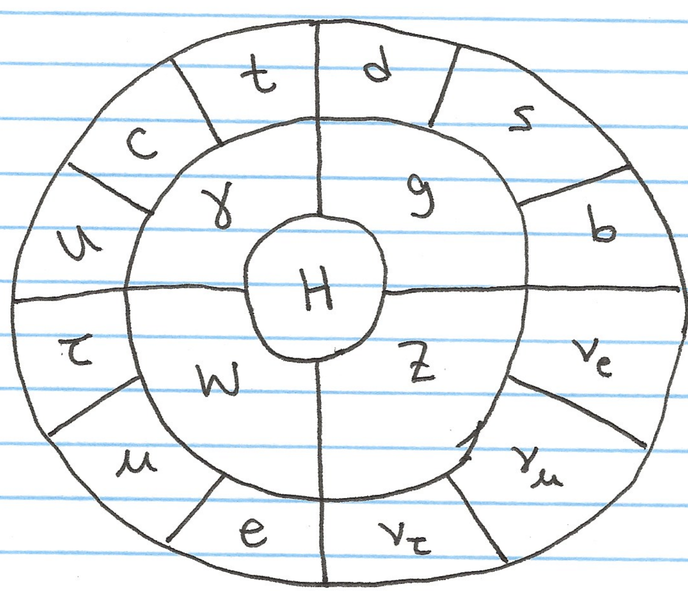
TRISEP Lectures: QCD

Hello! I am Andrew Larkoski, currently an Acting Assistant Professor at SLAC National Lab, but I also taught at Reed College (where David Griffiths is Emeritus Professor) for five years. I have been tasked with introducing the Standard Model of Particle Physics in the first 3 lectures of this school, in a total of 4.5 hours. The Standard Model encapsulates nearly everything we know about particle physics, so this is an impossible task. Instead of aiming for encyclopedic and mathematically dense, I will instead work to give you a flavor of how we know what we know in particle physics. There will be some Feynman diagrams here and there for illustration, but I will try to use the scientific method, intuition, and dimensional analysis as much as possible. Richard Feynman once remarked that if all of mathematics were lost, physics would be set back by a week, and I'll try to demonstrate why that is reasonable (if but a bit hyperbolic ☺).

The broad outline for these three lectures is as follows. Today (after a bit more introduction) we will discuss the strong nuclear force, quantum chromodynamics ~~of~~ or QCD. Particles charge under this force are called "quarks" and the force carrier is called the "gluon" and we will make significant progress in understanding QCD in analogy with electromagnetism. Tomorrow, we will introduce the weak nuclear force, and some of its strangeness like parity violation and relationship with electromagnetism.

Finally, on Wednesday, we will discuss the Higgs boson and the methods for its discovery at particle collider experiments. Again, this review/preview will be very superficial and only cover selected topics. For more details, and more about my approach to particle physics, you can check out the textbook I wrote, "Elementary Particle Physics: An Intuitive Introduction" published by Cambridge University Press in 2019. Further, if you have any questions about what I cover here, other aspects of particle physics, teaching physics, advice about academia, etc., please catch me during breaks or send me an email at larkoski@slac.stanford.edu.

Before starting in earnest, let's orient ourselves with the standard Model. My personal favorite representation of the Standard Model was created by Mark Levinson for the documentary "Particle Fever" and arranges particles in concentric circles:



The outermost ring consists of the <u>matter</u> of the Standard Model: spin-½ fermions. The top semi-ring are the quarks and the bottom semi-ring are the leptons. Quarks interact through every force, while leptons only interact through the electro-weak force. The next ring are the spin-1 force carrying bosons: the gluon g, the photon γ, and the W and Z bosons. The W and Z bosons mediate the weak force, and we will discover that ~~neig~~ neither QCD nor the weak force have classical limits. Finally, the Higgs boson lies at the center, it being responsible for much of what makes the Standard Model both a powerful description of nature and also hides many of its mysteries and, dare I say, magic.

