Phase Transition and Gravitational Wave Signatures

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I will discuss the conformal and non-conformal two Higgs doublet models with a focus on their phase transition and gravitational wave signatures. The construction of the finite temperature effective potential of both models will be discussed in detail. Compared to the non-conformal case, the conformal model yields a very interesting phase diagram in the 2-dimensional parameter space corresponding to the phase transition. An exploration of other conformal hidden sector models (such as the well-established real singlet and two real singlet models) suggests that the special shape of the phase diagram could be a universal feature in a generic class of conformal models.

Phase transition dynamics [1]

The essential quantity of phase transition dynamics is the bubble nucleation rate (the decay rate of the false vacuum) per unit time per unit volume:

$$\Gamma \approx \Gamma_0 e^{-S}, \qquad \Gamma(T) \approx \begin{cases} T^4 \left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} \exp\left(-\frac{S_3}{T}\right), & T > T_{\rm div} \\ T_{\rm div}^4 \left(\frac{S_4}{2\pi}\right)^2 \exp(-S_4), & T < T_{\rm div} \end{cases},$$

where S is the bounce action

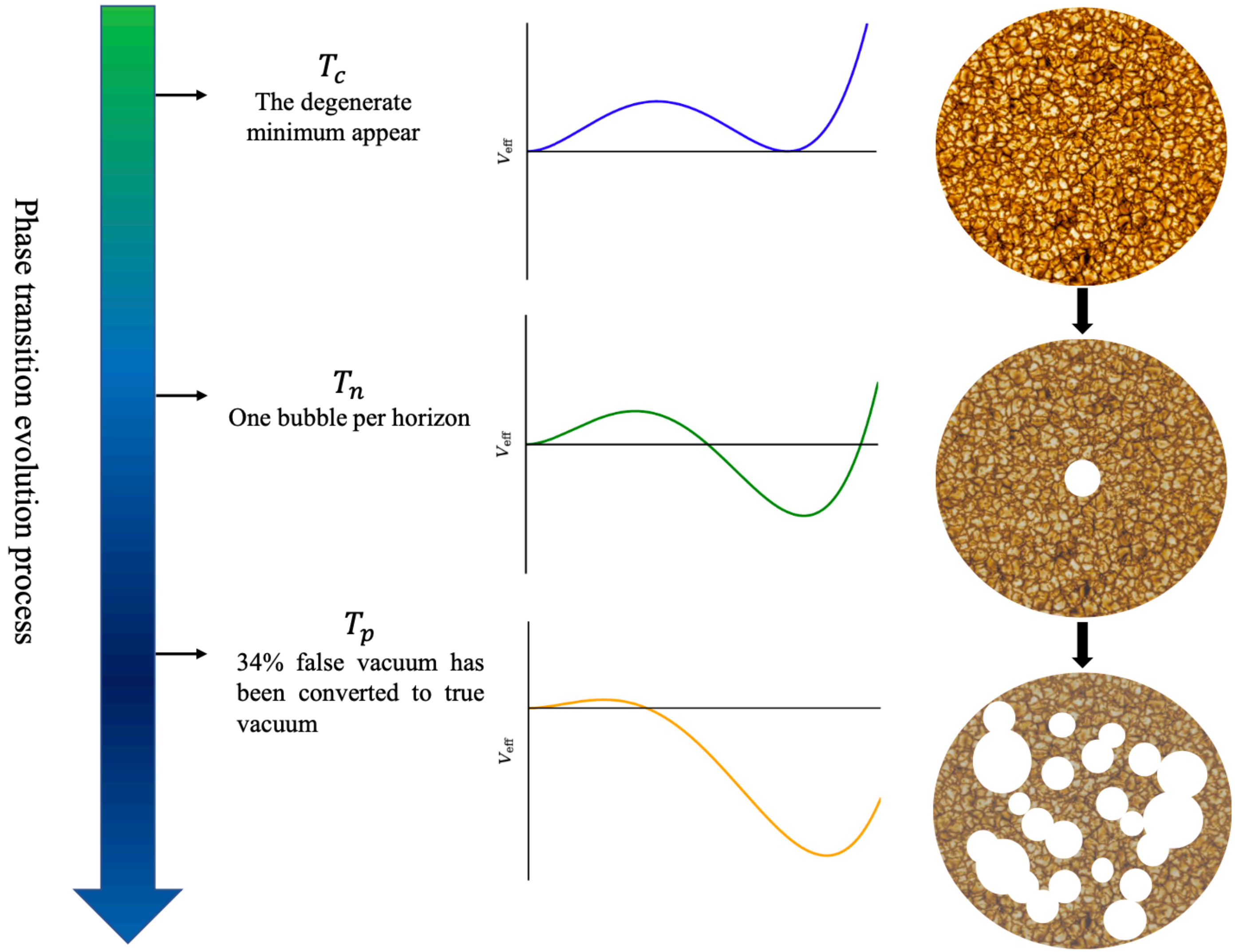
The most general form of the bounce action should be

$$S(T) = 4\pi \int_{1/T}^{0} d\tau \int_0^\infty dr r^2 \left[\frac{1}{2}\left(\frac{\partial \phi}{\partial \tau}\right)^2 + \frac{1}{2}\left(\frac{\partial \phi}{\partial r}\right)^2 + V_{\rm eff}(\phi, T)\right],$$

where $\tau = it$ is the Euclidean time.

From the finite-temperature effective potential, we can derive the bounce action by solving the following equation-of-motion (EOM) with the boundary conditions:

$$\frac{\partial^2 \phi}{\partial \tau^2} + \frac{\partial^2 \phi}{\partial r^2} + \frac{2}{r}\frac{\partial \phi}{\partial r} = \frac{\partial V_{\rm eff}}{\partial \phi} \qquad \left.\frac{\partial \phi}{\partial \tau}\right|_{\tau=0,\pm\frac{1}{2T}} = 0, \quad \left.\frac{\partial \phi}{\partial r}\right|_{r=0} = 0, \quad \lim_{r\to\infty} \phi(r) = \phi_{\rm false}\,.$$

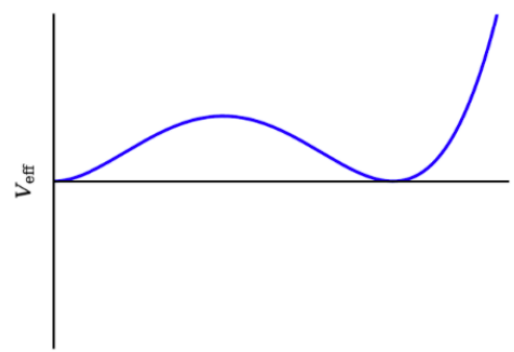

Phase transition evolution process
T_c
The degenerate minimum appear
T_n
One bubble per horizon
T_p
34% false vacuum has been converted to true vacuum
V_{eff}
V_{eff}
V_{eff}

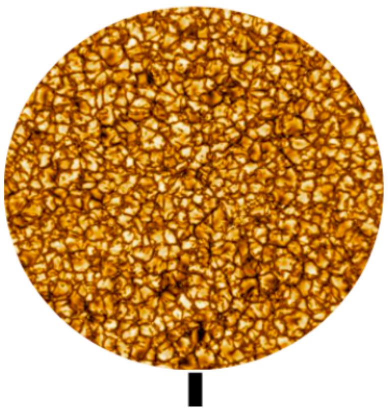
Critical Temperature

Critical temperature: the temperature at which the effective potential has two degenerate minimums.

T_c

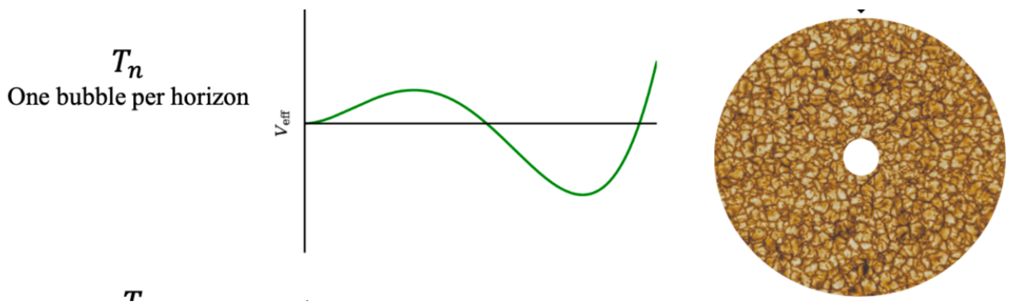
The degenerate minimum appear





Nucleation Temperature

Nucleation temperature: the temperature at which one bubble is nucleated in one casual Hubble volume.



