Bayesian Analysis in Parity Violating Electron Scattering Experiments

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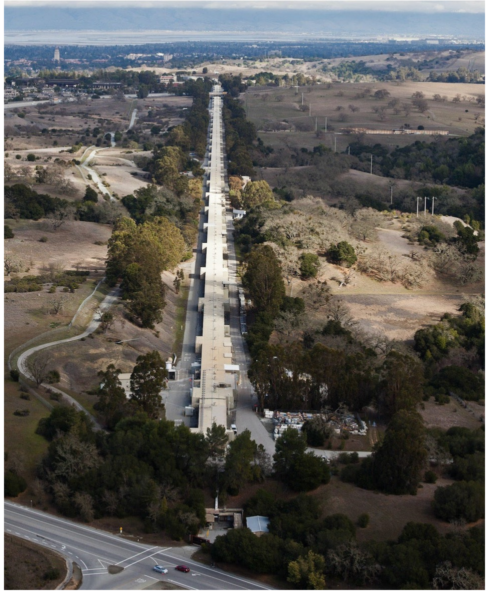
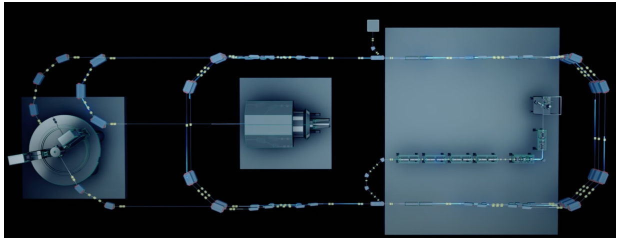
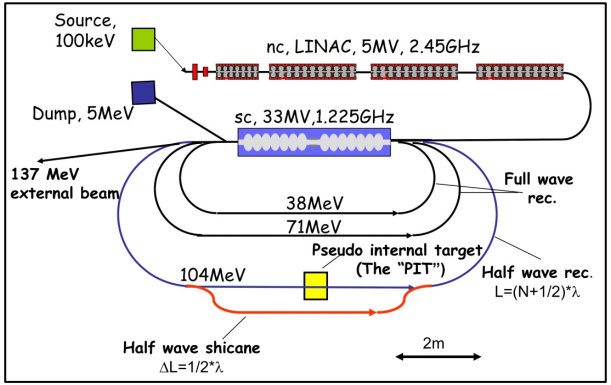
Parity Violating Electron Scattering (PVES) Experiment



https://www.cnet.com/pictures/slac-a-2-mile-particle-accelerator-next-to-stanford/





