Theoretical overview of neutrino oscillation

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Three-Neutrino Mixing Paradigm

$$\boxed{\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k} \nu_{kL}} \qquad \alpha = e, \mu, \tau$$

$$P_{\nu_\alpha \to \nu_\beta}(L,E) = \delta_{\alpha\beta} \underbrace{- 4 \sum_{k>j} \text{Re}\left[U^*_{\alpha k} \, U_{\beta k} \, U_{\alpha j} \, U^*_{\beta j}\right] \sin^2\left(\frac{\Delta m^2_{kj} L}{4E}\right)}_{\text{CP conserving}}$$

$$\underbrace{+ 2 \sum_{k>j} \text{Im}\left[U^*_{\alpha k} \, U_{\beta k} \, U_{\alpha j} \, U^*_{\beta j}\right] \sin\left(\frac{\Delta m^2_{kj} L}{2E}\right)}_{\text{CP violating}}$$

- Squared-mass differences: $\Delta m^2_{kj} = m^2_k - m^2_j$
- Mixing: $U^*_{\alpha k} \, U_{\beta k} \, U_{\alpha j} \, U^*_{\beta j}$ quartic rephasing invariants
- Jarlskog invariant: $J_{\text{CP}} = \text{Im}\left[U^*_{\alpha k} \, U_{\beta k} \, U_{\alpha j} \, U^*_{\beta j}\right]$

Standard Parameterization of Mixing Matrix

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

ATM
Acc LBL $\nu_\mu \to \nu_\mu$

Rea LBL $\bar{\nu}_e \to \bar{\nu}_e$
Acc LBL $\nu_\mu \to \nu_e$

SOL
KamLAND

$\beta\beta_{0\nu}$

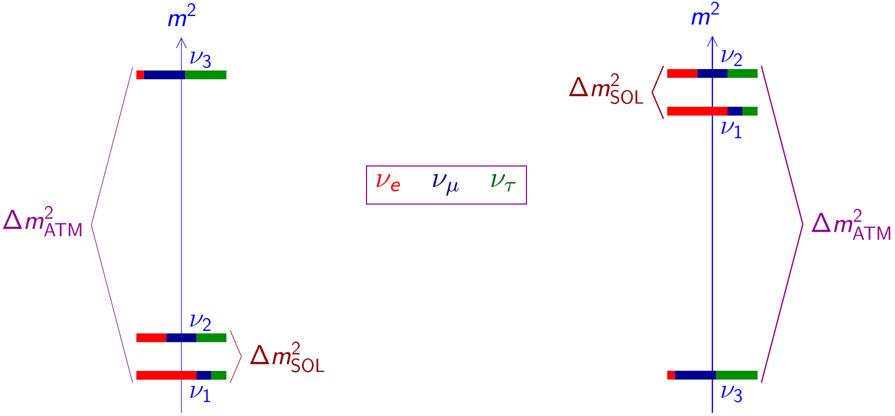
$$= \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

$c_{ab} \equiv \cos\vartheta_{ab}$ $s_{ab} \equiv \sin\vartheta_{ab}$ $0 \leq \vartheta_{ab} \leq \frac{\pi}{2}$ $0 \leq \delta_{13}, \lambda_{21}, \lambda_{31} < 2\pi$

OSCILLATION PARAMETERS:
- 3 Mixing Angles: ϑ_{12}, ϑ_{23}, ϑ_{13}
- 1 CPV Dirac Phase: δ_{13}
- 2 independent Δm^2_{kj}: Δm^2_{21}, Δm^2_{31}

2 CPV Majorana Phases: λ_{21}, λ_{31} $\Longleftrightarrow$ $|\Delta L| = 2$ processes $(\beta\beta_{0\nu})$

Mass Ordering



Normal Ordering

$\Delta m^2_{31} > \Delta m^2_{32} > 0$

Inverted Ordering

$\Delta m^2_{32} < \Delta m^2_{31} < 0$

- Absolute mass scale is not determined by neutrino oscillations
- β decay $\Longrightarrow$ $\boxed{m_\nu < 2.1\,\text{eV}\,(95\%\ \text{CL})}$ Cosmology $\Longrightarrow$ $\boxed{m_\nu \lesssim 0.5\,\text{eV}}$