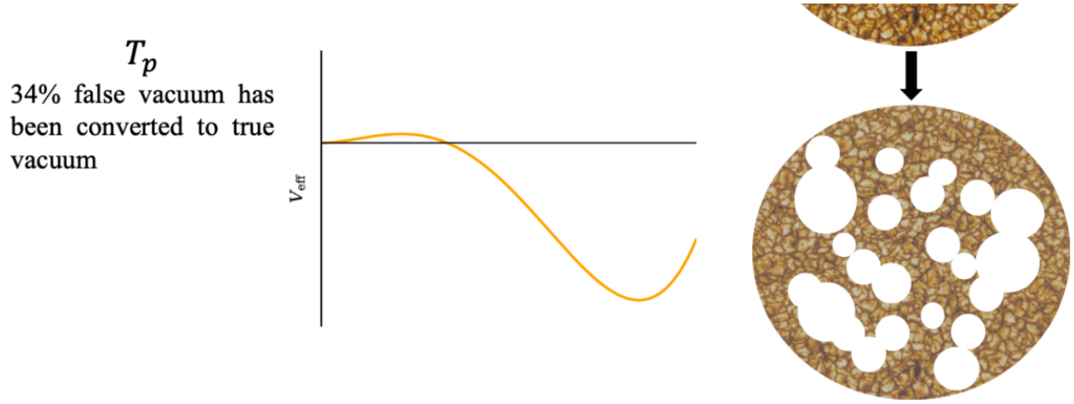
$$N(T_n) = \int_{t_c}^{t_n} dt \frac{\Gamma(t)}{H(t)^3} = \int_{T_n}^{T_c} \frac{dT}{T} \frac{\Gamma}{H^4} = 1 \,, \qquad H^2 = \frac{1}{3M_{\text{pl}}^2} (\rho_R + \rho_V) \,,$$

For the electroweak scale phase transition, this roughly corresponds to $S(T_n) \approx 140$.

Note: this is not true for supercooling cases**

Percolation Temperature

Percolation temperature: the temperature at which the probability of finding a point still in the false vacuum is 0.7 or 34% false vacuum has been converted to the true vacuum.



$$I(T) = \frac{4\pi v_b^3}{3} \int_T^{T_c} \frac{dT' \Gamma(T')}{H(T') T'^4} \left(\int_T^{T'} \frac{d\tilde{T}}{H(\tilde{T})} \right)^3 .$$

We assume the bubble wall achieves the terminal velocity very fast, so we can set Vb as a constant, which can be a good approximation.

Two Important Parameters for Gravitational Waves [2]

α: the ratio of the latent heat released by the phase transition normalized against the radiation density

$$\alpha = \frac{\epsilon}{\rho_{\mathrm{rad}}} = \frac{1}{\frac{\pi^2}{30} g_* T_n^4} \left(-\Delta V + T_n \Delta s\right)$$

$$\Delta V = V\left(v_{T_n}, T_n\right) - V\left(0, T_n\right)$$

$$\Delta s = \frac{\partial V}{\partial T}\left(v_{T_n}, T_n\right) - \frac{\partial V}{\partial T}\left(0, T_n\right)\ ,$$

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β/H*: the inverse duration of the phase transition β relative to the Hubble rate H* at the nucleation temperature T_n

$$\frac{\beta}{H_*} = \left[T \frac{d}{dT} \left(\frac{S_3(T)}{T} \right) \right] \bigg|_{T=T_n} .$$

Gravitational Waves [2]

The power spectrum of the acoustic gravitational wave is given by:

$$h^2\Omega_{sw}(f) = 8.5*10^{-6}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\Gamma_{AI}^2\overline{U}_f^4\left(\frac{H_*}{\beta}\right)v_w S_{sw}(f)$$

where the adiabatic index $\Gamma_{AI} = \omega/\varepsilon \simeq 4/3$. ω and ε denote respectively the volume-averaged enthalpy and energy density respectively. U_f is a measure of the root-mean-square (rms) fluid velocity and is given by:

$$\overline{U}_f^2 \simeq \frac{3}{4}\kappa_f \alpha_{T_n},$$

where $\varkappa_f$ is the efficiency parameter and it is well approximated by

$$\kappa_f \sim \frac{\alpha}{0.73 + 0.083\sqrt{\alpha} + \alpha}$$

when v_ω (wall speed) → 1. The spectral shape S_{sw} (f) is given by:

$$S_{sw}(f) = \left(\frac{f}{f_{sw}}\right)^3\left(\frac{7}{4+3\left(f/f_{sw}\right)^2}\right)^{\frac{7}{2}}$$

with peak frequency f_{sw} approximated by:

$$f_{sw} = 8.9\mu\text{Hz}\,\frac{1}{v_\omega}\left(\frac{\beta}{H_*}\right)\left(\frac{z_p}{10}\right)\left(\frac{T_n}{100\,\text{GeV}}\right)\left(\frac{g_*}{100}\right)^{\frac{1}{6}}$$

with z_p a simulation-derived factor that is of order 10, and we take it to be 6.9.

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Non-conformal 2 Higgs Doublet modle [3]

The effective potential:

$$V_{eff} = V_{tree} + V_{loop} + V_{CT} + V_{FT} + V_{RC}$$

	Type I	Type II	Lepton-Specific	Flipped
Up-type quarks	Φ_2	Φ_2	Φ_2	Φ_2
Down-type quarks	Φ_2	Φ_1	Φ_2	Φ_1
Leptons	Φ_2	Φ_1	Φ_1	Φ_2

Table 1. Classification of the Yukawa sector in the 2HDM according to the couplings of the fermions to the Higgs doublets.

tree level potential

In terms of the two $\mathrm{SU}(2)_L$ Higgs doublets Φ_1 and Φ_2,

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \phi_1^0 \end{pmatrix} \quad \text{and} \quad \Phi_1 = \begin{pmatrix} \phi_2^+ \\ \phi_2^0 \end{pmatrix}, \tag{2.1}$$

the tree-level potential of the 2HDM with a softly broken $\mathbb{Z}_2$ symmetry, under which the doublets transform as $\Phi_1 \to \Phi_1,\ \Phi_2 \to -\Phi_2$, reads

$$\begin{aligned} V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] + \frac{1}{2}\lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2 (\Phi_2^\dagger \Phi_2)^2 \\ + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \left[\frac{1}{2}\lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{h.c.} \right]. \end{aligned} \tag{2.2}$$

In the minimum of the potential eq. (2.2) the following minimum conditions have to be fulfilled,

$$\left.\frac{\partial V_{\text{tree}}}{\partial \Phi_a^\dagger}\right|_{\Phi_i = \langle \Phi_i \rangle} \stackrel{!}{=} 0 \qquad a, i \in \{1, 2\}\,, \tag{2.14}$$

with the brackets denoting the Higgs field values in the minimum, i.e. $\langle \Phi_i \rangle = (0, v_i/\sqrt{2})$ at $T = 0$. This results in two equations

$$m_{11}^2 = m_{12}^2 \frac{v_2}{v_1} - \frac{v_1^2}{2}\lambda_1 - \frac{v_2^2}{2}(\lambda_3 + \lambda_4 + \lambda_5) \tag{2.15a}$$

$$m_{22}^2 = m_{12}^2 \frac{v_1}{v_2} - \frac{v_2^2}{2}\lambda_2 - \frac{v_1^2}{2}(\lambda_3 + \lambda_4 + \lambda_5)\,. \tag{2.15b}$$

Exploiting the minimum conditions of the potential at zero temperature, we use the following set of independent parameters of the model,

$$m_h,\ m_H,\ m_A,\ m_{H^\pm},\ m_{12}^2,\ \alpha,\ \tan\beta,\ v\,. \tag{2.16}$$

we fix M_h = 125 GeV and v = 246 GeV, hence we are left with M_H, M_A, $M_{H^{+-}}$, β, α, m12 → the parameter space has 6 degrees of freedom

Fermions

The fermion mass at $T = 0$ is given by the tree-level VEV v_k of the doublet Φ_k^c as

$$m_f(T=0) = \frac{y_f}{\sqrt{2}} v_k \,.$$

where y_f is the tree-level Yukawa coupling and $k = 1, 2$ denotes the classical constant field configuration doublet $\Phi^c{}_k$ to which the fermion couples.

