

Proton Radius: A Puzzle or a Solution!?

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New Scientific Opportunities and the TRIUMF ARIEL e-linac
26 May 2022



Research Supported by NSF

Abstract

The *proton radius puzzle* is known as the discrepancy of the proton radius, obtained from muonic hydrogen spectroscopy (about 0.84 fm), and the proton radius obtained from (ordinary) hydrogen spectroscopy (mainly by the Paris group), who measured a number of transitions in atomic hydrogen, involving highly excited states (which led to a radius of 0.88 fm). Recently, a number of measurements of hydrogen transitions by the *Munich (Garching)* groups ($2S-4P$), by the spectroscopy group at the *University of Toronto* ($2S-2P_{1/2}$), and by the group at *Colorado State University* ($2S-8D$), have led to transition frequency data consistent with the smaller proton radius, pointing to a possible, purely experimental, resolution of the proton radius puzzle. In the talk, we will discuss a *complete reevaluation of the irreducible two-loop vacuum-polarization correction* to muonic hydrogen energy levels. This calculation addresses one of the most challenging contributions relevant for the proton radius puzzle. A comparison of the raw data for the Sachs G_E form factor of the proton, from the *PRad* and *Mainz* collaborations, reveals that the situation in regard to scattering experiments might be less clear than currently thought, raising the question whether or not the proton radius puzzle has been conclusively solved, and opening up interesting *experimental possibilities at TRIUMF ARIEL*.

Forgotten Physical Review Letter from 1969

PRL 23, 153 (1969) published on 21–JUL–1969
(coincidentally, the **precise** day when mankind set foot on the moon)

VOLUME 23, NUMBER 3

PHYSICAL REVIEW LETTERS

21 JULY 1969

HIGH-ENERGY MUON-PROTON SCATTERING: MUON-ELECTRON UNIVERSALITY*

L. Camilleri,† J. H. Christenson, M. Kramer,‡ and L. M. Lederman

Columbia University, New York, New York 10027, and Brookhaven National Laboratory, Upton, New York 11973

and

Y. Nagashima and T. Yamanouchi

University of Rochester, Rochester, New York 14627,
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(Received 10 April 1969)

Measurements of the μ - p elastic cross section in the range $0.15 < q^2 < 0.85$ (GeV/c)² are compared with similar e - p data. We find an apparent disagreement between the muon and electron experiments which can possibly be accounted for by a combination of systematic normalization errors.

Could this be a hint for new physics?

The signal seen in 1969 matches the discrepancy seen in the proton radius derived from hydrogen versus muonic hydrogen spectroscopy (both sign and magnitude).

Non-Universality of about 4% Seen in 1969

Plot from PRL 23, 153 (1969)

[Sachs form factor, muon versus electron scattering]:

VOLUME 23, NUMBER 3 PHYSICAL REVIEW

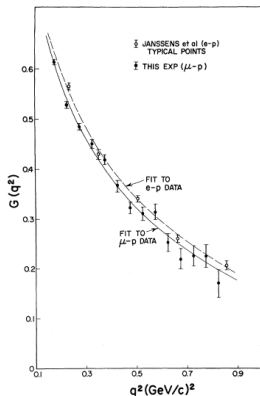


FIG. 1. Measurements of the form factor $G(q^2)$ vs q^2 for this experiment and for the $e-p$ data of Janssens et al. Not all of the electron data are shown. The solid and dashed curves represent fits to the muon and electron data, respectively.

Two (Perhaps, Three) Approaches to the Proton Radius Determination

Way #1: Scattering Experiments

$$\langle r^2 \rangle_P = r_P^2 = 6\hbar^2 \left. \frac{\partial G_E(q^2)}{\partial q^2} \right|_{q^2=0}$$

Way #2A: Muonic Hydrogen Spectroscopy

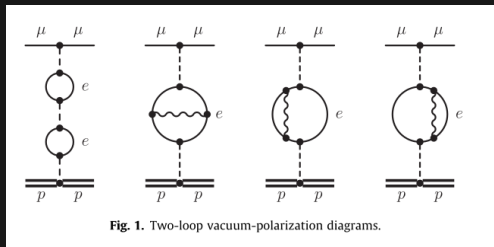
$$\Delta E = \frac{2}{3} \frac{(Z\alpha)^4 \mu c^2}{\pi n^3} \left(\frac{\mu c r_P}{\hbar} \right)^2$$

Way #2B: Hydrogen Spectroscopy

We have $Z = 1$. The reduced mass μ is roughly 200 times larger for muonic bound systems as compared to ordinary hydrogen. The finite-size effect is proportional to μ^3 , and thus, muonic hydrogen is very sensitive probe of the proton radius.

