Spontaneous Symmetry Breaking and its effects

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contents

Symmetry breaking can be distinguished into two types, explicit symmetry breaking and spontaneous symmetry breaking, characterized by whether the equations of motion fail to be invariant or the ground state fails to be invariant.

Spontaneous symmetry breaking is a spontaneous process of symmetry breaking, by which a physical system in a symmetric state ends up in an asymmetric state. In particular, it can describe systems where the equations of motion or the Lagrangian obey symmetries, but the lowest-energy vacuum solutions do not exhibit that same symmetry.

ϕ^4 theory- Discrete symmetry

- We begin with an analysis of spontaneous symmetry breaking in classical field theory. Consider first the familiar ϕ^4 theory Lagrangian,

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

but with m^2 replaced by a negative parameter, $-\mu^2$:

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 + \frac{1}{2}\mu^2\phi^2 - \frac{\lambda}{4!}\phi^4 .$$

The minimum-energy classical configuration is a uniform field $\phi(x) = \phi_0$, with ϕ_0 chosen to minimize the potential

$$V(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{\lambda}{4!}\phi^4 .$$

This potential has two minima, given by

$$\phi_0 = \pm v = \pm\sqrt{\frac{6}{\lambda}}\mu$$

The constant v is called the vacuum expectation value of ϕ.

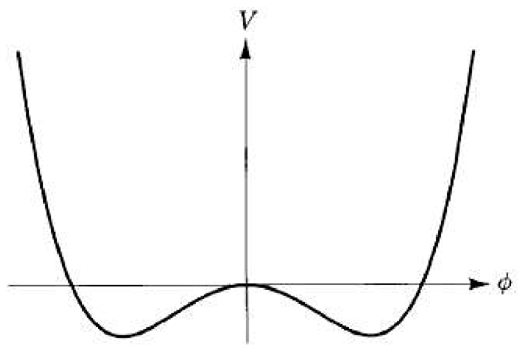
ϕ^4 theory- Discrete symmetry

- To interpret this theory, suppose that the system is near one of the minima (say the positive one). Then it is convenient to define

$$\phi(x) = v + \sigma(x)\,,$$

and rewrite $\mathcal{L}$ in terms of $\sigma(x)$.
Dropping the constant term as well we obtain the Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \sigma^2) - \frac{1}{2}(2\mu^2)\sigma^2 - \sqrt{\frac{\lambda}{6}}\mu\sigma^3 - \frac{\lambda}{4!}\sigma^4\,.$$



2HDM

- I am working on the spontaneous symmetry breaking of the non-conformal CP-conserving 2-Higgs-Doublet Model (2HDM).
- To study this, We need to provide the loop-corrected effective potential of for a non-vanishing temperature.
- The full potential can be expressed as

$$V_{eff} = V_{Tree} + V_{CW} + V_{CT} + V_T$$

where V_{Tree} is the tree-level potential, V_{CW} is the one-loop Coleman-Weinberg (CW) potential, V_{CT} are counter-terms, and V_T is the thermal contribution.

Tree-level potential

- In terms of the two $SU(2)_L$ Higgs doublets Φ_1 and Φ_2,

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad \text{and} \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix},$$

- the tree-level potential of the 2HDM with a softly broken $\mathbb{Z}_2$ symmetry, under which the doublets transform as $\Phi_1 \to \Phi_1$; $\Phi_2 \to -\Phi_2$ reads

$$\begin{aligned} V_{tree} = m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 - m_{12}^2 \left[\phi_1^\dagger \phi_2 + \phi_2^\dagger \phi_1 \right] + \frac{\lambda_1}{2} \left(\phi_1^\dagger \phi_1 \right)^2 \\ + \frac{\lambda_2}{2} \left(\phi_2^\dagger \phi_2 \right)^2 + \lambda_3 \phi_1^\dagger \phi_1 \phi_2^\dagger \phi_2 + \lambda_4 \phi_1^\dagger \phi_2 \phi_2^\dagger \phi_1 \\ + \frac{\lambda_5}{2} \left[\left(\phi_1^\dagger \phi_2 \right)^2 + \left(\phi_2^\dagger \phi_1 \right)^2 \right]. \end{aligned}$$

The mass parameters m_{11}^2 and m_{22}^2 and the couplings $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are real parameters of the model.

