TRISEP Lectures Higgs

We were inevitably ledd to the conclusion that the W and Z bosons were both spin-1 as well as massive particles from our analysis yesterday. This should trouble you, because the photon, the other spin-1 boson you know about, is mass<u>less.</u> Why? What's the big deal? So what, just add a mass and move on. Indeed, you can do that, but if you do you nearly completely lose the power of symmetries, conservation laws, and Noether's theorem for systematically constructing the mathematical framework of the weak force.

So, let's dive a bit deeper to see what the matter is. What is the physical consequence of the fact that the photon is massless? We had argued that for massless particles, helicity is a good quantum number. A massless particle's spin can be projected along its momentum axis and there are only two possibilities: either spin points parallel or anti-parallel to the momentum. No Lorentz transformation can mix these two helicity states. Equivalently, the helicity specifies the eigenvalue of spin under a <u>little group</u> transformation: a ~~the~~ subset of Lorentz transformations that leave the momentum of the particle unchanged. For a massless particle, we can never boost to a rest frame and so the only transformations that leave the momentum unchanged are rotations about its direction. These rotations mix two, orthogonal, dimensions and thus the little group is isomorphic to SO(2) or U(1), just moving a point around the unit circle. Representations of U(1) are all one-dimensional, but we can rotate around the circle either in a right- or left-handed sense, hence two spin states. We of course know that photons/electromagnetic radiation/light has only two polarization states by, e.g., using polarizing filters.

Why is the masslessness of the photon important? From an empirical perspective, a massless particle has no lowest energy, or no longest wavelenth/compton wavelength. This is extremely important for electromagnetism because we observe coherent electromagnetic effects at enormous distances (e.g., galactic magnetic fields). So, simply from the fact that electromagnetism has a classical limit tells us that the photon must have an amazingly small mass. For example, the existence of Maxwellian magnetic fields at the scale of our galaxy means that the Compton wavelength of the photon must be no smaller than the radius of the galaxy, corresponding to a mass bounded from above of $m_\gamma < 10^{-28}$ eV.

From a mathematical perspective, the masslessness of the photon is required by the structure of Maxwell's equations. We empirically observe that electric charge is conserved in all reactions, which is of course a consequence of Maxwell's equations. By Noether's theorem, conservation of electric charge means there is a symmetry (really, a gauge invariance) of the Lagrangian of E+M. If the photon were massive, then the electric charge conservation law would be modified:

$$\nabla \cdot \vec{J} + \frac{\partial \rho}{\partial t} \simeq m_\gamma^2 \partial \cdot A$$, where A_μ is the vector potential.

This wouldn't be good. Because of electric charge conservation (<u>locally</u>), we can freely modify the vector potential by a divergence of a scalar function $\lambda(x)$ and there is no effect to the mathematical description of E+M:

$$A_\mu \to A_\mu + \partial_\mu \lambda .$$

This is of course the familiar gauge invariance of E+M and it affects the degrees of freedom of the photon. Naively, the photon as described by A_μ should have four degrees of freedom. However, A_0 has no time derivative in Maxwell's equations, and is completely constrained by Gauss's Law. So there are at most three degrees of freedom. However, gauge invariance eliminates the degree of freedom in the direction of photon momentum. The derivative ∂_μ becomes the photon momentum under Fourier transformation, and we can fix this completely arbitrarily and there is no physical consequence. Thus, the vector potential or photon only has two degrees of freedom.

Symmetries highly constrain the possible ~~ph~~ mathematical descriptions of a system, and this is why the potential formulation of E+M is more "obviously" powerful than with $\vec{E}$ + $\vec{B}$ fields. A_μ is more constrained, so there are much less we can write down to describe E+M (c.f., Ampère's original equation for relating $\partial \vec{E}/\partial t$ to $\vec{\nabla} \times \vec{B}$). With E+M as a guide, we had proposed a $U(2) \simeq SU(2) \otimes U(1)$ symmetry that relates γ, W^+, W^-, and Z bosons to one another. For maximal predictivity, we ~~wa~~ want this symmetry to imply gauge transformations for all of these particles, so that we can use Noether's

Theorem to its fullest extent. But, as we just argued, this would mean that all of γ, W^+, W^-, Z must be massless, badly violating our empirical knowledge. So, we seem to have hit a dead-end. Do we give up on Noether?

