

Electrodisintegration of ^{16}O and determination of astrophysical S-factors of the inverse reaction

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New Scientific Opportunities with the TRIUMF ARIEL e-linac



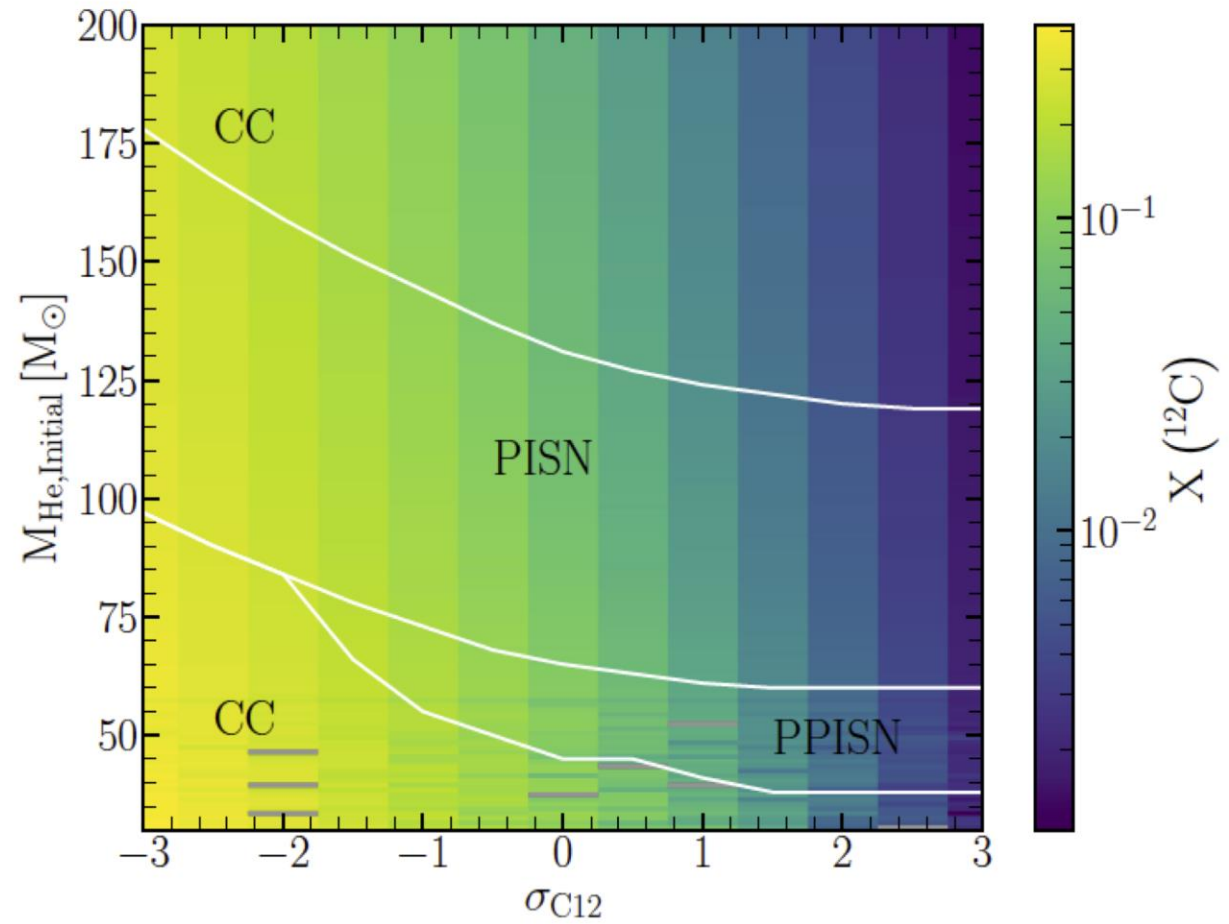
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Modeling of stellar evolution

- Complicated part: transport of a material inside a star
- Easy part: nucleosynthesis of a burning stage is described by a system of differential equations which are very easy to solve
 - > nuclear input enters in form of reaction rates at E_G
 - > $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ has the largest uncertainty compared to other rates
 - > with triple-alpha reaction part of He-burning stage
 - > affects C/O abundance, subsequent nucleosynthesis and end of live of massive stars

Example: R. Farmer et al., 2020 ApJL 902 L36

- Evolution of He-core in mass range between 30 and 200 M_{\odot} and $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate in $\pm 3\sigma_{\text{C12}}$ range
- Steps: 1 M_{\odot} and 0.5 σ_{C12} -> 2210 simulations
- For He-core > 40 M_{\odot} gamma rays can produce e^-e^+ pairs, radiation pressure drops, leading to gravitational collapse
 - > CC – core collapse stars
 - > PISN – pair instability supernovae
 - > PPISN – pulsational pair instability supernovae
- Using LIGO/Virgo gravitational wave data from binary black hole mergers to determine their masses and subsequently $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ rate



Constraints on $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ S-Factor

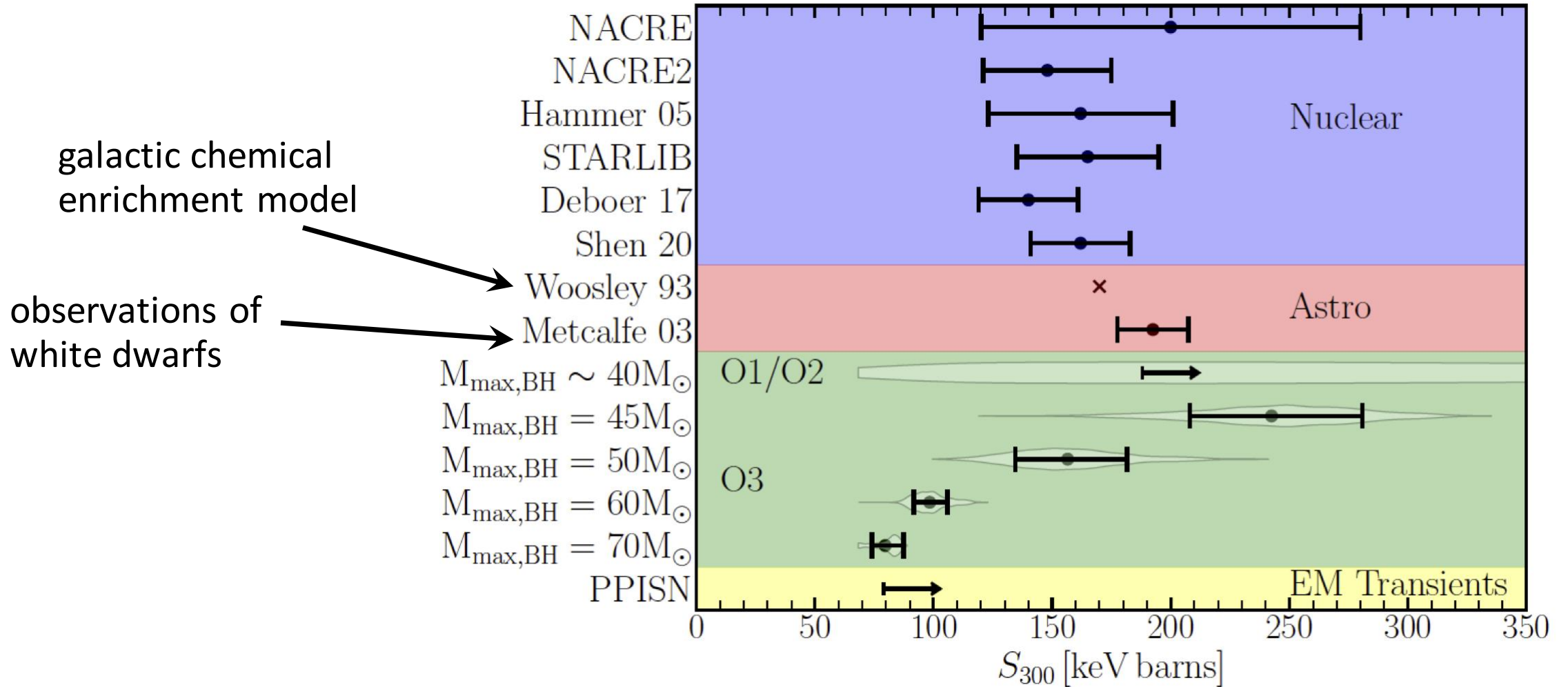
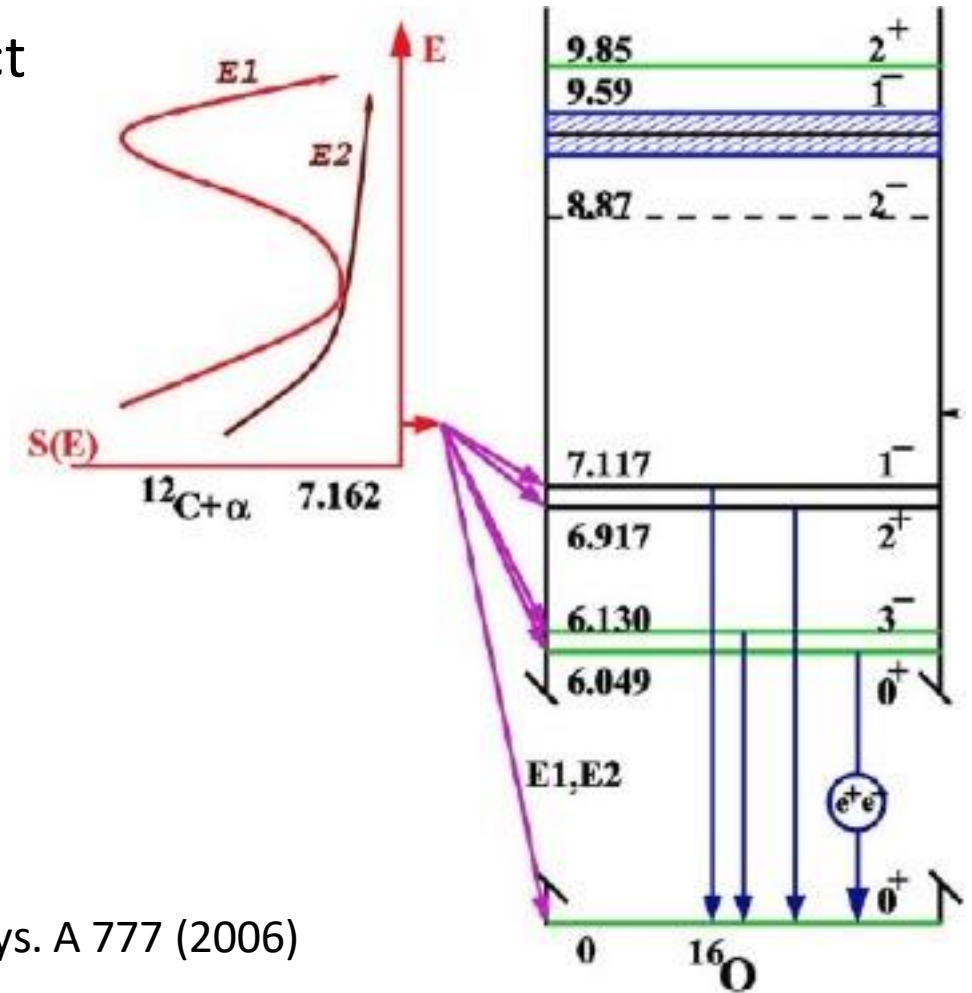