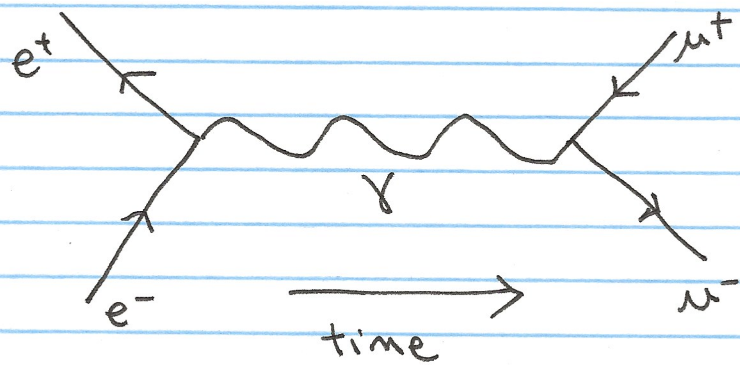
Speaking of "Particle Fever" and the Higgs boson, today, July 4, 2022, is the 10 year anniversary of the discovery of the Higgs, which was announced at CERN in 2012. While we won't discuss the Higgs in detail until Wednesday, I feel it is only proper to have a brief celebration today, and then we will make sense of it on Wednesday. So, I want to show a clip from "Particle Fever" that summarizes CERN on July 4, 2012. Be on the look out for some TRIUMF experimentalists...

(show clip)

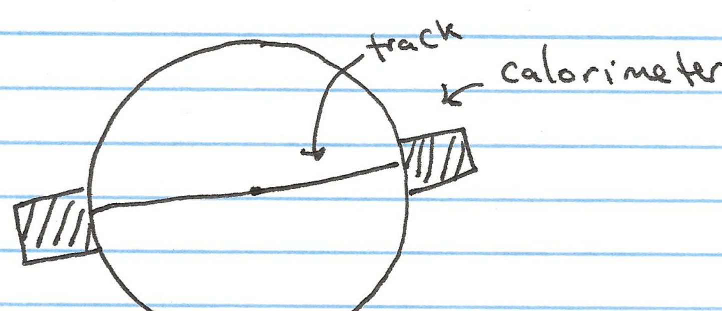
Now, I actually was at CERN at that time, but had a terrible cold, so I only made it in line by about 6 am. I was just beyond where the tracking shot following the line ended. I also took my badge from that week which proved I was there. But don't tell anyone! :)

Okay, enough silliness; onto the main event for this lecture: QCD. My research is mostly focused around QCD, so I could talk forever about it, but we will attempt to stay focused here, just providing details about why we know quarks and gluons exist. Our invitation to this topic will be the following event display from the OPAL experiment at the Large Electron-Positron Collider (LEP), which occupied the ring that now houses the Large Hadron Collider. What you are seeing here is the detector response to particles produced in the collision of electron and ~~proton~~ positron beams (in and out of page). The curved lines in the center represent the trajectories of charged particles, so-called tracks, and the blocks at larger radii are proportional to the energy deposited in the calorimetry. This particular collision event occurred at a center-of-mass enery of almost 100 GeV, about 100 times the mass of the proton. What is so striking about this image is that the particles travel along highly collimated directions, which are called "jets". This is a two-jet event, but is a contribution from a more general class of events in which hadrons, protons, pions, kaons, etc, are produced and detected. The fact that the hadrons are dominantly produced in two jets suggests there is some simpler, fundamental, process responsible, so let's see if we can make sense of it.

Let's think about a process that seems at least superficially similar, which will help guide our intuition. We are colliding electrons and positrons, which are electrically charged, and so obviously interact via electromagnetism. Muons also carry electric charge, so can be produced through electromagnetism, so we can consider the process: $e^+e^- \to \mu^+\mu^-$ which has a Feynman diagram of:



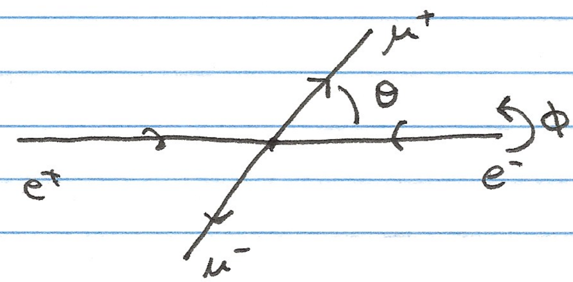
If you haven't seen Feynman diagrams, you read it in the direction of the flow of time. Arrows represent fermion number flow: arrows with time are particles (e^-, μ^-), while arrows against time are anti-particles (e^+, μ^+). The photon, the force carrier of electro magnetism, mediates the interaction. What we would see in our experiment is just two tracks, back-to-back with equal energy (by momentum conservation, just like colliding carts on ramps):



Hmm, this looks very similar to our jets. Can we leverage the similarity to make a hypothesis for the fundamental interaction that is responsible for $e^+e^- \to$ hadrons?

