Equivalence and Classification of 4D Adinkras

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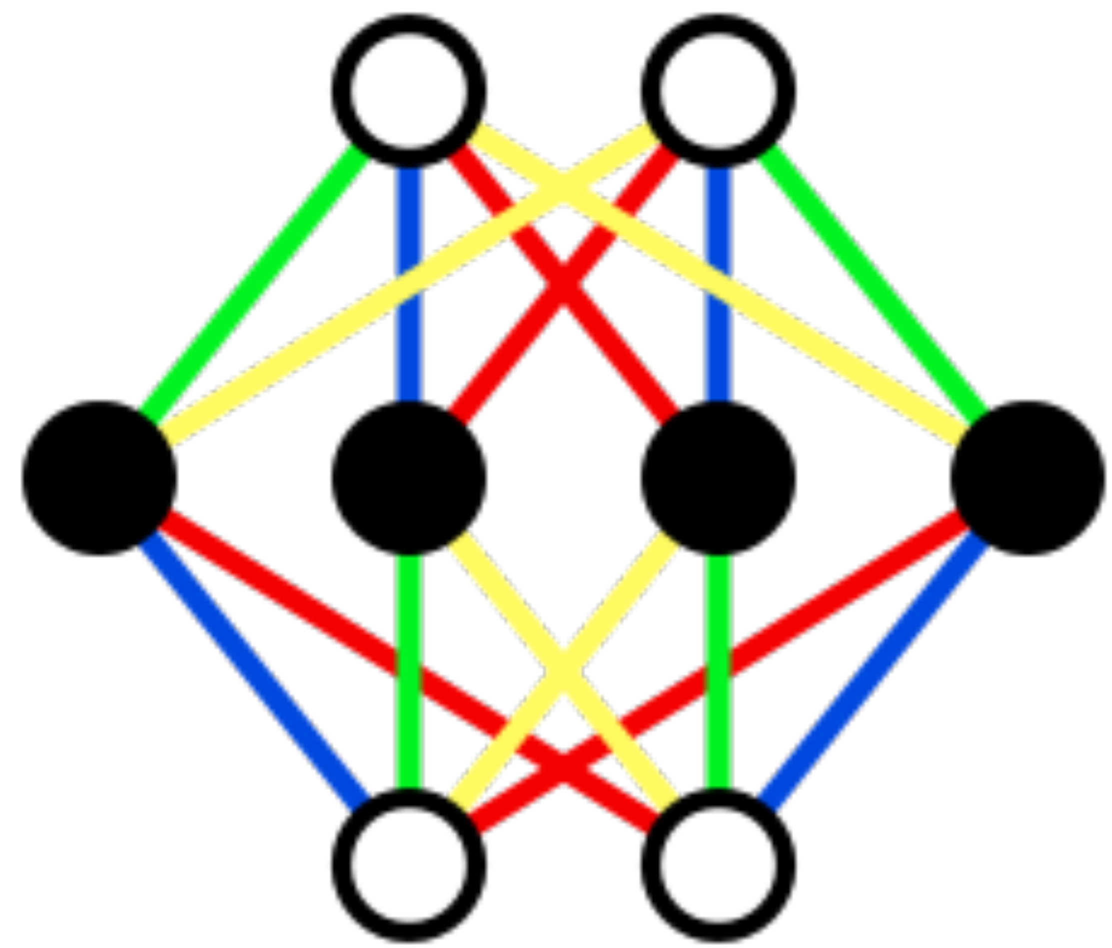
What is an Adinkra?

- Graphical Representations of Super Symmetric Algebras
- Encode information about Super-tino partners with complimentary spin
- Analogically, predict coordinates like operators
- Distinct equivalence classes of a Coxeter group

What is an Adinkra?

- Nodal Graphs
- Bipartite
- Coloured
- N-regular

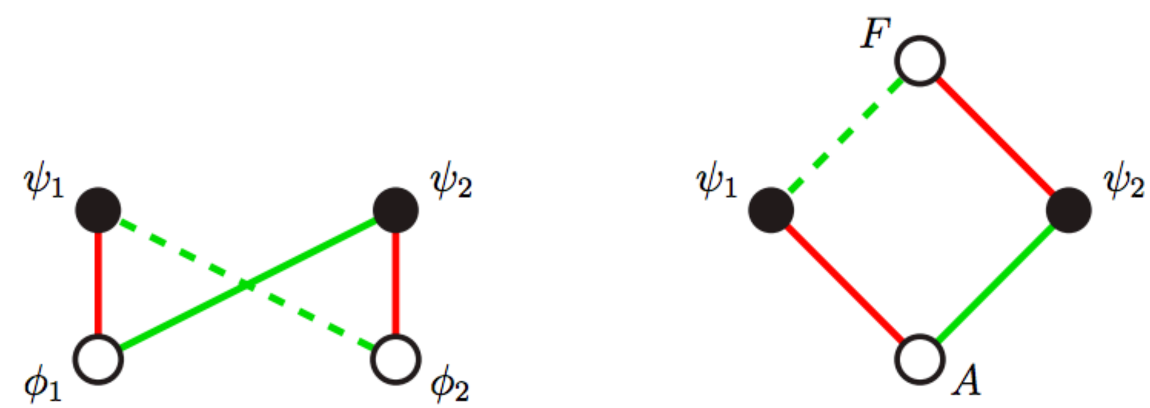


- Specifically, we are concerned with Valise representations

What is an Adinkra?