Tree-level potential- Minimum condition

- After EW symmetry breaking the two Higgs doublets acquire vacuum expectation values (VEVs) $\bar{\omega}_i \in \mathbb{R}$ $(i = 1, 2, 3)$.
- We have to satisfy minimum conditions

$$\left.\frac{\partial V_{tree}}{\partial \Phi_a^\dagger}\right|_{\Phi_i = \langle \Phi_i \rangle} = 0 \qquad a, i \in \{1, 2\},$$

with the brackets denoting the Higgs field values in the minimum, i.e.
$\langle \Phi_i \rangle = (0, \frac{v_i}{\sqrt{2}})$ at
T = 0. This results in two equations

$$m_{11}^2 = m_{12}^2 \frac{v_2}{v_1} - \frac{v_1^2}{2}\lambda_1 - \frac{v_2^2}{2}(\lambda_3 + \lambda_4 + \lambda_5)\,.$$

$$m_{22}^2 = m_{12}^2 \frac{v_1}{v_2} - \frac{v_2^2}{2}\lambda_2 - \frac{v_1^2}{2}(\lambda_3 + \lambda_4 + \lambda_5)\,.$$

Theoretical constraints

- Copositivity criteria, in order to ensure that the scalar potential is bounded from below, the following conditions from copositivity criteria should be satisfied

$$\lambda_1 \geq 0, \qquad \lambda_2 \geq 0, \qquad \lambda_3 + \sqrt{\lambda_1 \lambda_2} \geq 0,$$
$$\lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} \geq 0$$

- Unitarity constraints, exact $\mathbb{Z}_2$ symmetry $m_{12} = 0$ leads the upper limits on the masses.

$$0 < \lambda_1 + \lambda_2 < \frac{32\pi}{3} \qquad |m_A^2 - \frac{m_{12}^2}{\sin\beta\cos\beta}| < \frac{16\pi v^2}{3} = 1\ \text{TeV}^2$$
$$|m_{H^\pm}^2 - \frac{m_{12}^2}{\sin\beta\cos\beta}| < \frac{16\pi v^2}{3} \qquad m_A, m_{H^\pm} < 1\ \text{TeV}\,.$$

One-loop CW potential- General formula

- The Coleman-Weinberg potential in the $\overline{MS}$ scheme is given by

$$V_{CW}(\{\omega\}) = \sum_i \frac{n_i}{64\pi^2}(-1)^{2s_i} m_i^4(\{\omega\})\left[log\Big(m_i^2(\{\omega\})\Big) - c_i\right]$$

where the sum extends over the Higgs and Goldstone bosons, the massive gauge bosons, the longitudinal photon and the fermions, $i = h, H, A, H^{\pm}, G^0, G^{\pm}, W^{\pm}, Z, \gamma_L, f, (f = e, \mu, \tau, u, c, t, d, s, b)$. The m_i^2 is the respective eigenvalue for the particle i of the mass matrix squared expressed through the tree-level relations in terms of ω_i $(i = 1, 2, 3)$. The variable s_i denotes the spin of the particle, n_i represents the number of degrees of freedom.

One-loop CW potential- Constants

- These are the neutral scalars $\Phi^0 \equiv h, H, A, G^0$, the charged scalars $\Phi^\pm \equiv H^\pm, G^\pm$, the leptons l, the quarks q and the longitudinal and transversal gauge bosons, $V_L \equiv Z_L, W_L, \gamma_L$ and $V_T \equiv Z_T, W_T, \gamma_T$, with the respective n_i,

$$\begin{aligned} n_{\Phi^0} &= 1, \quad n_{\Phi^\pm} = 2, \quad n_l = 4, \quad n_q = 12, \\ n_{W_L=2}, &\quad n_{W_T} = 4, \quad n_{Z_L} = 2, \quad n_{Z_T} = 4, \\ & \qquad n_{\gamma_T} = 2, \quad n_{\gamma_L} = 1. \end{aligned}$$

In the $\overline{MS}$ scheme employed here, the constants c_i read

$$c_i = \begin{cases} \frac{5}{6}, & i = W^\pm, Z \\ \frac{3}{2}, & \text{otherwise} \end{cases} .$$

We fix the renormalisation scale μ by $\mu = v = 246.22$ GeV.

Counter-terms- General formula

- Introducing counterterms for each of the parameters of the tree-level potential, the counterterm potential V_{CT} is the last term of one-loop effective potential

$$V_{CT} = \delta m_{11}^2 \frac{\omega_1^2}{2} + \delta m_{22}^2 \frac{\omega_2^2 + \omega_3^2}{2} - \delta m_{12}^2 \omega_1 \omega_2 + \frac{\delta\lambda_1}{8}\omega_1^4 + \frac{\delta\lambda_2}{8}(\omega_2^2 + \omega_3^2)^2 + (\delta\lambda_3 + \delta\lambda_4)\frac{\omega_1^2(\omega_2^2 + \omega_3^2)}{4} + \delta\lambda_5 \frac{\omega_1^2(\omega_2^2 - \omega_3^2)}{4}$$

For simplicity we assume $\delta m_{12}^2 = 0$ and $\omega_3 = 0$, then we have

$$V_{CT} = \delta m_{11}^2 \frac{\omega_1^2}{2} + \delta m_{22}^2 \frac{\omega_2^2}{2} + \frac{\delta\lambda_1}{8}\omega_1^4 + \frac{\delta\lambda_2}{8}\omega_2^4 + (\delta\lambda_3 + \delta\lambda_4 + \delta\lambda_5)\frac{\omega_1^2 \omega_2^2}{4} .$$