Fig 7. from R. Farmer et al., 2020 ApJL 902 L36

The cross section of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ at E_G

- $\sigma \simeq 10^{-5}$ pb, due to large Coulomb barrier direct measurement would not be feasible
- The cross section at E_G is dominated by two components:
 - E1 component, $J^\pi = 1^-$: subthreshold state at 7.117 MeV and broad resonance at 9.59 MeV
 - E2 component, $J^\pi = 2^+$: subthreshold state at 6.917 MeV and narrow resonance at 9.85 MeV



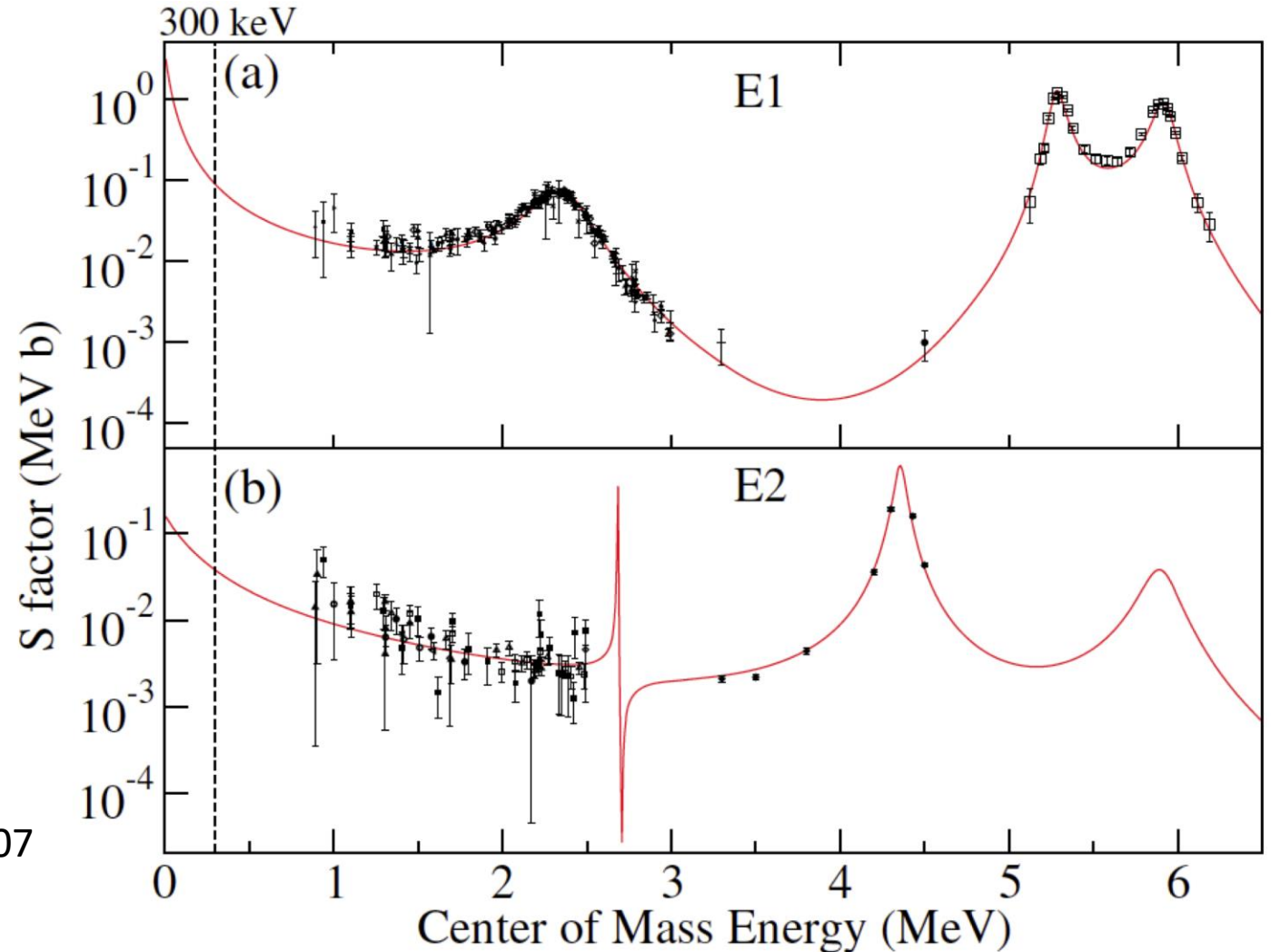
L.R. Buchmann, C.A. Barnes, Nucl. Phys. A 777 (2006)

$^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ S-factors components

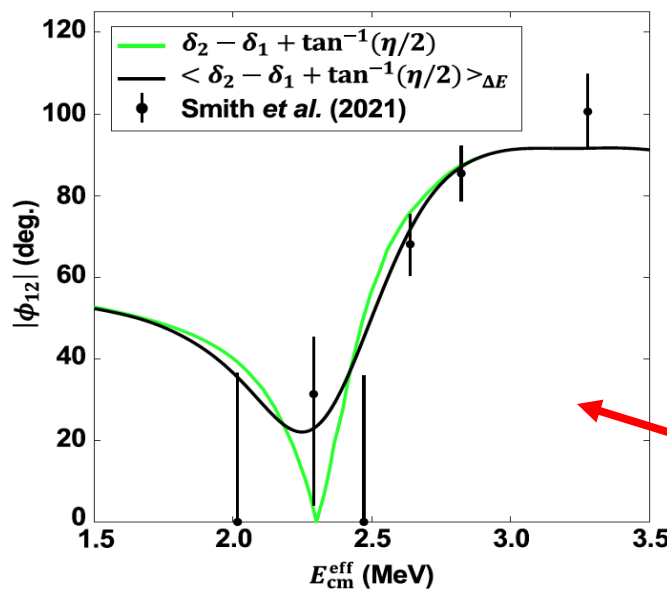
$$\sigma(E) = \frac{S(E)}{E} e^{-2\pi\eta}$$