Gauge Bosons

$$\overline{m}_W^2 = \frac{g^2}{4}\omega^2 + 2g^2T^2 \tag{A.5}$$

$$\overline{m}_\gamma^2 = (g^2+g'^2)\left(T^2+\frac{\omega^2}{8}\right) - \frac{1}{8}\sqrt{(g^2-g'^2)^2\,(64T^4+16T^2\omega^2)+(g^2+g'^2)^2\,\omega^4} \tag{A.6}$$

$$\overline{m}_Z^2 = (g^2+g'^2)\left(T^2+\frac{\omega^2}{8}\right) + \frac{1}{8}\sqrt{(g^2-g'^2)^2\,(64T^4+16T^2\omega^2)+(g^2+g'^2)^2\,\omega^4}\,, \tag{A.7}$$

where g and g' denote the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge couplings, respectively, and

$$\omega^2 = \sum_{i=1,2,3} \omega_i^2\,. \tag{A.8}$$

Again, the physical masses are obtained for $\omega_i \equiv \bar{\omega}_i$, and at $T = 0$ we recover the well-known relations for the physical gauge boson masses ($v^2 = v_1^2 + v_2^2 = \sum_{i=1,2,3} \bar{\omega}_i^2\big|_{T=0}$)

$$m_W^2 = \frac{g^2}{4}v^2\,, \quad m_Z^2 = \frac{g^2+g'^2}{4}v^2 \quad \text{and} \quad m_\gamma^2 = 0\,. \tag{A.9}$$

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Higgs Bosons

The tree-level relations for the mass matrices of the Higgs bosons in the interaction basis in terms of the ω_k are obtained by differentiating the tree-level Higgs potential V_{tree} eq. (2.2) twice with respect to the real interaction fields

$$\phi_i \equiv \{\rho_1, \eta_1, \rho_2, \eta_2, \zeta_1, \psi_1, \zeta_2, \psi_2\} \tag{A.10}$$

and replacing the fields with their classical constant field configurations

$$\phi_i^c \equiv \{0, 0, 0, 0, \omega_1, 0, \omega_2, \omega_3\}\,, \tag{A.11}$$

leading to the mass matrix

$$(\mathcal{M})_{ij} = \frac{1}{2} \left. \frac{\partial^2 V_{\text{tree}}}{\partial \phi_i \partial \phi_j} \right|_{\phi=\phi^c} . \tag{A.12}$$

The physical masses are given by the field values in the global minimum of the potential where $\omega_k \equiv \bar{\omega}_k$, which at $T = 0$ reduces to $\bar{\omega}_{1,2}|_{T=0} = v_{1,2}$ and $\bar{\omega}_3|_{T=0} = 0$. Because of charge conservation the mass matrix of eq. (A.12) decomposes into a 4×4 matrix $\mathcal{M}^C$ for the charged fields $\rho_1, \eta_1, \rho_2, \eta_2$ and a 4×4 matrix $\mathcal{M}^N$ for the neutral states $\zeta_1, \psi_1, \zeta_2, \psi_2$. In the CP-conserving 2HDM the neutral CP-even and CP-odd fields do not mix so that the latter matrix further decomposes into two 2×2 matrices, one for the CP-even Higgs states $\zeta_{1,2}$ and one for the pseudoscalar states $\psi_{1,2}$.

```
In[∘]:= Eigenvalues[{{A, C}, {C, B}}] // FullSimplify
```

$$\left\{\frac{1}{2}\left(A+B-\sqrt{(A-B)^2+4C^2}\right),\ \frac{1}{2}\left(A+B+\sqrt{(A-B)^2+4C^2}\right)\right\}$$

```
In[∘]:= Eigenvalues[{{A, C}, {C, B}} + T^2 {{c1, 0}, {0, c2}}] // FullSimplify
```

$$\left\{\frac{1}{2}\left(A+B-\sqrt{(A-B)^2+4C^2+T^2\left(2(A-B)+T^2(c_1-c_2)\right)(c_1-c_2)}+T^2(c_1+c_2)\right),\right.$$
$$\left.\frac{1}{2}\left(A+B+\sqrt{(A-B)^2+4C^2+T^2\left(2(A-B)+T^2(c_1-c_2)\right)(c_1-c_2)}+T^2(c_1+c_2)\right)\right\}$$

```
MH =
 1/2 (A + B + Sqrt[(A - B)^2 + 4 C^2]) /. A → m11^2 + 1/2 * (3 * λ1 * ω1^2 + (λ3 + λ4 + λ5) * ω2^2) /.
    B → m22^2 + 1/2 * (3 * λ2 * ω2^2 + (λ3 + λ4 + λ5) * ω1^2) /. C → -m12^2 + (λ3 + λ4 + λ5) * ω1 * ω2 /.
   v1 → 246 * Cos[β] /. v2 → 246 * Sin[β] /. ω1 → ϕ * Cos[β] /. ω2 → ϕ * Sin[β] // FullSimplify;

Mh =
 1/2 (A + B - Sqrt[(A - B)^2 + 4 C^2]) /. A → m11^2 + 1/2 * (3 * λ1 * ω1^2 + (λ3 + λ4 + λ5) * ω2^2) /.
    B → m22^2 + 1/2 * (3 * λ2 * ω2^2 + (λ3 + λ4 + λ5) * ω1^2) /. C → -m12^2 + (λ3 + λ4 + λ5) * ω1 * ω2 /.
   v1 → 246 * Cos[β] /. v2 → 246 * Sin[β] /. ω1 → ϕ * Cos[β] /. ω2 → ϕ * Sin[β] // FullSimplify;
```

Loop level potential

The Coleman-Weinberg potential in the MS scheme is given by:

$$V_{\mathrm{CW}}(\{\omega\}) = \sum_i \frac{n_i}{64\pi^2}(-1)^{2s_i}\, m_i^4(\{\omega\}) \left[\log\left(\frac{m_i^2(\{\omega\})}{\mu^2}\right) - c_i\right],$$

$$\begin{aligned} n_{\Phi^0} &= 1\,, & n_{\Phi^\pm} &= 2\,, & n_l &= 4\,, & n_q &= 12\,, \\ n_{W_T} &= 4\,, & n_{W_L} &= 2\,, & n_{Z_T} &= 2\,, & n_{Z_L} &= 1\,, \\ n_{\gamma_T} &= 2\,, & n_{\gamma_L} &= 1\,. \end{aligned}$$

In the $\overline{\mathrm{MS}}$ scheme employed here, the constants c_i read

$$c_i = \begin{cases} \frac{5}{6}\,, & i = W^\pm, Z, \gamma \\ \frac{3}{2}\,, & \text{otherwise}\,. \end{cases}$$

We fix the renormalisation scale μ by $\mu = v = 246.22\,\mathrm{GeV}$.

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Counter Term Potential [4]

Notice that loop corrections generally shift the values of the VEVs as well as the renormalized mass-squared matrix of the CP-even neutral scalar bosons. To keep them intact, we introduce the following counterterms [76, 77],

$$V_{\rm CT}(\tilde{\rho}_1,\tilde{\rho}_2,\tilde{s}) = \delta m_1^2\tilde{\rho}_1^2 + \delta m_2^2\tilde{\rho}_2^2 + \delta m_s^2\tilde{s}^2 + \delta\lambda_1\tilde{\rho}_1^4 + \delta\lambda_2\tilde{\rho}_2^4 + \delta\lambda_s\tilde{s}^4 + \delta\lambda_{12}\tilde{\rho}_1^2\tilde{\rho}_2^2 + \delta\lambda_{1s}\tilde{\rho}_1^2\tilde{s}^2 + \delta\lambda_{2s}\tilde{\rho}_2^2\tilde{s}^2. \tag{45}$$