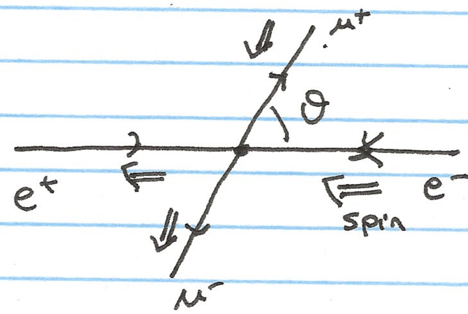
Our goal will be to have a particle description for the creation of jets. A "particle" of particle physics is defined through Wigner's Classification: as an irreducible representation of the Lorentz group (+ any charges of "internal" forces). Because we see jets in e^+e^- collisions, we will assume that these particles are electrically charged. Irreducible representations of the Lorentz group are defined by their mass (the Casimir of the momentum operator) and the spin (the Casimir of the Lorentz boost/rotation operator). How can we get a handle on the spin of the particle that produces jets?

Well, how can we determine the spin of the μs produced in $e^+e^- \to \mu^+\mu^-$? Let's determine the number of degrees of freedom of the muons in these collisions. First each muon has 3 momentum components, and the energy is then fixed by the mass of the muon and the momentum: $E = \sqrt{m_\mu^2 + |\vec{p}|^2}$. The total momentum is 0 in the center-of-mass, and the total energy is fixed, $E_{tot} = E_{cm}$. So, there are 6 variables and 4 constraints, leaving two variables, which can be described by scattering angles; θ, ϕ:



If the e^+e^- beams are unpolarized, there is an azimuthal symmetry, and so the distribution of events is flat in ϕ. So, there is only one interesting variable: the scattering angle θ.

What is the distribution of θ? Let's draw some vectors representing the helicities of the particles, the projection of their spin along the direction of momentum. All particles are spin-$1/2$, and so the helicity can be + (along $\vec{p}$) or − (against $\vec{p}$). Further, the photon is spin 1 and so the helicities of directly interacting particles with the photon must sum to 1 (i.e., point in the same direction). With these constraints, one possible configuration of spins is:



$\Rightarrow$ = direction of spin/helicity

Note that when $\theta = 0$ there is perfect alignment of e^+e^-, $\mu^+\mu^-$ spins. These spin states then have large overlap, in the bra-ket/wavefunction sense. However, if $\theta \to \pi$, the spins are anti-aligned, and are therefore orthogonal. There is no intermediate angle at which the spins are also orthogonal, therefore the amplitude for this configuration is proportional to:

$$\langle \mu^- \Uparrow, \mu^+ \Downarrow | e^- \Uparrow, e^+ \Downarrow \rangle \propto 1 + \cos\theta.$$

One can do the same analysis if the spins of the muons are flipped, and all that changes is $\theta \to \pi - \theta$ or

$$\langle \mu^- \Downarrow, \mu^+ \Uparrow | e^- \Uparrow, e^+ \Downarrow \rangle \propto 1 - \cos\theta.$$

Electromagnetism couples to electric charge, and not spin, so the constants of proportionality of these two amplitudes is identical. Further, all other possible spin configurations produce these two scattering angle dependencies (or 0). Now, we don't directly measure particle spin, so we need to sum over all spins. Spin/helicity is an observable, and so this sum occurs at the level of probability, not amplitude, just like how probabilities of distinct energy eigenstates in quantum mechanics sum incoherently. That is, distinct experimental outcomes are classical. So, the dependence on the scattering angle we would observe in data is:

$$p(\theta) \propto |\langle \mu^-\Uparrow, \mu^+\Downarrow | e^-\Uparrow, e^+\Downarrow\rangle|^2 + |\langle \mu^-\Downarrow, \mu^+\Uparrow | e^-\Uparrow, e^+\Downarrow\rangle|^2$$

$$\propto (1+\cos\theta)^2 + (1-\cos\theta)^2 \propto 1+\cos^2\theta.$$

That is, the production of spin-1/2 particles in e^+e^- collisions has a distinct $1+\cos^2\theta$ dependence on the scattering angle.