$$\eta = \frac{2\pi Z_1 Z_2 e^2}{h\nu_{rel}}$$

R. J. deBoer et al., Rev. Mod. Phys. 89, 035007 (2017) and references therein

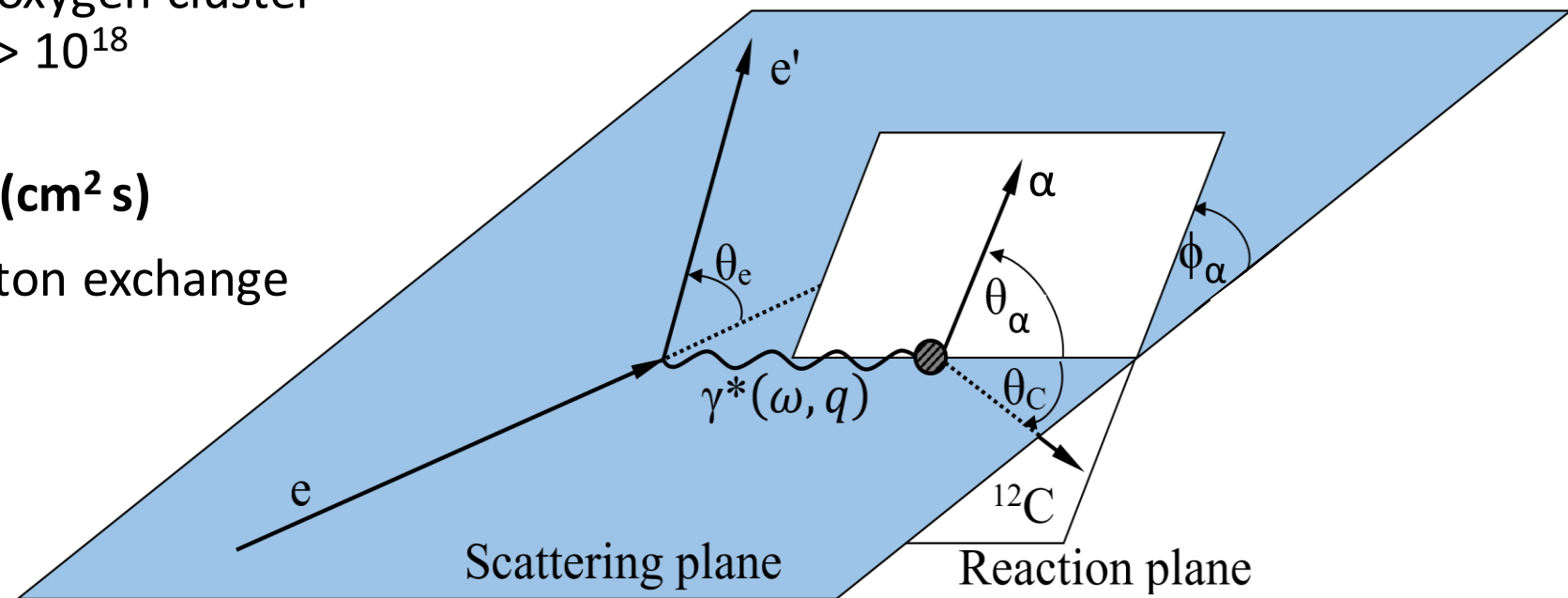


Nuclear measurements

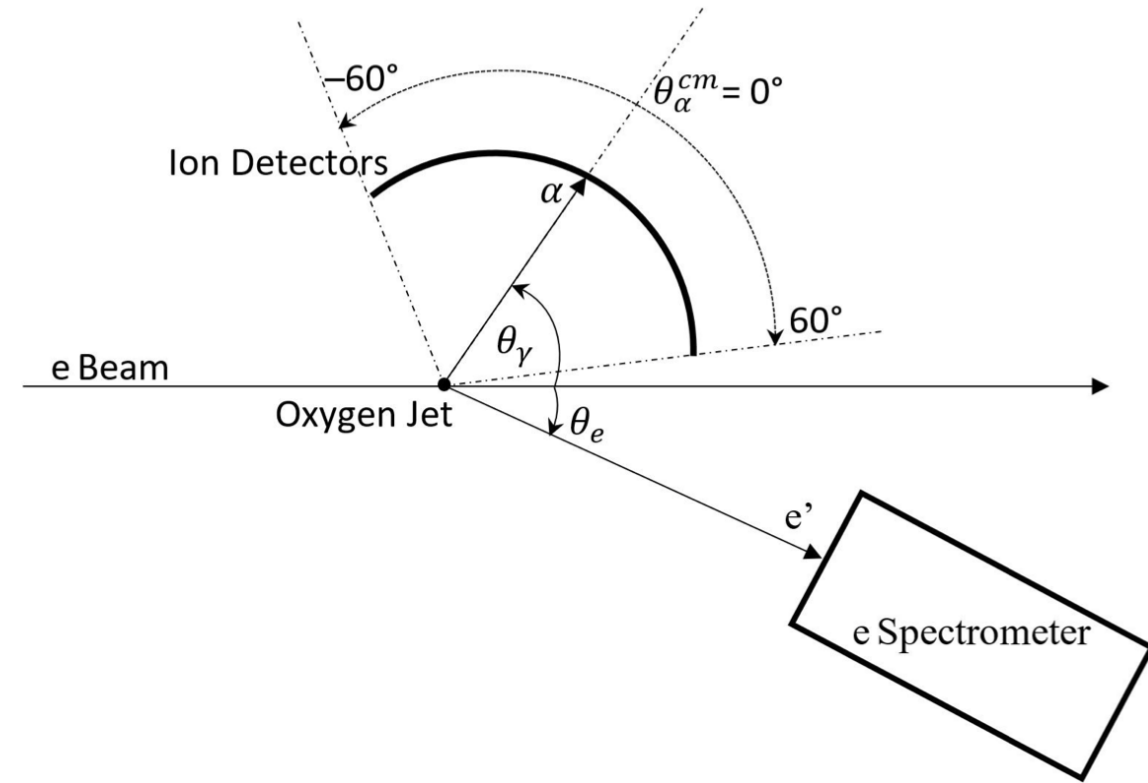
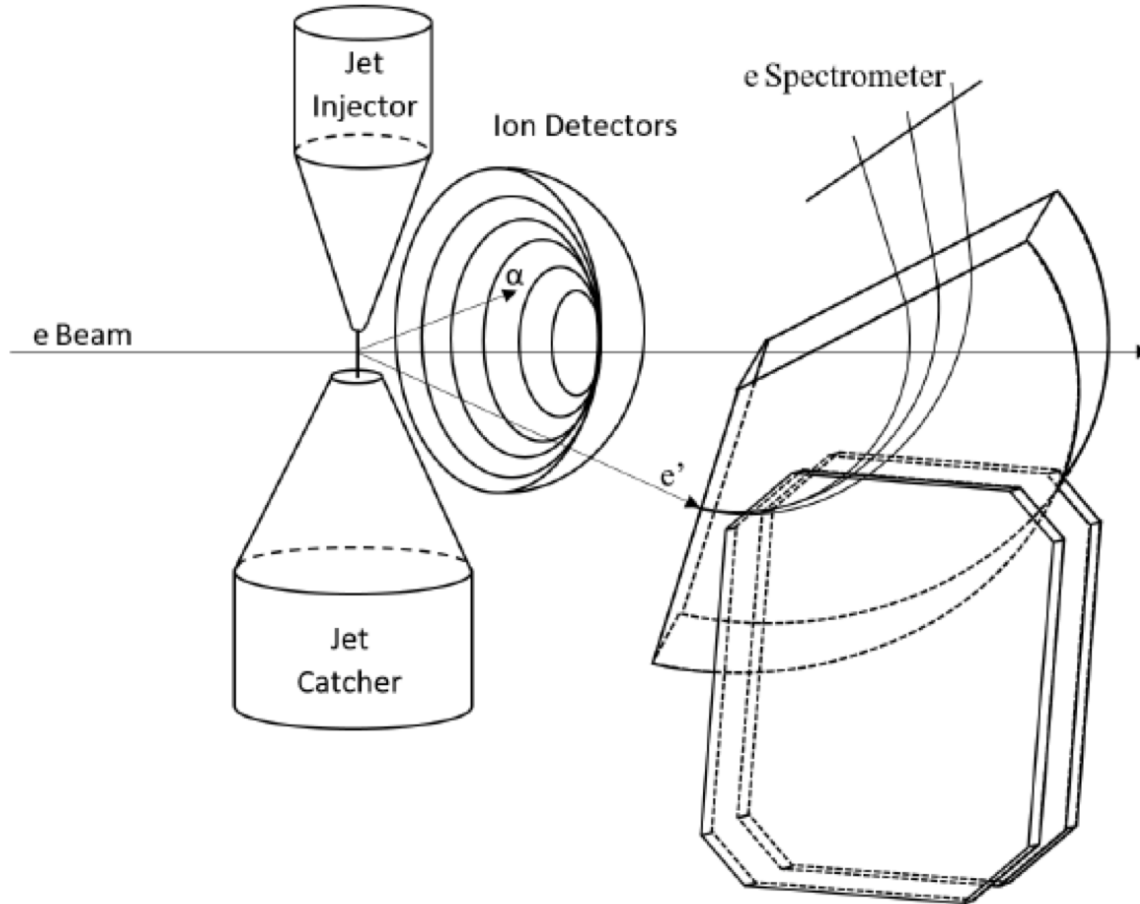
Direct measurements	Indirect measurements	
$^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$; α beam: detection of angular distribution of γ ; $\rightarrow S_{E1}$ and S_{E2}	β decay of ^{16}N : $^{16}\text{O}^* \rightarrow \alpha + ^{12}\text{C}$; $\rightarrow S_{E1}$	
$\alpha(^{12}\text{C}, ^{16}\text{O}) \gamma$; ^{12}C beam (inverse kinematic): detection of ^{16}O recoils; $\rightarrow S_{\text{tot}}$	Inverse reaction	
	Photodisintegration of ^{16}O: $^{16}\text{O}(\gamma, \alpha)^{12}\text{C}$	Electrodisintegration of ^{16}O: $^{16}\text{O}(e, e'\alpha)^{12}\text{C}$
	Bubble chamber, R. J. Holt et al., (2018), arXiv:1809.10176 Time projection chamber, M. Gai et al., JINST 5, P12004 (2010), R. Smith et al., Nat. Commun. 12, 5920 (2021)	I. F., W. T. Donnelly and R. G. Milner, Phys. Rev. C 100, (2019) 025804 S. Lunkenheimer, PhD Thesis 2022, University of Mainz, Germany, MAGIX @Mainz

Advantage of $^{16}\text{O}(e,e'\alpha)^{12}\text{C}$

- Inverse reaction: larger cross section than direct reaction
- New generation of energy recovery linear (ERL) accelerators with $I \geq 10$ mA (MESA @Mainz, CBETA @Cornell) + oxygen cluster gas-jet target with thickness $> 10^{18}$ atoms/cm² (MAGIX @Mainz)
 - ⇒ **Luminosity** $> 10^{35}$ 1/(cm² s)
- Reaction involves virtual photon exchange



