B field non-uniformity

1. UCN depolarization
2. Polynomial parameterization
3. Motional false EDM
4. Magic field

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workshop nEDM2017 **Harrison Hot Springs**

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Transverse UCN depolarization during precession

$$\frac{d\alpha}{dT} = -\frac{\alpha}{T_{\text{wall}}} - \frac{\alpha}{T_{2,\text{mag}}} + \dot{\alpha}_{\text{grav}}$$

Wall collision

Intrinsic depolarization

Random trajectories in a gradient.

Intuitive theory gives:

$$\frac{1}{T_{2,\text{mag}}} \approx \frac{D^3\gamma_n^2}{9\pi v}\left(\left(\frac{dB_z}{dx}\right)^2 + \left(\frac{dB_z}{dy}\right)^2\right) + \frac{\text{H}^3\gamma_n^2}{16v}\left(\frac{dB_z}{dz}\right)^2$$

Gravitational depolarization

Different energy groups have different height z

$$\dot{\alpha}_{\text{grav}} = -\gamma_n^2\left(\frac{dB_z}{dz}\right)^2 \text{Var}[z]\, T$$

Vertical gradient: UCN gravitational depolarization

We apply a vertical gradient

$$G_{1,0} = dB_z/dz$$

- Longitudinal depolarization: no RF pulse
- Ramsey: normal cycles with π/2 pulses. We fit $\Delta\alpha = \gamma_n^2\, G_{1,0}^2\, \mathrm{Var}[z]\, T^2/2$

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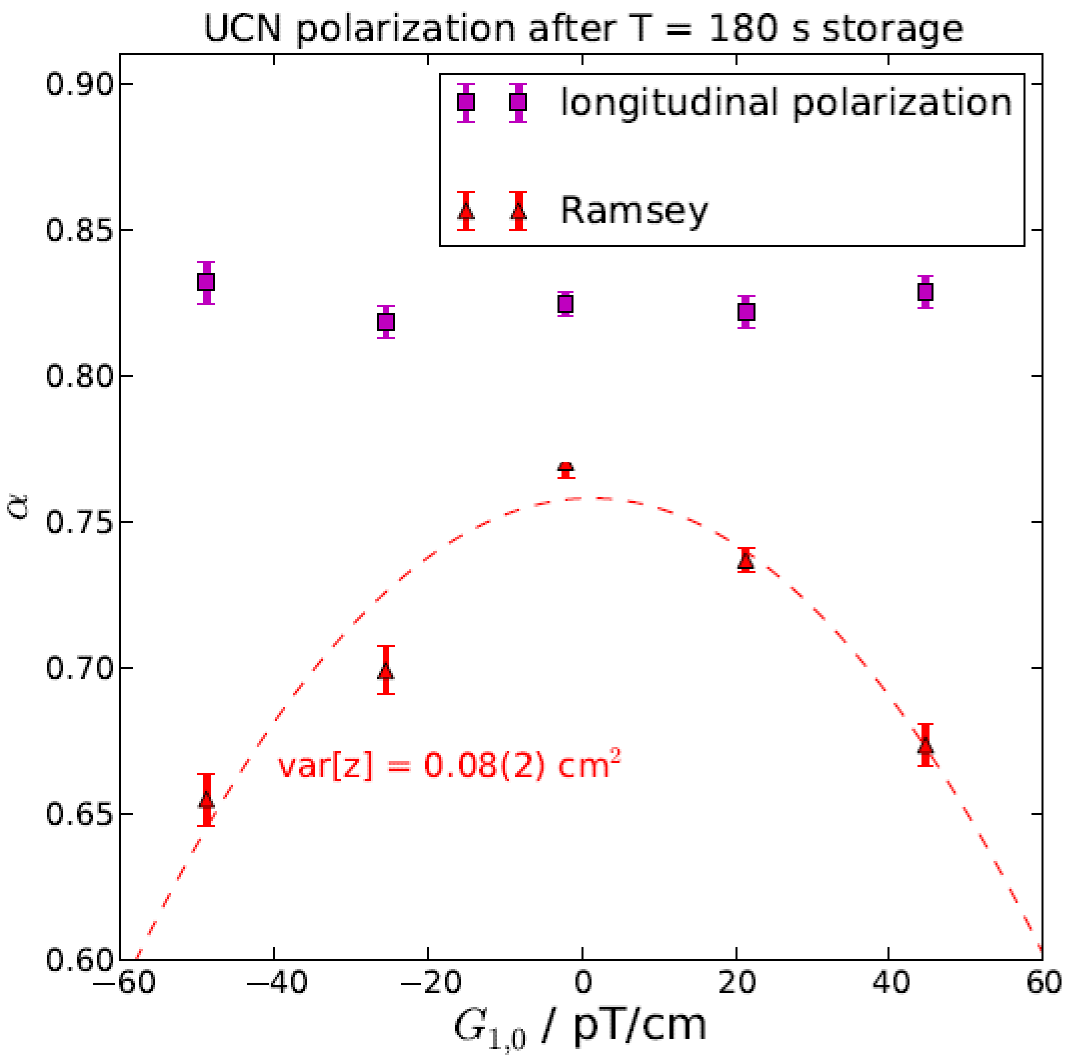
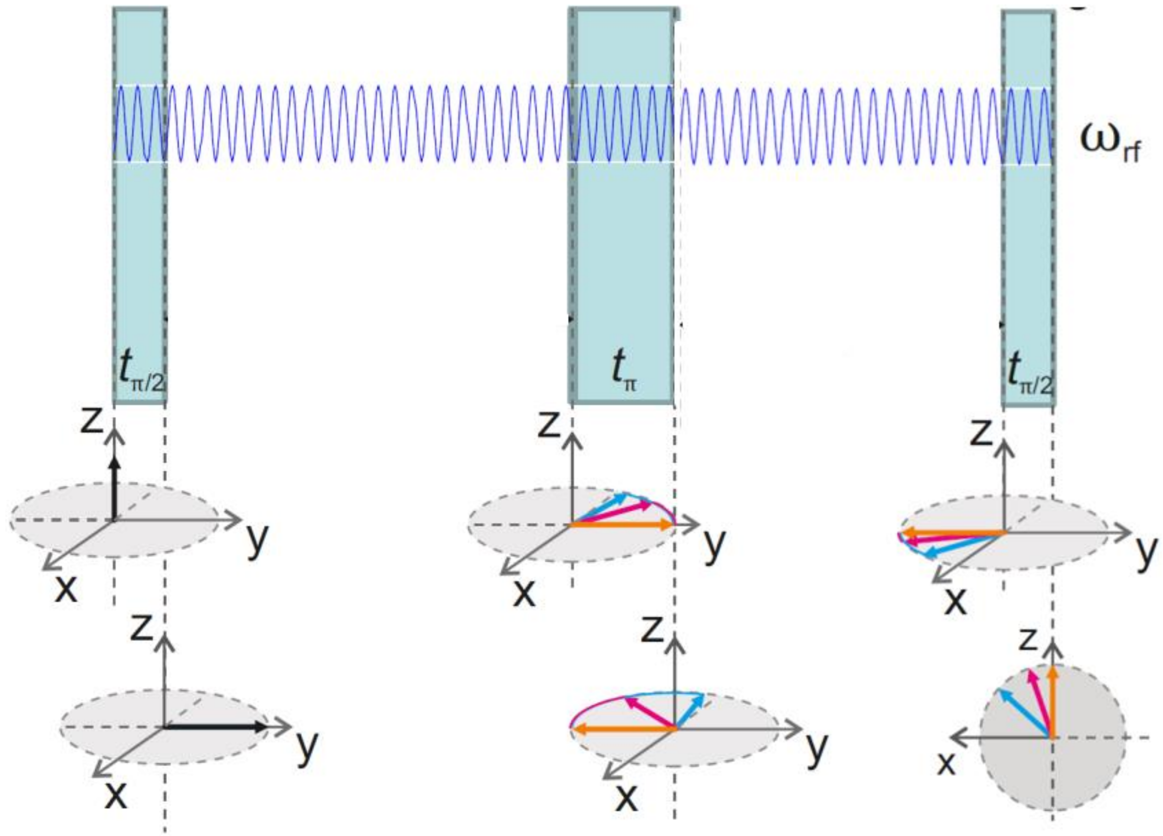