MESA accelerator layout
https://www.mesa.uni-mainz.de/eng/

https://bateslab.mit.edu

https://en.wikipedia.org/wiki/Thomas_Jefferson_National_Accelerator_Facility

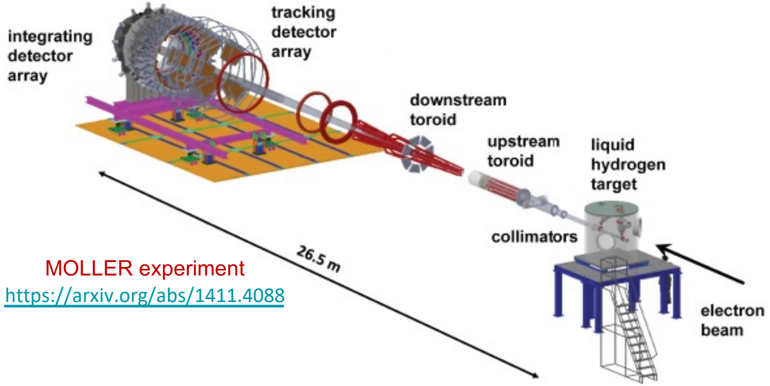
Parity Violating Asymmetry

$$A_{PV} = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}$$

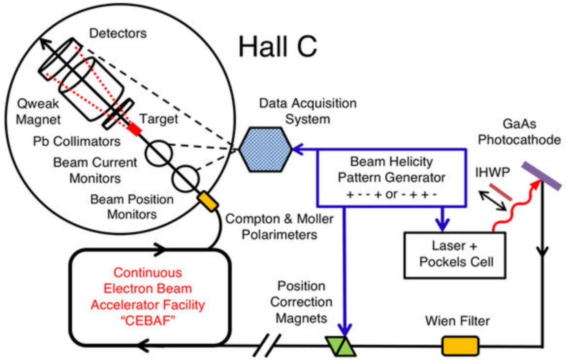
σ_R and σ_L are the probability cross-sections for incident right and left-handed electrons.

A_{PV} arises from the interference of the weak and electromagnetic amplitudes

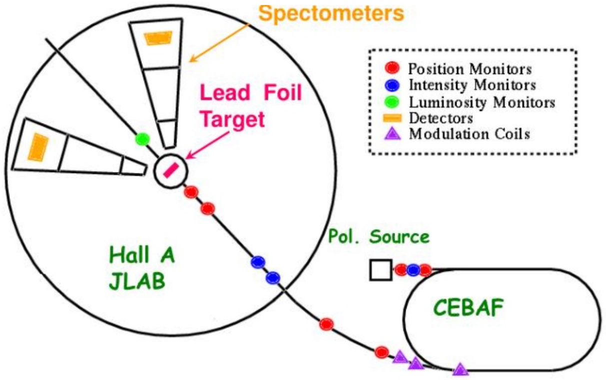
Values of A_{PV} in the range from 10^{-4} to 10^{-8} can be measured with good accuracy



MOLLER experiment
https://arxiv.org/abs/1411.4088

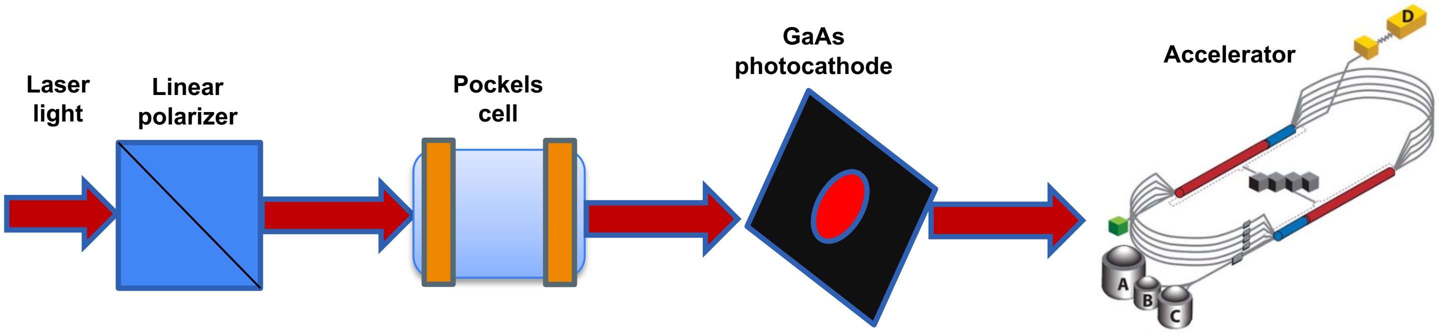


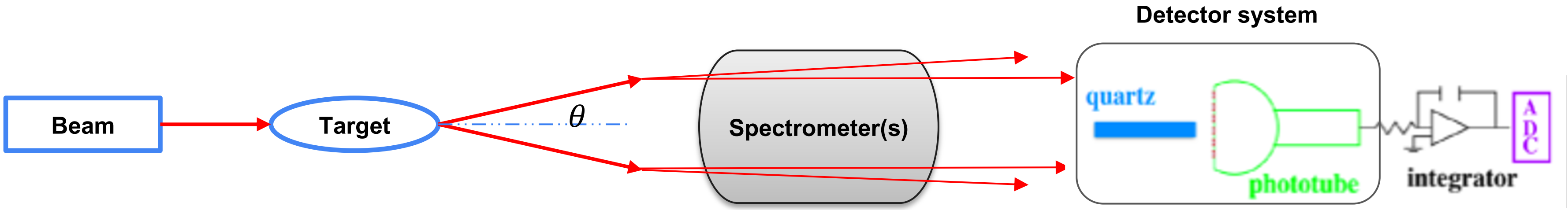
Qweak experiment
https://arxiv.org/abs/1202.1255