No other *inequivalent* adinkra can be constructed to satisfy the rules except those that obey the following automorphisms:

- Edge-Colour Swapping
- Dashing Flipping
- Node Swapping
- Sign Changing
- Klein Flipping



Two $N = 2$ Adinkras depicting two Supermuliplets

$$\mathrm{D}_1 \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = i \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}, \qquad (\mathrm{L}_1)_i{}^{\hat{k}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

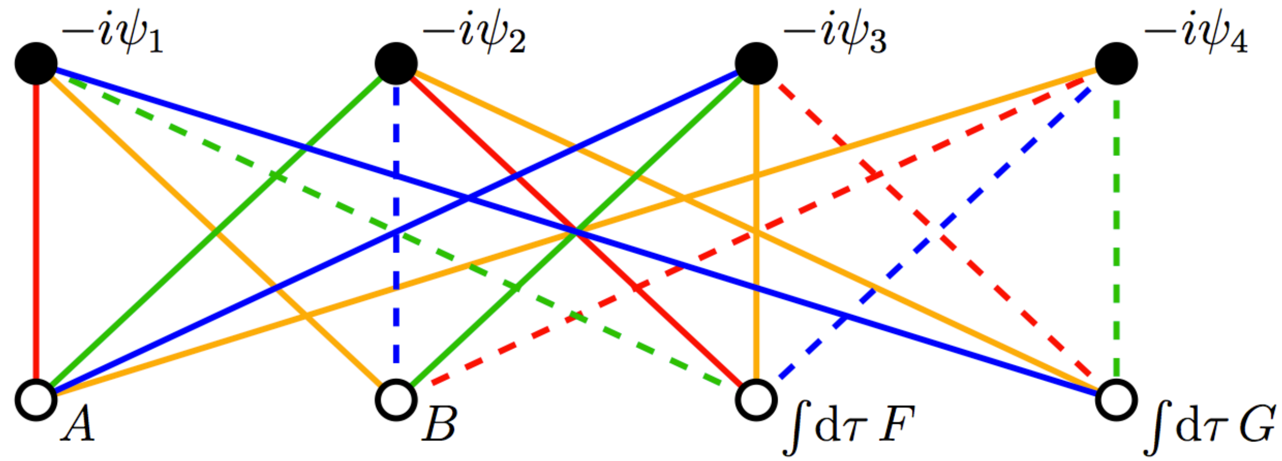
$$\mathrm{D}_2 \begin{bmatrix} \phi_1 \\ \phi_2 \end{bmatrix} = i \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix}, \qquad (\mathrm{L}_2)_i{}^{\hat{k}} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

L-Matrices for left-hand Adinkra

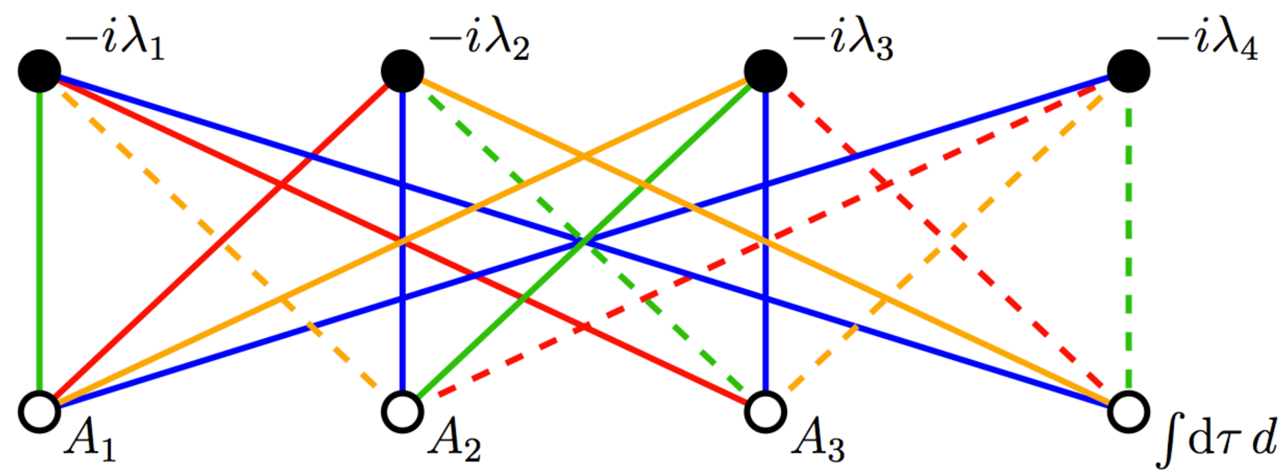
Adinkra (In)Equivalence From Coxeter Group Representations: A Case Study
Isaac Chappell II, S. James Gates Jr, T. Hubsch https://arxiv.org/abs/1210.0478