Global Neutrino Oscillation Fits

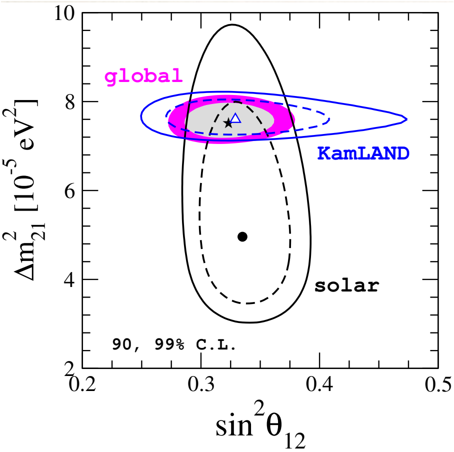
Bari: Capozzi, Lisi, Marrone, Palazzo, PPNP 102 (2018) 48;
Lisi @ NuInt 18, 15 October

NuFit: Esteban, Gonzalez-Garcia, Hernandez-Cabezudo, Maltoni,
Martinez-Soler, Schwetz, `http://www.nu-fit.org`;
NuFit 4.0: Gonzalez-Garcia @ NuTown 2018, 23 October

Valencia: de Salas, Forero, Ternes, Tortola, Valle, PLB 782 (2018) 633;
`http://globalfit.astroparticles.es`;
Tortola @ Neutrino 2018, 5 June

(Alphabetic Order)

The Solar Sector



[M. Tortola @ Neutrino 2018]

- Solar – KamLAND Δm^2_{21} tension ($\approx 1.5\sigma$)
- The KamLAND Δm^2_{21} is in tension with the absence of a SK+SNO low-energy spectrum up-turn

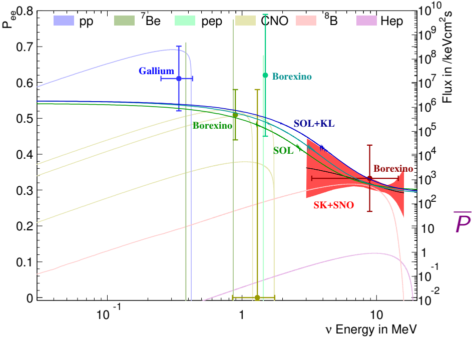
Solar Neutrino Spectrum

$$\overline{P}_{ee}^{\text{SOL}} = \sum_{k=1}^{3} |U_{ek}|^2 |U_{ek}^0|^2 = \left(\frac{1}{2} + \frac{1}{2} \cos 2\vartheta_{12}^0 \, \cos 2\vartheta_{12} \right) \cos^4\vartheta_{13} + \sin^4\vartheta_{13}$$

Averaged Vacuum Oscillations

$$\theta_{12}^0 \simeq \theta_{12}$$

$$\overline{P}_{ee}^{\text{SOL}} \simeq \left(1 - \frac{1}{2} \sin^2\vartheta_{12} \right) \times \left(1 - \sin^2\vartheta_{13} \right)$$



[SK, PRD 94 (2016) 052010, arXiv:1606.07538]

