Magnetic Field Coils for the MSR Interior

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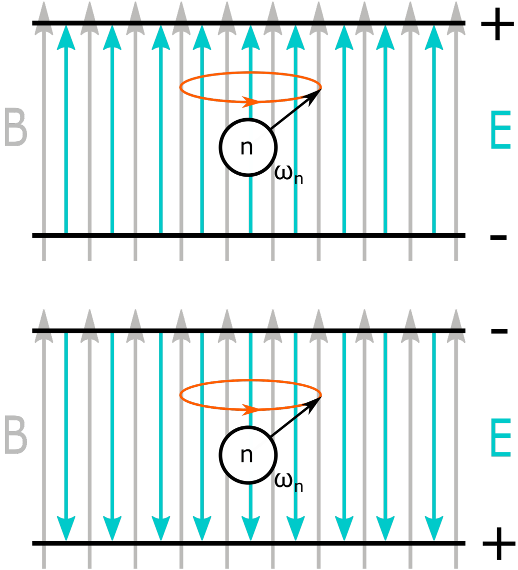
TUCAN nEDM Mesurement Method



- The neutron electric dipole (nEDM) is extracted from the spin precession rate of the neutrons by taking the difference in the precession rates for polarized neutrons in configurations with the magnetic and electric fields parallel and anti-parallel.
- The goal of the TUCAN Collaboration is to measure the nEDM to a precision of better than $10^{-27} e\,cm$.

Magnetic Field Requirements

- As the ultra-cold neutrons (UCN) will travel through out the measurement cells during the measurement process, any inhomogeneities in the magnetic field will change the instantaneous precession rate of neutrons at each new location.
- Some magnetic field shapes will not only depolarize the neutrons in the cells, but also contribute to a false nEDM measurement. For example when using a mercury comagnetometer,

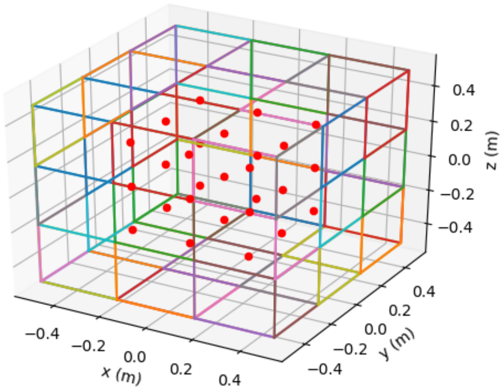
$$d_{Hg}^{false} = -\frac{\hbar\gamma_n\gamma_{Hg199}}{2c^2}\langle xB_x + yB_y\rangle. \tag{1}$$

- Due to these effects a highly uniform magnetic field is required in the measurement cells.
- The degree of uniformity that is required is determined from the desired accuracy of the measurement.

Setting the Uniformity Limits

- As the nEDM measurement process requires polarized neutrons, the length of time that the neutron polarization must be maintained sets the limits on the required magnetic field homogeneity.
- One metric of the time that neutrons will remain polarized in a magnetic field is the T_2 relaxation time, which is a time constant related to the time for a fully polarized sample to completely depolarize.
- To reach a neutron $T_2 > 10,000\ \mathrm{s}$ the following requirements can be placed on the field uniformity:
  - $\Delta B_z < 140\ \mathrm{pT}$, where ΔB_z is the difference in the B_z minimum and maximum in a cell.
  - $\sigma(B_z) < 40\ \mathrm{pT}$, where $\sigma(B_z)$ is the standard deviation of the field over a cell.

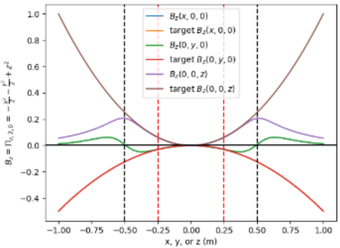
$n \times n$ Coil Array



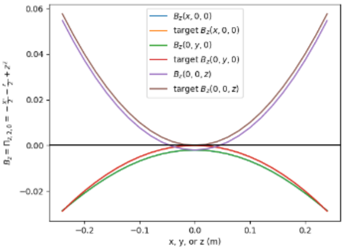
An example coil array shown with an 3×3 array of 9 coils on each cube face. Adjacent coils are drawn with different colors.