Classes of Adinkras

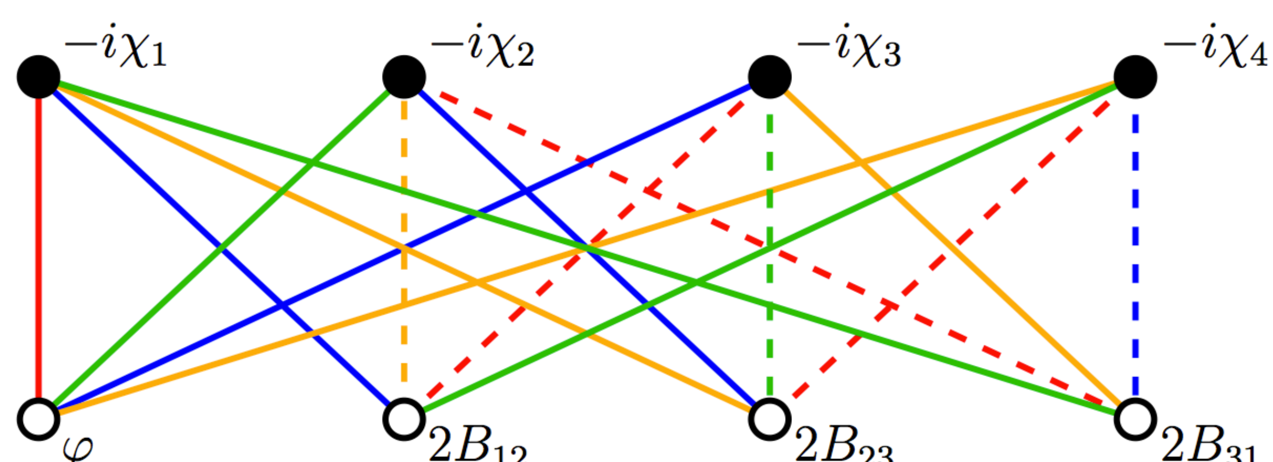
Chiral



Vector



Tensor



Supermultiplets

Chiral Supermultiplet : $(A,\ B,\ \psi_a,\ F,\ G)$

$$\mathrm{D}_a A = \psi_a \quad , \quad \mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \quad ,$$

$$\mathrm{D}_a \psi_b = i\,(\gamma^\mu)_{a\,b}\,\partial_\mu A - (\gamma^5\gamma^\mu)_{a\,b}\,\partial_\mu B - i\,C_{a\,b}\,F + (\gamma^5)_{a\,b} G \ ,$$

$$\mathrm{D}_a F = (\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \quad , \quad \mathrm{D}_a G = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\,\psi_b \quad ,$$

Vector Supermultiplet : $(A_\mu,\ \lambda_b,\ \mathrm{d})$

$$\mathrm{D}_a\,A_\mu = (\gamma_\mu)_a{}^b\,\lambda_b \quad ,$$

$$\mathrm{D}_a \lambda_b = -\,i\,\tfrac{1}{4}([\,\gamma^\mu\,,\,\gamma^\nu\,])_{ab}\,(\,\partial_\mu\,A_\nu - \partial_\nu\,A_\mu\,) + (\gamma^5)_{a\,b}\,\mathrm{d} \ ,$$

$$\mathrm{D}_a\,\mathrm{d} = i\,(\gamma^5\gamma^\mu)_a{}^b\,\partial_\mu\lambda_b \quad ,$$

Tensor Supermultiplet : $(\varphi,\ B_{\mu\,\nu},\ \chi_a)$

$$\mathrm{D}_a\varphi = \chi_a \quad , \quad \mathrm{D}_a B_{\mu\,\nu} = -\,\tfrac{1}{4}([\,\gamma_\mu\,,\,\gamma_\nu\,])_a{}^b\,\chi_b \quad ,$$

$$\mathrm{D}_a\chi_b = i\,(\gamma^\mu)_{a\,b}\,\partial_\mu\varphi - (\gamma^5\gamma^\mu)_{a\,b}\,\epsilon_\mu{}^{\rho\,\sigma\,\tau}\partial_\rho B_{\sigma\,\tau} \ .$$

A Lorentz Covariant Holoraumy-Induced “Gadget” From Minimal Off-Shell 4D, N = 1 Supermultiplets
S. J. Gates, Jr., T. Grover, M. D. Miller-Dickson, B. A. Mondal, A. Oskoui, S. Regmi, E. Ross, R. Shetty
https://arxiv.org/abs/1508.07546

Supermultiplets

Chiral Supermultiplet

$$[\mathrm{D}_a\ ,\ \mathrm{D}_b]A \ = \ -i\,2\,C_{ab}F + 2(\gamma^5)_{ab}G - 2(\gamma^5\gamma^\mu)_{ab}\partial_\mu B \quad ,$$

$$[\mathrm{D}_a\ ,\ \mathrm{D}_b]B \ = \ i\,2\,C_{ab}G + 2(\gamma^5)_{ab}F + 2(\gamma^5\gamma^\mu)_{ab}\partial_\mu A \quad ,$$

$$[\mathrm{D}_a\ ,\ \mathrm{D}_b]\psi_c \ = \ -i(\gamma^5\gamma^\mu)_{ab}(\gamma^5[\gamma_\mu\ ,\ \gamma^\sigma])_c{}^d\partial_\sigma\psi_d \quad ,$$

$$[\mathrm{D}_a\ ,\ \mathrm{D}_b]F \ = \ -i\,2\,C_{ab}\eta^{\mu\sigma}\partial_\mu\partial_\sigma A + 2(\gamma^5)_{ab}\eta^{\mu\sigma}\partial_\mu\partial_\sigma B - 2(\gamma^5\gamma^\mu)_{ab}\partial_\mu G \quad ,$$

$$[\mathrm{D}_a\ ,\ \mathrm{D}_b]G \ = \ i\,2\,C_{ab}\eta^{\mu\sigma}\partial_\mu\partial_\sigma B + 2(\gamma^5)_{ab}\eta^{\mu\sigma}\partial_\mu\partial_\sigma A + 2(\gamma^5\gamma^\mu)_{ab}\partial_\mu F \quad ,$$