Spin echo method



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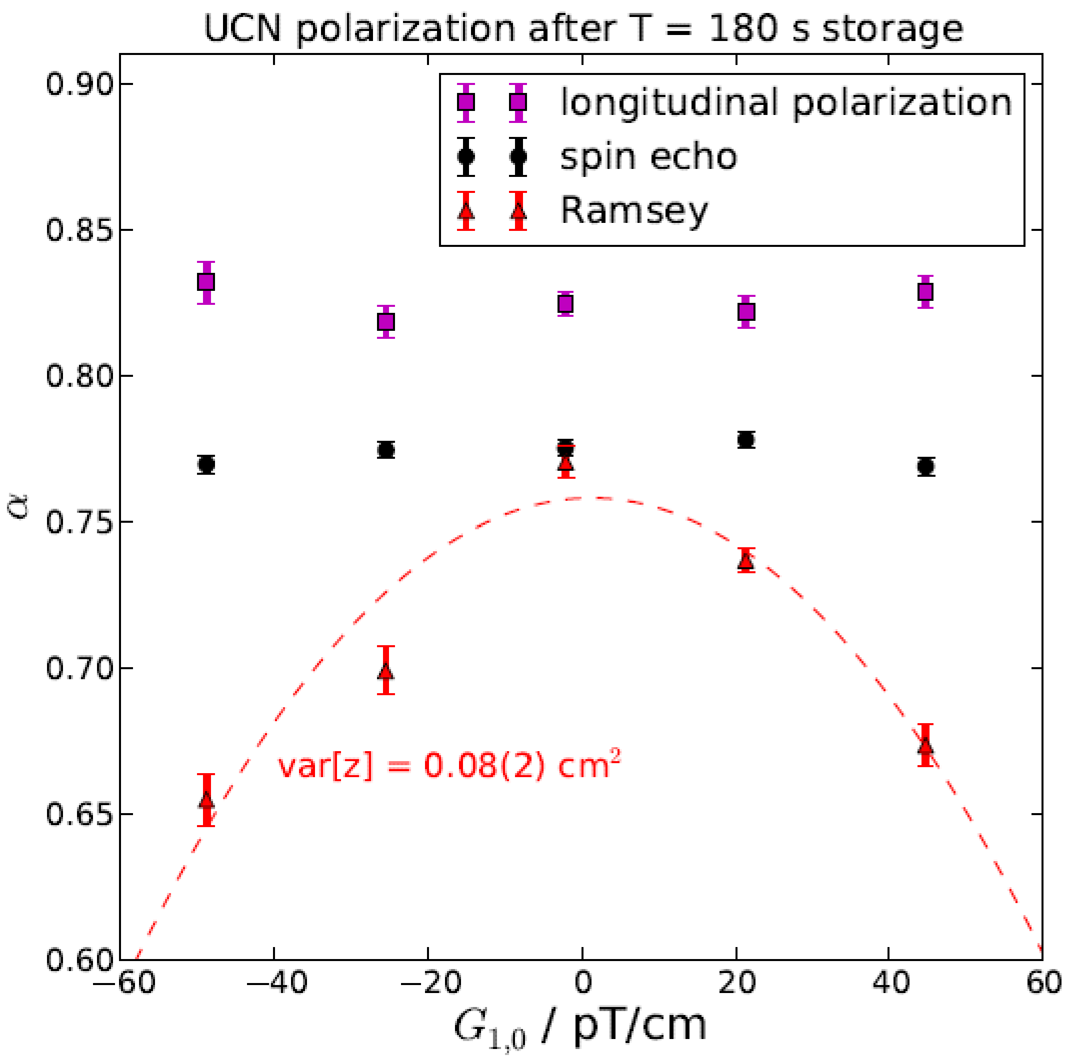
- Spin echo: additional π pulse in the middle of the precession. It **cancels the gravitational depolarization**.