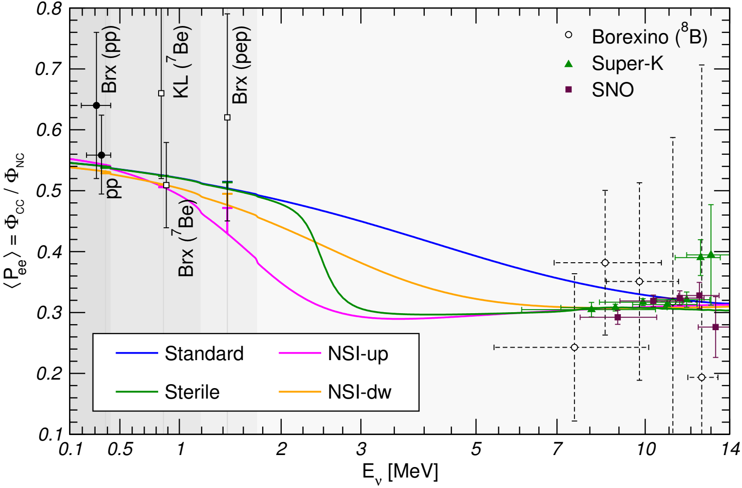
Adiabatic MSW Transitions

$$\theta_{12}^0 \simeq \pi/2$$

$$\overline{P}_{ee}^{\text{SOL}} \simeq \sin^2\vartheta_{12} \times \left(1 - \sin^2\vartheta_{13} \right)$$

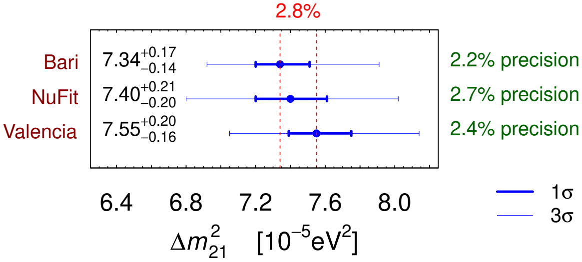
$$\tan 2\vartheta_{12}^0 = \frac{\tan 2\vartheta_{12}}{1 - \dfrac{2EV_{\text{CC}}^0}{\Delta m_{21}^2 \cos\vartheta_{12}}}$$

- Small (solar) $\Delta m_{21}^2 \Longrightarrow$ Low-E transition
- Large (KamLAND) $\Delta m_{21}^2 \Longrightarrow$ High-E transition
- Non-Standard Interactions (NSI)?
- Very light sterile neutrinos?

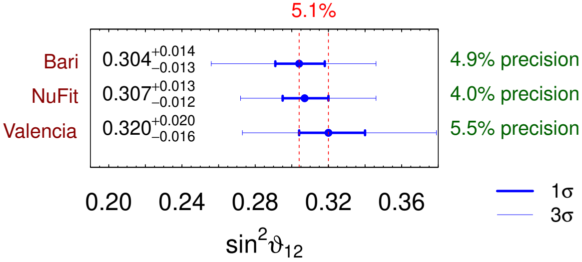


[Maltoni, Smirnov, EPJA 52 (2016) 87, arXiv:1507.05287]

Towards Precision Neutrino Physics?



Nominal Precision + 2.8% Systematic Uncertainty !



Nominal Precision + 5.1% Systematic Uncertainty !

LBL $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$

$$\Delta = \frac{\Delta m^2_{31} L}{4E} \qquad A = \frac{2EV}{\Delta m^2_{31}} \qquad V = \sqrt{2} G_F N_e$$

$$\sin\theta_{13} \ll 1 \qquad\qquad \Delta m^2_{21}/\Delta m^2_{31} \ll 1$$

$$\begin{aligned} P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq {} & \sin^2 \overset{\overset{\vartheta_{13}}{\downarrow}}{2\vartheta_{13}} \sin^2 \overset{\overset{\vartheta_{23}\ \text{octant}}{\downarrow}}{\vartheta_{23}} \frac{\sin^2[(1-A)\Delta]}{(1-A)^2} \\ & + \frac{\Delta m^2_{21}}{\Delta m^2_{31}} \sin 2\vartheta_{13} \sin 2\vartheta_{12} \sin 2\vartheta_{23} \cos(\Delta + \underset{\underset{\text{CPV}}{\uparrow}}{\delta_{13}}) \frac{\sin(A\Delta)}{A} \frac{\sin[(1-A)\Delta]}{1-A} \\ & + \left(\frac{\Delta m^2_{21}}{\Delta m^2_{31}}\right)^2 \sin^2 2\vartheta_{12} \cos^2\vartheta_{23} \frac{\sin^2(A\Delta)}{A^2} \end{aligned}$$