(Not Yet Published) Reevaluation of Two-Loop Vacuum Polarization

Muonic bound systems are sensitive to vacuum-polarization corrections.
For a theoretical overview: [UDJ, *Ann. Phys. (N.Y.)* **326**, 500 (2011)]



The two-loop vacuum-polarization correction contributes an energy shift of about 1.5081 meV to the muonic hydrogen Lamb shift and had never been reevaluated beyond the classic works of Kallen and Sabry (1955), and Barbieri and Remiddi (1973). A reevaluation using dimensional regularization and integration-by-parts techniques (S. Laporta and UDJ, in preparation) sheds additional light on the problem, in view of a comparison to a proton size puzzle of 0.3 meV in the muonic hydrogen Lamb shift.

2010 measurement: $r_p = 0.84184(67) \text{ fm} \approx 0.84 \text{ fm}$.

Nature **466**, 213 (2010):

The size of the proton

Randolf Pohl¹, Aldo Antognini¹, François Nez², Fernando D. Amaro³, François Biraben², João M. R. Cardoso³, Daniel S. Covita^{3,4}, Andreas Dax⁵, Satish Dhawan⁵, Luis M. P. Fernandes³, Adolf Giesen^{6†}, Thomas Graf⁶, Theodor W. Hänsch¹, Paul Indelicato³, Lucile Julien², Cheng-Yang Kao⁷, Paul Knowles⁸, Eric-Olivier Le Bigot², Yi-Wei Liu⁷, José A. M. Lopes³, Livia Ludhova⁸, Cristina M. B. Monteiro³, Françoise Mulhauser^{8†}, Tobias Nebel¹, Paul Rabinowitz⁹, Joaquim M. F. dos Santos³, Lukas A. Schaller⁸, Karsten Schuhmann¹⁰, Catherine Schwob², David Taqq¹¹, João F. C. A. Veloso⁴ & Franz Kottmann¹²

present calculations^{11–15} of fine and hyperfine splittings and QED terms, we find $r_p = 0.84184(67) \text{ fm}$, which differs by 5.0 standard deviations from the CODATA value³ of 0.8768(69) fm. Our result implies that either the Rydberg constant has to be shifted by $-110 \text{ kHz}/c$ (4.9 standard deviations), or the calculations of the QED effects in atomic hydrogen or muonic hydrogen atoms are insufficient.

2010 measurement: $r_p = 0.879(8)$ fm ≈ 0.88 fm.**High-Precision Determination of the Electric and Magnetic Form Factors of the Proton**

J. C. Bernauer,^{1,*} P. Achenbach,¹ C. Ayerbe Gayoso,¹ R. Böhm,¹ D. Bosnar,² L. Debenjak,³ M. O. Distler,^{1,†} L. Doria,¹ A. Esser,¹ H. Fonvieille,⁴ J. M. Friedrich,⁵ J. Friedrich,¹ M. Gómez Rodríguez de la Paz,¹ M. Makek,² H. Merkel,¹ D. G. Middleton,¹ U. Müller,¹ L. Nungesser,¹ J. Pochodzalla,¹ M. Potokar,³ S. Sánchez Majos,¹ B. S. Schlimme,¹ S. Širca,^{6,3} Th. Walcher,¹ and M. Weinriefer¹

(A1 Collaboration)

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(Received 29 July 2010; published 10 December 2010)

New precise results of a measurement of the elastic electron-proton scattering cross section performed at the Mainz Microtron MAMI are presented. About 1400 cross sections were measured with negative four-momentum transfers squared up to $Q^2 = 1$ (GeV/c)² with statistical errors below 0.2%. The electric and magnetic form factors of the proton were extracted by fits of a large variety of form factor models directly to the cross sections. The form factors show some features at the scale of the pion cloud. The charge and magnetic radii are determined to be $\langle r_E^2 \rangle^{1/2} = 0.879(5)_{\text{stat}}(4)_{\text{syst}}(2)_{\text{model}}(4)_{\text{group}}$ fm and $\langle r_M^2 \rangle^{1/2} = 0.777(13)_{\text{stat}}(9)_{\text{syst}}(5)_{\text{model}}(2)_{\text{group}}$ fm.

DOI: 10.1103/PhysRevLett.105.242001

PACS numbers: 14.20.Dh, 13.40.Gp, 25.30.Bf

Proton radius puzzle (simplified): $r_p = 0.84$ fm or $r_p = 0.88$ fm?