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Supermultiplets

Vector Supermultiplet

$$[\mathrm{D}_a\ ,\ \mathrm{D}_b]A_\mu \ =\ -2\epsilon^{\sigma\nu}{}_{\mu\alpha}(\gamma^5\gamma^\alpha)_{ab}\partial_\sigma A_\nu - 2(\gamma^5\gamma_\mu)_{ab}\mathrm{d} \quad ,$$

$$[\mathrm{D}_a\ ,\ \mathrm{D}_b]\mathrm{d} \ =\ 2(\gamma^5\gamma^\mu)_{ab}\partial^\nu\left(\partial_\mu A_\nu \ -\ \partial_\nu A_\mu\right) \quad ,$$

$$[\mathrm{D}_a\ ,\ \mathrm{D}_b]\lambda_c \ =\ -i\,2\,C_{ab}(\gamma^\mu)_c{}^d\partial_\mu\lambda_d - i\,2\,(\gamma^5)_{ab}(\gamma^5\gamma^\mu)_c{}^d\partial_\mu\lambda_d$$

$$-i\,2\,(\gamma^5\gamma^\mu)_{ab}(\gamma^5)_c{}^d\partial_\mu\lambda_d \quad ,$$

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Supermultiplets

Tensor Supermultiplet

$$[\mathrm{D}_a \, , \, \mathrm{D}_b]\varphi \; = \; 2(\gamma^5\gamma^\mu)_{ab}\epsilon^{\rho}{}_{\mu}{}^{\alpha\beta}\partial_\rho B_{\alpha\beta} \quad ,$$

$$[\mathrm{D}_a \, , \, \mathrm{D}_b]B_{\mu\nu} \; = \; -\,\epsilon_{\mu\nu}{}^{\alpha}{}_{\beta}(\gamma^5\gamma^\beta)_{ab}\partial_\alpha\varphi + 4(\gamma^5\gamma_{[\nu})_{ab}\epsilon^{\rho}{}_{\mu]}{}^{\alpha\beta}\partial_\rho B_{\alpha\beta} \quad ,$$

$$[\mathrm{D}_a \, , \, \mathrm{D}_b]\chi_c \; = \; i\,2\,C_{ab}(\gamma^\mu)_c{}^d\partial_\mu\chi_d - i\,2\,(\gamma^5)_{ab}(\gamma^5\gamma^\mu)_c{}^d\partial_\mu\chi_d + i\,2\,(\gamma^5\gamma^\mu)_{ab}(\gamma^5)_c{}^d\partial_\mu\chi_d \quad .$$

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Supermultiplet Fermions

Chiral Supermultiplet Fermion

$$\begin{aligned}[\mathrm{D}_a\,,\,\mathrm{D}_b]\,\psi_c &= -\,i\,(\gamma^5\gamma^\nu)_{ab}(\gamma^5[\,\gamma_\nu\,,\,\gamma^\mu\,])_c{}^d\partial_\mu\psi_d \\ &\equiv \left[\boldsymbol{H}^{\mu(CS)}\right]_{abc}{}^d\,(\partial_\mu\psi_d\,) \quad ,\end{aligned}$$

Vector Supermultiplet Fermion

$$\begin{aligned}[\mathrm{D}_a\,,\,\mathrm{D}_b]\,\lambda_c &= -\,i2\,C_{ab}(\gamma^\mu)_c{}^d\partial_\mu\lambda_d \;-\; i2\,(\gamma^5)_{ab}(\gamma^5\gamma^\mu)_c{}^d\partial_\mu\lambda_d \\ &\quad -\,i2\,(\gamma^5\gamma^\mu)_{ab}(\gamma^5)_c{}^d\partial_\mu\lambda_d \\ &\equiv \left[\boldsymbol{H}^{\mu(VS)}\right]_{abc}{}^d\,(\partial_\mu\lambda_d\,) \quad ,\end{aligned}$$

Tensor Supermultiplet Fermion

$$\begin{aligned}[\mathrm{D}_a\,,\,\mathrm{D}_b]\,\chi_c &= i2\,C_{ab}(\gamma^\mu)_c{}^d\partial_\mu\chi_d \;-\; i2\,(\gamma^5)_{ab}(\gamma^5\gamma^\mu)_c{}^d\partial_\mu\chi_d \\ &\quad +\,i2\,(\gamma^5\gamma^\mu)_{ab}(\gamma^5)_c{}^d\partial_\mu\chi_d \\ &\equiv \left[\boldsymbol{H}^{\mu(TS)}\right]_{abc}{}^d\,(\partial_\mu\chi_d\,) \quad .\end{aligned}$$

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Chiral Supermultiplet: L-Matrices