What about data? What do we see? Here is a plot of $e^+e^- \to \mu^+\mu^-$ collisions at 29 GeV, and the $1+\cos^2\theta$ dependence is clearly observed.

So, If these jets are created by spin-1/2 particles the scattering angle of the jets with respect to the collision axis should still exhibit the $1+\cos^2\theta$ dependence. What do we see? Well, this is a bit more subtle because jets are composite objects, so there is no unique direction we can say is its momentum vector. Nevertheless, just summing the momentum of nearby, collimated particles works well as observed in this plot from ALEPH, another on LEP. The dependence on the scattering angle is very close to the $1+\cos^2\theta$ dependence, but differs a bit (cf dashed line). These data are collected at a center-of-mass energy of 91 GeV, where the Z boson is very important (but more on that tomorrow). However, this gives us confidence that the particles responsible for jets are spin-1/2.

Another property we can study in these collisions is the number of and masses of these spin-1/2 particles. To do this, we will simply count the number of $e^+e^- \to$ hadrons events as a function of collision energy. In general, the rate of production from collisions is measured in cross sectional area, as it represents the amount of area of the wavefunction overlap of the colliding e^+e^- devoted to the process of interest. We can use dimensional analysis and some properties of quantum mechanics to estimate this cross section.

Let's first consider $e^+e^- \to \mu^+\mu^-$ collisions at energies well above the masses of the electrons and muons. First, from quantum mechanics, the probability amplitude of $e^+e^- \to \mu^+\mu^-$, $\langle \mu^+\mu^- | e^+e^- \rangle$ is proportional to the strength of their electric potential energy, simply from the structure of the Schrödinger equation, for example. Both electrons and muons carry the fundamental electric charge e, and so

$$\langle \mu^+\mu^- | e^+e^- \rangle \propto e^2,$$

as their electric potential $V(r) = \frac{e^2}{4\pi\epsilon_0}\frac{1}{r}$.

Because the cross section is a measurable probability, it is proportional to the square of this:

$$\sigma \propto |\langle \mu^+\mu^- | e^+e^- \rangle|^2 \propto e^4 \propto \alpha^2,$$

where α is the fine-structure constant (from Sommerfeld in old quantum theory). Further, we need the correct units/dimensions for σ. σ is an area, or squared length, and in natural units lengths are inverse energy (by, e.g., de Broglie wavelength, for example). The only relevant energy is the center-of-mass collision energy, E_{cm}, and so

$\sigma \propto \frac{\alpha^2}{E_{cm}^2}$. This means that collisions are <u>less</u> likely at higher energies because the de Broglie wavelengths are smaller and so there is a smaller region of wavefunction overlap of the initial and final states. So, our muon production cross section is:

$$\sigma(e^+e^- \to \mu^+\mu^-) \propto \frac{\alpha^2}{E_{cm}^2}.$$

We can do all of the same analysis for our jet production particles, which we will call "quarks". For a given energy E_{cm}, we only produce quarks if twice their mass is less than E_{cm}, and the cross section is weighted by their squared electric charges. So, the cross section for quark production takes the form:

$$\sum_i^{E_{cm}>2m_i} \sigma(e^+e^- \to q_i\bar{q}_i) \propto \frac{\alpha^2}{E_{cm}^2} \sum_i^{E_{cm}>2m_i} Q_i^2,$$ where the sum

runs over "active quarks" for which $E_{cm} > 2m_i$.

and Q_i is the i^{th} quark electric charge. What is typically studied is the ratio of these cross sections, called R:

$$R \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \frac{\sum_i \sigma(e^+e^- \to q_i \bar{q}_i)}{\sigma(e^+e^- \to \mu^+\mu^-)} \overset{E_{cm} > 2m_i}{=} \sum_i Q_i^2 .$$

So, this R ratio directly is sensitive to the number and electric charge of the quarks. So, if we plot it as a function of E_{cm}, we should see it increase whenever thresholds are passed, when a new quark can be produced. Let's look at the data!