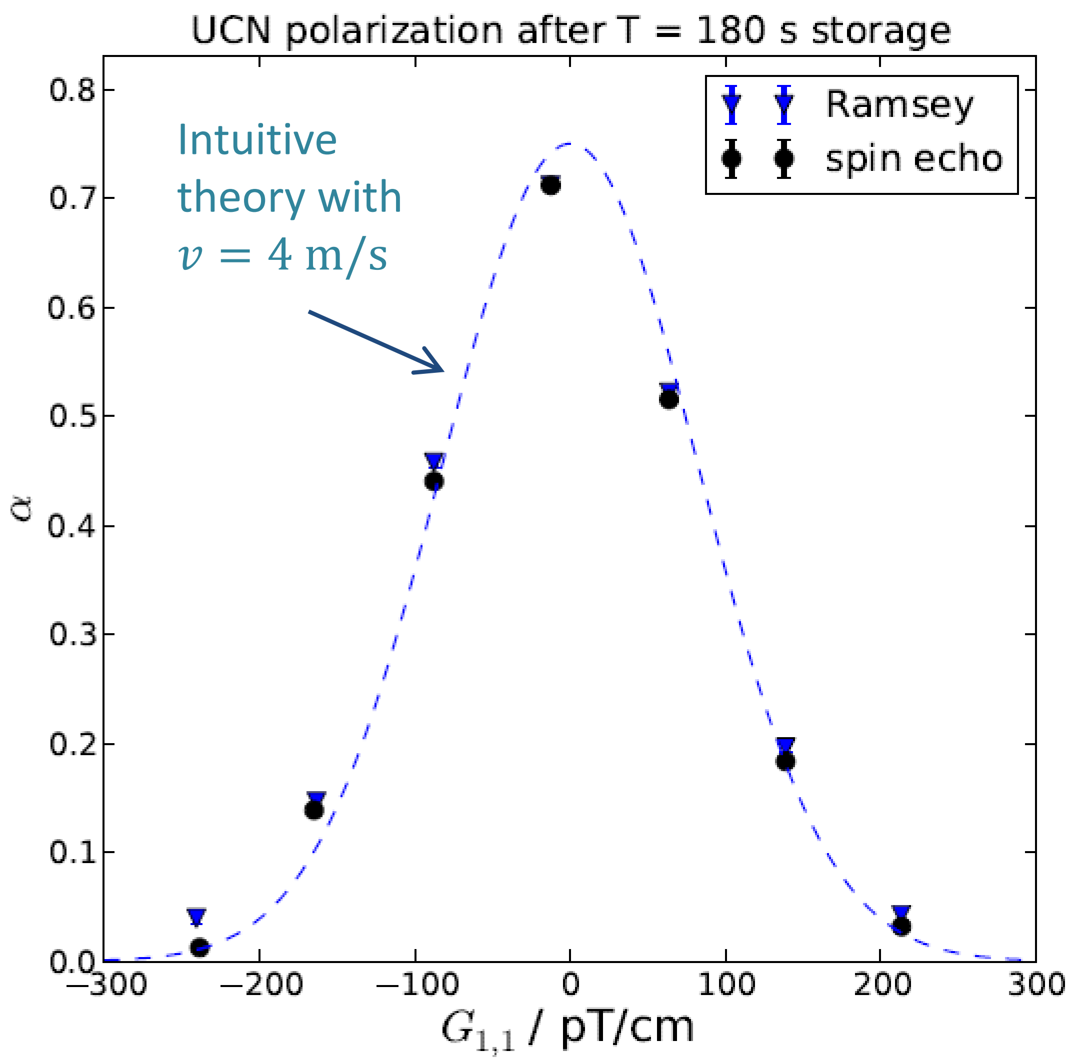


Horizontal gradient: UCN intrinsic depolarization

We apply a horizontal gradient

$$G_{1,1} = dB_z/dx$$

- Ramsey: normal cycles with π/2 pulses.
- Spin echo: additional π pulse in the middle of the precession. It **does not cancels the intrinsic depolarization**.



Systematics: Mind the high order modes

Harmonic polynomial expansion

$$\vec{B}(\vec{r}) = \sum_{l,m} G_{l,m} \begin{pmatrix} \Pi_{x,l,m}(\vec{r}) \\ \Pi_{y,l,m}(\vec{r}) \\ \Pi_{z,l,m}(\vec{r}) \end{pmatrix}$$

Construction of the basis of polynomials:

- Field as the gradient of a potential $\vec{B} = \vec{\nabla}\Phi$
- The potential is harmonic $\Delta\Phi = 0$
- The spherical harmonics form a basis of harmonic polynomials

$$\Phi = \sum_{l,m} G_{l,m}\, \Phi_{l,m}(x, y, z)$$

- We define the mode l, m as $\vec{\Pi}_{l,m} = \vec{\nabla}\Phi_{\mathrm{l,m}}$

	l, m	Π_x	Π_y	Π_z
	$0, -1$	0	1	0
	$0, 0$	0	0	1
	$0, 1$	1	0	0
UNIF. GRADIENTS	$1, -2$	y	x	0
UNIF. GRADIENTS	$1, -1$	0	z	y
UNIF. GRADIENTS	$1, 0$	$-\frac{1}{2}x$	$-\frac{1}{2}y$	z
UNIF. GRADIENTS	$1, 1$	z	0	x
UNIF. GRADIENTS	$1, 2$	x	$-y$	0
QUADRATIC MODES	$2, -3$	$2xy$	$x^2 - y^2$	0
QUADRATIC MODES	$2, -2$	$2yz$	$2xz$	$2xy$
QUADRATIC MODES	$2, -1$	$-\frac{1}{2}xy$	$-\frac{1}{4}(x^2 + 3y^2 - 4z^2)$	$2yz$
QUADRATIC MODES	$2, 0$	$-xz$	$-yz$	$z^2 - \frac{1}{2}(x^2 + y^2)$
QUADRATIC MODES	$2, 1$	$-\frac{1}{4}(3x^2 + y^2 - 4z^2)$	$-\frac{1}{2}xy$	$2xz$
QUADRATIC MODES	$2, 2$	$2xz$	$-2yz$	$x^2 - y^2$
QUADRATIC MODES	$2, 3$	$x^2 - y^2$	$-2xy$	0

Frequency shifts & false EDM in a non-uniform field

Frequency shift for a spin ½ from a transverse magnetic noise $\underline{B}$ (Redfield theory)

$$\delta f = \frac{\gamma^2}{4\pi} \int_0^\infty d\tau \, \mathrm{Im}\, e^{-i\omega\tau} \left\langle \underline{B}(0)\underline{B}^*(\tau) \right\rangle$$

A false EDM arise due to the combination of the relativistic motional field $\vec{E} \times \vec{v}\,/c^2$ and the non-uniform field.

$$d^{\mathrm{false}} = \frac{\hbar\gamma^2}{2c^2} \int_0^\infty d\tau \cos\omega\tau \, \left\langle B_x(0)v_x(\tau) + B_y(0)v_y(\tau) \right\rangle$$

In the non-adiabatic (low field) limit ω=0, the false EDM is

$$d^{\mathrm{false}} = -\frac{\hbar\gamma^2}{2c^2} \left\langle xB_x + yB_y \right\rangle$$
$$= -\frac{\hbar\gamma^2}{32c^2} D^2 \left[G_{1,0} - G_{3,0} \left(\frac{D^2}{8} - \frac{H^2}{4} \right) + \cdots \right]$$