Counter-term- Coefficients

We have five different equations (five first and second order derivatives with respect to ω_1 and ω_2) and five coefficients. After solving these equations we obtain

$$\delta m_{11}^2 = -\frac{3}{2v_1}\frac{\partial V_{CW}}{\partial \omega_1} + \frac{1}{2}\frac{\partial^2 V_{CW}}{\partial \omega_1^2} + \frac{v_2}{v_1}\frac{\partial^2 V_{CW}}{\partial \omega_2 \partial \omega_1}$$

$$\delta m_{22}^2 = -\frac{3}{2v_2}\frac{\partial V_{CW}}{\partial \omega_2} + \frac{1}{2}\frac{\partial^2 V_{CW}}{\partial \omega_2^2} + \frac{v_1}{v_2}\frac{\partial^2 V_{CW}}{\partial \omega_2 \partial \omega_1}$$

$$\delta \lambda_1 = \frac{1}{v_1^3}\frac{\partial V_{CW}}{\partial \omega_1} - \frac{1}{v_1^2}\frac{\partial^2 V_{CW}}{\partial \omega_1^2}$$

$$\delta \lambda_2 = \frac{1}{v_2^3}\frac{\partial V_{CW}}{\partial \omega_2} - \frac{1}{v_2^2}\frac{\partial^2 V_{CW}}{\partial \omega_2^2}$$

$$\delta \lambda_{3,4,5} = -\frac{1}{v_1 v_2}\frac{\partial^2 V_{CW}}{\partial \omega_2 \partial \omega_1} .$$

GW Mechanism- Flatness condition

- Gildener-Weinberg (GW) mechanism extends the CW mechanism to a much larger class of gauge theories, theories in which there may be arbitrary numbers of scalar fields with more or less arbitrary interactions.
- We choose the renormalization scale Λ to have a value Λ_w at which $V_0(\Phi)$ does have a nontrivial minimum on some ray $\Phi_i = n_i\phi$

$$\min_{N_i N_i = 1} (f_{ijkl} N_i N_j N_k N_l) = 0$$

$V_0(N)$ on the unit sphere $N_i N_i = 1$ is zero. The one loop potential has a form like

$$\delta V(n\phi) = A\phi^4 + B\phi^4 \ln\left(\frac{\phi^2}{\Lambda_w^2}\right)$$

GW Mechanism- Condition to have minimum

- where A and B are dimensionless constants.

$$A = \frac{1}{64\pi^2 \langle\phi\rangle^4}\Big\{3\,Tr\big[\mu^4 \ln\Big(\frac{\mu^2}{\langle\phi\rangle^2}\Big)\big]$$

$$+Tr\big[M^4 \ln\Big(\frac{M^2}{\langle\phi\rangle^2}\Big)\big] - 4\,Tr\big[m^4 \ln\Big(\frac{m^2}{\langle\phi\rangle^2}\Big)\big]\Big\}$$

$$B = \frac{1}{64\pi^2 \langle\phi\rangle^4}\Big(3\,Tr\mu^4 + TrM^4 - 4\,Trm^4\Big)\,.$$

- We see that the potential has a nontrivial stationary point, at a value of $\langle\phi\rangle$ given by

$$\ln\Big(\frac{\langle\phi\rangle^2}{\Lambda_w^2}\Big) = -\frac{1}{2} - \frac{A}{B}\,.$$

We will assume that in this case $B > 0$. We see immediately that the potential at its stationary point is less than its value $V(0) = 0$ at the origin

$$V(n\langle\phi\rangle) = \delta V(n\langle\phi\rangle) = -\frac{1}{2}B\langle\phi\rangle^4 < 0\,.$$

Conformal 2HDM

- In the Weinberg-Salam model, the masses of gauge bosons and fermions are taken to be generated by the Higgs mechanism.
- In conformal model, spontaneous symmetry breakdown does not occur at the tree level. We have to consider the potential at least up to the one-loop level.
- We follow the method of Gildener and Weinberg and set a condition that V_0 has a minimum value of zero on some ray $n\rho = n_0\rho$ for arbitrary ρ in order to make the loop expansion of V reliable.
- A necessary condition is

$$\frac{\partial}{\partial n_k}\left(\frac{V_0}{\rho^4}\right) = 0. \qquad (k = 1, 2, 3, 4)$$

Conformal 2HDM

- The flatness conditions are as follows

$$\begin{cases} n = (n_1, n_2, 0, 0), \\ n_1^2 = (\frac{\sqrt{\lambda_2}}{\sqrt{\lambda_1}+\sqrt{\lambda_2}}), n_2^2 = (\frac{\sqrt{\lambda_1}}{\sqrt{\lambda_1}+\sqrt{\lambda_2}}) \\ \sqrt{\lambda_1 \lambda_2} + \lambda_3 + \lambda_4 + \lambda_5 = 0 \end{cases}$$

- The effective potential in conformal 2HDM is given by

$$V_{eff} = V_0 + V_{CW}$$

Summary

- Differences between conformal and non-conformal 2HDM
- Type of spontaneous symmetry breaking
- mass matrix
- effective potential
- β angel

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Thank you !