The nine counterterm coefficients are determined by the following nine equations at $(\tilde{\rho}_1,\tilde{\rho}_2,\tilde{s}) = (v_1, v_2, v_s)$,

$$\frac{\partial V_{\rm CT}}{\partial\tilde{\rho}_1} = -\frac{\partial V_1}{\partial\tilde{\rho}_1}, \qquad \frac{\partial V_{\rm CT}}{\partial\tilde{\rho}_2} = -\frac{\partial V_1}{\partial\tilde{\rho}_2}, \qquad \frac{\partial V_1}{\partial\tilde{s}} = -\frac{\partial V_1}{\partial\tilde{s}}, \tag{46}$$

$$\frac{\partial^2 V_{\rm CT}}{\partial\tilde{\rho}_1^2} = -\frac{\partial^2 V_1}{\partial\tilde{\rho}_1^2}, \qquad \frac{\partial^2 V_{\rm CT}}{\partial\tilde{\rho}_2^2} = -\frac{\partial^2 V_1}{\partial\tilde{\rho}_2^2}, \qquad \frac{\partial^2 V_{\rm CT}}{\partial\tilde{s}^2} = -\frac{\partial^2 V_1}{\partial\tilde{s}^2}, \tag{47}$$

$$\frac{\partial^2 V_{\rm CT}}{\partial\tilde{\rho}_2\partial\tilde{\rho}_1} = -\frac{\partial^2 V_1}{\partial\tilde{\rho}_2\partial\tilde{\rho}_1}, \qquad \frac{\partial^2 V_{\rm CT}}{\partial\tilde{s}\partial\tilde{\rho}_1} = -\frac{\partial^2 V_1}{\partial\tilde{s}\partial\tilde{\rho}_1}, \qquad \frac{\partial^2 V_{\rm CT}}{\partial\tilde{s}\partial\tilde{\rho}_2} = -\frac{\partial^2 V_1}{\partial\tilde{s}\partial\tilde{\rho}_2}. \tag{48}$$

The masses of the Nambu-Goldstone bosons G^0 and $G^\pm$ vanish at $(\tilde{\rho}_1,\tilde{\rho}_2,\tilde{s}) = (v_1, v_2, v_s)$ in the Landau gauge, inducing logarithmic IR divergence terms in Eqs. (47) and (48) proportional to

$$\frac{\partial\tilde{m}_G^2}{\partial\phi_i}\frac{\partial\tilde{m}_G^2}{\partial\phi_j}\ln\frac{\tilde{m}_G^2}{\mu^2}, \qquad \phi_i = \tilde{\rho}_1,\tilde{\rho}_2,\tilde{s}. \tag{49}$$

$$\delta m_1^2 = -\frac{3}{4v_1}\frac{\partial V_1}{\partial \tilde{\rho}_1} + \frac{1}{4}\frac{\partial^2 V_1}{\partial \tilde{\rho}_1^2} + \frac{v_2}{4v_1}\frac{\partial^2 V_1}{\partial \tilde{\rho}_2 \partial \tilde{\rho}_1} + \frac{v_s}{4v_1}\frac{\partial^2 V_1}{\partial \tilde{s} \partial \tilde{\rho}_1}, \tag{50}$$

$$\delta m_2^2 = -\frac{3}{4v_2}\frac{\partial V_1}{\partial \tilde{\rho}_2} + \frac{1}{4}\frac{\partial^2 V_1}{\partial \tilde{\rho}_2^2} + \frac{v_1}{4v_2}\frac{\partial^2 V_1}{\partial \tilde{\rho}_2 \partial \tilde{\rho}_1} + \frac{v_s}{4v_2}\frac{\partial^2 V_1}{\partial \tilde{s} \partial \tilde{\rho}_2}, \tag{51}$$

$$\delta m_s^2 = -\frac{3}{4v_s}\frac{\partial V_1}{\partial \tilde{s}} + \frac{1}{4}\frac{\partial^2 V_1}{\partial \tilde{s}^2} + \frac{v_1}{4v_s}\frac{\partial^2 V_1}{\partial \tilde{s} \partial \tilde{\rho}_1} + \frac{v_2}{4v_s}\frac{\partial^2 V_1}{\partial \tilde{s} \partial \tilde{\rho}_2}, \tag{52}$$

$$\delta \lambda_1 = \frac{1}{8v_1^3}\frac{\partial V_1}{\partial \tilde{\rho}_1} - \frac{1}{8v_1^2}\frac{\partial^2 V_1}{\partial \tilde{\rho}_1^2}, \quad \delta \lambda_2 = \frac{1}{8v_2^3}\frac{\partial V_1}{\partial \tilde{\rho}_2} - \frac{1}{8v_2^2}\frac{\partial^2 V_1}{\partial \tilde{\rho}_2^2}, \tag{53}$$

$$\delta \lambda_s = \frac{1}{8v_s^3}\frac{\partial V_1}{\partial \tilde{s}} - \frac{1}{8v_s^2}\frac{\partial^2 V_1}{\partial \tilde{s}^2}, \quad \delta \lambda_{12} = -\frac{1}{4v_1 v_2}\frac{\partial^2 V_1}{\partial \tilde{\rho}_2 \partial \tilde{\rho}_1}, \tag{54}$$

$$\delta \lambda_{1s} = -\frac{1}{4v_1 v_s}\frac{\partial^2 V_1}{\partial \tilde{s} \partial \tilde{\rho}_1}, \quad \delta \lambda_{2s} = -\frac{1}{4v_2 v_s}\frac{\partial^2 V_1}{\partial \tilde{s} \partial \tilde{\rho}_2}, \tag{55}$$

at $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}) = (v_1, v_2, v_s)$.

Finite T Potential and Ring Corrections [3]

$$V^T = \sum_k n_k \frac{T^4}{2\pi^2} J_{\pm}^{(k)} . \qquad (2.21)$$

The sum extends over $k = W_L, Z_L, \gamma_L, W_T, Z_T, \Phi^0, \Phi^{\pm}, f$. Note, that the Goldstone bosons and the longitudinal part of the photon, which are massless at $T = 0$, acquire a mass at finite temperature and are included in the sum. Denoting the mass eigenvalue including the thermal corrections for the particle k by $\overline{m}_k$, $J_{\pm}^{(k)}$ is given by (see e.g. [70])

$$J_{\pm}^{(k)} = \begin{cases} J_- \left(\frac{m_k^2}{T^2}\right) - \frac{\pi}{6}\left(\frac{\overline{m}_k^3}{T^3} - \frac{m_k^3}{T^3}\right) & k = W_L, Z_L, \gamma_L, \Phi^0, \Phi^{\pm} \\ J_- \left(\frac{m_k^2}{T^2}\right) & k = W_T, Z_T \\ J_+ \left(\frac{m_k^2}{T^2}\right) & k = f \end{cases} \qquad (2.22)$$

with the thermal integrals

$$J_{\pm}\left(\frac{m_k^2}{T^2}\right) = \mp \int_0^{\infty} \mathrm{d}x \, x^2 \log\left[1 \pm e^{-\sqrt{x^2 + m_k^2/T^2}}\right] ,$$

where J_+ (J_-) applies for k being a fermion (boson)

Explore the parameter space

Unitarity Constraints

$$0 < \lambda_1 + \lambda_2 < \frac{32\pi}{3} = 35.51$$

$$\left| M_A{}^2 - \frac{\mathrm{m12}^2}{\mathrm{Sin}[\beta] * \mathrm{Cos}[\beta]} \right| < \frac{16\pi v^2}{3} = 1\,013\,960$$

$$\left| M_{H^+}{}^2 - \frac{\mathrm{m12}^2}{\mathrm{Sin}[\beta] * \mathrm{Cos}[\beta]} \right| < \frac{16\pi v^2}{3} = 1\,013\,960$$

$$M_A, M_{H^+} < 1\,\mathrm{TeV}$$

In[∘]:= λ1 + λ2

$$1.0 * \mathrm{Abs}\left[\mathrm{M_A}^2 - \frac{\mathrm{m12}^2}{\mathrm{Sin}[\beta] * \mathrm{Cos}[\beta]}\right]$$

$$1.0 * \mathrm{Abs}\left[\mathrm{M_{CH}}^2 - \frac{\mathrm{m12}^2}{\mathrm{Sin}[\beta] * \mathrm{Cos}[\beta]}\right]$$

Out[∘]= 7.01261

Out[∘]= 10 000.

Out[∘]= 92 500.