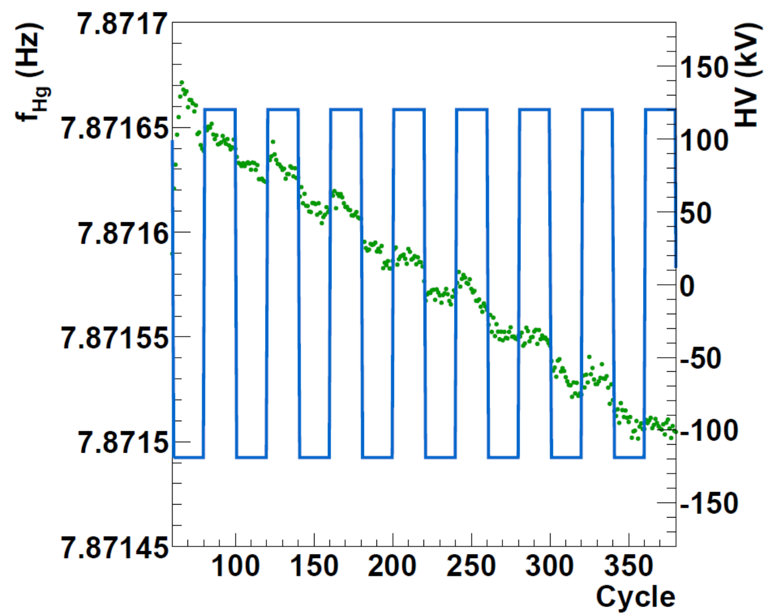
Direct verification at PSI, linear term

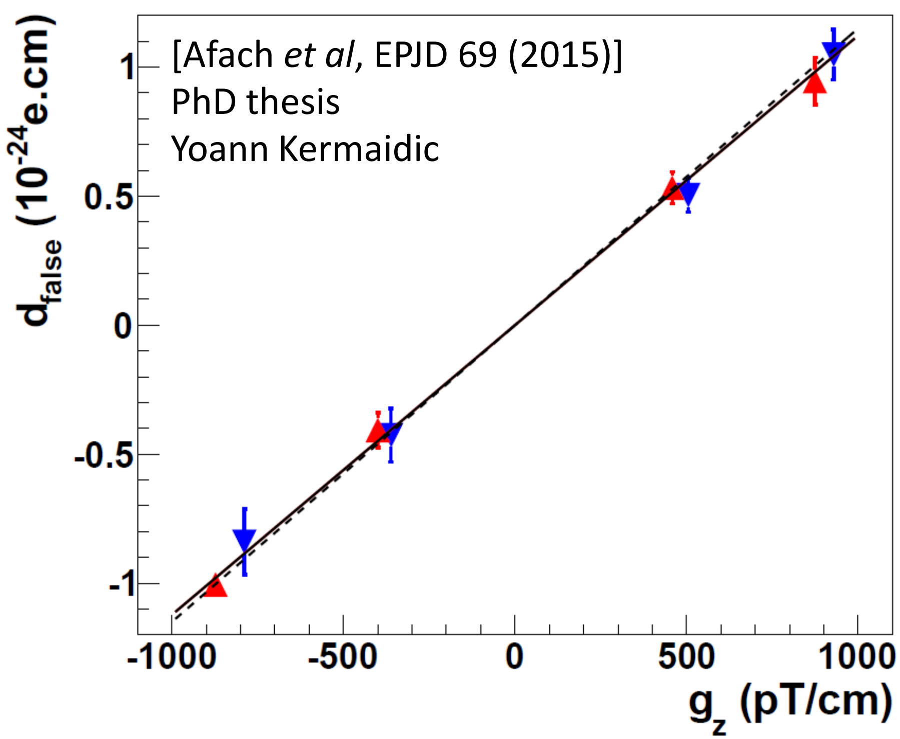
Linear mode $B_z = G_{1,0}\, z$

The theory:

$$d^{\text{false}} = -\frac{\hbar\gamma^2}{32c^2} D^2 G_{1,0}$$

is verified





Direct verification at PSI, cubic term

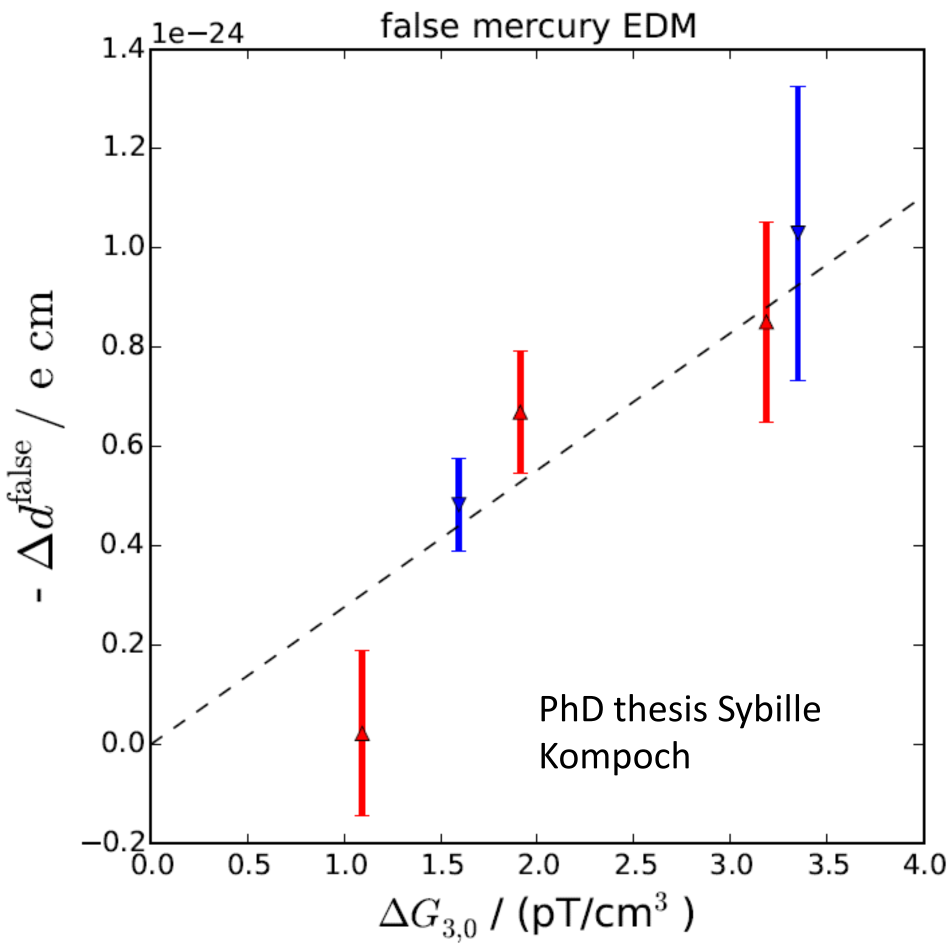
Cubic mode

$$B_z = G_{3,0}\left(z^3 - \frac{3}{2}z(x^2+y^2)\right)$$

The theory:

$$d^{\text{false}} = -\frac{\hbar\gamma^2}{32c^2}D^2\left(\frac{D^2}{8}-\frac{H^2}{4}\right)G_{3,0}$$

is verified



Actual higher order modes (field mapping)