Schematic layout of the ideal experiment

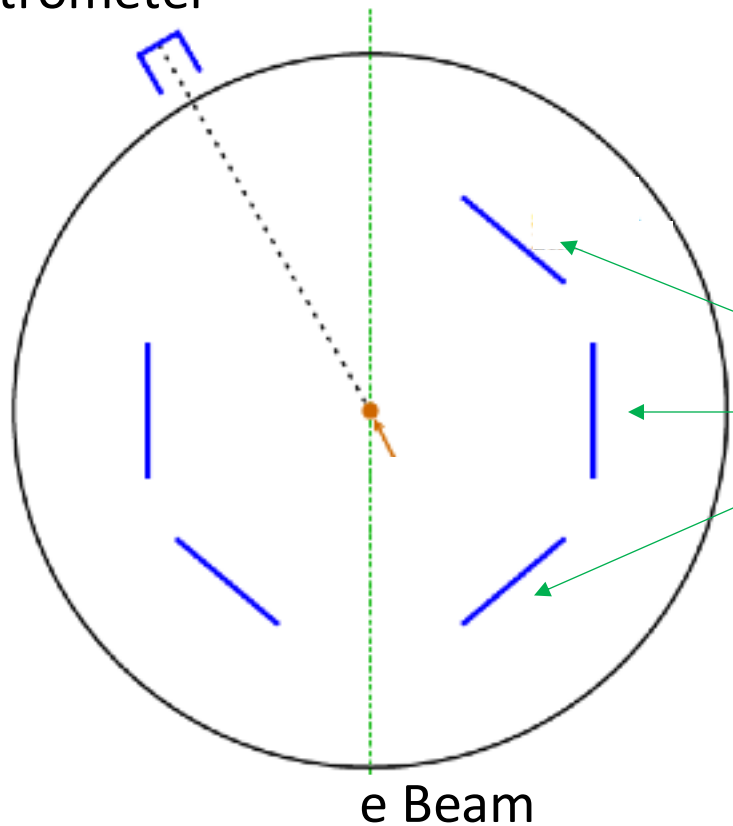


CBETA e-beam: 40 mA, $E_0 = 78, 114, 150$ MeV

Schematic layout of MAGIX approach

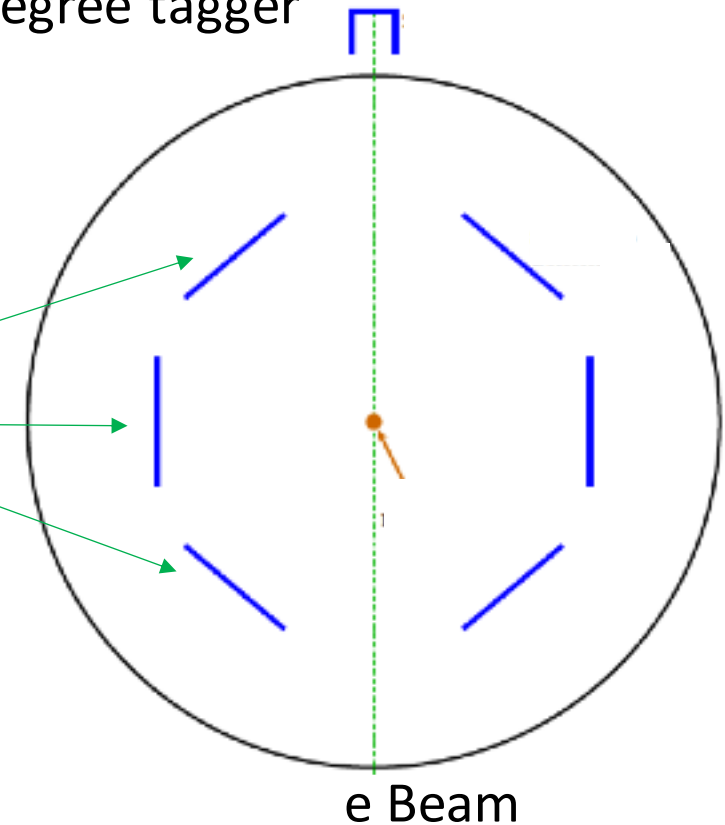
- Phase 0 and 1

Spectrometer



- Phase 2

Zero degree tagger



Silicon strip detectors

Systematics from oxygen isotopes

- Oxygen isotope abundance: ^{16}O 99.757%, ^{17}O 0.038% and ^{18}O 0.205%

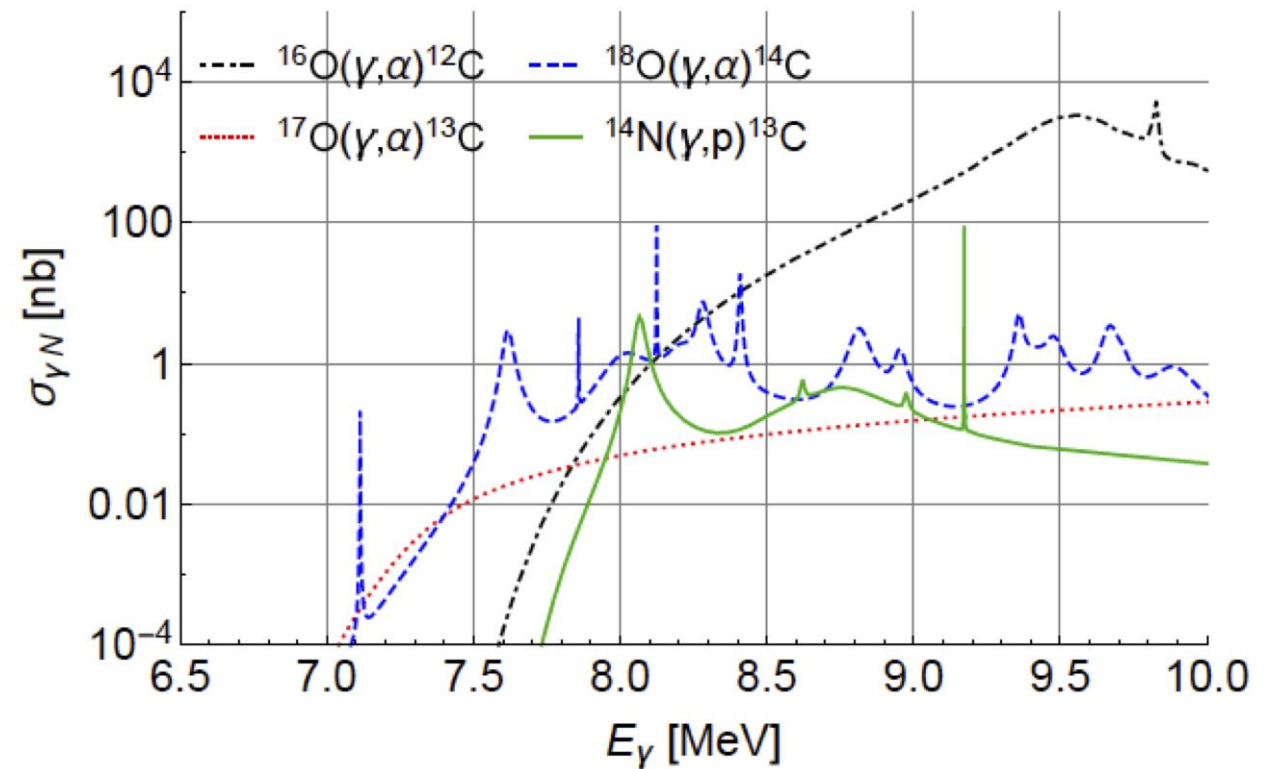
$$Q(^{16}\text{O} \rightarrow \alpha + ^{12}\text{C}) = -7.162 \text{ MeV}$$

$$Q(^{17}\text{O} \rightarrow \alpha + ^{13}\text{C}) = -6.359 \text{ MeV}$$

$$Q(^{18}\text{O} \rightarrow \alpha + ^{14}\text{C}) = -6.228 \text{ MeV}$$

- Photonuclear cross sections:
natural abundance of O isotopes +
depletion of ^{17}O and ^{18}O by factor
1000, and 5 ppmv for ^{14}N

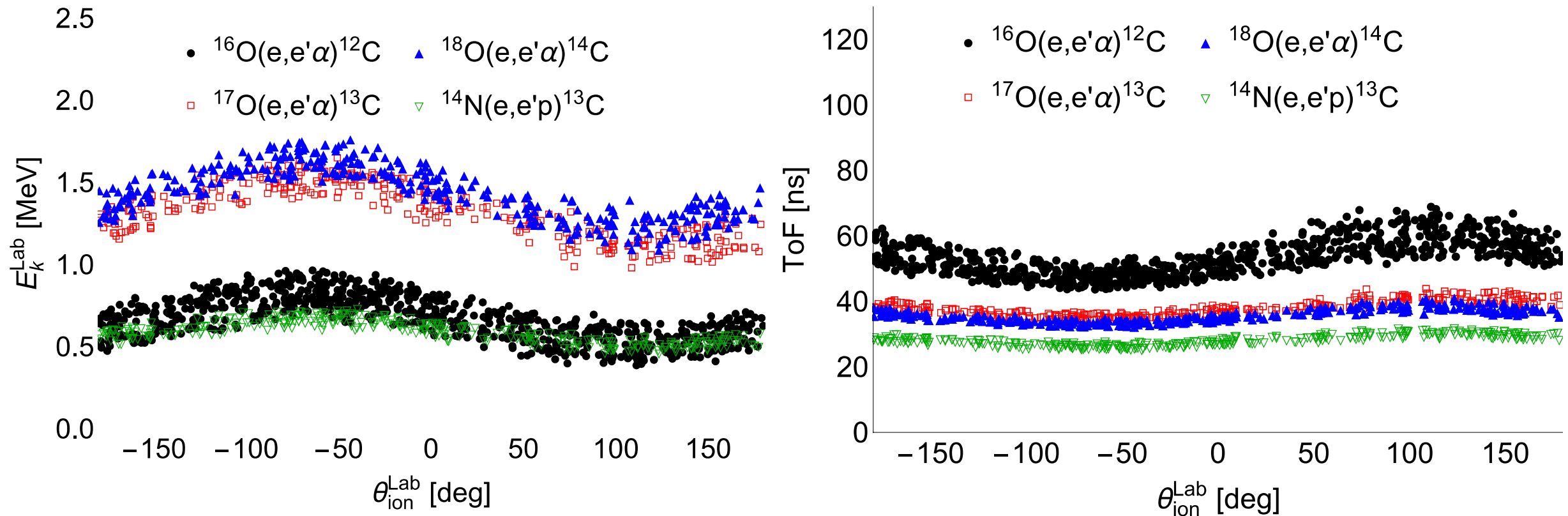
K. J. R. Rosman, P. D. P. Taylor, Pure Appl. Chem 71 (1999) 1593



https://wiki.jlab.org/ciswiki/index.php/Simulations_and_Backgrounds#Relevant_Theoretical_Cross_Sections

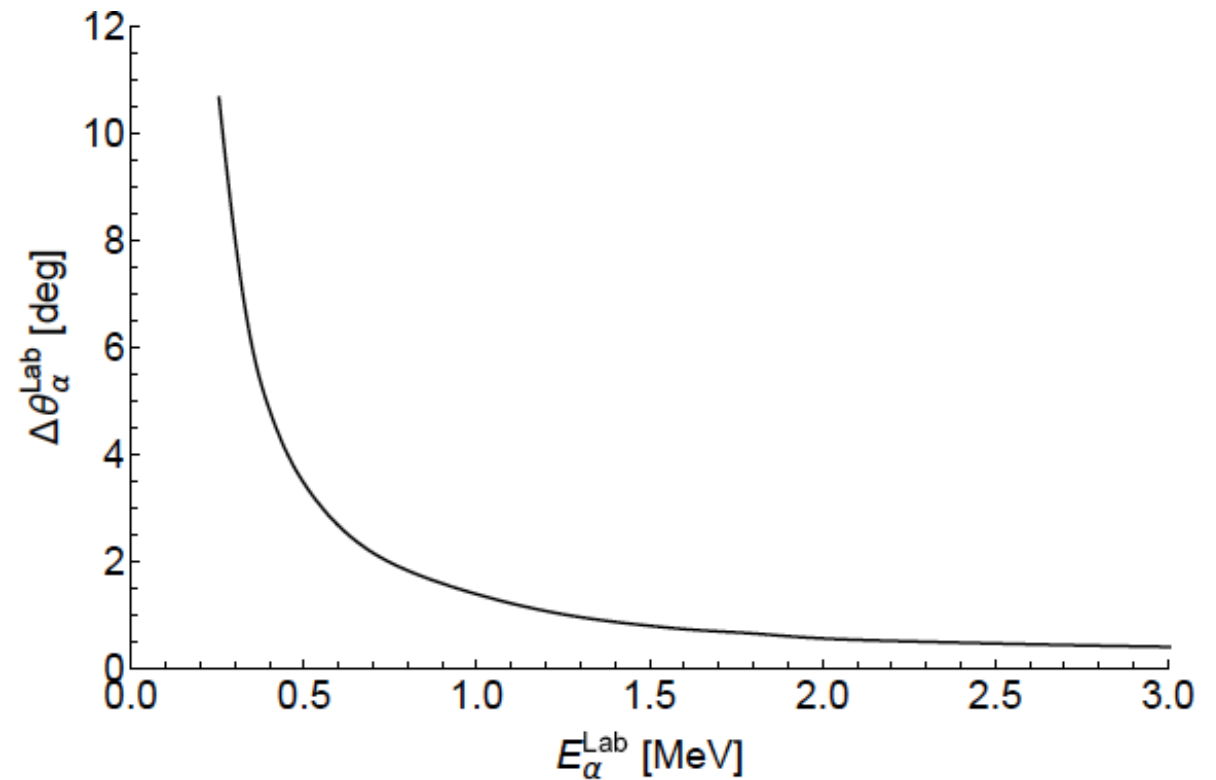
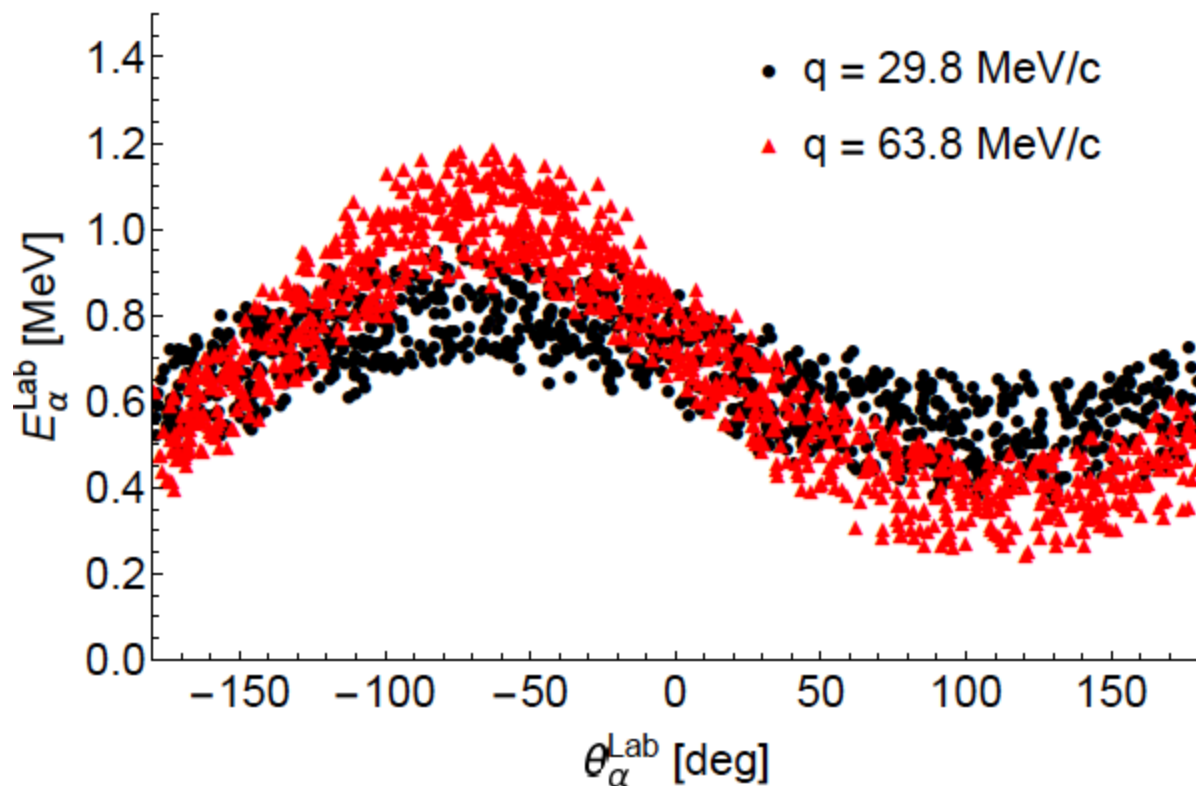
Systematics from oxygen isotopes: Solution