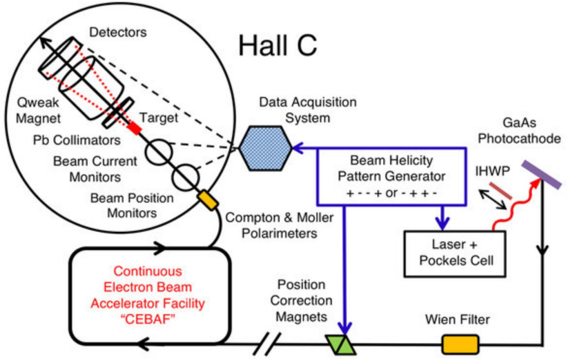
^{208}Pb Radius Experiment (PREx)
https://arxiv.org/abs/1209.3179

Measurements of Parity Violating Asymmetry





Measurements of Parity Violating Asymmetry



Qweak experiment
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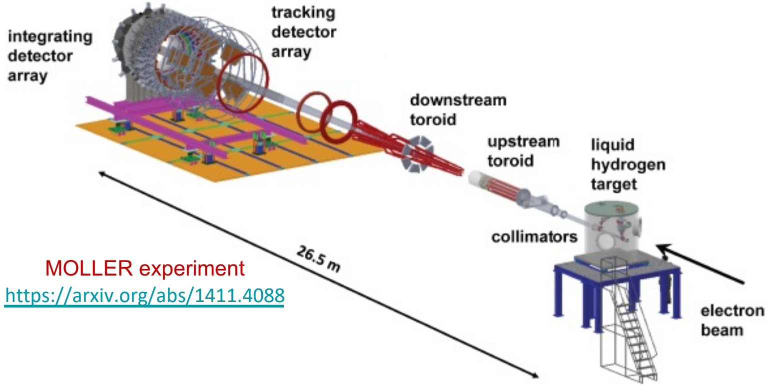

MOLLER experiment
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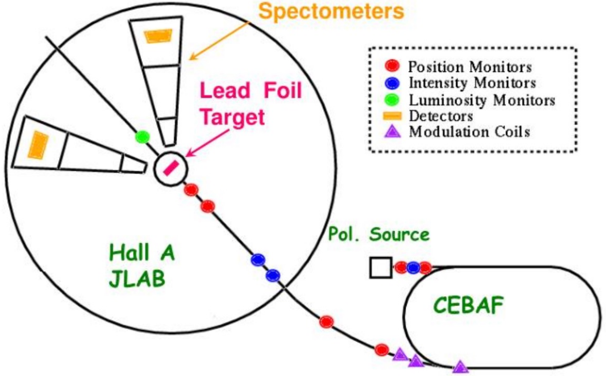
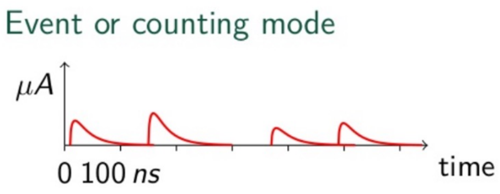