Linear mode

$G_{1,0} \approx 16\ \mathrm{pT/cm}$

$d^{\mathrm{false}}_{\mathrm{Hg}\to n} \approx 7 \times 10^{-26}\ e\ \mathrm{cm}$

Cubic mode

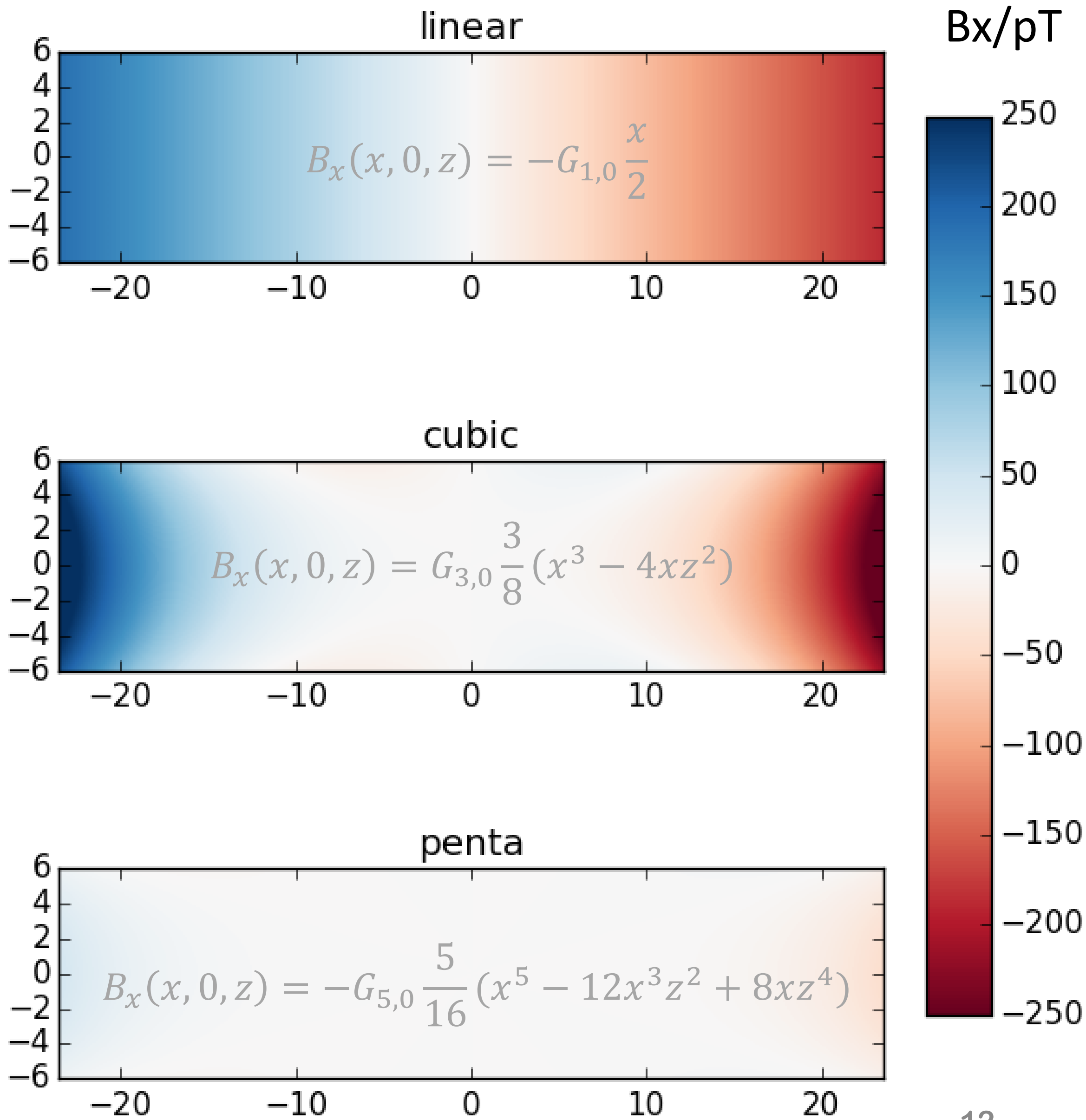
$G_{3,0} \approx -0.06\ \mathrm{pT/cm^3}$

$d^{\mathrm{false}}_{\mathrm{Hg}\to n} \approx 7 \times 10^{-26}\ e\ \mathrm{cm}$

Penta mode

$G_{5,0} \approx 2 \times 10^{-5}\ \mathrm{pT/cm^5}$

$d^{\mathrm{false}}_{\mathrm{Hg}\to n} \approx 0.8 \times 10^{-26}\ e\ \mathrm{cm}$



Correcting the false EDM: "crossing point"

Gravitational shift

$$R := \frac{f_n}{f_{\mathrm{Hg}}} = \frac{\gamma_n}{\gamma_{Hg}}\left(1 + \frac{G_{\mathrm{grav}}\langle z\rangle}{B_0} + \delta_{\mathrm{extra}}\right)$$