- SRIM simulation: energy loss of α -particles in 2 mm wide oxygen jet, with a density of $6.65 \cdot 10^{-4} \text{ g/cm}^3$, $E_e = 114 \text{ MeV}$, $\theta_e = 15^\circ$, $1.0 \leq E_\alpha^{cm} \leq 1.1 \text{ MeV}$



Virtual photon advantage

- SRIM simulation: angular spread of α -particles in 2 mm wide oxygen jet, with a density of $6.65 \cdot 10^{-4} \text{ g/cm}^3$, $E_e = 114 \text{ MeV}$, $\theta_e = 15^\circ$ and 35° , $1.0 \leq E_\alpha^{cm} \leq 1.1 \text{ MeV}$



The cross section formulas

- Electrodisintegration of ^{16}O :

$$\frac{d\sigma}{dE'_e d\Omega_e d\Omega_\alpha^{cm}} = \frac{M_\alpha M_{12C}}{8\pi^3 W} \frac{p_\alpha^{cm}}{(\hbar c)^3} \sigma_{Mott} (\tilde{v}_L R_L + \tilde{v}_T \mathbf{R}_T + \tilde{v}_{LT} R_{LT} + \tilde{v}_{TT} R_{TT})$$

A. S. Raskin and T. W. Donnelly, Ann. of Phys. 191 (1989)

- Direct reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$:

$$\left. \frac{d\sigma}{d\Omega_\gamma^{cm}} \right|_{(\alpha, \gamma)} = \frac{M_\alpha M_{12C}}{2\pi W} \frac{E_\gamma}{\hbar c} \frac{\alpha}{p_\alpha^{cm}} \mathbf{R}_{T,(\alpha, \gamma)}$$

\tilde{v}_k lepton kinematic factors

R_k response functions

$\mathbf{R}_{T,(\alpha, \gamma)} = \mathbf{R}_T$ in real photon limit

Response functions for $J^\pi = 0^+$ nuclei

$$\begin{aligned}
 R_L = & P_0(\cos \theta_\alpha) \left(|t_{C0}|^2 + |t_{C1}|^2 + |t_{C2}|^2 \right) \\
 & + P_1(\cos \theta_\alpha) \left(2\sqrt{3}|t_{C0}||t_{C1}| \cos(\delta_{C1} - \delta_{C0}) + 4\sqrt{\frac{3}{5}}|t_{C1}||t_{C2}| \cos(\delta_{C2} - \delta_{C1}) \right) \\
 & + P_2(\cos \theta_\alpha) \left(2|t_{C1}|^2 + \frac{10}{7}|t_{C2}|^2 + 2\sqrt{5}|t_{C0}||t_{C2}| \cos(\delta_{C2} - \delta_{C0}) \right) \\
 & + P_3(\cos \theta_\alpha) \left(6\sqrt{\frac{3}{5}}|t_{C1}||t_{C2}| \cos(\delta_{C2} - \delta_{C1}) \right) \\
 & + P_4(\cos \theta_\alpha) \left(\frac{18}{7}|t_{C2}|^2 \right)
 \end{aligned}$$

$$\begin{aligned}
 R_T = & P_0(\cos \theta_\alpha) \left(|t_{E1}|^2 + |t_{E2}|^2 \right) \\
 & + P_1(\cos \theta_\alpha) \left(\frac{6}{\sqrt{5}}|t_{E1}||t_{E2}| \cos(\delta_{E2} - \delta_{E1}) \right) \\
 & + P_2(\cos \theta_\alpha) \left(-|t_{E1}|^2 + \frac{5}{7}|t_{E2}|^2 \right) \\
 & + P_3(\cos \theta_\alpha) \left(-\frac{6}{\sqrt{5}}|t_{E1}||t_{E2}| \cos(\delta_{E2} - \delta_{E1}) \right) \\
 & + P_4(\cos \theta_\alpha) \left(-\frac{12}{7}|t_{E2}|^2 \right)
 \end{aligned}$$

$$R_{TT} = -R_T \cos(2\phi_\alpha)$$

Matrix elements and coefficients

- Multipole matrix elements ($q_0 = 1.2 \text{ fm}^{-1}$):

$$t_{EJ} = \frac{\omega}{q} \left(\frac{q}{q_0}\right)^J a'_{EJ} \left[1 + \left(\frac{q}{q_0}\right)^2 b'_{EJ}(q)\right] e^{-\left(\frac{q}{q_0}\right)^2} \quad t_{CJ} = \left(\frac{q}{q_0}\right)^J a'_{CJ} \left[1 + \left(\frac{q}{q_0}\right)^2 b'_{CJ}(q)\right] e^{-\left(\frac{q}{q_0}\right)^2}$$

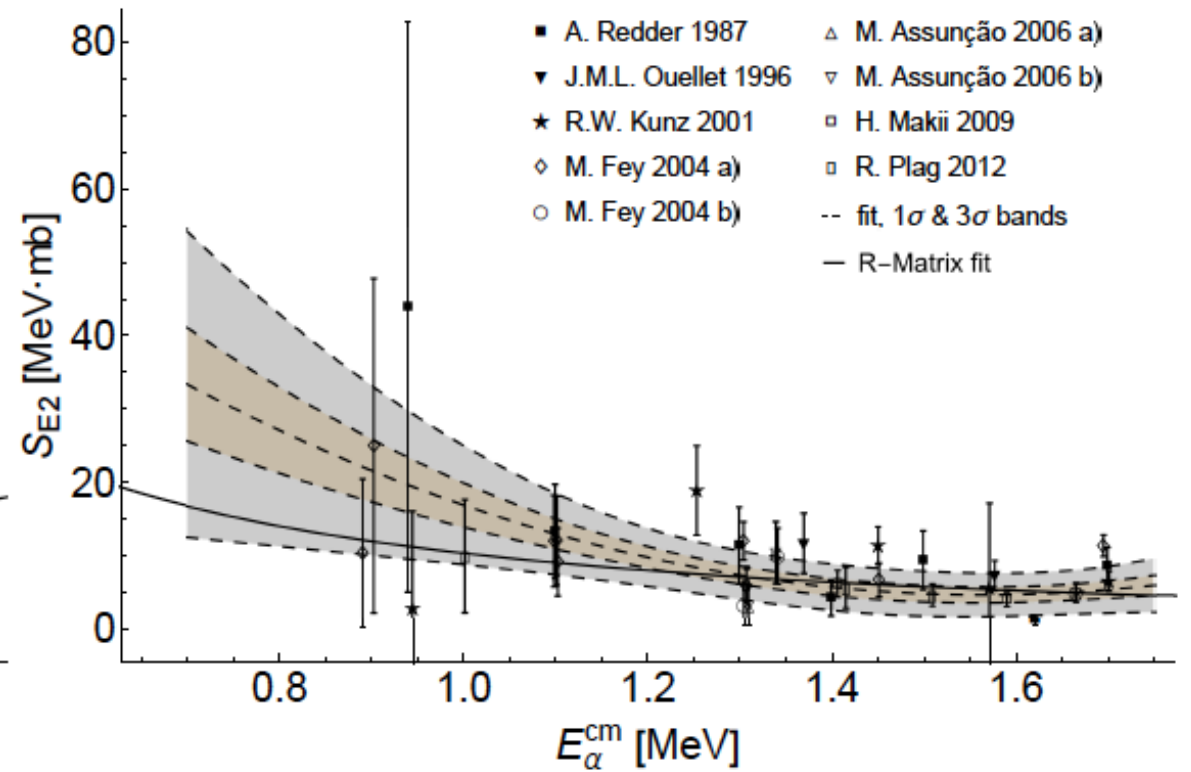
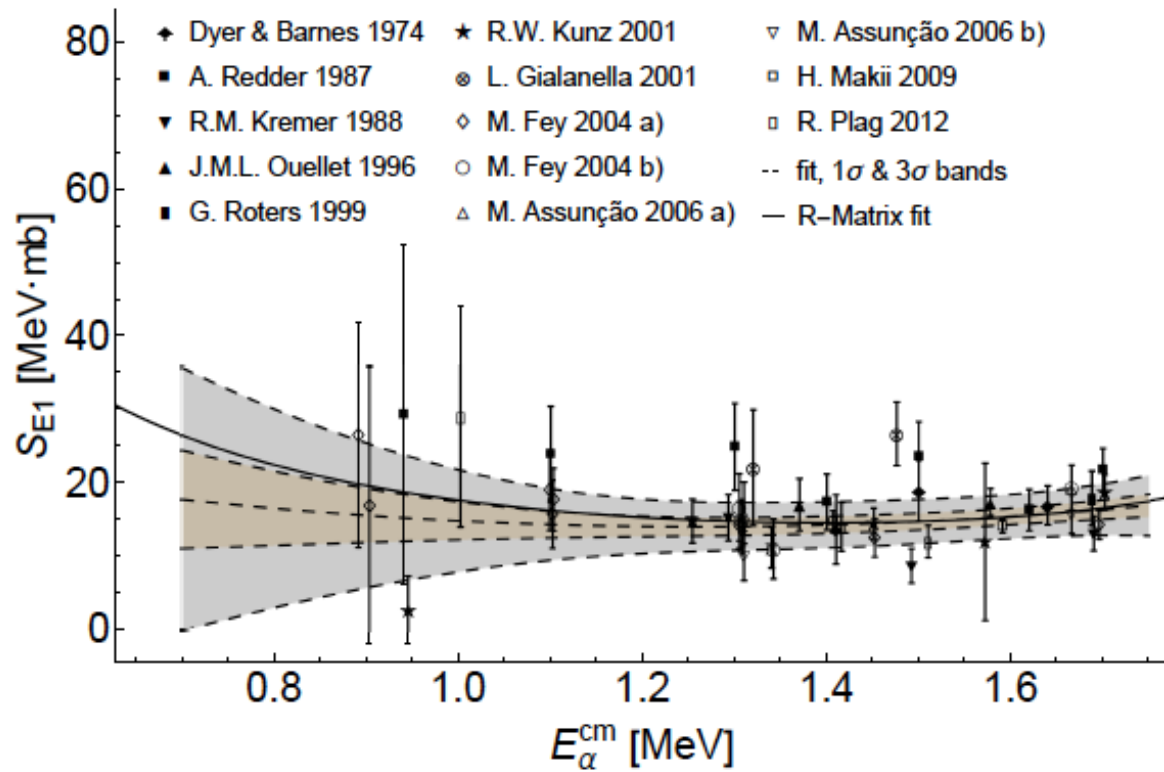
(t_{C0} leading dependence cannot occur due to orthogonality of initial and final state)