- The purpose of this coil array is to cancel arbitrary field inhomogeneities inside the magnetically shielded room(MSR).
- It is constructed as an array of square coils arranged symmetrically on the surface of a cube.
- The cancellation method is performed by using a fitting method to determine the required current in each coil to best cancel the measured fields at the selected measurement points, the 27 red circles in the example.

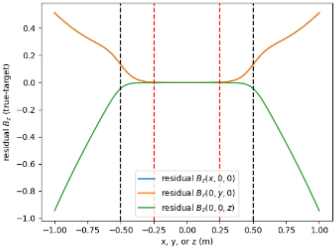
Example Field Cancellation



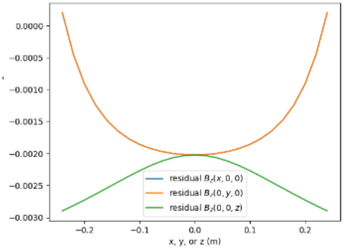
(a) B_z field.



(b) zoomed in B_z field.



(c) residual B_z field.



(d) zoomed in residual B_z field.

- $B_z(x, y, z) = z^2 - \frac{1}{2}(x^2 + y^2)$
- Black dashed lines are the coil array edges.
- Red dashed lines on left hand plots is zoom extents on right hand plots.

Table 1: Results from 3×3 Array on Applied Fields

(ℓ, m) or dipole	ΔB_z (pT) before correction	ΔB_z (pT) after correction	$\sigma(B_z)$ (pT) before correction	$\sigma(B_z)$ (pT) after correction	$\langle B_T^2 \rangle$ (nT^2) before correction	$\langle B_T^2 \rangle$ (nT^2) after correction
(1,0)	455	6	140	0.6	0.26	3×10^{-6}
(1,1)	1790	106	458	9	0.36	2×10^{-4}
(2,0)	500	58	106	7	0.04	4×10^{-4}
(2,1)	700	16	120	2	0.02	5×10^{-5}
(2,2)	1065	116	230	12	0.17	4×10^{-4}
dipole 1	1140	232	244	14	0.53	7×10^{-4}
dipole 2 upper	500	20	109	3	0.47	3×10^{-5}
dipole 2 lower	220	17	50	3	0.47	3×10^{-5}
dipole 3 upper	870	37	187	8	0.16	1×10^{-4}
dipole 3 lower	400	41	87	4	0.16	1×10^{-4}
(3,1)	200	61	31	6	0.004	1×10^{-4}
(3,3)	340	65	61	5	0.02	2×10^{-4}
(4,2)	123	59	14	6	0.0005	2×10^{-4}
goal		140		40		0.1

What are the $G_{\ell,m}$'s?

- For magnetic fields in volumes containing no electric currents it can be convenient to describe the field in the terms of a polynomial sum of the form:

$$B(x,y,z) = \sum_{l,m} G_{l,m} \begin{pmatrix} \Pi_{x,l,m}(x,y,z) \\ \Pi_{y,l,m}(x,y,z) \\ \Pi_{z,l,m}(x,y,z) \end{pmatrix} \tag{2}$$

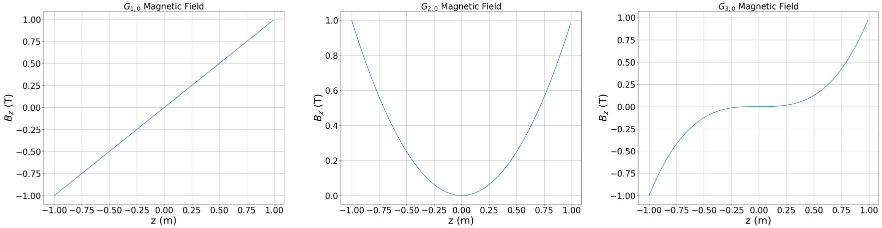
  where each term is a harmonic solution of the Laplace equation.
- See some example terms below:

l	m	Π_x	Π_y	Π_z
0	−1	0	1	0
0	0	0	0	1
0	1	1	0	0
1	−2	y	x	0
1	−1	0	z	y
1	0	$-\frac{1}{2}x$	$-\frac{1}{2}y$	z
1	1	z	0	x
1	2	x	$-y$	0

source: C. Abel et al., Phys. Rev. A 99, 042112 (2019)

