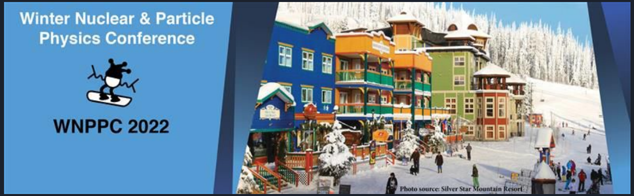
Numerical Description of High Energy Electron Propagating in a Coulomb Field

Syed Navid Reza

University of Alberta

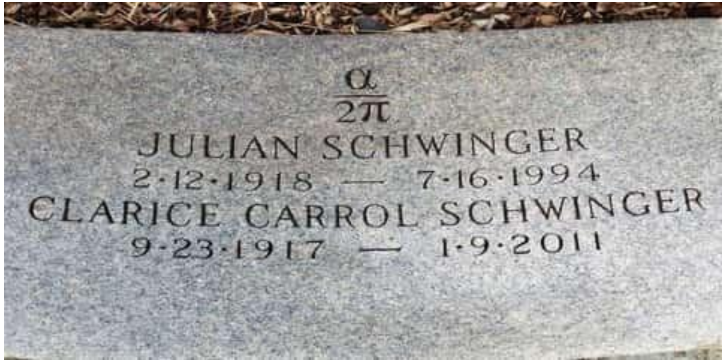
Supervisor: Dr. Andrzej Czarnecki



Anomalous Magnetic Moment of Electron

- Schrodinger's Equation predicts g=0
- From Dirac Equation coupled to an electromagnetic field we get g=2
- Schwinger first calculated the 1-loop correction and got

$$\frac{g-2}{2} = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$$



Anomalous Magnetic Moment of Electron(Cont.)

- R.Karplus and N.M. Kroll (1950) calculated the 2nd loop effect and Peterman (1997) corrected the estimation

$$\frac{g-2}{2} \approx \frac{\alpha}{2\pi} + \left(\frac{197}{144\pi^2} + \frac{1}{12} - \frac{\log 2}{2} + \frac{3\zeta(3)}{4\pi^2}\right)\alpha^2 + \mathcal{O}(\alpha^3)$$

- S.Laporta and E.Remiddi in 1996 found a closed expression for the next order
- T.Aoyma, M.Hayakawa, T.Kinoshita and M.Nio calculated the next two orders using computers. The fifth order contains 12672 diagrams
- Using numerical methodologies the g-2 has been calculated upto 10th order

Schwinger’s Motivation

Extraction of gauge invariant results from a gauge invariant theory by applying methods of solution involving only gauge covariant quantities without reference to coordinate system of the gauge.

- It is illustrated in the problem of vacuum polarization by a prescribed field (gauge covariant quantities)
- The vacuum current of a charged Dirac field is expressed in terms of Green’s Function
- The derived equation of motion depends only on the proper-time parameter using the EM field strength as the gauge invariant basis

So, he is looking for a equation of motions involving only the proper-time parameter and EM field strength serving as the gauge invariant basis

Schwinger's Formalism

The Dirac equation in external electromagnetic field

$$\gamma^\mu \left(-i\partial_\mu - eA_\mu(x)\right)\psi(x) + m\psi(x) = 0$$

The Green's Function and the charged current is

$$G = \frac{1}{\gamma\Pi + m} = (-\gamma\Pi + m)\left[m^2 - (\gamma\Pi)^2\right]^{-1}$$

$$\langle j_\mu(x)\rangle = ie\mathrm{tr}\gamma_\mu G(x,x)$$

The action integral then written by varying the EM field

$$\delta W^{(1)} = \int (dx)(\delta A_\mu(x))\langle j_\mu(x)\rangle = ie\mathrm{Tr}(\delta A)G$$

Schwinger's Formalism(Cont.)

The main trick to Schwinger's calculation is the the Schwinger's parameterization

$$G = (-\gamma\Pi + m)i\int_0^\infty ds \exp\left[-i(m^2 - (\gamma\Pi)^2)s\right]$$
$$= i\int_0^\infty ds \exp\left[-i(m^2 - (\gamma\Pi)^2)s\right](-\gamma\Pi + m)$$

$$\frac{1}{A} = i\int_0^\infty dse^{-isA}$$

Then the action integral and the effective lagrangian becomes

$$\delta W^{(1)} = \delta\left(\int (dx)\mathcal{L}^{(1)}(x)\right)$$

$$\mathcal{L}^{(1)}(x) = \frac{1}{2}i\int_0^\infty \frac{ds}{s}\exp(-im^2 s)\mathrm{tr}\langle x|U(s)|x\rangle$$

$$U(s) = \exp(-i\mathcal{H}s)$$

$$\mathcal{H} = -(\gamma\Pi)^2 = \Pi^2 - \tfrac{1}{2}e\sigma_{\mu\nu}F_{\mu\nu}$$

Schwinger's Formalism(Cont.)