From the quark model, quarks have fractional charges, $|Q_i| = 1/3$ or $2/3$ and so the first four quarks (up, down, strange, charm) would produce

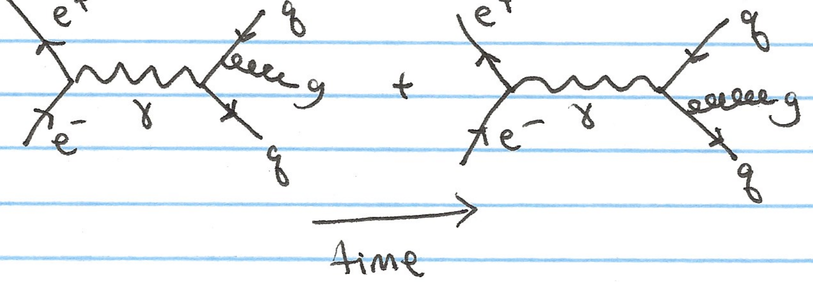
$$R = \underbrace{2 \cdot \left(\tfrac{1}{3}\right)^2}_{\text{down, strange}} + \underbrace{2\left(\tfrac{2}{3}\right)^2}_{\text{up, charm}} = \frac{10}{9}$$, but the data is closer to

$$R_{data} \cong 3\tfrac{1}{3} = \frac{10}{3}$$, three times larger! Hmm, so

for every quark we actually need three of them to account for this difference. What we can do for now is to use this as a hypothesis for our last part. Let's say for each type of quark there are three "colors", red, green, blue (but have nothing to do with visible light). Like electric charge, these colors can be the charge of a new force, and the force carrier will transform a quark of one color to a quark of a different color. This new force carrier could then be responsible for hadrons as bound states of quarks within the quark model. For example, electromagnetism cannot be responsible for hadronic bound states because there exist hadrons that are formed from the bound state of three quarks with the same electric charge (i.e., Ω^- baryon = bound state of three strange quarks).

Let's keep going and give this force carrier a name; the gluon, and assume it shares properties with the photon, like being a spin-1 boson. How can we experimentally observe this gluon and therefore establish that this new force is indeed a force? Let's go back to E&M and ask how we create photons. Photons radiate away from electric charges, and therefore are not created by static charges or constant currents. However, if electric charges accelerate, they produce electromagnetic radiation of which photons are its quanta. Then, one can predict the energy in photons through the Larmor formula or whatever.

In analogy with E&M, if we want to produce gluons we need to accelerate particles charged under color; i.e., quarks. How can we accelerate them? Well in e^+e^- collisions, when we produce a $q\bar{q}$ final state, the quarks experience <u>enormous</u> accelerations: they don't exist, then they do, so this environment is ripe for gluon production. In terms of Feynman diagrams, the process $e^+e^- \to q\bar{q}g$ is described by:



that is, the gluon could have been emitted from either the accelerate quark or anti-quark and there is no measurement we can perform to distinguish them, so we have to sum them coherently; i.e., at the probability amplitude level.

What would we observe in our experiment? Well if $e^+e^- \to q\bar{q}$ corresponds to two jets, $e^+e^- \to q\bar{q}g$ would be three jets! So if we see three jet events, this is direct evidence for the gluon as the force carrier between quarks of different colors. Further, like we did to establish the spin of quarks, we can establish the spin of the gluon from these three jet events. What we will do is measure the distributions of the fraction of the energies of the jets, with

$$x_i = \frac{2E_i}{E_{cm}} ,$$

where E_i is the energy of the i^{th} most energetic jet. We can then compare the data to the predicted distributions if the gluon were spin-0, 1, or 2 and see which matches. While I am glossing over a lot of subtleties, the result is striking, as analyzed by the SLD collaboration, an experiment on the linear collider at SLAC. Apparently the gluon is spin-1!

So, we have established the particle content of QCD: electrical and color-charged spin-½ quarks, and a spin-1 gluon that mediates the force. This theory is called quantum chromodynamics, as chromo = color in greek.

As for why they are called "quarks", Murray Gell-Mann appropriated the word from James Joyce's "Finngans Wake."