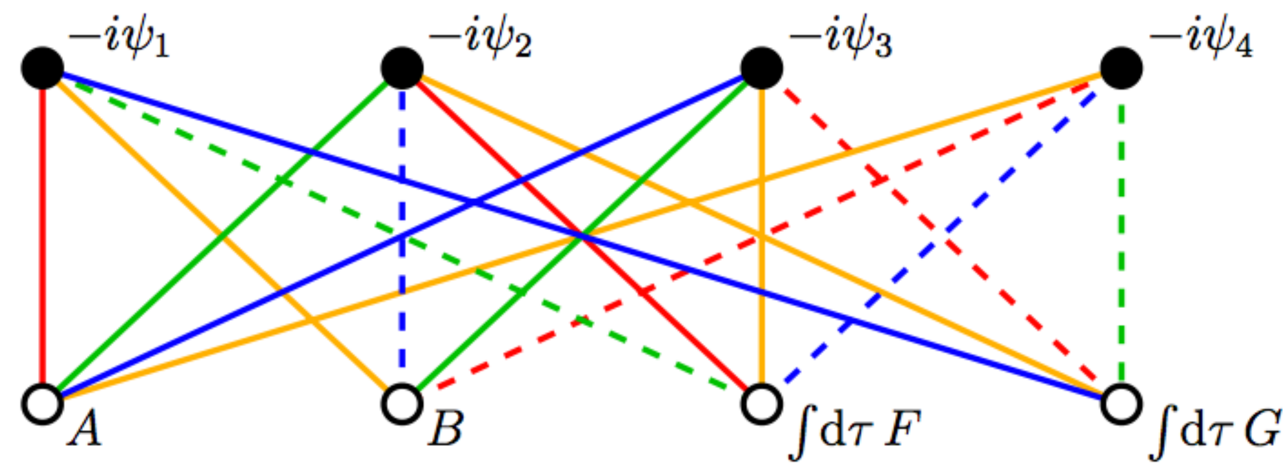
$$(\mathrm{L}_{\mathrm{I}})_i{}^{\hat{k}} \quad = \quad (\mathcal{C}^{(\mathrm{I})})_i{}^{\hat{\ell}} \quad \times \quad (\mathcal{P}_{(\mathrm{I})})_{\hat{\ell}}{}^{\hat{k}}$$

$$(\mathrm{L}_1)_i{}^{\hat{k}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$(\mathrm{L}_2)_i{}^{\hat{k}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$(\mathrm{L}_3)_i{}^{\hat{k}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$(\mathrm{L}_4)_i{}^{\hat{k}} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$



Chiral Supermultiplet

$$\begin{aligned} \mathrm{D}_2(a,b,c,d)^t &= i(\mathrm{L}_2)_i{}^{\hat{k}}\,(\kappa,\lambda,\mu,\nu)^t \\ &= i\,\langle 23\bar{1}\bar{4}\rangle\,(\kappa,\lambda,\mu,\nu)^t \qquad = i\,(\lambda,\mu,-\kappa,-\nu)^t, \end{aligned}$$

$$\mathrm{D}_2 \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = i \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \kappa \\ \lambda \\ \mu \\ \nu \end{bmatrix} \qquad = i \begin{bmatrix} \lambda \\ \mu \\ -\kappa \\ -\nu \end{bmatrix} .$$

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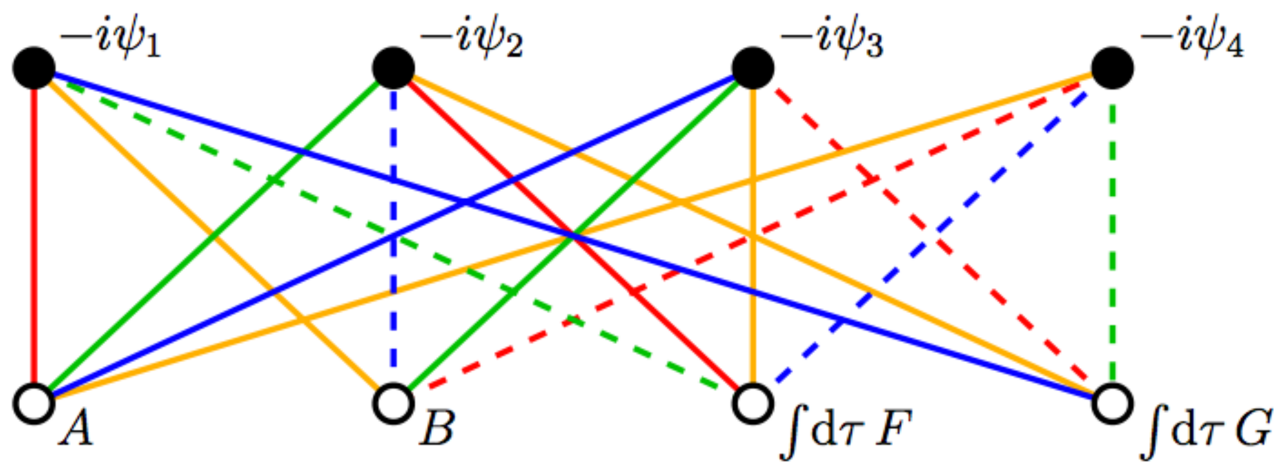
Chiral Supermultiplet

$$\mathrm{D}_a A = \psi_a \quad ,$$

$$\mathrm{D}_a B = i\,(\gamma^5)_a{}^b\,\psi_b \quad ,$$

$$\mathrm{D}_a \psi_b = i\,(\gamma^\mu)_{a\,b}\,\partial_\mu A \;-\; (\gamma^5\gamma^\mu)_{a\,b}\,\partial_\mu B$$



$$\{\, \mathrm{D}_a\,,\, \mathrm{D}_b \,\}\, A = i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\, A \quad , \quad \{\, \mathrm{D}_a\,,\, \mathrm{D}_b \,\}\, B = i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\, B \quad ,$$

$$\{\, \mathrm{D}_a\,,\, \mathrm{D}_b \,\}\, \psi_c = i\,2\,(\gamma^\mu)_{a\,b}\,\partial_\mu\,\psi_c + i\,2\,(\gamma^\mu)_{a\,b}\,(\gamma_\mu)_c{}^d\,\mathcal{K}_d(\psi) \quad ,$$

$$\mathcal{K}_c(\psi) = -\,\tfrac{1}{2}\,\mathrm{D}_c F \quad , \quad \mathcal{K}_c(\psi) = i\,\tfrac{1}{2}\,(\gamma^5)_c{}^d\,\mathrm{D}_d G$$

