

# Temperature-dependent cluster decay half-lives

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# What is cluster decay?

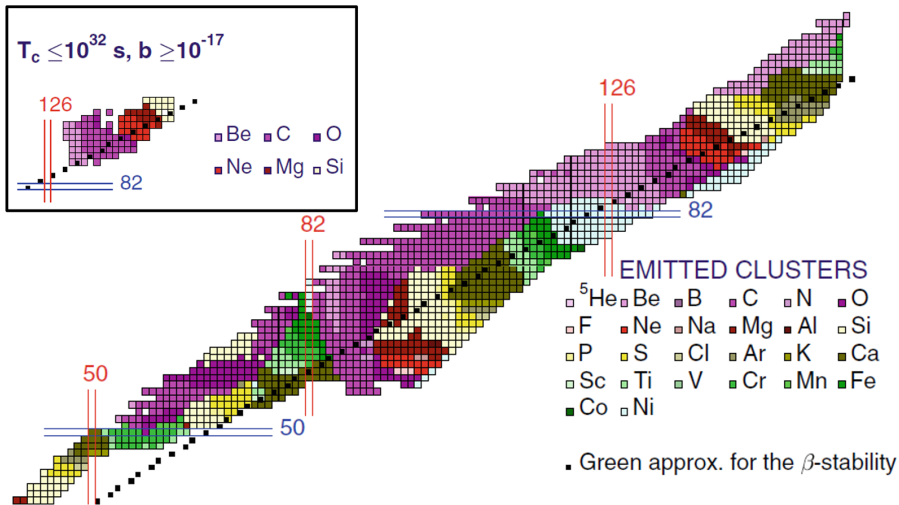


Figure – Chart of cluster emitters<sup>1</sup>.

# Radioactive family

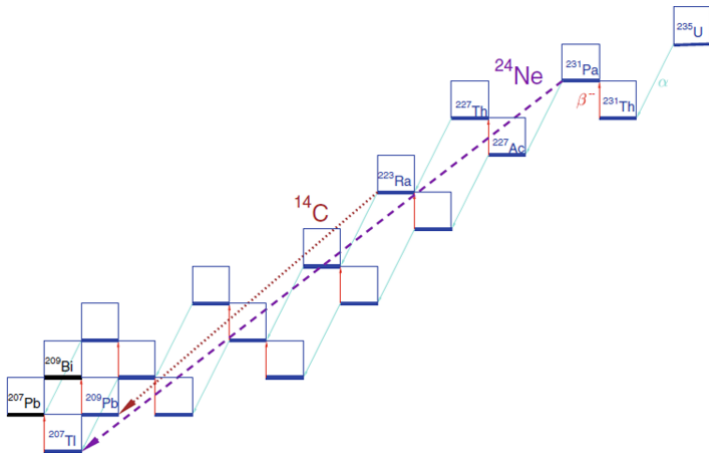


Figure – The  $^{235}\text{U}$  natural radioactive family with two cluster decay modes <sup>1</sup>.

1. N.Poenaru and W.Greiner. "Cluster Radioactivity". In : Clusters in Nuclei. Berlin Heidelberg : Springer-Verlag Berlin Heidelberg, 2010, p. 1-56.

# Semiempirical relations in radioactive decay

From microscopic mechanism of the radioactive decay ,

$$t_{1/2} = \frac{\hbar \ln 2}{\Gamma} = \frac{\ln 2}{\nu} \left| \frac{H_{\ell}^{+}(\chi, \rho)}{R F_c(R)} \right|^2, \quad (1)$$

Qi et al. present a linear relation for charged-particle emissions,

$$\log t_{1/2} = a \chi' + b \rho' + c, \quad (2)$$

where  $\rho' = \sqrt{Z_c Z_d \frac{A_c A_d}{A_p} \left( A_c^{1/3} + A_d^{1/3} \right)}$ , and  $\chi' = Z_c Z_d \sqrt{\frac{A_c A_d}{A_p E}}$ .