Co-positivity Criteria

$$\lambda_1 \geq 0;$$

$$\lambda_2 \geq 0$$

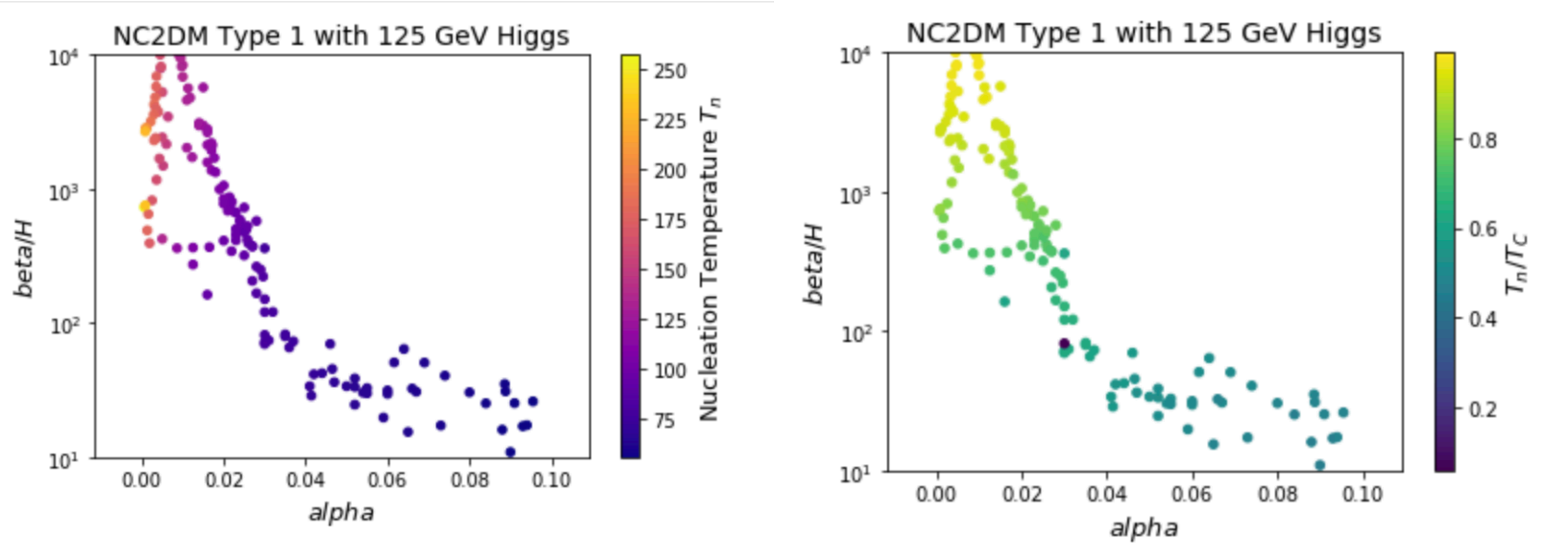
$$\lambda_3 + \sqrt{\lambda_1 \lambda_2} \geq 0$$

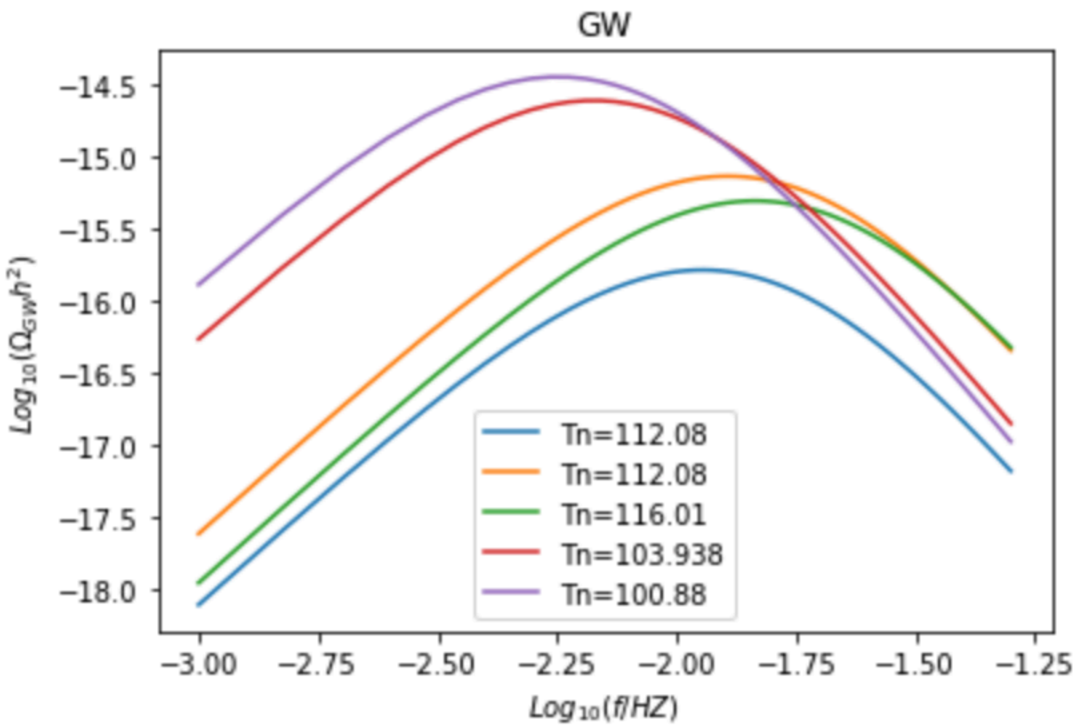
$$\lambda_3 + \lambda_4 - |\lambda_5| + \sqrt{\lambda_1 \lambda_2} \geq 0$$

In[∘]:= $\lambda3 + \sqrt{\lambda1 * \lambda2}$

$\lambda3 + \lambda4 - \mathrm{Abs}[\lambda5] + \sqrt{\lambda1 * \lambda2}$

Out[∘]= 3.8067


NC2DM Type 1 with 125 GeV Higgs
beta/H
alpha
Nucleation Temperature T_n
NC2DM Type 1 with 125 GeV Higgs
beta/H
alpha
T_n/T_C



GW
−14.5
−15.0
−15.5
−16.0
−16.5
−17.0
−17.5
−18.0
$Log_{10}(\Omega_{GW}h^2)$
−3.00
−2.75
−2.50
−2.25
−2.00
−1.75
−1.50
−1.25
$Log_{10}(f/HZ)$
Tn=112.08
Tn=112.08
Tn=116.01
Tn=103.938
Tn=100.88

Conformal 2 Higgs Doublet Modle

Fields and tree level V

```
In[]:= $Assumptions = True;
       $Assumptions = { {ω1, ω2, ω3} ∈ Reals};
       $Assumptions = { {λ1, λ2, λ3, λ4, λ5} ∈ Reals};
```

In[]:= $H_1 = \begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix};$

In[]:= $\begin{pmatrix} \phi_{11} \\ \phi_{12} \end{pmatrix} = \frac{1}{\sqrt{2}} * \begin{pmatrix} \rho_1 + I * \eta_1 \\ \omega 1 + \xi_1 + I * \psi_1 \end{pmatrix};$

```
In[]:= H1dag = ComplexExpand[ConjugateTranspose[H₁]];
```

In[]:= $H_2 = \begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix};$

In[]:= $\begin{pmatrix} \phi_{21} \\ \phi_{22} \end{pmatrix} = (1/(\sqrt{2})) * \begin{pmatrix} \rho_2 + I * \eta_2 \\ \omega 2 + I * \omega 3 + \xi_2 + I * \psi_2 \end{pmatrix};$

```
In[]:= H2dag = ComplexExpand[ConjugateTranspose[H₂]];
```

In[]:= $V_{tree} = \text{Part}\Big[\frac{\lambda 1}{2} * (\text{H1dag}.H_1)^2 + \frac{\lambda 2}{2} * (\text{H2dag}.H_2)^2 + \lambda 3 * (\text{H1dag}.H_1) * (\text{H2dag}.H_2) + \lambda 4 * (\text{H1dag}.H_2) * (\text{H2dag}.H_1) + \left(\frac{\lambda 5}{2} * (\text{H1dag}.H_2)^2 + \frac{\lambda 5}{2} * (\text{H2dag}.H_1)^2\right) \text{ // FullSimplify}, 1, 1\Big]$

Flat Direction

Flat direction

Now, we work on the VEV conditions, which will choose a flat direction for the background field.