4D, $N = 1$ Supersymmetry Genomics (I)
S.J. Gates Jr., J. Gonzales, B. MacGregor, J. Parker, R. Polo-Sherk, V.G.J. Rodgers, L. Wassink
https://arxiv.org/abs/0902.3830

Gadget Construction: Chiral Supermultiplet

All permutations with determinant {-1,-1,-1,-1} and trace {2,0,2,0}

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}\quad\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}\quad\begin{pmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{pmatrix}\quad\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}\quad\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{pmatrix}\quad\begin{pmatrix}0&1&0&0\\0&0&0&1\\1&0&0&0\\0&0&1&0\end{pmatrix}$$

Det $= -1$	Det $= -1$	Det $= -1$	Det $= -1$	Det $= -1$	Det $= -1$
Tra $= 2$	Tra $= 2$	Tra $= 2$	Tra $= 2$	Tra $= 0$	Tra $= 0$

$$\begin{pmatrix}0&0&1&0\\1&0&0&0\\0&0&0&1\\0&1&0&0\end{pmatrix}\quad\begin{pmatrix}0&0&1&0\\0&1&0&0\\1&0&0&0\\0&0&0&1\end{pmatrix}\quad\begin{pmatrix}0&0&1&0\\0&0&0&1\\0&1&0&0\\1&0&0&0\end{pmatrix}\quad\begin{pmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{pmatrix}\quad\begin{pmatrix}0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0\end{pmatrix}\quad\begin{pmatrix}0&0&0&1\\0&0&1&0\\1&0&0&0\\0&1&0&0\end{pmatrix}$$

Det $= -1$	Det $= -1$	Det $= -1$	Det $= -1$	Det $= -1$	Det $= -1$
Tra $= 0$	Tra $= 2$	Tra $= 0$	Tra $= 0$	Tra $= 2$	Tra $= 0$

Gadget Construction: Chiral Supermultiplet

Super Adjacency matrices as permutations of the 4x4 identity matrix

$$\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix}+\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix}+\begin{pmatrix}0&0&1&0\\0&0&0&1\\0&1&0&0\\1&0&0&0\end{pmatrix}+\begin{pmatrix}0&0&0&1\\0&0&1&0\\1&0&0&0\\0&1&0&0\end{pmatrix}=\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{pmatrix}+\begin{pmatrix}0&1&0&0\\0&0&0&1\\1&0&0&0\\0&0&1&0\end{pmatrix}+\begin{pmatrix}0&0&1&0\\1&0&0&0\\0&0&0&1\\0&1&0&0\end{pmatrix}+\begin{pmatrix}0&0&0&1\\0&1&0&0\\0&0&1&0\\1&0&0&0\end{pmatrix}=\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix}$$

$$\begin{pmatrix}1&0&0&0\\0&0&0&1\\0&0&1&0\\0&1&0&0\end{pmatrix}+\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\1&0&0&0\end{pmatrix}+\begin{pmatrix}0&0&1&0\\0&1&0&0\\1&0&0&0\\0&0&0&1\end{pmatrix}+\begin{pmatrix}0&0&0&1\\1&0&0&0\\0&1&0&0\\0&0&1&0\end{pmatrix}=\begin{pmatrix}1&1&1&1\\1&1&1&1\\1&1&1&1\\1&1&1&1\end{pmatrix}$$

Gadget Results: Chiral Supermultiplet

All possible configurations of sign-in matrix components

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Generation of 6 uncoloured super-adjacency matrices

$$\begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & -1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 \\ 1 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 & 1 \\ 1 & -1 & -1 & -1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

Gadget Results: Chiral Supermultiplet

The space of all possible Chiral graphs contains 18 Colour-Sign Super Adjacency matrices