PHYSICAL REVIEW C **80**, 044326 (2009)

## Microscopic mechanism of charged-particle radioactivity and generalization of the Geiger-Nuttall law

C. Qi,<sup>1,2</sup> F. R. Xu,<sup>1,3</sup> R. J. Liotta,<sup>2</sup> R. Wyss,<sup>2</sup> M. Y. Zhang,<sup>1</sup> C. Asawatangtrakuldee,<sup>1</sup> and D. Hu<sup>1</sup>

<sup>1</sup>*School of Physics, and State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871,*

# Semiempirical relations in radioactive decay

In a recent paper, the authors imposed the angular momentum and isospin dependence as

$$\log t_{1/2} = a\chi' + b\rho' + c + d\rho'\sqrt{l(l+1)} + e\sqrt{I(I+1)} + fA(1 - (-1)^l). \quad (3)$$



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Extended universal decay law formula for the  $\alpha$  and cluster decays

Asim Soylu<sup>a,\*</sup>, Chong Qi<sup>b</sup>

# Semiclassical relation

The spontaneous emission of a charged particle can be described as a quantum tunneling phenomenon in which the penetration probability, which is necessary to compute the half-life, is calculated within the semiclassical JWKB approximation as

$$P = \exp \left[ -2 \int_{R_0}^R k(r) dr \right], \quad (4)$$

where  $k(r) = \sqrt{\frac{2\mu}{\hbar^2} |V(r) - E|}$ .  $E$  is the energy of the tunneling particle.

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where  $k(r) = \sqrt{\frac{2\mu}{\hbar^2} |V(r) - E|}$ .  $E$  is the energy of the tunneling particle. The interaction potential  $V(r)$  has contributions from both the Coulomb and centrifugal potentials, i.e.,

$$V(r) = Z_c Z_d \frac{e^2}{r} + \frac{\hbar^2}{2\mu} \frac{\ell(\ell + 1)}{r^2}. \quad (5)$$

## T-dependent UDL

The penetration probability (Eq. 4) is calculated considering :

- The centrifugal potential is very small compared with the Coulomb potential <sup>2</sup>,

$$\sigma = \frac{\hbar^2}{2\mu E} \ell(\ell + 1) \ll 1. \quad (6)$$

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2. G. Gamow and C. L. Critchfield. Theory of atomic nucleus and nuclear energy sources. Oxford, Clarendon Press., 1949; X. Zhang, C. Xu, and Z. Ren. Phys. Rev. C, 84 :044312, Oct 2011.
  3. R. K. Gupta, D. Singh, and W. Greiner. Phys. Rev. C, 75 :024603, Feb 2007.



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- The potential barrier is relatively wide, i.e.,  $R \gg R_0$ .
- The  $T$  dependence is introduced through the turning points assuming <sup>3</sup>,

$$R(T) = R_0 (1 + 0.0005 T^2), \quad (7)$$

where  $R_0 = 1.07A^{1/3}$  fm.

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where

$$\log [\nu_0 t'_{1/2}] = a_1 \chi' + b_1 \rho' + c_1 \frac{\ell(\ell + 1)}{\rho'}, \quad (9)$$

where the parameters  $a_1$ ,  $b_1$ , and  $c_1$  are considered free parameters to be fitted, and  $\epsilon = 0.0005 \text{ MeV}^{-2}$ .

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Some important features of the  $T$ -dependent UDL are :

- The zero temperature term could be taken to be the same as the **experimental half-life**.
- The temperature-dependent term just depends on **basic information** of the parent nucleus, the daughter nucleus, and the emitted cluster.
- Standard Double Folding model (DFM) is modified to include  $T$ -**dependent** cluster and daughter **densities** as<sup>2</sup>

$$\rho(r, T) = \rho_0(T) \left[ 1 + \exp \left( \frac{r - R(T)}{\alpha(T)} \right) \right]^{-1}, \quad (10)$$

to fit the parameters  $a_1$ ,  $b_1$ , and  $c_1$ .

# T-dependent UDL

Decay mode	$a_1$	$b_1$	$c_1$
Cluster decay	-11.59659	9.33728	-3.43742

Table – Parameters  $a_1$ ,  $b_1$ , and  $c_1$  of  $T$ -dependent term of the UDL.

