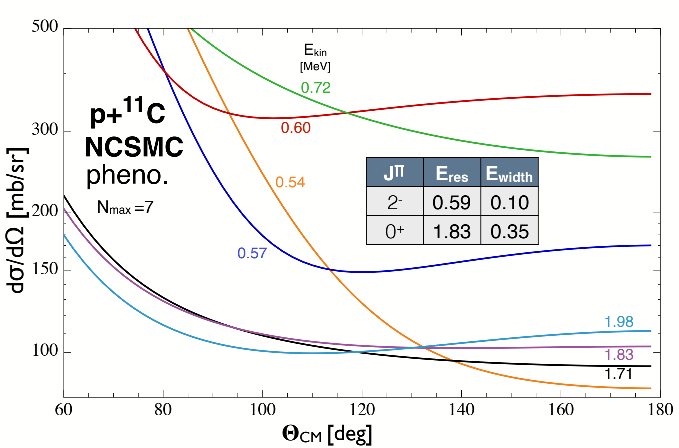
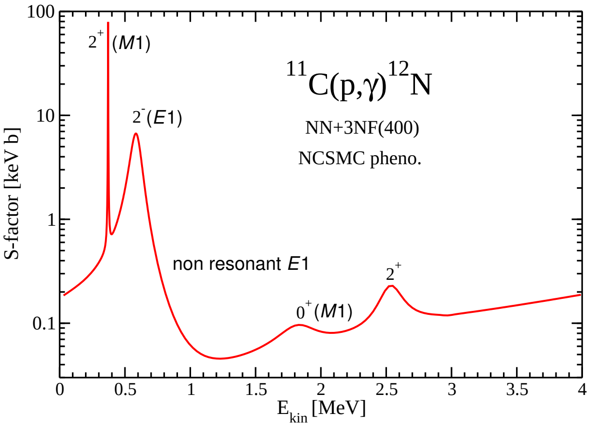


NCSMC calculations will be validated by measuring cross sections

$^{11}C(p,\gamma)^{12}N$



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TRIUMF

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Thank you for your attention!