Recent Hydrogen Measurement [2017, German]: $2S-4P$

2017 measurement [Garching]: $r_p = 0.8335(95)$ fm.

Emphasis on so-called cross-damping terms.

See also [UDJ and P.J.Mohr, Can. J. Phys. **80**, 633 (2002)].

RESEARCH

RESEARCH ARTICLE

ATOMIC PHYSICS

The Rydberg constant and proton size from atomic hydrogen

Axel Beyer,¹ Lothar Maisenbacher,^{1*} Arthur Matveev,¹ Randolph Pohl,^{1†}
Ksenia Khabarova,^{2,3} Alexey Grinin,¹ Tobias Lamour,¹ Dylan C. Yost,^{1‡}
Theodor W. Hänsch,^{1,4} Nikolai Kolachevsky,^{2,3} Thomas Udem^{1,4}

At the core of the “proton radius puzzle” is a four-standard deviation discrepancy between the proton root-mean-square charge radii (r_p) determined from the regular hydrogen (H) and the muonic hydrogen (μp) atoms. Using a cryogenic beam of H atoms, we measured the $2S-4P$ transition frequency in H, yielding the values of the Rydberg constant $R_\infty = 10973731.568076(96)$ per meter and $r_p = 0.8335(95)$ femtometer. Our r_p value is 3.3 combined standard deviations smaller than the previous H world data, but in good agreement with the μp value. We motivate an asymmetric fit function, which eliminates line shifts from quantum interference of neighboring atomic resonances.

Beyer *et al.*, *Science* **358**, 79–85 (2017) 6 October 2017

Recent Hydrogen Measurement [2018, French]: $1S-3S$

2018 measurement [Paris]: $r_p = 0.877(13)$ fm. Oops...

PHYSICAL REVIEW LETTERS **120**, 183001 (2018)

New Measurement of the $1S-3S$ Transition Frequency of Hydrogen: Contribution to the Proton Charge Radius Puzzle

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Michel Abgrall and Jocelyne Guéna
*LNE-SYRTE, Observatoire de Paris, Université PSL, CNRS,
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 (Received 8 December 2017; revised manuscript received 9 March 2018; published 4 May 2018)

We present a new measurement of the $1S-3S$ two-photon transition frequency of hydrogen, realized with a continuous-wave excitation laser at 205 nm on a room-temperature atomic beam, with a relative uncertainty of 9×10^{-13} . The proton charge radius deduced from this measurement, $r_p = 0.877(13)$ fm, is in very good agreement with the current CODATA-recommended value. This result contributes to the ongoing search to solve the proton charge radius puzzle, which arose from a discrepancy between the CODATA value and a more precise determination of r_p from muonic hydrogen spectroscopy.

[The experimental approach taken by the Paris group should be largely independent of cross-damping terms.]

Recent Hydrogen Measurement [2019, Canada]: $2S-2P_{1/2}$

2019 measurement [Toronto]: $r_p = 0.833(10)$ fm.

Leads to a small “Canadian proton”.

ATOMIC PHYSICS

A measurement of the atomic hydrogen Lamb shift and the proton charge radius

N. Bezginov¹, T. Valdez¹, M. Horbatsch¹, A. Marsman¹, A. C. Vutha², E. A. Hessels^{1*}

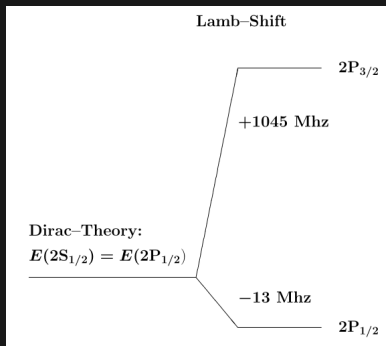
The surprising discrepancy between results from different methods for measuring the proton charge radius is referred to as the proton radius puzzle. In particular, measurements using electrons seem to lead to a different radius compared with those using muons. Here, a direct measurement of the $n = 2$ Lamb shift of atomic hydrogen is presented. Our measurement determines the proton radius to be $r_p = 0.833$ femtometers, with an uncertainty of ± 0.010 femtometers. This electron-based measurement of r_p agrees with that obtained from the analogous muon-based Lamb shift measurement but is not consistent with the larger radius that was obtained from the averaging of previous electron-based measurements.

Bezginov *et al.*, *Science* **365**, 1007–1012 (2019) 6 September 2019

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Lamb Shift and Fine Structure [Canada, Perhaps a Caveat]

Hydrogen spectrum ($n = 2$ manifold, without hyperfine structure):



Perhaps a little caveat: The $2P_{1/2}-2P_{3/2}$ fine-structure is nearly independent of the proton radius and can be calculated to very high precision; its measurement would constitute an important consistency check for the smallness of the “Canadian protons”.