In[∘]:= $D[V_{tree}, \{\zeta_1, 1\}] /. \omega 3 \to 0 /. \rho_1 \to 0 /. \eta_1 \to 0 /. \rho_2 \to 0 /. \eta_2 \to 0 /. \zeta_1 \to 0 /. \psi_1 \to 0 /. \zeta_2 \to 0 /. \psi_2 \to 0$ // Simplify

$D[V_{tree}, \{\zeta_2, 1\}] /. \omega 3 \to 0 /. \rho_1 \to 0 /. \eta_1 \to 0 /. \rho_2 \to 0 /. \eta_2 \to 0 /. \zeta_1 \to 0 /. \psi_1 \to 0 /. \zeta_2 \to 0 /. \psi_2 \to 0$ // Simplify

Out[∘]= $\frac{1}{2}\left(\lambda 1\, \omega 1^3 + (\lambda 3 + \lambda 4 + \lambda 5)\, \omega 1\, \omega 2^2\right)$

Out[∘]= $\frac{1}{2}\, \omega 2 \left(\lambda 3\, \omega 1^2 + \lambda 4\, \omega 1^2 + \lambda 5\, \omega 1^2 + \lambda 2\, \omega 2^2\right)$

In[∘]:= $\text{Solve}\left[\frac{1}{2}\left(\lambda 1\, \omega 1^3 + \lambda 345 * \omega 1\, \omega 2^2\right) == 0, \lambda 345\right]$ // FullSimplify

$\text{Solve}\left[\frac{1}{2}\, \omega 2 \left(\lambda 345 * \omega 1^2 + \lambda 2\, \omega 2^2\right) == 0, \lambda 345\right]$ // FullSimplify

Out[∘]= $\left\{\left\{\lambda 345 \to -\frac{\lambda 1\, \omega 1^2}{\omega 2^2}\right\}\right\}$

Out[∘]= $\left\{\left\{\lambda 345 \to -\frac{\lambda 2\, \omega 2^2}{\omega 1^2}\right\}\right\}$

By Multiplying $\lambda 345 \to -\frac{\lambda 1\ \omega 1^2}{\omega 2^2}$ and $\lambda 345 \to -\frac{\lambda 2\ \omega 2^2}{\omega 1^2}$, we can get rid of the $\frac{\omega 2^2}{\omega 1^2}$ and obtain: ${\lambda_{345}}^2 - \lambda 1 * \lambda 2 = 0$

On the other hand we can write these two equations as: $\frac{\omega 2^2}{\omega 1^2} = -\frac{\lambda 1}{\lambda_{345}}$ and $\frac{\omega 2^2}{\omega 1^2} = -\frac{\lambda_{345}}{\lambda 2}$.

These two equations can be further written as $\frac{\omega 2^2}{\omega 1^2} = \frac{\lambda 1 - \lambda_{345}}{\lambda 2 - \lambda_{345}}$ which corresponds to the angle notation $\tan^2\theta = \frac{\lambda 1 - \lambda_{345}}{\lambda 2 - \lambda_{345}}$.

one-loop improved tadpole conditions

tree mass

Fields »

mass »

flatness condition from tree potential

$$T_{\phi 1} = \frac{1}{2}\left(\lambda 1\,\omega 1^3 + \lambda 345 * \omega 1\,\omega 2^2\right) = 0$$
$$T_{\phi 2} = \frac{1}{2}\omega 2\left(\lambda 345 * \omega 1^2 + \lambda 2\,\omega 2^2\right) = 0$$

loop potential »

loop mass

one-loop improved tadpole conditions

previous from tree level potential, we have flatness conditions from tree level potential:

$$\frac{1}{2}\left(\lambda 1\,\omega 1^3 + (\lambda 3 + \lambda 4 + \lambda 5) * \omega 1\,\omega 2^2\right)$$
$$\frac{1}{2}\omega 2\left(\lambda 3\,\omega 1^2 + \lambda 4\,\omega 1^2 + \lambda 5\,\omega 1^2 + \lambda 2\,\omega 2^2\right)$$

we write:

In[∘]:=

```
treeflat1 = 1/2 (λ1 ω1^3 + (λ3 + λ4 + λ5) * ω1 ω2^2)
treeflat2 = 1/2 ω2 (λ3 ω1^2 + λ4 ω1^2 + λ5 ω1^2 + λ2 ω2^2)
```

Out[∘]= $\frac{1}{2}\left(\lambda 1\,\omega 1^3 + (\lambda 3 + \lambda 4 + \lambda 5)\,\omega 1\,\omega 2^2\right)$

Out[∘]= $\frac{1}{2}\omega 2\left(\lambda 3\,\omega 1^2 + \lambda 4\,\omega 1^2 + \lambda 5\,\omega 1^2 + \lambda 2\,\omega 2^2\right)$

we have the following from loop potential:

```
In[∘]:= loopϕ1 = D[V_loop1, {ξ1, 1}] /. ω3 → 0 /. ρ1 → 0 /. η1 → 0 /. ρ2 → 0 /. η2 → 0 /. ξ1 → 0 /. ψ1 → 0 /. ξ2 → 0 /.
    ψ2 → 0 // FullSimplify
loopϕ2 = D[V_loop1, {ξ2, 1}] /. ω3 → 0 /. ρ1 → 0 /. η1 → 0 /. ρ2 → 0 /. η2 → 0 /. ξ1 → 0 /. ψ1 → 0 /. ξ2 → 0 /.
    ψ2 → 0 // FullSimplify
```

Out[∘]=

$$\left\{\left\{\frac{1}{256\pi^2}\,\omega1\,(\omega1^2+\omega2^2)\left(-16\,\lambda5^2+16\,(\lambda4+\lambda5)^2+8\,(\lambda3+\lambda4+\lambda5)^2+16\,\lambda5^2\,\mathrm{Log}\left[-\frac{\lambda5\,(\omega1^2+\omega2^2)}{\mu^2}\right]+16\,(\lambda4+\lambda5)^2\left(-3+2\,\mathrm{Log}\left[-\frac{(\lambda4+\lambda5)\,(\omega1^2+\omega2^2)}{\mu^2}\right]\right)+8\,(\lambda3+\lambda4+\lambda5)^2\left(-3+2\,\mathrm{Log}\left[-\frac{(\lambda3+\lambda4+\lambda5)\,(\omega1^2+\omega2^2)}{\mu^2}\right]\right)-2\,g_l^4+6\,\mathrm{Log}\left[\frac{(\omega1^2+\omega2^2)\,g_l^2}{4\,\mu^2}\right]g_l^4-(g_l^2+g_y^2)^2+3\,\mathrm{Log}\left[\frac{(\omega1^2+\omega2^2)\,(g_l^2+g_y^2)}{4\,\mu^2}\right](g_l^2+g_y^2)^2\right)\right\}\right\}$$

Out[∘]=

$$\left\{\left\{\frac{1}{256\pi^2}\,\omega2\left(16\,(\omega1^2+\omega2^2)\left(-\lambda3^2-2\,\lambda3\,\lambda4-3\,\lambda4^2-2\,(\lambda3+3\,\lambda4)\,\lambda5-4\,\lambda5^2+\lambda5^2\,\mathrm{Log}\left[-\frac{\lambda5\,(\omega1^2+\omega2^2)}{\mu^2}\right]+2\,(\lambda4+\lambda5)^2\,\mathrm{Log}\left[-\frac{(\lambda4+\lambda5)\,(\omega1^2+\omega2^2)}{\mu^2}\right]+(\lambda3+\lambda4+\lambda5)^2\,\mathrm{Log}\left[-\frac{(\lambda3+\lambda4+\lambda5)\,(\omega1^2+\omega2^2)}{\mu^2}\right]\right)+3\,(\omega1^2+\omega2^2)\left(-1+2\,\mathrm{Log}\left[\frac{(\omega1^2+\omega2^2)\,g_l^2}{\mu^2}\right]+\mathrm{Log}\left[\frac{(\omega1^2+\omega2^2)\,(g_l^2+g_y^2)}{64\,\mu^2}\right]\right)g_l^4+2\,(\omega1^2+\omega2^2)\left(-1+3\,\mathrm{Log}\left[\frac{(\omega1^2+\omega2^2)\,(g_l^2+g_y^2)}{4\,\mu^2}\right]\right)g_l^2\,g_y^2+(\omega1^2+\omega2^2)\left(-1+3\,\mathrm{Log}\left[\frac{(\omega1^2+\omega2^2)\,(g_l^2+g_y^2)}{4\,\mu^2}\right]\right)g_y^4+24\,\omega2^2\left(\left(2+\mathrm{Log}[4]-2\,\mathrm{Log}\left[\frac{\omega2^2\,y_{b1}^2}{\mu^2}\right]\right)y_{b1}^4+\left(2+\mathrm{Log}[4]-2\,\mathrm{Log}\left[\frac{\omega2^2\,y_t^2}{\mu^2}\right]\right)y_t^4\right)\right)\right\}\right\}$$

our one-loop improved tadpole conditions now read:

$$\frac{\text{treeflat1}}{\omega1}+\frac{\text{loop}\phi1}{\omega1}=0\ \&\ \frac{\text{treeflat2}}{\omega2}+\frac{\text{loop}\phi2}{\omega2}=0$$