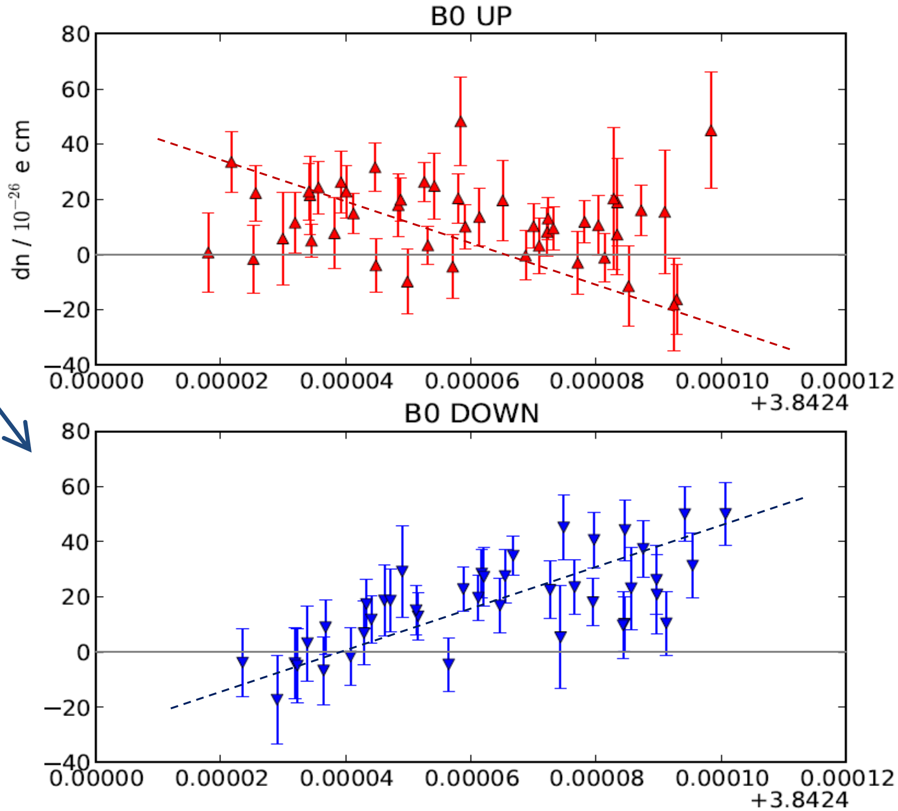
$$G_{\mathrm{grav}} = G_{1,0} + G_{3,0}\left(\frac{3H^2}{20} - \frac{3D^2}{16}\right)$$

False EDM

$$d_n^{\mathrm{false}} = \frac{\hbar\gamma_n\gamma_{\mathrm{Hg}}}{32c^2}D^2\left[G_{\mathrm{grav}} + G_{3,0}\left(\frac{D^2}{16} + \frac{H^2}{10}\right)\right]$$

INVISIBLE

Y axis:
One needs to correct the d value from the invisible cubic contribution, using field maps as input



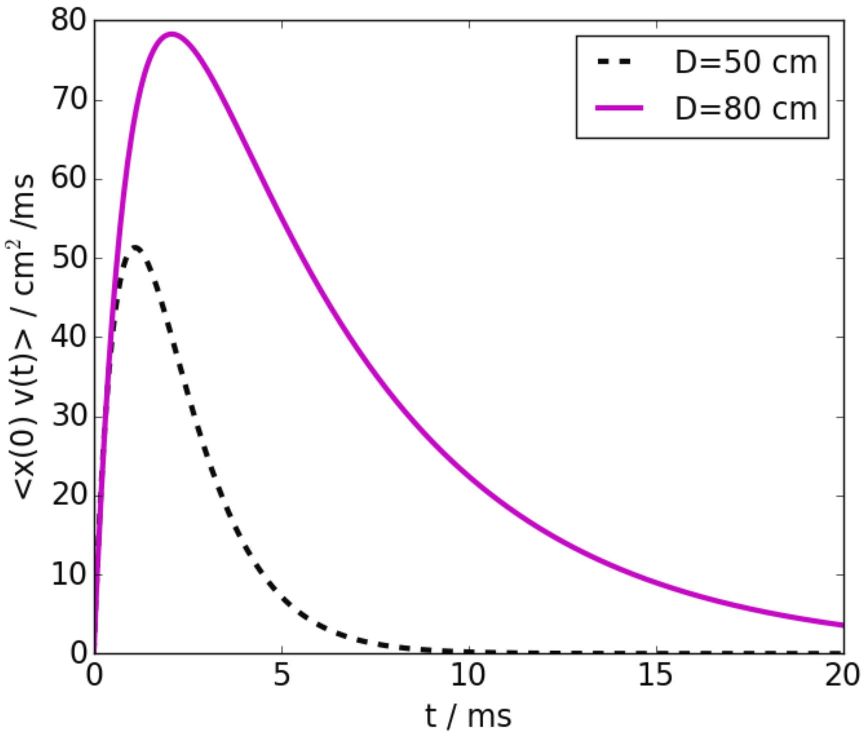
X axis:
One needs to correct the R value from the extra shifts

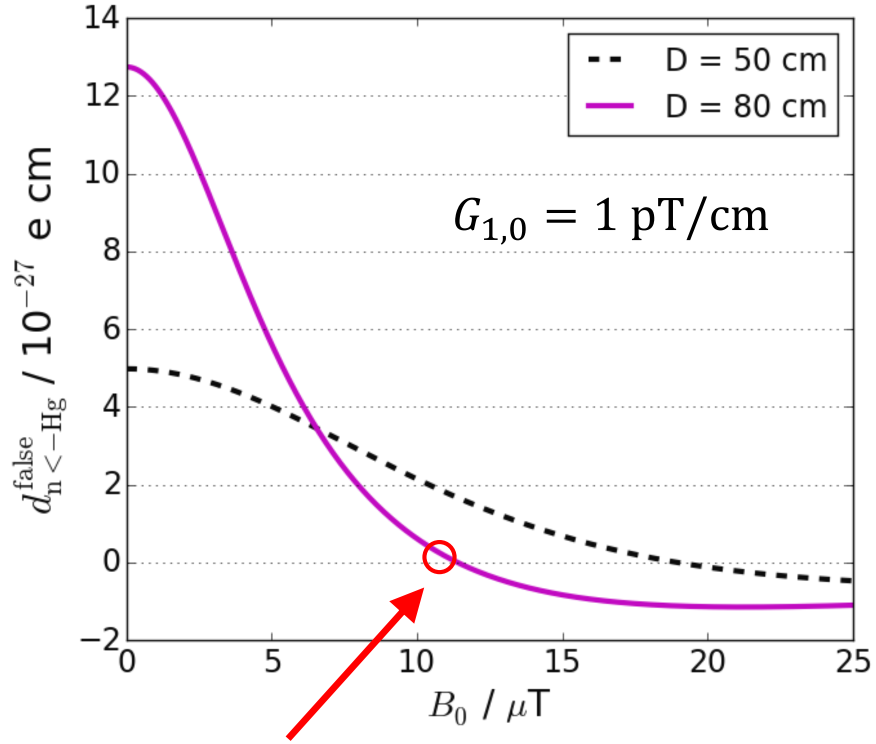
- Transverse shift
- Earth rotation
- etc

A magic B0 value

$$d^{\text{false}} = \frac{\hbar\gamma_n\gamma_{Hg}}{2c^2}\int_0^\infty d\tau \cos\omega\tau \, \langle B_x(0)v_x(\tau) + B_y(0)v_y(\tau)\rangle \qquad \omega = \gamma_{Hg}B_0$$