“French versus Canadian and German Protons”

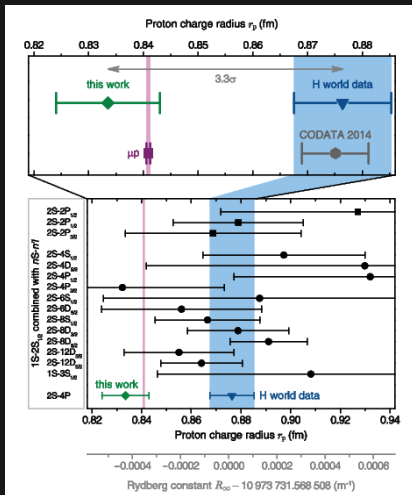
One might ask, jokingly:

“Are French protons larger than German and Canadian protons?”

Blue: *Decades* of work of the French [Paris] group

Green: Result of the 2017 measurement of the Garching group

[*Science* **358**, 79 (2017)] (in agreement with the Toronto measurement)



2022 measurement: $r_p = 0.8584(51)$ fm.

Colorado State University with support from the Russian Quantum Center.
“Size of American protons between German/Canadian and French ones.”


PHYSICAL REVIEW LETTERS **128**, 023001 (2022)

Measurement of the $2S_{1/2} - 8D_{5/2}$ Transition in Hydrogen

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 (Received 26 September 2021; revised 15 November 2021; accepted 7 December 2021; published 13 January 2022)

We present a measurement of the hydrogen $2S_{1/2} - 8D_{5/2}$ transition performed with a cryogenic atomic beam. The measured resonance frequency is $\nu = 770649561570.9(2.0)$ kHz, which corresponds to a relative uncertainty of 2.6×10^{-12} . Combining our result with the most recent measurement of the $1S - 2S$ transition, we find a proton radius of $r_p = 0.8584(51)$ fm and a Rydberg constant of $R_\infty = 10973731.568332(52)$ m⁻¹. This result has a combined 3.1σ disagreement with the Committee on Data for Science and Technology (CODATA) 2018 recommended value.

DOI: 10.1103/PhysRevLett.128.023001

Article

A small proton charge radius from an electron–proton scattering experiment

<https://doi.org/10.1038/s41586-019-1721-2>

Received: 17 June 2019

Accepted: 19 September 2019

Published online: 6 November 2019

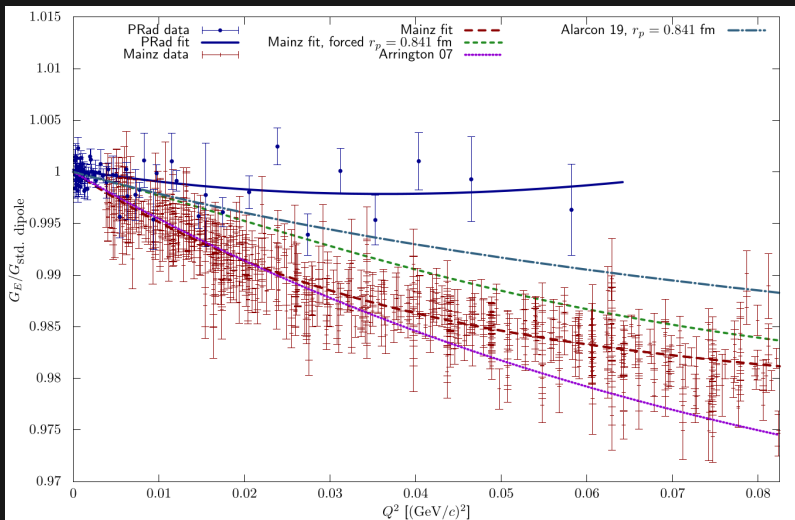
W. Xiong¹, A. Gasparian^{2*}, H. Gao¹, D. Dutta^{3*}, M. Khandaker⁴, N. Liyanage⁵, E. Pasyuk⁶, C. Peng¹, X. Bai⁵, L. Ye³, K. Gnanvo⁵, C. Gu¹, M. Levillain², X. Yan¹, D. W. Higinbotham⁵, M. Meziane¹, Z. Ye¹⁷, K. Adhikari³, B. Aljawrneh², H. Bhatt³, D. Bhetuwal³, J. Brock⁶, V. Burkert⁶, C. Carlin⁶, A. Deur⁶, D. Di³, J. Dunne³, P. Ekanayaka³, L. El-Fassi³, B. Emmich³, L. Gan⁸, O. Glamazdin⁹, M. L. Kabir³, A. Karki³, C. Keith⁹, S. Kowalski¹⁰, V. Lagerquist¹¹, I. Larin^{12,13}, T. Liu¹, A. Liyanage¹⁴, J. Maxwell⁶, D. Meekins⁶, S. J. Nazeer¹⁴, V. Nelyubin⁵, H. Nguyen⁵, R. Pedroni², C. Perdrisat¹⁵, J. Pierce⁶, V. Punjabi¹⁶, M. Shabestari³, A. Shahinyan¹⁷, R. Silwal¹⁰, S. Stepanyan⁹, A. Subedi³, V. V. Tarasov¹², N. Ton⁹, Y. Zhang¹ & Z. W. Zhao¹