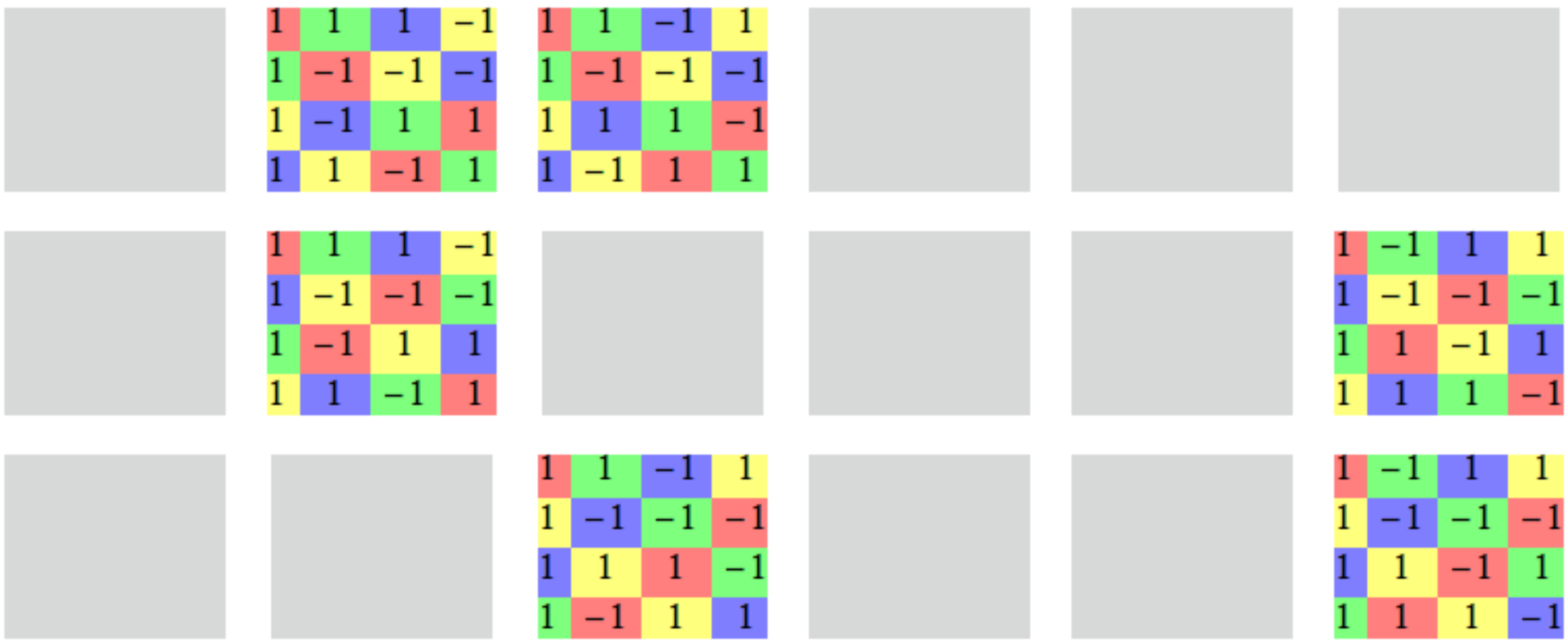
$$\begin{bmatrix}1&1&1&-1\\1&-1&-1&-1\\1&1&-1&1\\1&-1&1&1\end{bmatrix}\begin{bmatrix}1&1&1&-1\\1&-1&-1&-1\\1&-1&1&1\\1&1&-1&1\end{bmatrix}\begin{bmatrix}1&1&-1&1\\1&-1&-1&-1\\1&1&1&-1\\1&-1&1&1\end{bmatrix}\begin{bmatrix}1&1&-1&1\\1&-1&-1&-1\\1&-1&1&1\\1&1&1&-1\end{bmatrix}\begin{bmatrix}1&-1&1&1\\1&-1&-1&-1\\1&1&1&-1\\1&1&-1&1\end{bmatrix}\begin{bmatrix}1&-1&1&1\\1&-1&-1&-1\\1&1&-1&1\\1&1&1&-1\end{bmatrix}$$

$$\begin{bmatrix}1&1&1&-1\\1&-1&-1&-1\\1&1&-1&1\\1&-1&1&1\end{bmatrix}\begin{bmatrix}1&1&1&-1\\1&-1&-1&-1\\1&-1&1&1\\1&1&-1&1\end{bmatrix}\begin{bmatrix}1&1&-1&1\\1&-1&-1&-1\\1&1&1&-1\\1&-1&1&1\end{bmatrix}\begin{bmatrix}1&1&-1&1\\1&-1&-1&-1\\1&-1&1&1\\1&1&1&-1\end{bmatrix}\begin{bmatrix}1&-1&1&1\\1&-1&-1&-1\\1&1&1&-1\\1&1&-1&1\end{bmatrix}\begin{bmatrix}1&-1&1&1\\1&-1&-1&-1\\1&1&-1&1\\1&1&1&-1\end{bmatrix}$$

$$\begin{bmatrix}1&1&1&-1\\1&-1&-1&-1\\1&1&-1&1\\1&-1&1&1\end{bmatrix}\begin{bmatrix}1&1&1&-1\\1&-1&-1&-1\\1&-1&1&1\\1&1&-1&1\end{bmatrix}\begin{bmatrix}1&1&-1&1\\1&-1&-1&-1\\1&1&1&-1\\1&-1&1&1\end{bmatrix}\begin{bmatrix}1&1&-1&1\\1&-1&-1&-1\\1&-1&1&1\\1&1&1&-1\end{bmatrix}\begin{bmatrix}1&-1&1&1\\1&-1&-1&-1\\1&1&1&-1\\1&1&-1&1\end{bmatrix}\begin{bmatrix}1&-1&1&1\\1&-1&-1&-1\\1&1&-1&1\\1&1&1&-1\end{bmatrix}$$

Gadget Results: Chiral Supermultiplet

The space of all possible Chiral Adinkras contains 6 Colour-Sign Super Adjacency matrices