^{208}Pb Radius Experiment (PREx)
https://arxiv.org/abs/1209.3179

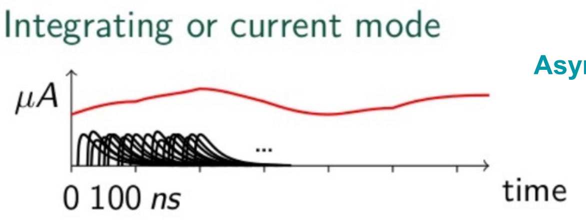
Data acquisition in PVES Experiment

✓ Event data taking mode:



Background measurements (f_i^{bkgd})

✓ Integrating data taking mode :



Asymmetry measurements (A_{expt}, A_i^{bkgd})

$$A_{PV} = R_{tot} \frac{\frac{A_{expt}}{P_b} - \sum_i f_i^{bkgd} A_i^{bkgd}}{1 - \sum_i f_i^{bkgd}}$$

R_{tot}: overall normalization factor,

P_b: beam polarization

A_{expt}: experimentally measured asymmetry

f_i^{bkgd}: fractional dilution factors

A_i^{bkgd}: asymmetries

i : number of the detectors

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Qweak experiment and signal correction

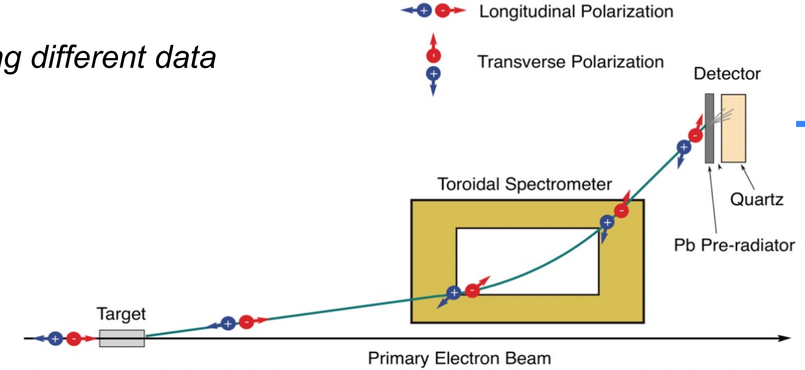
$$A^{ij}_{calc} = (1 - f^{i}_{NB})[(1 - f^{i}_{\pi})(A^{L}_{e}cos\theta^{j}_{P} + A^{T}_{e}cos\theta^{j}_{P}sin\phi^{i}) + f^{i}_{\pi}(A^{L}_{\pi}cos\theta^{j}_{P} + A^{T}_{\pi}cos\theta^{j}_{P}sin\phi^{i})]$$

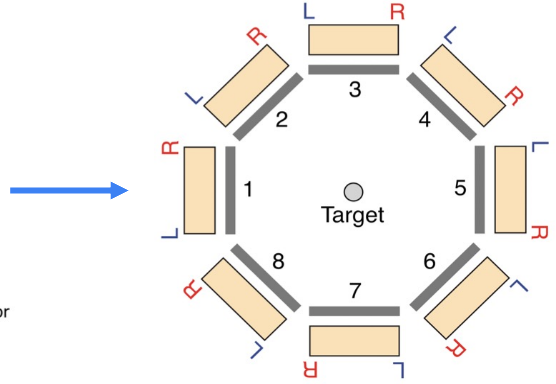
i: detector index

j: data set index (2 data sets)

ϕ*:* azimuthal angle:

θ_P*: electron-spin angle during different data taking modes*





Bayesian analysis properties:

- ✓ Using probability statements
- ✓ Treating the parameters in a statistical model as random
- ✓ Using a prior distribution to quantify our knowledge about the parameter
- ✓ Using the conditional distribution of parameters, given the data to update our prior knowledge
- ✓ Update from the prior to the posterior via the Bayes theorem

Bayes' Law: $p(A \mid B) = \dfrac{p(B \mid A) * p(A)}{p(B)}$

Bayesian analysis of experimental data

Bayes rule: $\mathrm{p}(\boldsymbol{a}|\boldsymbol{d},I) = \frac{p(\boldsymbol{d}|\boldsymbol{a},I)\,p(\boldsymbol{a}|I)}{p(\boldsymbol{d}|I)}$

- ✓ We want to infer the parameters $\boldsymbol{a}$ of a model M, based on data $\boldsymbol{d}$
- ✓ Use Bayes rule, which gives the *posterior*: $p(\boldsymbol{a}|\boldsymbol{d},M,I) \propto p(\boldsymbol{d}|\boldsymbol{a},M,I)\ p(\boldsymbol{a}|M,I)$.