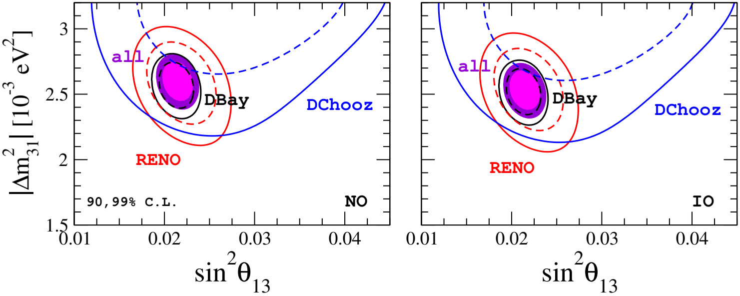
NO: $\Delta m^2_{31} > 0$ IO: $\Delta m^2_{31} < 0$

For antineutrinos: $\delta_{13} \to -\delta_{13}$ (CPV) and $A \to -A$ (Matter Effect)

[see: Mezzetto, Schwetz, JPG 37 (2010) 103001]

ϑ_{13} is determined mainly by Daya Bay

$\sin^2\vartheta_{13} = 0.0219 \pm 0.0007$ [Daya Bay, arXiv:1809.02261]



[de Salas, Forero, Ternes, Tortola, Valle, PLB 782 (2018) 633, arXiv:1708.01186]

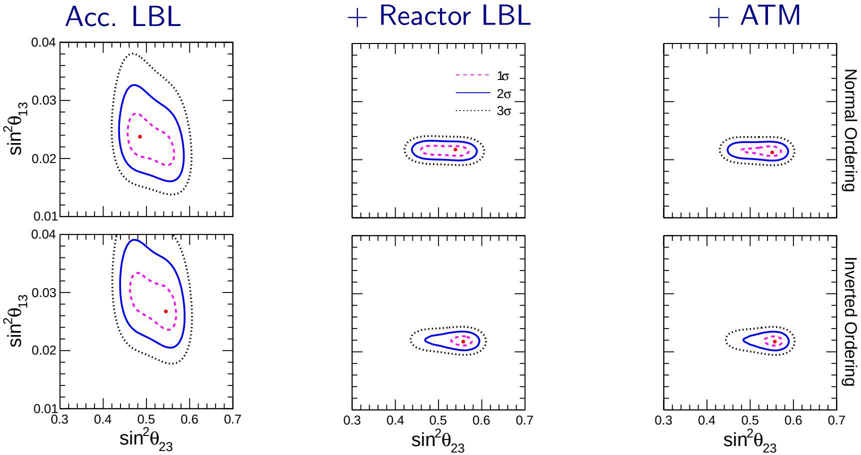
$$P^{\rm LBL}_{\bar\nu_e\to\bar\nu_e} \simeq 1 - \sin^2 2\vartheta_{13} \left[\cos^2\vartheta_{12}\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right) + \sin^2\vartheta_{12}\sin^2\left(\frac{\Delta m^2_{32}L}{4E}\right)\right]$$

Mass Ordering: Normal?

Bari: $\chi^2_{\text{IO}} - \chi^2_{\text{NO}} = 9.5 \quad (\approx 3.1\sigma)$

NuFit: $\chi^2_{\text{IO}} - \chi^2_{\text{NO}} = 9.1 \quad (\approx 3.0\sigma)$

Valencia: $\chi^2_{\text{IO}} - \chi^2_{\text{NO}} = 11.7 \quad (\approx 3.4\sigma)$



[Capozzi, Lisi, Marrone, Palazzo, PPNP 102 (2018) 48, arXiv:1804.09678]

Mass Ordering: Normal?