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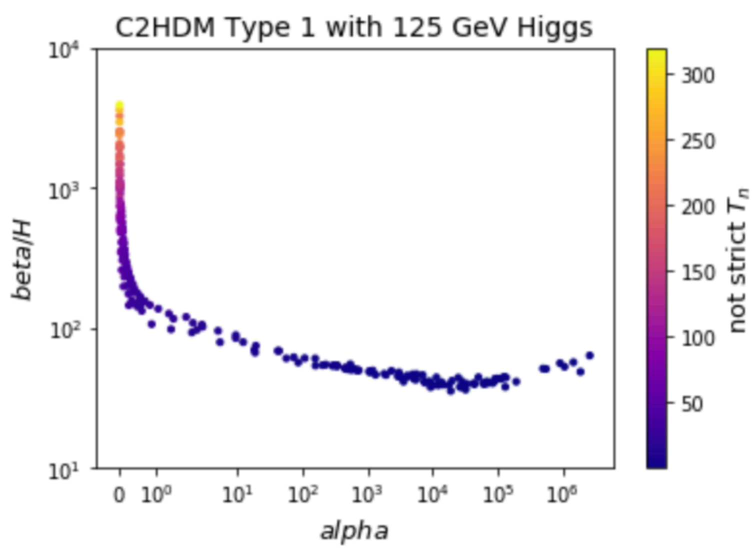
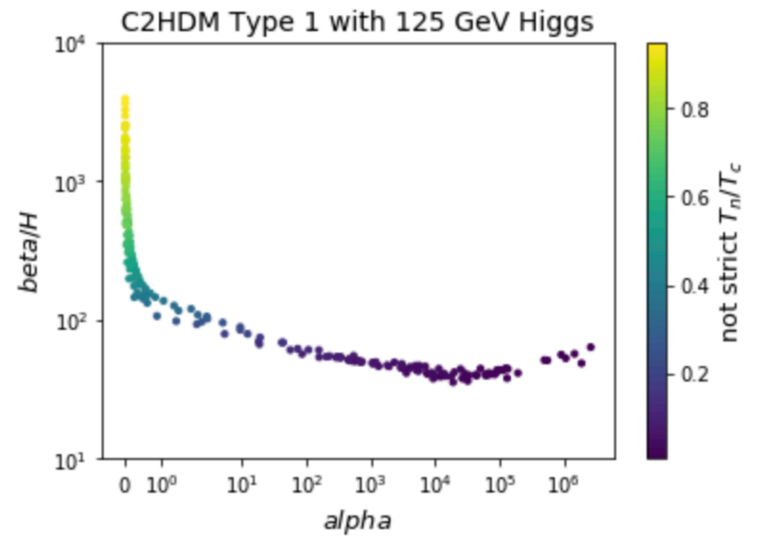
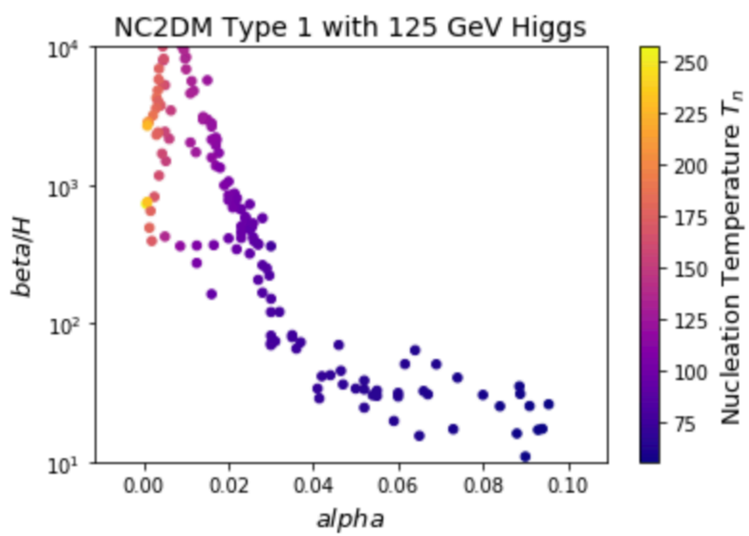
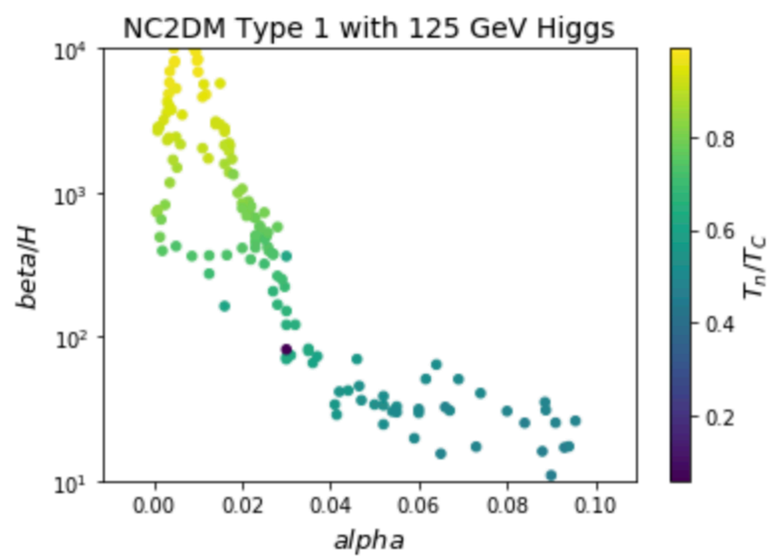
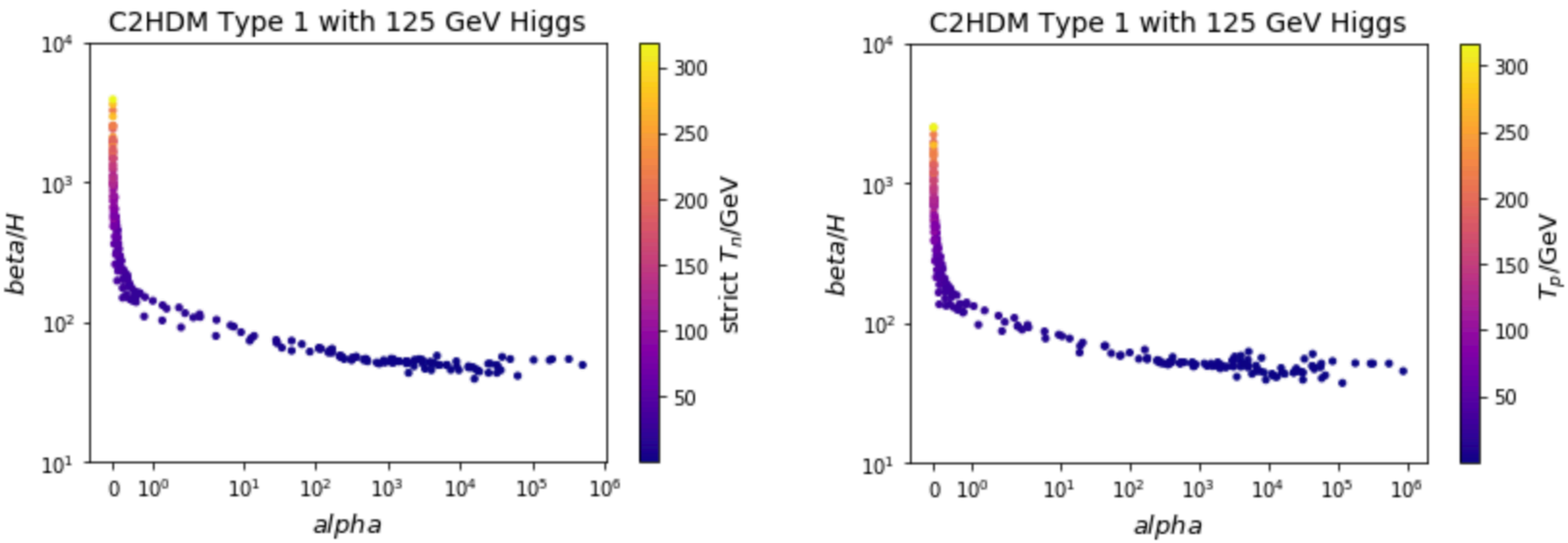

C2HDM Type 1 with 125 GeV Higgs
beta/H
alpha
not strict T_n
300
250
200
150
100
50