Nature | Vol 575 | 7 November 2019 | **147**

experiment at Jefferson Laboratory (PRad), a high-precision e–p experiment that was established after the discrepancy was identified. We used a magnetic-spectrometer-free method along with a windowless hydrogen gas target, which overcame several limitations of previous e–p experiments and enabled measurements at very small forward-scattering angles. Our result, $r_p = 0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}}$ femtometres, is smaller than the most recent high-precision e–p measurement⁵ and 2.7 standard deviations smaller than the average of all e–p experimental results⁶. The smaller r_p we

Scattering Experiments: Mainz [2010] and PRad [2019]

Problem: 2010 Mainz raw data and 2019 raw PRad data are discrepant.
[Graphics courtesy of J. Bernauer]



Question

In view of interesting possibilities with muon physics:

Should one add a muonic beam to TRIUMF ARIEL
and compare electron and muon scattering on the same apparatus?

In some sense, confirm or refute the 1969 measurements?

[See the “forgotten” PRL mentioned at the beginning of this talk.]

Low-Cost Muon Source: Idea # 1

Lithium lens = magnetic lens.

Proceedings of the 2001 Particle Accelerator Conference, Chicago

LOW-BUDGET MUON SOURCE*

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Abstract

Generation of muon beams with protons on a current-carrying target followed by a lithium lens and a quadrupole decay channel is considered. A 8 GeV proton beam from the Fermilab Booster is used to provide a muon beam for the MUCOOL experiment for ionization cooling demonstration. The proposed scheme can also be used to create muon beams with a fraction of a 1 GeV proton beam of the Spallation Neutron Source. Monte Carlo simulations of the entire system are performed. For both cases optimization of the target and matching lithium lens is done. It is shown that such a set followed by an inexpensive decay channel based on quadrupole magnets with and without RF cavities provides a rather intense bunched muon beam.



Figure 1: Layout of the target station.

Table 1: Parameters of the target station

Parameter	Symbol	Target	Li lens
Material		Cu	Li
Length (cm)	L	15.0	40.0
Radius (cm)	R	1.60	8.50
Current (kA)	J	560	560
Maximal field (T)	B	7.00	1.33

Maybe not so practical at TRIUMF, but still worth to be mentioned. . .

Nuclear Inst. and Methods in Physics Research, A 909 (2018) 309–313



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Contents lists available at [ScienceDirect](https://www.sciencedirect.com)

Nuclear Inst. and Methods in Physics Research, A

journal homepage: www.elsevier.com/locate/nima



A muon source based on plasma accelerators

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ARTICLE INFO

Keywords:

Muon
Compton
Plasma
Source

ABSTRACT

The conceptual design of a compact source of GeV-class muons is presented, based on a plasma based electron-gamma collider. Evaluations of muon flux, spectra and brilliance are presented, carried out with ad-hoc Monte Carlo simulations of the electron-gamma collisions. These are analyzed in the context of a large spread of the invariant mass in the e-gamma interaction, due to the typical characteristics of plasma self-injected GeV electron beams, carrying large bunch charges with huge energy spread. The availability of a compact point-like muon source, triggerable at nsec level, may open a completely new scenario in the muon radiography application field.

Conclusions

- ▶ Muonic hydrogen theory, and Lamb shift theory, are well under control.
- ▶ Proton radius situation may be less clear than commonly thought.
- ▶ Electron versus muon scattering experiments could shed light.
- ▶ Confirmation of the 1969 experiment would be very desirable.
- ▶ Add a muon option to TRIUMF/ARIEL!?
- ▶ *There actually are theoretical considerations which could lead to an electron-muon non-universality.* My e-mail: ulj@mst.edu.