The Lagrangian evolves as

$$\langle x'|U(s)|x\rangle = \langle x(s)'|x(0)''\rangle$$

The transformation function only depends on the proper-time parameter. This dynamical problem in the Heisenberg picture

$$\frac{dx_\mu}{ds} = 2\Pi_\mu$$
$$\frac{d\Pi_\mu}{ds} = 2eF_{\mu\nu}\Pi_\nu - ie\partial_\nu F_{\mu\nu} + \frac{e}{2}\sigma_{\lambda\nu}\partial_\mu F_{\lambda\nu}$$

The transformation function is characterized by the differential equations

$$i\partial_s\langle x(s)'|x(0)''\rangle = \langle x(s)'|\mathcal{H}|x(0)''\rangle$$
$$(-i\partial'_\mu - eA_\mu(x'))\langle x(s)'|x(0)''\rangle = \langle x(s)'|\Pi_\mu(s)|x(0)''\rangle$$
$$(i\partial''_\mu - eA_\mu(x''))\langle x(s)'|x(0)''\rangle = \langle x(s)'|\Pi_\mu(0)|x(0)''\rangle$$

Constant Field

$$\frac{dx}{ds} = 2\Pi$$
$$\frac{d\Pi}{ds} = 2eF\Pi$$

$$i\partial_s \langle x(s)'|x(0)''\rangle = \left[-\frac{1}{2}e\sigma F + (x'-x'')K(x'-x'') - \frac{1}{2}i\mathrm{tr}eF\coth(eFs)\right]\langle x(s)'|x(0)''\rangle$$

$$\langle x(s)'|\Pi(s)|x(0)''\rangle = \frac{1}{2}\left[eF\coth(eFs) + eF\right](x'-x'')\langle x(s)'|x(0)''\rangle$$
$$\langle x(s)'|\Pi(0)|x(0)''\rangle = \frac{1}{2}\left[eF\coth(eFs) - eF\right](x'-x'')\langle x(s)'|x(0)''\rangle$$

$$\left[-i\partial'_\mu - eA_\mu(x') - \frac{1}{2}eF_{\mu\nu}(x'-x'')_\nu\right]C(x',x'') = 0$$
$$\left[i\partial''_\mu - eA_\mu(x'') - \frac{1}{2}eF_{\mu\nu}(x'-x'')_\nu\right]C(x',x'') = 0$$

$$K = \tfrac{1}{2}e^2F^2\sinh^{-2}(eFs)$$
$$C(x',x'') = C\Phi(x',x'')$$
$$C = -i(4\pi)^2$$
$$\Phi(x',x'') = \exp\left[ie\int_{x''}^{x'} dx A(x)\right]$$

Anomalous Magnetic Moment of Electron(Cont.)

The modified Dirac equation for an electron interacting with its own radiation field

$$\gamma_\mu(-i\partial_\mu - eA_\mu(x))\psi(x) + \int (dx')M(x,x')\psi(x') = 0$$

The mass operator

$$M(x,x') = m_0\delta(x-x') + ie^2\gamma_\mu G(x,x')\gamma_\mu D_+(x-x')$$

The analogous equation of motion in weak field approximation

$$\langle x(s)|x(0)'\rangle \simeq -i(4\pi)^{-2}\Phi(x,x')s^{-2}\exp\left[i\frac{1}{4}(x-x')^2/s\right]\exp(i\frac{1}{2}e\sigma Fs)$$

Anomalous Magnetic Moment of Electron(Cont.)

$$\left[\gamma(-i\partial - eA) + m - \mu' \frac{1}{2}\sigma F\right]\psi = 0$$

$$m = m_0 + \frac{\alpha}{2\pi} m \int_0^\infty ds s^{-1} \int_0^1 du(1+u)\exp(-m^2 us)$$

$$\mu' = \frac{\alpha}{2\pi} em \int_0^\infty ds \int_0^1 duu(1-u)\exp(-m^2 us)$$

$$\mu' = \frac{\alpha}{2\pi} em \frac{1}{2m^2}$$

$$= \frac{\alpha}{2\pi}\left(\frac{e}{2m}\right)$$

Why Schwinger?

- The system only propagates using the proper-time
- It is efficient to obtain the effective Euler-Heisenberg lagrangian
- The formalism is itself gauge invariant
- In Schwinger’s own word his formalism is more focused on fields than particles
- Most of all his formalism is more intuitive as it is based on differential viewpoint
- Removing divergences is comparatively easier