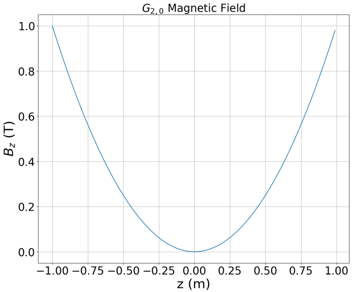
Purpose of G_{lm} Coils

- The goal of the the G_{lm} coils is to cancel or selectively induce specific multipole moments that are related to significant systematics effects in the nEDM measurement.
- Being able to produce these magnetic multipole moments at a high purity will allow dedicated systematic runs to be performed.
- It is believed that 3 such coils are required corresponding to the three lowest order multipole moments, $G_{1,0}$, $G_{2,0}$, and $G_{3,0}$, should be sufficient for the experiment.

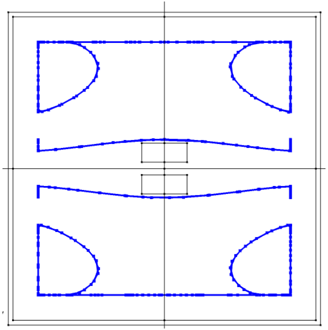


$G_{\ell,m}$ Coil Shapes

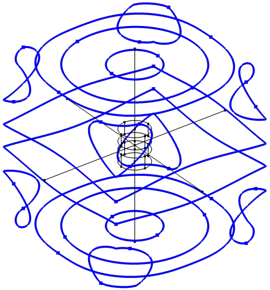
Field Shape



Coil Side View



Coil Isometric View

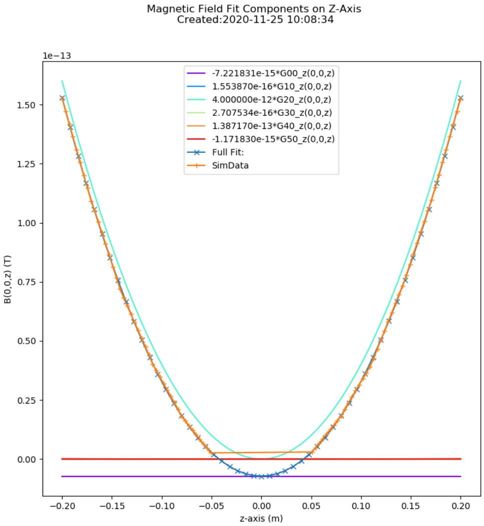


$G_{\ell,m}$ Coil Metrics

The primary metric for these coils is then how well the specific field shape is recreated over the measurement cells.

- Relative Harmonic Contributions
  - By fitting a series of $G_{\ell,m}$ terms to the field produced from a coil configuration the purity of the mode can be examined.
- Calculate the specific systematics that would be produced by a pure multipole to determine the cancellation efficiency.
  - For example, $d_{Hg}^{false} = -\frac{\hbar\gamma_n\gamma_{Hg199}}{2c^2}\langle xB_x + yB_y\rangle$

Example $G_{\ell,m}$ Evaluation



Relative $G_{\ell,0}$	
$G_{\ell,0}$	$G_{\ell,0}/G_{2,0}$
$G_{0,0}$	0.002
$G_{1,0}$	0.00004
$G_{2,0}$	1.0
$G_{3,0}$	0.00007
$G_{4,0}$	0.03
$G_{5,0}$	0.0003

Comparision to Ideal		
B_z	Simulated	Ideal
mean	3.03e-14	3.79e-14
$\sigma(B_z)$	4.87e-14	4.78e-14
ΔB_z	21.69e-14	15e-14
d_{Hg}^{false}	-8.3013e-15	-8.16e-15

Conclusions

- Based on the required measurement accuracy metrics were set for the magnetic field uniformity and shapes to determine when a coil would function as intended for the experiment.
- An $n \times n$ array of square coils was designed to remove arbitrary field inhomogeneities.
- This coil configuration was shown to be able to cancel a wide range of field shapes.
- A set of $G_{\ell,m}$ coils were designed to create specific field shapes in the measurement cell volumes.
- It has been shown that it is possible to create the required field shapes with a sufficient quality.
- For each type coil configurations have been simulated that meet the field uniformity requirements.