Bari: $\chi^2_{\text{IO}} - \chi^2_{\text{NO}} = 9.5 \quad (\approx 3.1\sigma)$

NuFit: $\chi^2_{\text{IO}} - \chi^2_{\text{NO}} = 9.1 \quad (\approx 3.0\sigma)$

Valencia: $\chi^2_{\text{IO}} - \chi^2_{\text{NO}} = 11.7 \quad (\approx 3.4\sigma)$

Matter Effect

- $\nu_e \leftrightarrows \nu_\mu$ MSW resonance: $V = \frac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \iff \Delta m^2_{13} > 0$ NO
- $\bar{\nu}_e \leftrightarrows \bar{\nu}_\mu$ MSW resonance: $V = -\frac{\Delta m^2_{13} \cos 2\vartheta_{13}}{2E} \iff \Delta m^2_{13} < 0$ IO

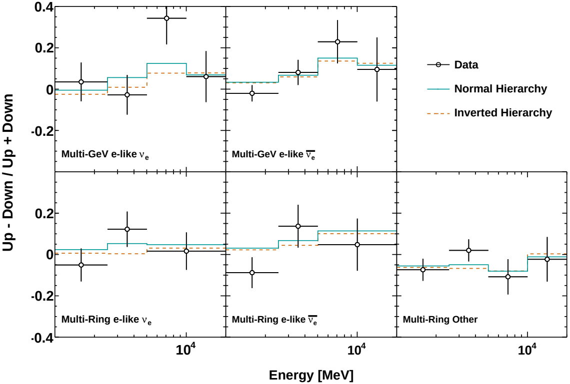
Mass Ordering: Normal?

Bari: $\chi^2_{\mathrm{IO}} - \chi^2_{\mathrm{NO}} = 9.5 \quad (\approx 3.1\sigma)$

NuFit: $\chi^2_{\mathrm{IO}} - \chi^2_{\mathrm{NO}} = 9.1 \quad (\approx 3.0\sigma)$

Valencia: $\chi^2_{\mathrm{IO}} - \chi^2_{\mathrm{NO}} = 11.7 \quad (\approx 3.4\sigma)$

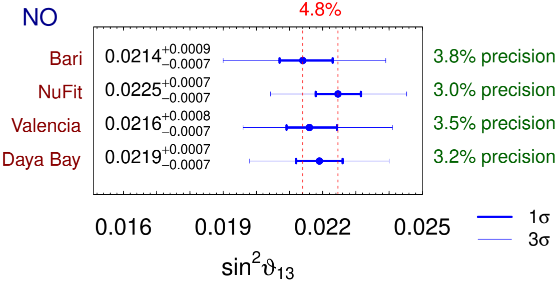
SK atmospheric preference for NO due to excess of *e*-like events



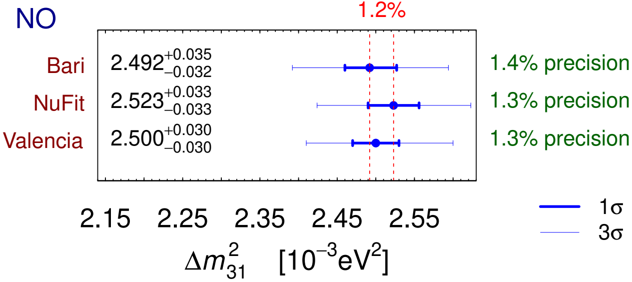
Super-Kamiokande

$$\chi^2_{\mathrm{IO}} - \chi^2_{\mathrm{NO}} = 4.33$$

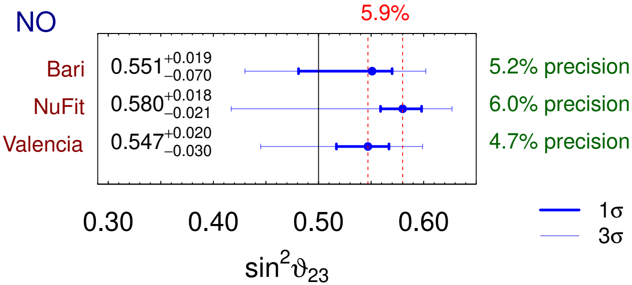
[PRD 97 (2018) 072001]



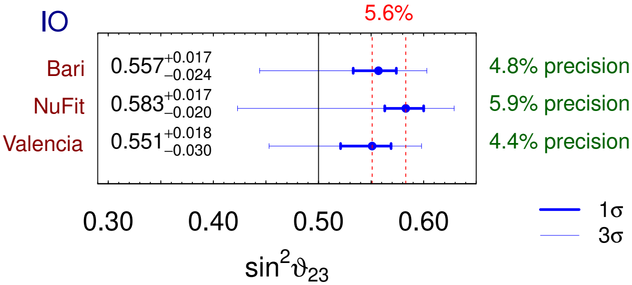
Nominal Precision + 4.8% Systematic Uncertainty !