I represents general information

$p(\boldsymbol{d} \mid \boldsymbol{a}, M, I)$ is the *likelihood*, the probability of the observed data, given the parameters, model, and general info

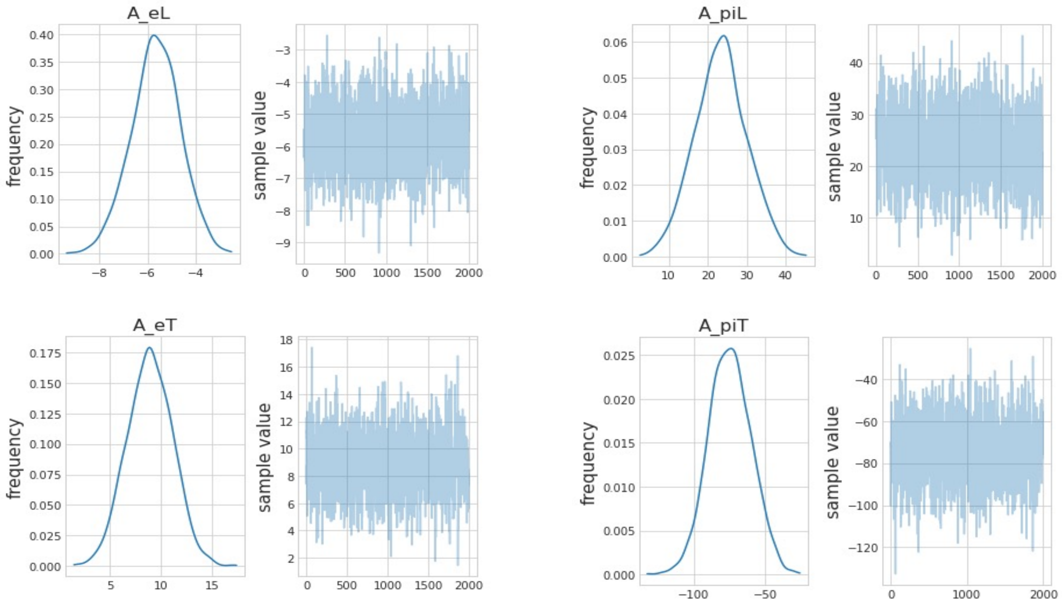
$p(\boldsymbol{a} \mid M, I)$ is the *prior*, which represents what we know about the parameters exclusive of the data

Generating data based on model

- ✓ **Model:**

$$A^{ij}_{calc} = (1 - f^{i}_{NB})[(1 - f^{i}_{\pi})(A^{L}_{e} cos\theta^{j}_{P} + A^{T}_{e} cos\theta^{j}_{P} sin\phi^{i}) + f^{i}_{\pi}(A^{L}_{\pi} cos\theta^{j}_{P} + A^{T}_{\pi} cos\theta^{j}_{P} sin\phi^{i})]$$

- ✓ **Component asymmetry distributions**



Comparison of results

	Value	Uncertainty
A_e^L	-5.25	1.49
A_e^T	12.3	3.6
A_π^L	25.4	9.0
A_π^T	-60.1	19.3

Results from Qweak experiment

	Value	Uncertainty
A_e^L	-5.65	1.01
A_e^T	8.92	2.23
A_π^L	23.42	6.69
A_π^T	-74.56	14.84

Example of an extraction with Bayesian method

Thank you