C2HDM Type 1 with 125 GeV Higgs
beta/H
alpha
not strict T_n/T_c
0.8
0.6
0.4
0.2


NC2DM Type 1 with 125 GeV Higgs
beta/H
alpha
Nucleation Temperature T_n
250
225
200
175
150
125
100
75


NC2DM Type 1 with 125 GeV Higgs
beta/H
alpha
T_n/T_C
0.8
0.6
0.4
0.2

Consider supercooling for conformal 2HDM with strictly calculated Tn



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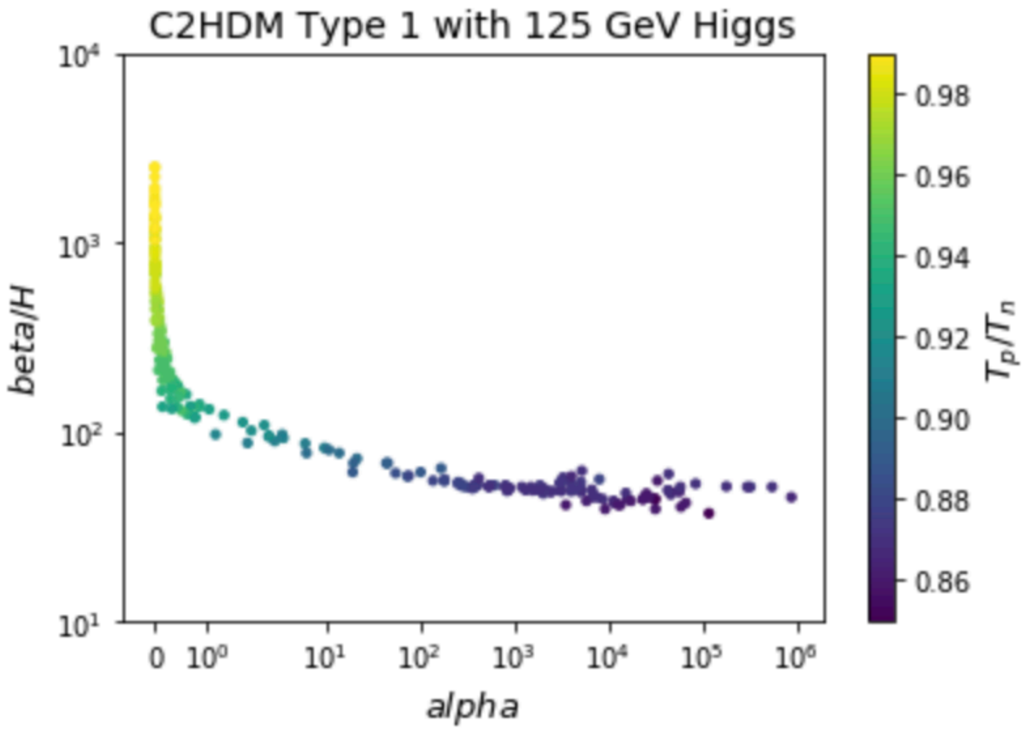

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Strict calculation of Tn is required for large alpha

The comparison of old Tn and correct Tn for some points (with large alpha values)

CH	H	A	I1	I2	I3	I4	I5	Tc/ GeV	Xi	Tn/ GeV	Tn'/ GeV	Tp/ GeV	Tp/ Tn	Tp/ Tn'	Tn/ Tc
268	268	200	.9620	.2671	.6497	1.7127	.6609	22. 46	6.35	0.507	0.715	0.62	1.22	0.87	0.023
269	269	200	.9978	.2654	.6632	1.7304	.6609	23.6	6.3	0.624	0.857	0.742	1.19	0.86	0.026
270	270	200	.0341	.26377	.6767	1.748	.6609	24. 72	6.2	0.736	1.06	0.909	1.22	0.86	0.03
271	271	200	.0709	.26211	.6903	1.766	.6609	24. 716	6.17	0.768	1.26	1.093	1.4	0.87	0.031
272	272	200	.1081	.26048	.7040	1.784	.6609	26. 86	6	1.092	1.47	1.284	1.16	0.873	0.04

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CH	H	A	I1	I2	I3	I4	I5	Tc/GeV	Xi	Tn/GeV	Tn/Tc	alpha	beta	Tp	Tp/Tn	Tp/Tc	Alpha’	Beta’
200	200	300	45678	46363	86176	16524	.4872											
220	220	300	.6971	.4014	.0705	0.1123	.4872	18.82	6.5	0.567	0.03	33042	48	0.49	0.87	0.026	58552	51
221	221	300	0.7252	39114	.0815	0.1269	.4872	19.53	6.44	0.63	0.032	38309	56	0.548	0.87	0.028	43952	59.7
222	222	300	75216	38223	.0926	0.1415	.4872	20.25	6.4	0.7	0.035	18542	44	0.61	0.86	0.03	33483	55
223	223	300	77838	37436	.1036	0.1562	.4872	20.97	6.34	0.795	0.038	13107	47	0.687	0.86	0.033	23527	44
224	224	300	80408	36729	.1148	0.1710	.4872	21.68	6.3	0.888	0.04	9621	45	0.764	0.86	0.035	17513	43

CH	H	A	I1	I2	I3	I4	I5	Tc/GeV	Xi	Tn/GeV	Tn/Tc	alpha	beta	Tp	Tp/Tn	Tp/Tc	Alpha’	Beta’
300	300	300	.6054	20462	.1154	.4872	.4872	67.59	3.7	33.73	0.5	0.27	201.6	31.85	0.94	0.47	0.33	171
310	310	300	.2659	19218	.2705	.6888	.4872	74.15	3.38	41.56	0.56	0.16	249	39.35	0.95	0.53	0.19	209
320	320	300	.0573	17956	.4312	.8970	.4872	81.9	3.06	50.76	0.62	0.094	314	48.35	0.96	0.59	0.1	269
330	330	300	.0205	16653	.5977	.1118	.4872	91.66	2.73	62.66	0.68	0.052	407	60	0.96	0.65	0.059	328
340	340	300	.2161	15293	.7699	.3332	.4872	107.2	2.36	80	0.75	0.024	571	77.7	0.97	0.73	0.026	389
343	343	300	.6329	14871	.8227	.4009	.4872	114.9	2.2	88.44	0.77	0.02	639	85.5	0.97	0.74	0.021	484
347	347	300	.2395	14297	.8940	2.492	.4872	129.5	1.9	103.2	0.8	0.012	794	100	0.98	0.77	0.014	584
350	350	300	.7378	13858	.9480	.5613	.4872	146	1.635	120.3	0.824	0.0062	996	117	0.98	0.8	0.007	684
351	351	300	.9130	13710	.9662	.5844	.4872	153.2	1.58	127	0.83	0.0049	947	124.8	0.98	0.82	0.0052	731

Conclusions:

1. The difference made by supercooling is important, especially for extremely large alpha values
2. Generally, for alpha values inside [0.0001, 1], supercooling has quite small contributions which can be neglected.
3. Supercooling makes alpha bigger and beta smaller in general (for all points), again, for extremely large alpha values, the supercooling is very strong since we have very small Tn/Tc for these cases, hence the difference can be big which means it is indeed necessary to consider supercooling in our calculation and do this comparison.
4. An interesting thing we need to be careful about is the calculation of Tn, the community usually uses S(Tn) = 140 (or 150, 160) as an approximation numerically, but our calculations show that it holds only inside the range [0.0001,1]. The approximation can be terrible for big alpha values, calculations directly from the definition of Tn is required.

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Reference

- [1] Wang, Xiao, Fa Peng Huang, and Xinmin Zhang. "Phase transition dynamics and gravitational wave spectra of strong first-order phase transition in supercooled universe." *Journal of Cosmology and Astroparticle Physics* 2020.05 (2020): 045.
- [2] Huang, Wei-Chih, Francesco Sannino, and Zhi-Wei Wang. "Gravitational waves from Pati-Salam dynamics." *Physical Review D* 102.9 (2020): 095025.
- [3] Basler, Philipp, et al. "Strong first order electroweak phase transition in the CP-conserving 2HDM revisited." *Journal of High Energy Physics* 2017.2 (2017): 1-38.
- [4] Zhang, Zhao, et al. "Phase transition gravitational waves from pseudo-Nambu-Goldstone dark matter and two Higgs doublets." *Journal of High Energy Physics* 2021.5 (2021): 1-31.[3] F. Sannino and J. Virkaja¨rvi, Phys. Rev. D92, 045015 (2015), arXiv:1505.05872 [hep-ph].

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THANKS!