Nominal Precision + 1.2% Systematic Uncertainty !



Nominal Precision + 5.9% Systematic Uncertainty !

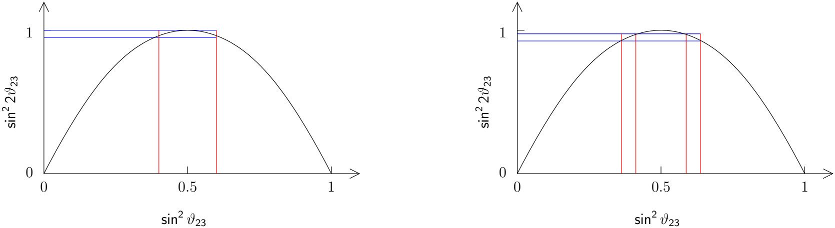


Nominal Precision + 5.6% Systematic Uncertainty !

Difficulty of measuring precisely ϑ_{23}

$$P^{\text{LBL}}_{\nu_\mu\to\nu_\mu} \simeq 1 - \sin^2 2\vartheta_{23} \, \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

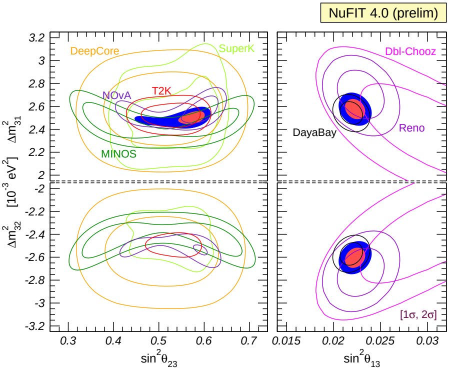
$$\sin^2 2\vartheta_{23} = 4 \sin^2\vartheta_{23} \left(1 - \sin^2\vartheta_{23}\right)$$

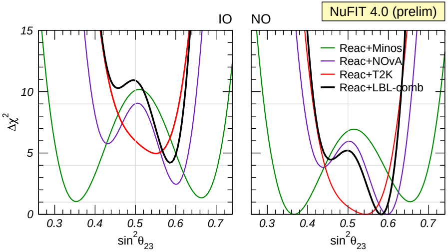


The octant degeneracy is resolved by small ϑ_{13} effects:

$$P^{\text{LBL}}_{\nu_\mu\to\nu_\mu} \simeq 1 - \left[\sin^2 2\vartheta_{23} \cos^2\vartheta_{13} + \sin^4\vartheta_{23} \sin^2 2\vartheta_{13}\right] \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$

$$P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq \sin^2\vartheta_{23} \sin^2 2\vartheta_{13} \, \sin^2\left(\frac{\Delta m^2_{31} L}{4E}\right)$$





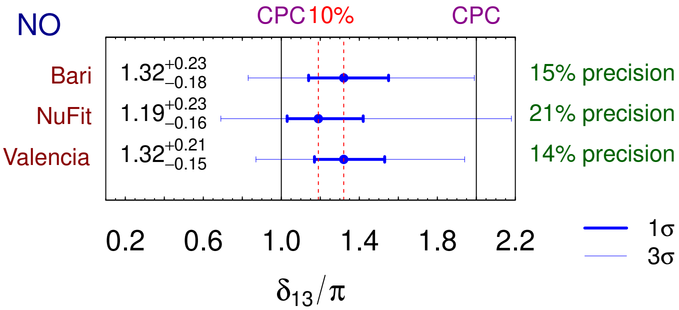
[C. Gonzalez-Garcia @ NuTown 2018]