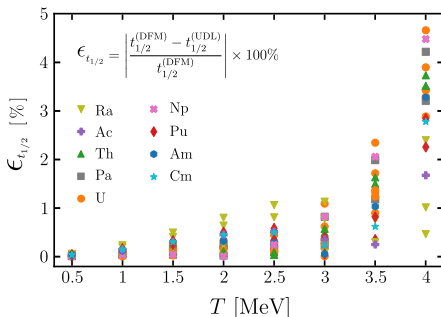


Figure – Percentage error between the half-lives calculated with the UDL and those obtained with the  $T$ -dependent DFM.

# T-dependent UDL

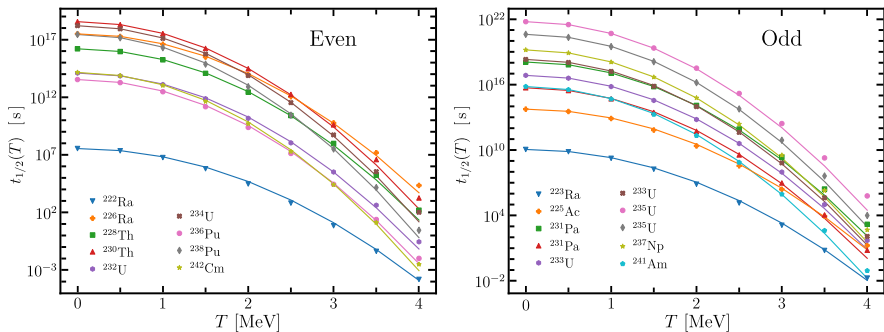


Figure – Logarithm of the cluster decay half-lives in seconds as a function of temperature. The symbols correspond to DFM calculations and the lines correspond to the UDL fitted from it.

# T-dependent UDL

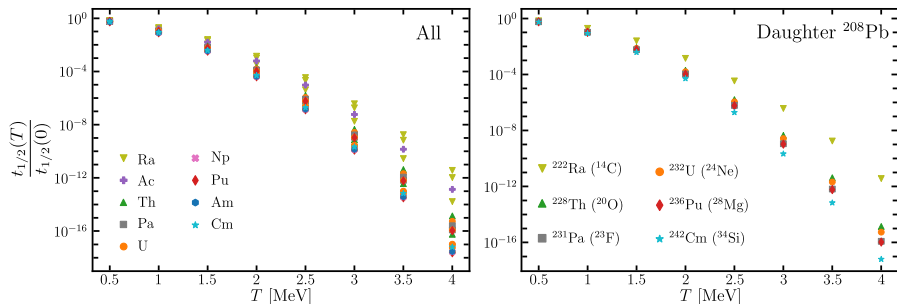


Figure – Ratio between the half-lives at  $T > 0$  MeV and those at  $T = 0$  MeV.

# Results and discussions

- The  **$T$ -dependent Universal Decay Law (UDL)** is formulated for cluster radioactivity.
- Cluster decay **half-lives decrease with increasing  $T$**  and may become dominant at high temperatures.
- The results of this study can have implications in studies on **heavy ion collisions** and **r-process nucleosynthesis**.

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PHYSICAL REVIEW LETTERS

week ending  
5 AUGUST 2011

## Heavy-Particle Radioactivity of Superheavy Nuclei

D. N. Poenaru,<sup>1,2,\*</sup> R. A. Gherghescu,<sup>1,2</sup> and W. Greiner<sup>1</sup><sup>1</sup>*Frankfurt Institute for Advanced Studies (FIAS), Ruth-Moufang-Str. 1, 60438 Frankfurt am Main, Germany*<sup>2</sup>*Horia Hulubei National Institute of Physics and Nuclear Engineering (IFIN-HH),**P.O. Box MG-6, RO-077125 Bucharest-Magurele, Romania*

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The concept of heavy-particle radioactivity (HPR) is changed to allow emitted particles with  $Z_e > 28$  from parents with  $Z > 110$  and daughter around  $^{208}\text{Pb}$ . Calculations for superheavy (SH) nuclei with  $Z = 104\text{--}124$  are showing a trend toward shorter half-lives and larger branching ratio relative to  $\alpha$  decay for heavier SHs. It is possible to find regions in which HPR is stronger than alpha decay. The new mass table AME11 and the theoretical KTUY05 and FRDM95 masses are used to determine the released energy. For 124 we found isotopes with half-lives in the range of ns to ps.

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- *Collaborator* : **Liliana Caballero**, University of Guelph, Canada.

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