- Long wavelength limit and continuity equation:

$$t_{EJ} \rightarrow -\sqrt{\frac{J+1}{J}} \left(\frac{\omega}{q}\right) t_{CJ} \quad a'_{EJ} = -\sqrt{\frac{J+1}{J}} a'_{CJ}$$

Leading order coefficients

- Second order polynomial fit to data $E_{\alpha}^{cm} < 1.7$ MeV



Next-to-leading order coefficients

- No knowledge about next to leading order coefficients $b'_{EJ,CJ}$ with $J = 1, 2$
→ Assuming $b'_{EJ,CJ} \approx 1$ and “+” sign
- No knowledge about C0 multipole and $b'_{C0} \cdot a'_{C0}$
→ Assuming $b'_{C0} \approx 1$ and “+” sign, **Case A** $a'_{C0} = a'_{E2}$ and **Case B** $a'_{C0} = 0.5a'_{E2}$
- For $E_{\alpha}^{cm} < 1.7$ MeV only Coulomb phase contributes:

$$\delta_{Cl} - \delta_{C0} = \delta_{El} - \delta_{E0} = \sum_{n=1}^l \arctan \frac{\eta}{l}$$

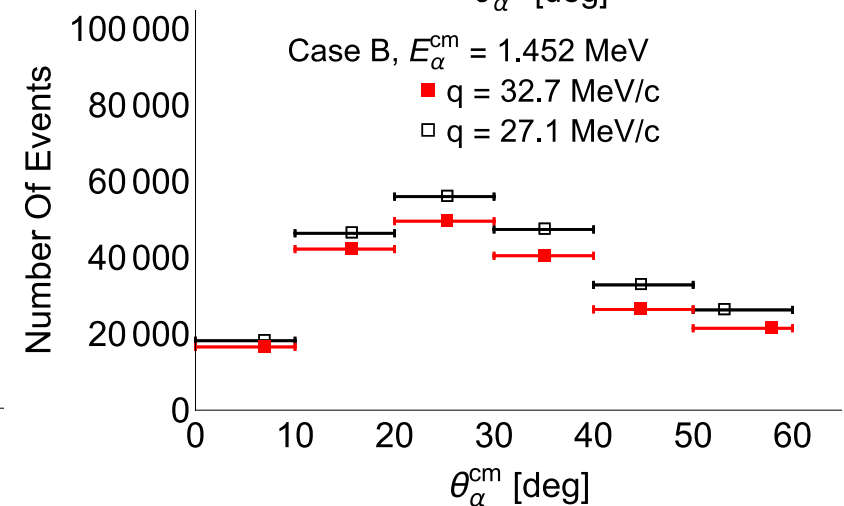
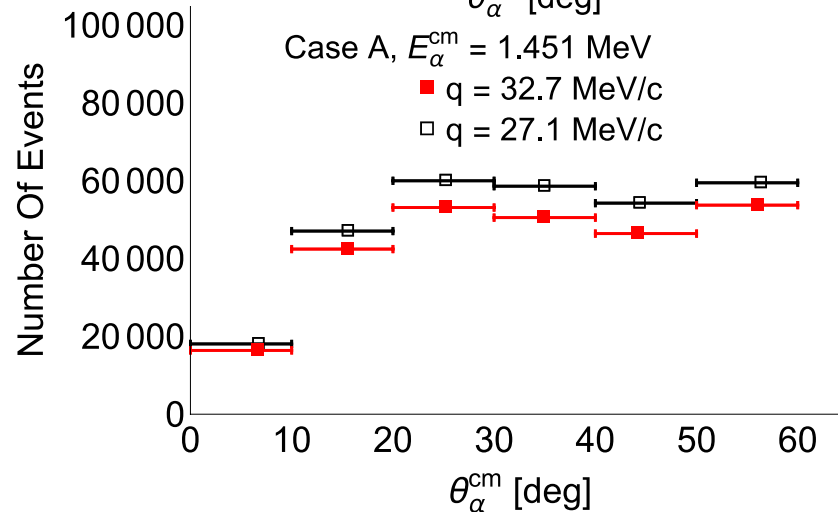
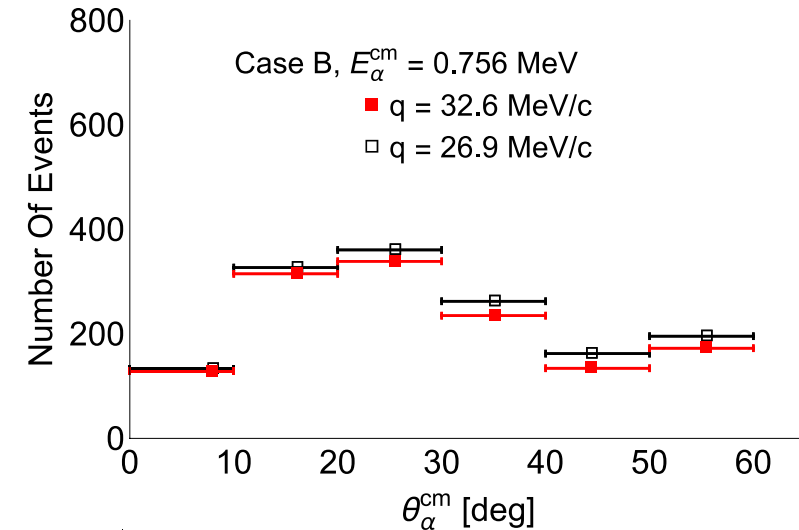
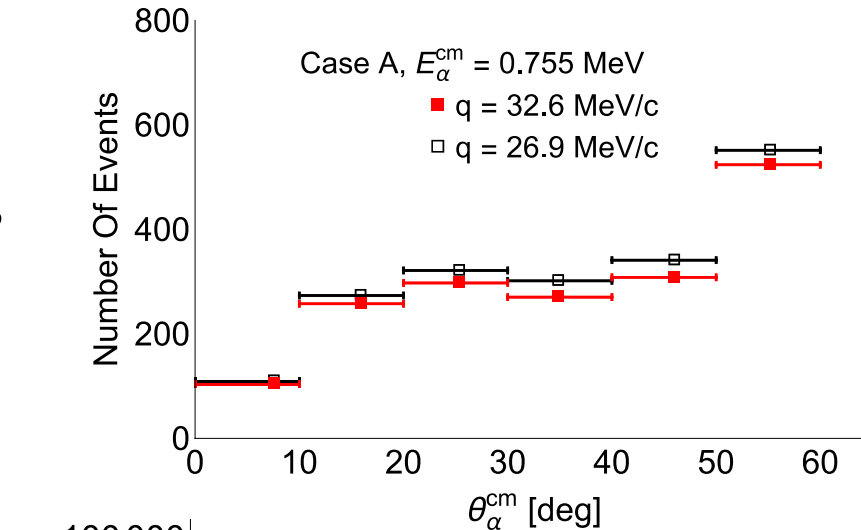
Number of events after 100 days

- Events were sorted in:
 - four 1.91 MeV wide q -bins
 - ten 100 keV wide E_α^{cm} -bins
 - six 10° wide θ_α^{cm} -bins
- $E_e = 114$ MeV, $\theta_e = 15^\circ$,
Case A and Case B

• Now we can compute statistical uncertainties

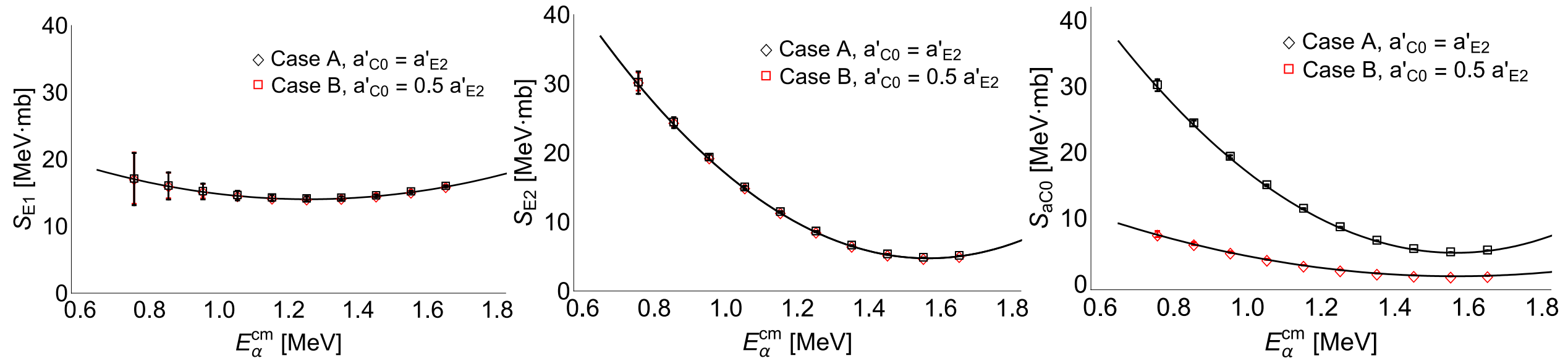
- Horizontal placement of data points according to:

G. D. Lafferty and T. R. Wyatt, Nucl. Instrum. Methods Phys. Res. A 355, 541 (1995).



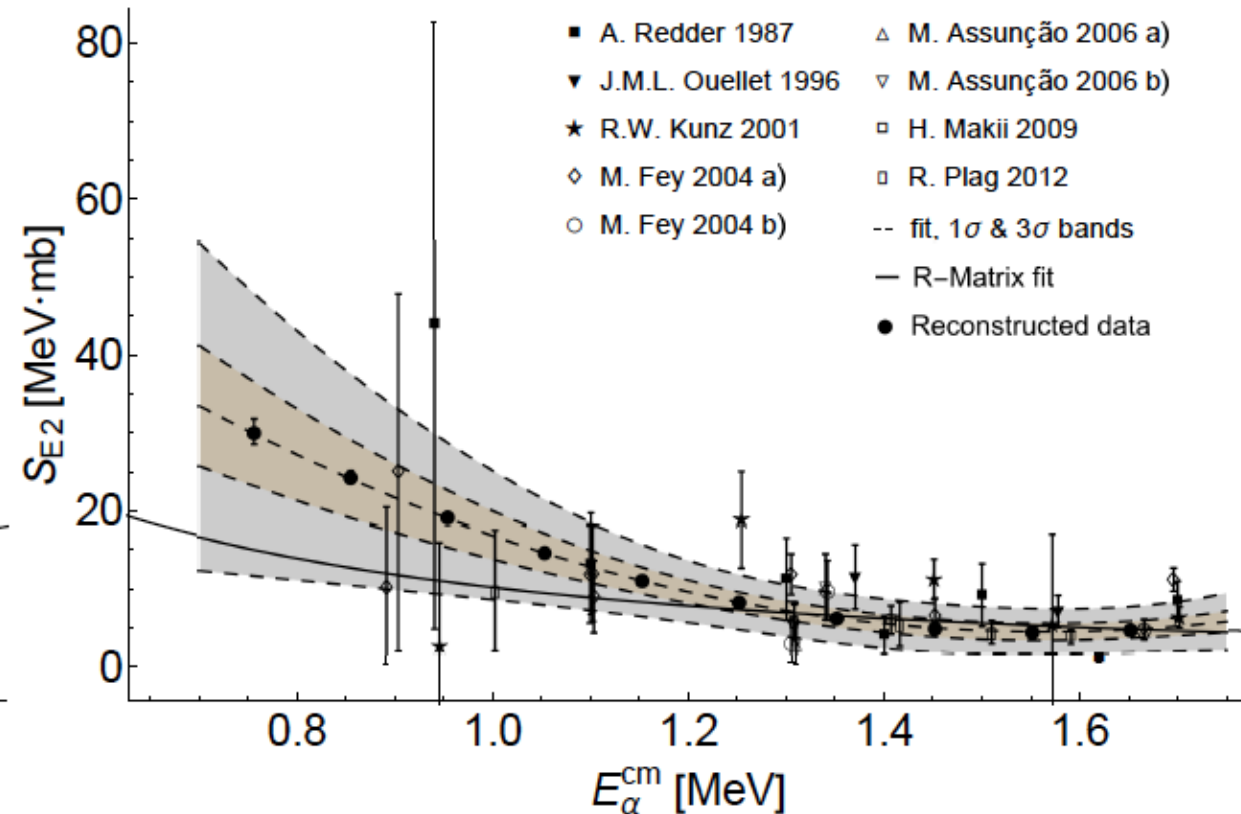
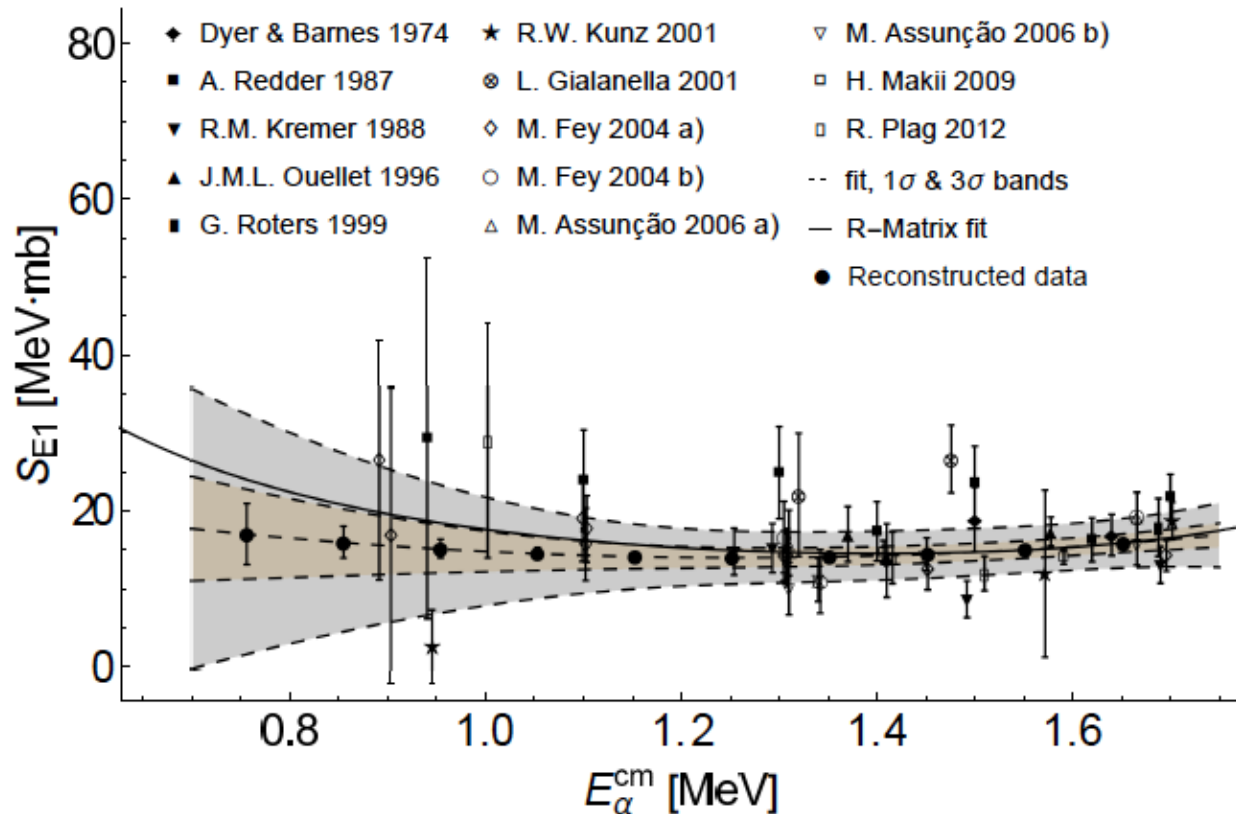
S-factors with projected statistical uncertainties

- $E_e = 114$ MeV, $\theta_e = 15^\circ$, Case A and Case B
- Three fitting parameters a'_{E1} , a'_{E2} and a'_{C0} \rightarrow S_{E1} , S_{E2} and S_{aC0} non-astrophysical factor