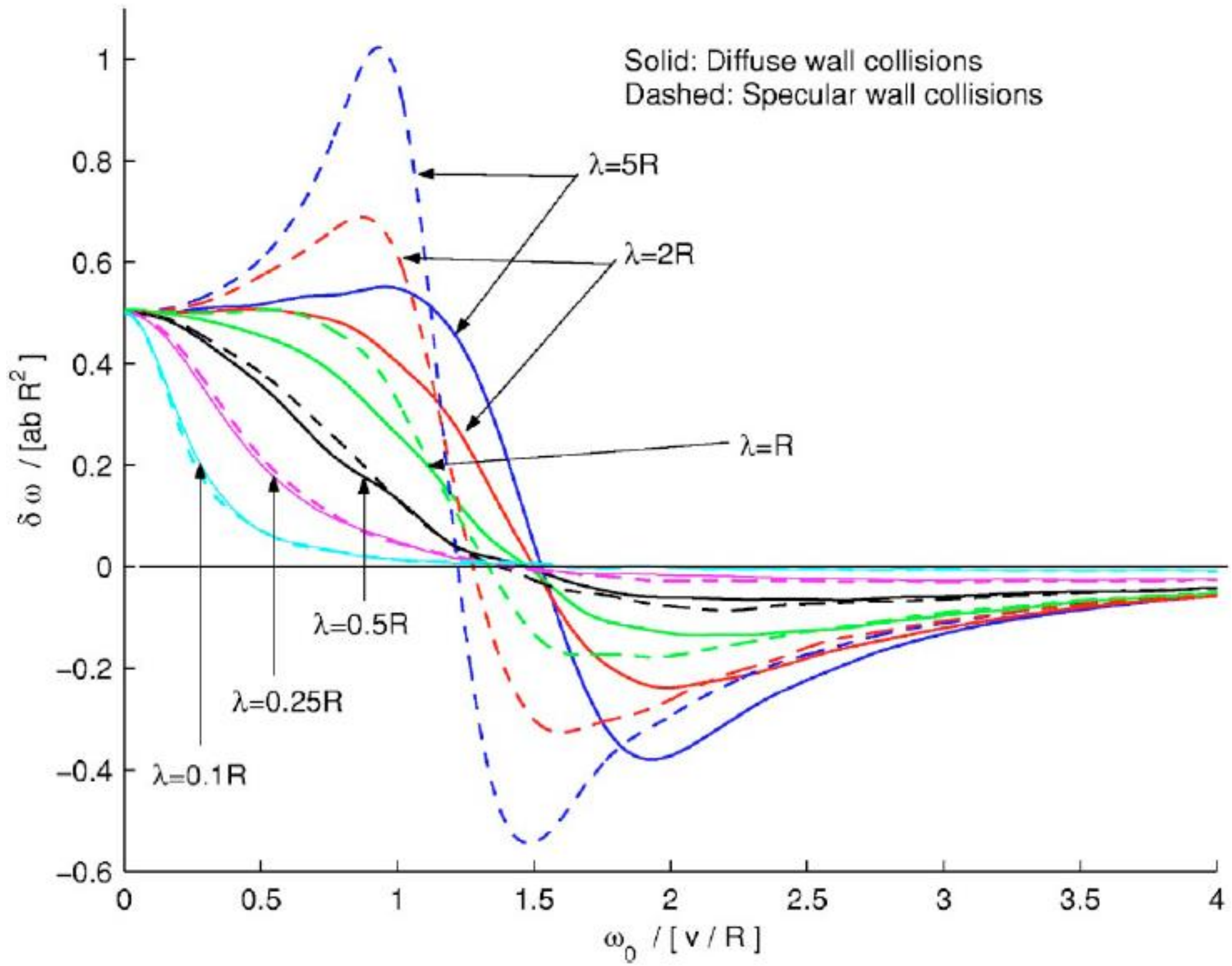
MC simulation ballistic thermal motion of mercury atoms