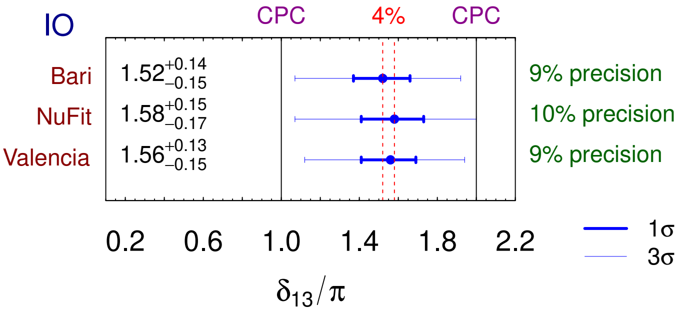
The octant degeneracy is resolved by small ϑ_{13} effects:

$$P^{\text{LBL}}_{\nu_\mu\to\nu_\mu} \simeq 1 - \left[\sin^2 2\vartheta_{23}\cos^2\vartheta_{13} + \sin^4\vartheta_{23}\sin^2 2\vartheta_{13}\right]\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

$$P^{\text{LBL}}_{\nu_\mu\to\nu_e} \simeq \sin^2\vartheta_{23}\sin^2 2\vartheta_{13}\,\sin^2\left(\frac{\Delta m^2_{31}L}{4E}\right)$$

CP Violation?





Towards a precise determination of the mixing matrix

well determined

totally unknown

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\lambda_{21}} & 0 \\ 0 & 0 & e^{i\lambda_{31}} \end{pmatrix}$$

large uncertainty due to ϑ_{23} and δ_{13}

medium uncertainty due to ϑ_{23}

$|U|_{3\sigma} =$

only the mass composition of ν_e is well determined

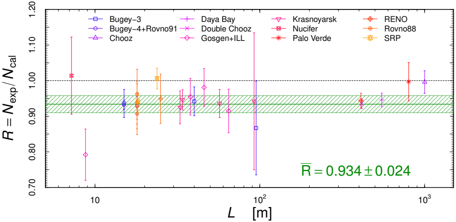
Why it is important to measure accurately the mixing parameters?

- They are fundamental parameters
- They lead to selection in huge model space. Examples:
  - Deviation from Tribimaximal Mixing $U \simeq \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\ 1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$
  - Violation of μ-τ symmetry $(|U_{\mu k}| = |U_{\tau k}|)$
- They have phenomenological usefulness (e.g. to determine the initial flavor composition of astrophysical neutrinos)
- CP:
  - CP conservation would need an explanation (a new symmetry?)
  - CP violation may be linked to the CP violation in the sector of heavy neutrinos which generate the matter-antimatter asymmetry in the Universe through leptogenesis (CP-violating decay of heavy neutrinos)

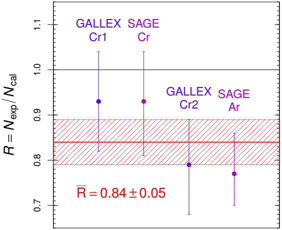
Light Sterile Neutrinos

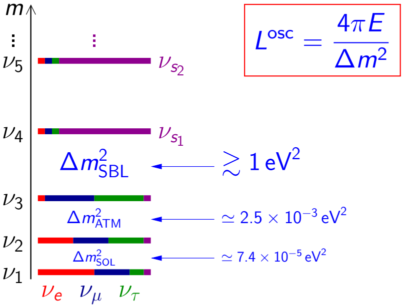
Short-Baseline Anomalies

Reactor Anomaly: $\bar{\nu}_e \to \bar{\nu}_x$ ($\sim 3\sigma$)



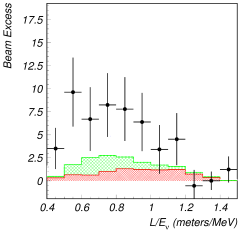
Gallium Anomaly: $\nu_e \to \nu_x$ ($\sim 3\sigma$)





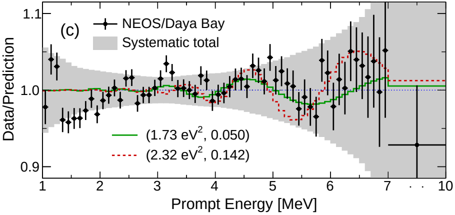
$$\boxed{L^{\text{osc}} = \frac{4\pi E}{\Delta m^2}}$$