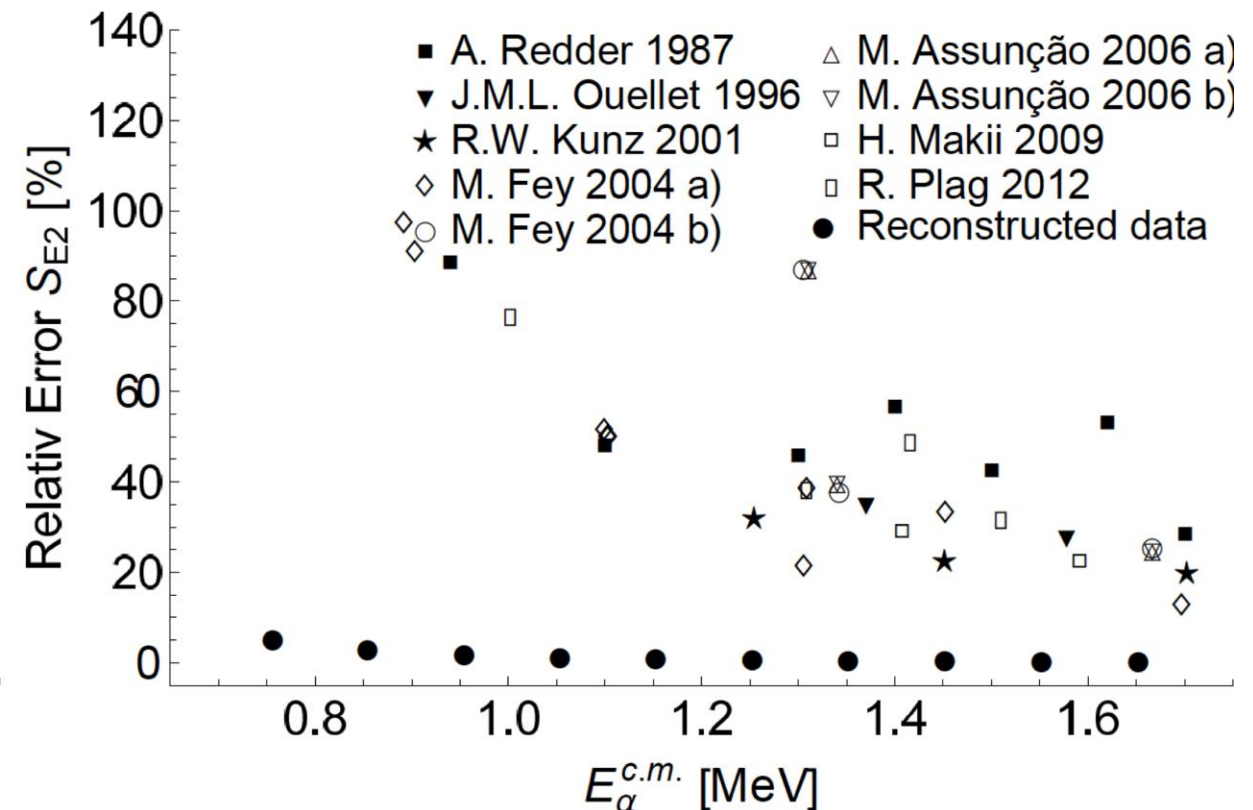
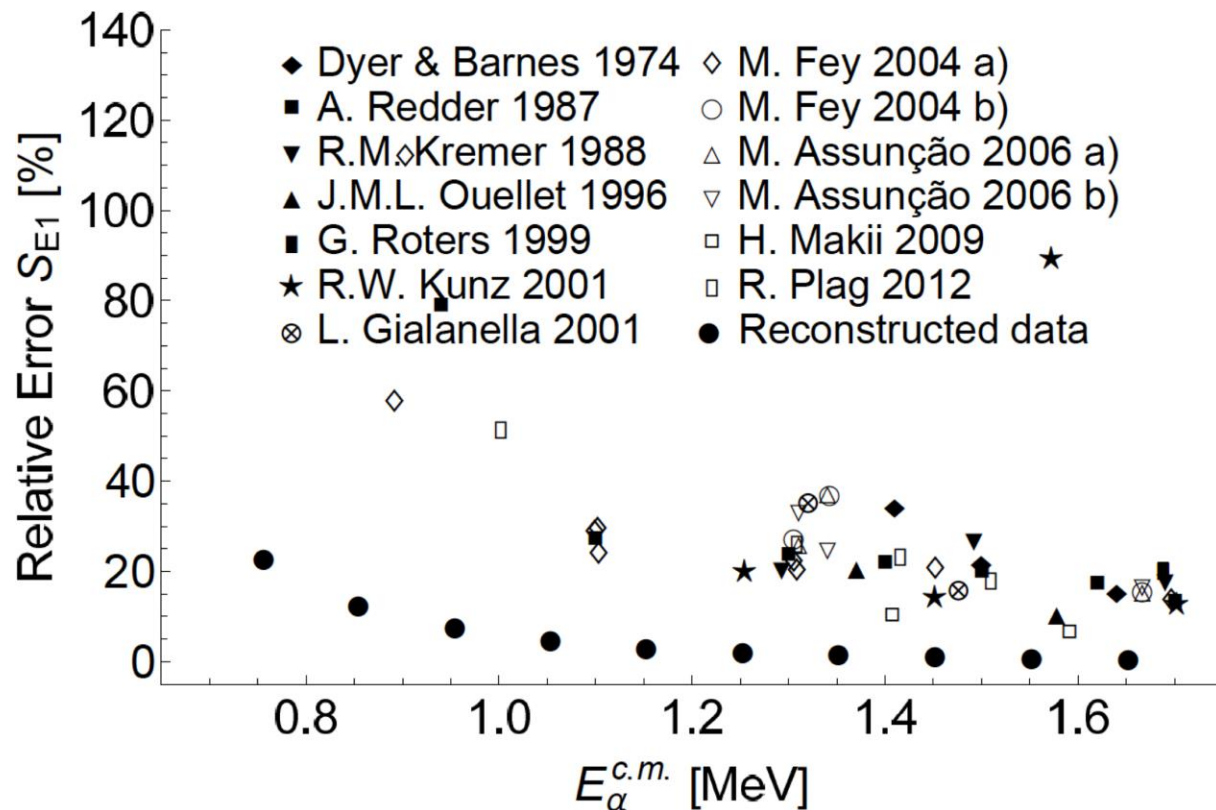
S-factors with projected statistical uncertainties

- $E_e = 114 \text{ MeV}$, $\theta_e = 15^\circ$, Case A



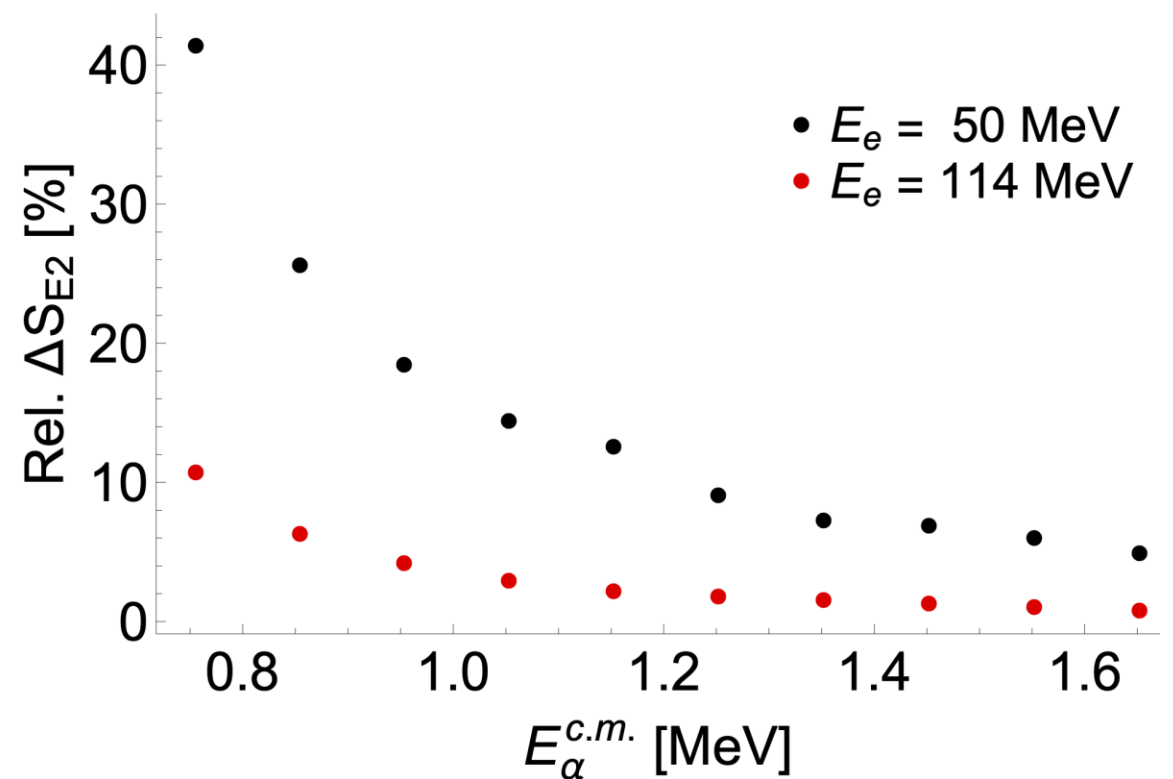
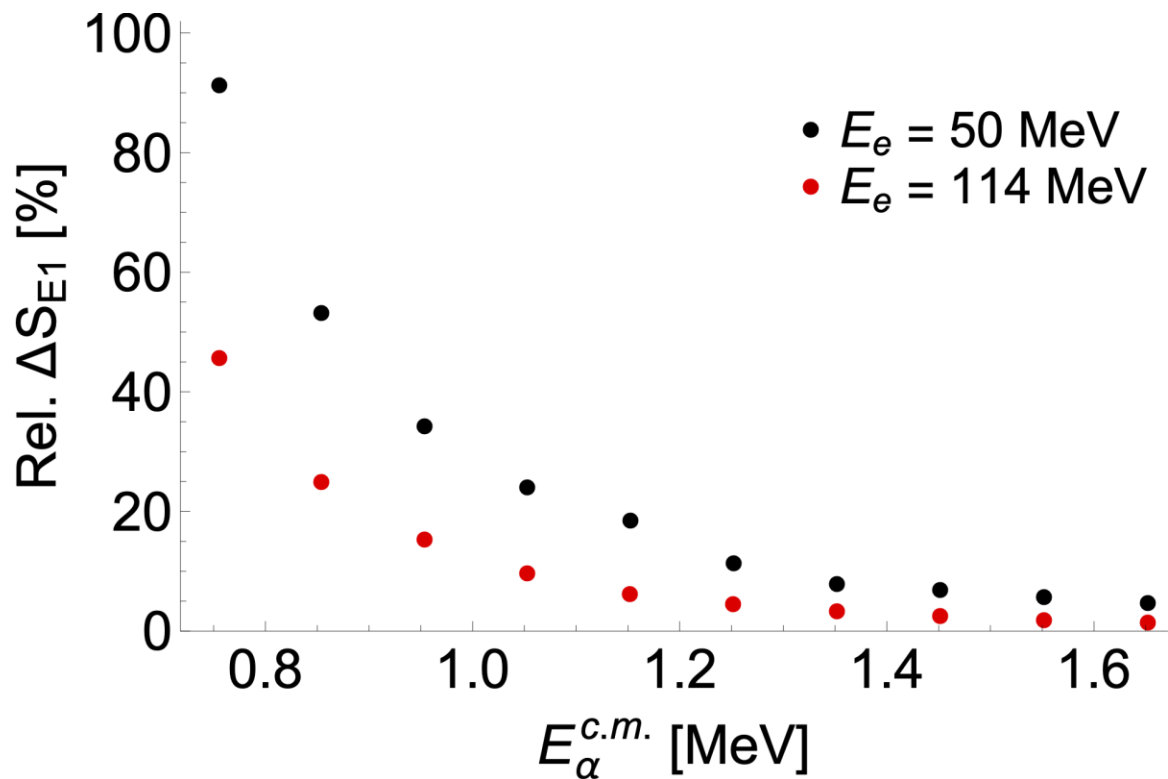
S-factors with projected statistical uncertainties

- $E_e = 114 \text{ MeV}$, $\theta_e = 15^\circ$, Case A
- Compared to most accurate measurements, statistical uncertainties of S_{E1} and S_{E2} are improved at least by factors 5.6 and 23.9, respectively



S-factors with projected statistical uncertainties

- $E_e = 50$ MeV, $\theta_e = 15^\circ$, Case A, 10 mA for 100 Days
- $E_e = 114$ MeV, $\theta_e = 15^\circ$, Case A, 10 mA for 100 Days



Conclusion

- Using a simple model, possibilities of new ERL accelerators and the gas-jet target, calculations of $^{16}\text{O}(e,e'\alpha)^{12}\text{C}$ reaction rate in range $0.7 < E_{\alpha}^{cm} < 1.7$ MeV and showed that one would be able to determine $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction rate with unprecedented statistical precision
- At $E_e = 114$ MeV and spectrometer with 10% E_e' acceptance the full range $0. < E_{\alpha}^{cm} < 10.2$ MeV is accessible from one experiment
- Shorter run at higher E_{α}^{cm} to test the particle identification (α from different Oxygen isotopes), systematics and all assumptions (next-to-leading order coefficients, q_0)
- For more details: I. F., W. T. Donnelly and R. G. Milner, Phys. Rev. C 100, (2019) 025804

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Thank you