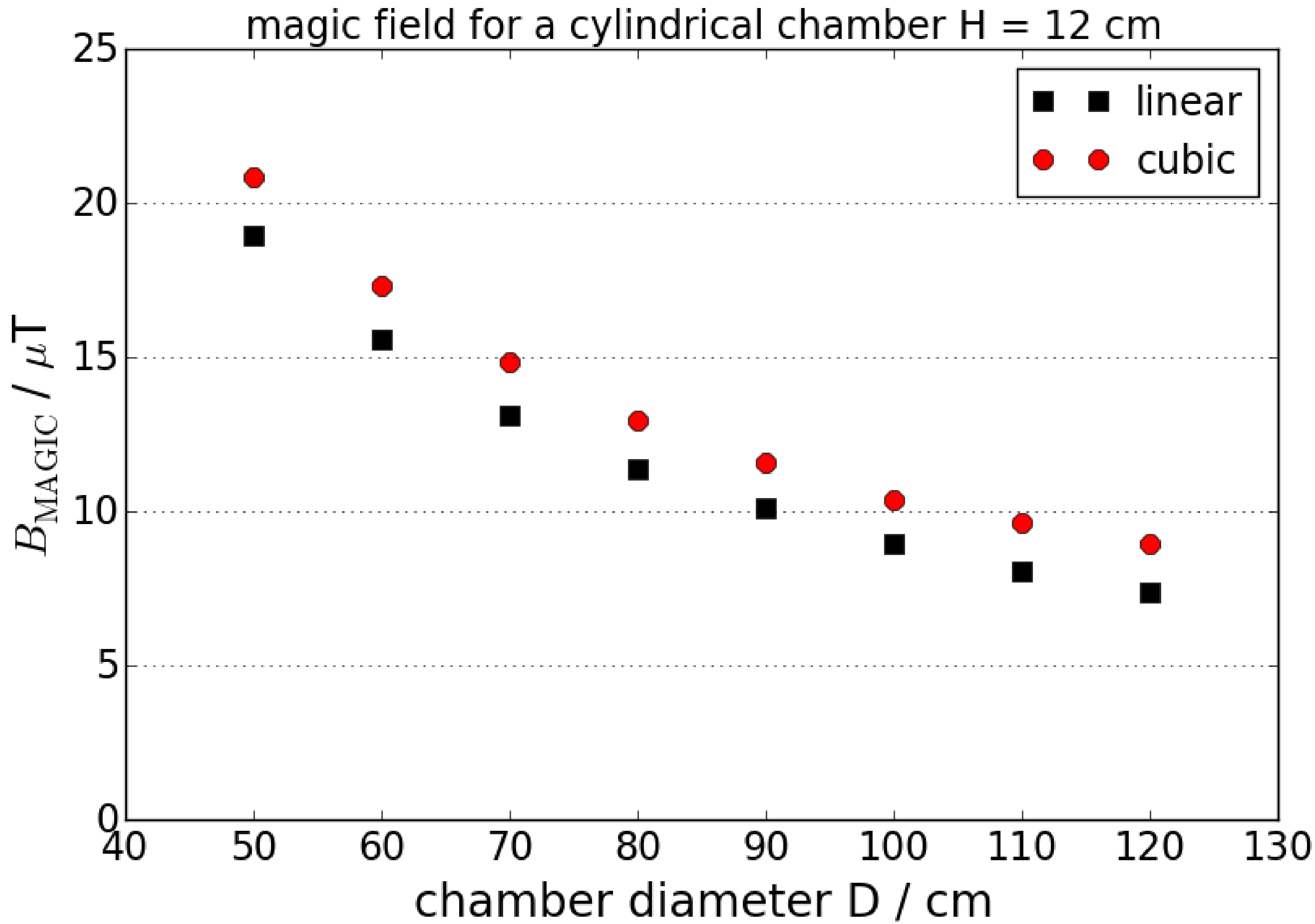


ZERO FALSE EDM FOR THE MAGIC VALUE B0 = 11 μT

Previous work, 3He comagnetometer

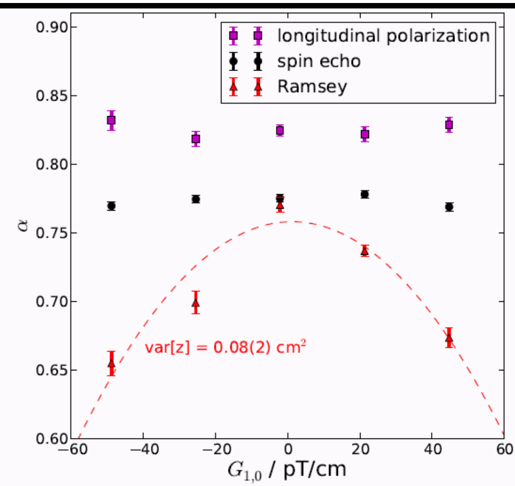


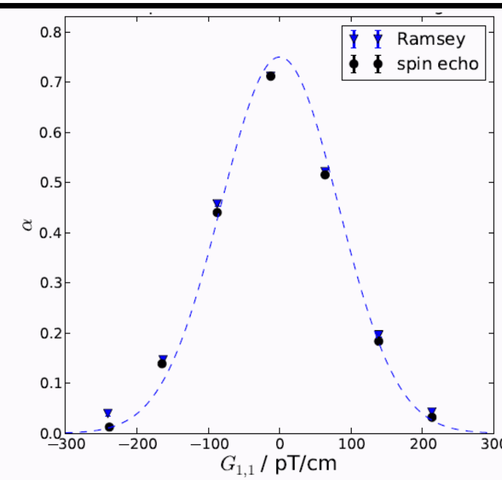
Lamoreaux and Golub, PRA 71 032104 (2005)


magic field for a cylindrical chamber H = 12 cm
linear
cubic
B_{MAGIC} / μT
chamber diameter D / cm
0
5
10
15
20
25
40
50
60
70
80
90
100
110
120
130

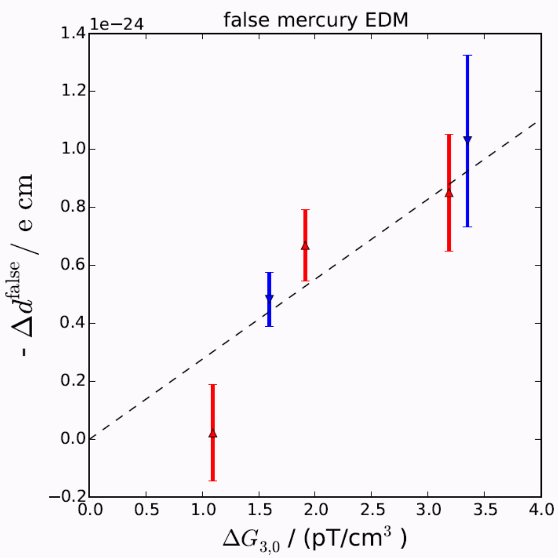
Wrap up

We understand better UCN depolarization due to gradients

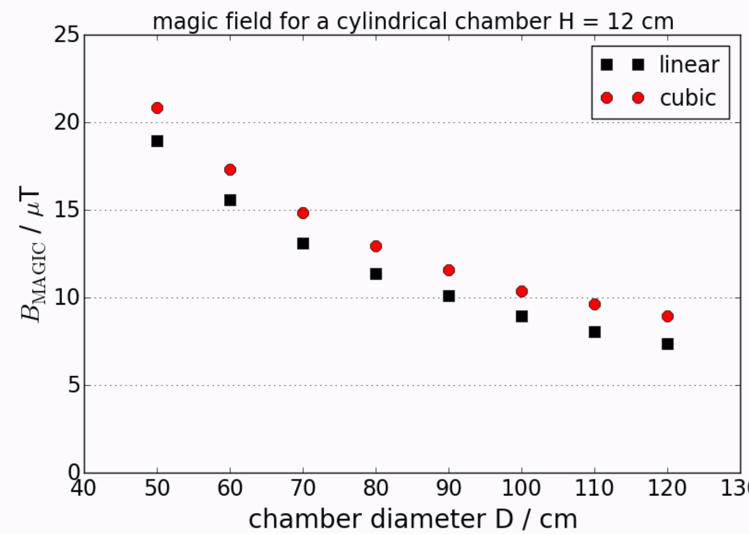




Mind the higher order modes!
We verified the cubic false EDM



Operating at the magic B0 in n2EDM with big chambers?



Thank you

Details on the harmonic expansion

Construction of the basis of polynomials:

- Field as the gradient of a potential $\vec{B} = \vec{\nabla}\Phi$, the potential is harmonic $\Delta\Phi = 0$
- We define the custom harmonic polynomials as $\Phi_{l,m} = C_{l,m}(\phi) r^l P_l^{|m|}(\cos\theta)$ with

$$C_{l,m}(\phi) = \frac{(l-1)!\,(-2)^m}{(l+m)!}\cos m\phi \ \text{ for } m \geq 0$$

$$C_{l,m}(\phi) = \frac{(l-1)!\,(-2)^{|m|}}{(l+|m|)!}\sin |m|\phi \ \text{ for } m < 0$$

- They form a basis of harmonic polynomials: $\Phi = \sum_{l,m} G_{l,m}\,\Phi_{l,m}(x,y,z)$
- The field is therefore $\vec{B} = \sum_{l,m} G_{l,m}\,\vec{\Pi}_{l,m}(x,y,z)$ with $\vec{\Pi}_{l,m} = \vec{\nabla}\Phi_{\mathrm{l,m}}$