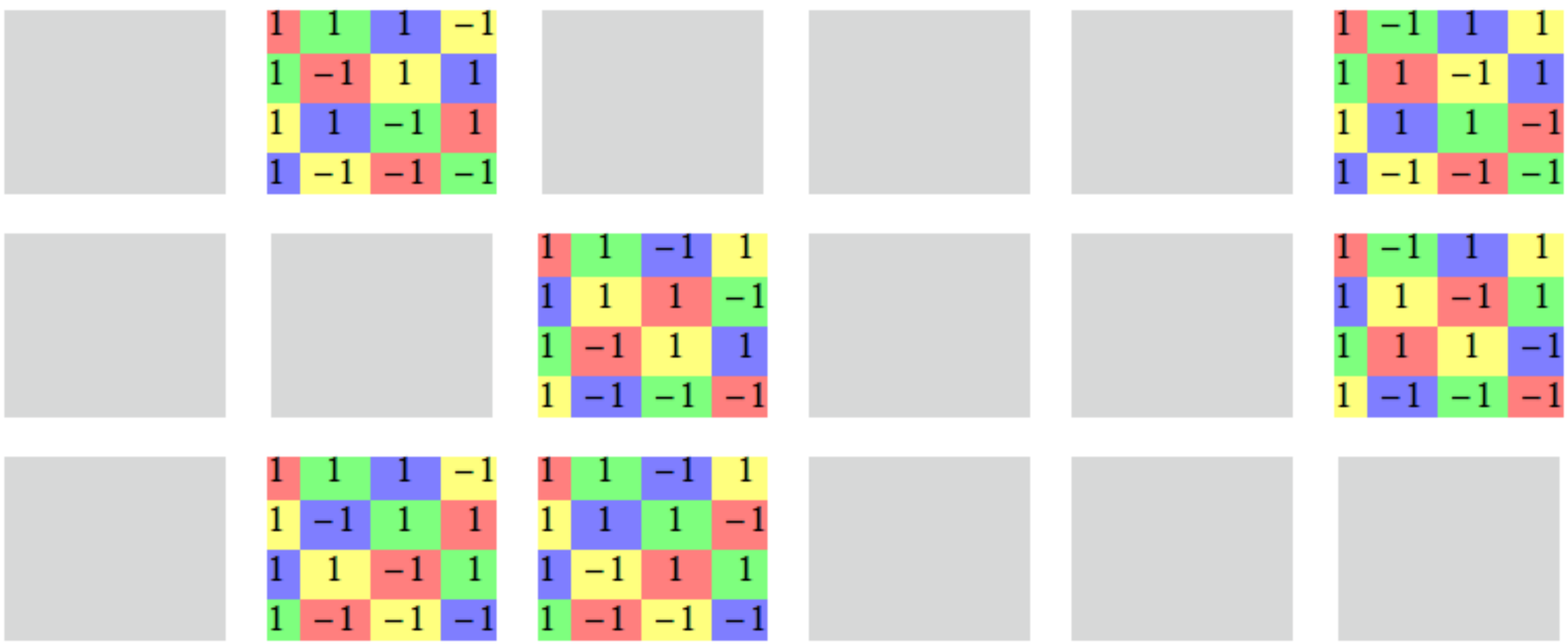
The entire equivalence class of Chiral Adinkras can be generated with a single identifier

Base 4 : 110100110

Base 10: 82964

Gadget Results: Vector Supermultiplet

The space of all possible Vector Adinkras contains 6 Colour-Sign Super Adjacency matrices



The entire equivalence class of Vector Adinkras can be generated with a single identifier

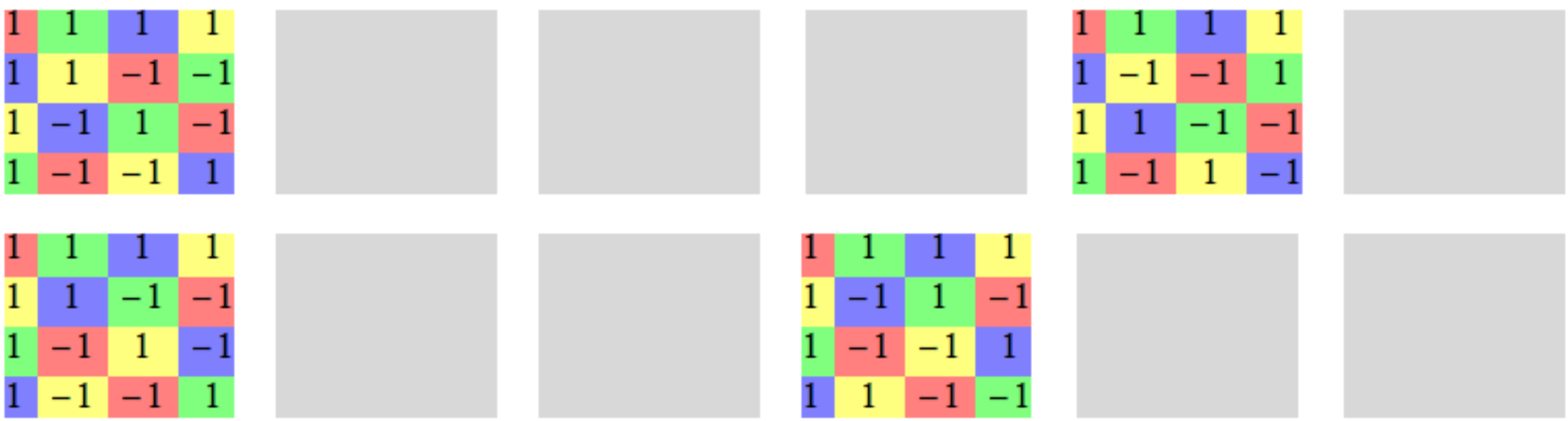
Base 4 : 111000310

Base 10: 86068

Vectorial L-Matrices of determinant {-1,-1,-1,-1} and trace {2,2,0,0}

Gadget Results: Tensor Supermultiplet

The space of all possible Tensor Adinkras contains 6 Colour-Sign Super Adjacency matrices



The entire equivalence class of Tensor Adinkras can be generated with a single identifier

Base 4 : 011111010

Base 10: 21828

Tensorial L-Matrices of determinant {1,1,1,1} and trace {1,1,1,1}

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