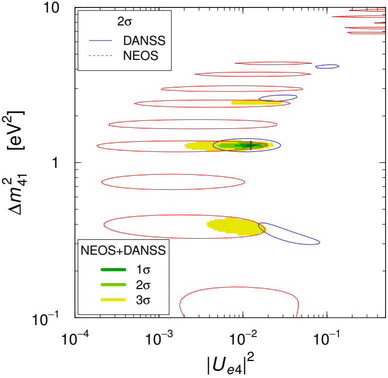
LSND Anomaly: $\bar{\nu}_\mu \to \bar{\nu}_e$ ($\sim 4\sigma$)



Reactor Spectral Ratios

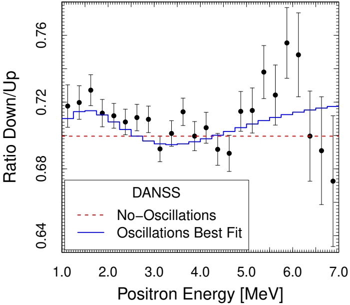
NEOS [PRL 118 (2017) 121802, arXiv:1610.05134]





DANSS

[PLB 787 (2018) 56, arXiv:1804.04046]



MODEL INDEPENDENT!

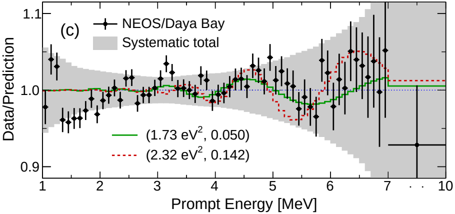
$\sim 3.5\sigma$

[Gariazzo, CG, Laveder, Li, PLB 782 (2018) 13, arXiv:1801.06467]

[See also: Dentler et al, JHEP 1808 (2018) 010, arXiv:1803.10661]

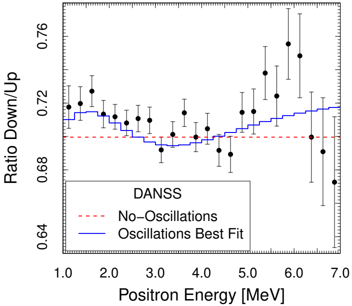
Reactor Spectral Ratios

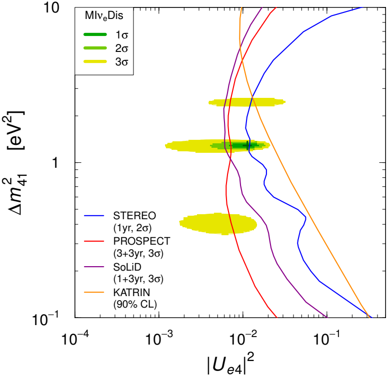
NEOS [PRL 118 (2017) 121802, arXiv:1610.05134]



DANSS

[PLB 787 (2018) 56, arXiv:1804.04046]





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$\sim 3.5\sigma$

[Gariazzo, CG, Laveder, Li, PLB 782 (2018) 13, arXiv:1801.06467]

[See also: Dentler et al, JHEP 1808 (2018) 010, arXiv:1803.10661]

Conclusions

- Mainstream 3ν-mixing research: precise measurements of masses, mixing angles and CP violating phases with neutrino oscillations, β decay, $\beta\beta_{0\nu}$ decay.

- Neutrinos provide a Window to the New Physics beyond the Standard Model through:
  - Small (Majorana) Masses.
  - Sterile Neutrinos.
  - Non-Standard Interactions. [see Ohlsson, RPP 76 (2013) 044201, arXiv:1209.2710]
  - Electromagnetic Interactions. [see CG, Studenikin, RMP 87 (2015) 531, arXiv:1403.6344]
  - ...

- Exciting model-independent indication of light sterile neutrinos at the eV scale from the NEOS and DANSS experiments, in agreement with the reactor and Gallium anomalies.

  It will be tested soon by several experiments.