Remember, in physiss the answer to any rhetorical "yes and no" question is always no! Through work by Stueckelberg, Nambu, Schwinger, Anderson, Englert, Bront, Higgs, Guralnik, Hagen, Kibble, Migdal, Polyakov, 't Hooft, and others found the loophole and in doing so predicted the existence of a new particle. The loophole is now called the Higgs mechanism and the new particle is the Higgs boson, so let's see if we can make sense of it. The fundamental idea is the following. We want to keep gauge invariance as a guiding principle, so γ, W^+, W^-, Z are massless, and form a representation of $SU(2) \otimes U(1)$. To make W^+, W^-, and Z massive, we need to somehow give them an extra degree of freedom, corresponding to longitudinal polarization along their direction of momentum. $W^{\pm}$ and Z are spin-1 particles, and so their spin about their direction of momentum can take one of three quantum eigenvalues: $+\hbar$, $-\hbar$ (parallel and anti-parallel), and $0\hbar$. The $0\hbar$ state is the one we are missing and does not exist in a photon because electric/magnetic fields are always transverse to momentum/Poynting vector. Thus, to make $W^{+/-}, Z$ massive they need to somehow absorb, or "eat", spin-0 states. So, the simplest thing we can do is to introduce a new, spin-0 particle, to accomplish this. This is the Higgs boson.

Let's see the way that this can happen, and what we will do here is just study an analogue in quantum mechanics, and then use that insight for interpreting what can happen with quantum fields. Let's define our goal again: we want 3 massless spin-0 particles that can be eaten by the W^+, W^-, and Z bosons to give them mass. A fourth spin-0 particle must itself be massive so that the photon can skip lunch and stay massless. Let's consider regular quantum mechanics in two-dimensions, in which the potential in position space is:

$$V(x,y) = \frac{\lambda}{4}\left(v^2 - x^2 - y^2\right)^2,$$

where λ is some coupling, $\lambda > 0$, and v is some characteristic distance. This potential has a maximum at $x = y = 0$ and a minimum, where $V = 0$, along the circle $x^2 + y^2 = v^2$. We can of course find energy eigenvalues in this coordinate system, but we note that the azimuthal symmetry suggests a better way. Let's use polar coodinates, in which we expand about the minimum of the potential:

$$x = (r+v)\cos\phi, \quad y = (r+v)\sin\phi, \quad \text{where } r \text{ is a radial}$$

fluctuation about the minimum, and ϕ is an azimuthal coordinate. In these coordinates, the potential simplifies:

$$V(r,\phi) = \frac{\lambda}{4}\left(v^2 - (r+v)^2\right)^2 = \lambda v^2 r^2 + \ldots,$$

ignoring cubic and quartic terms in r for now. At least close enough to the minimum, the potential is just a harmonic oscillator, where:

$$\frac{m\omega^2}{2} = \lambda v^2$$

and ~~so~~ its ground state energy is

$$E_0 = \frac{\hbar\omega}{2} = \frac{\hbar}{2}\sqrt{\frac{2\lambda v^2}{m}} = \hbar v\sqrt{\frac{\lambda}{2m}},$$

which is non-zero. When translated to quantum field theory, this minimal particle energy corresponds to a finite, non-zero mass particle, as the mass is the least energy a particle can carry. However, in performing this change of variables, the potential has no dependence on azimuthal angle ϕ, but is periodic; $0 \le \phi < 2\pi$. This is just like a particle on a ring, and if you recall, the particle on a ring ground state wave function is

$\psi(\phi) = \frac{1}{\sqrt{2\pi}}$, a constant in ϕ. Thus, its ground state energy is $E_0 = 0$. A 0 energy state in quantum mechanics corresponds in quantum field theory to a particle that can have 0 energy. The only way this is possible is if the particle is massless.

Thus, we have a procedure for a common origin for 3 massless and 1 massive scalar particles/fields. If we have a mechanism for constructing this "mexican hat" potential in four "Cartesian" scalar fields:

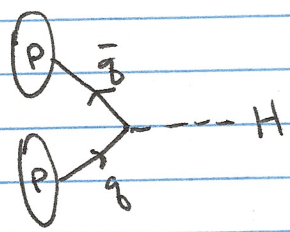
$$V(\vec{\Phi}) \sim \frac{\lambda}{4}\left(v^2 - |\vec{\Phi}|^2\right)^2,$$

then the potential will enforce that the low energy excitations are exactly what we need for weak force masses. In the Standard Model, we simply postulate this potential, with no explanation for its genesis.

To establish the veracity of this framework, we <u>need</u> to see the radial excitation particle, the Higgs boson. We noted that it must be a spin-0 scalar, which will have consequences for how it interacts, as we will see. We will just explore one more property of the Higgs here, generalizing what we required of it to give $W^\pm$, Z masses, but not of the photon. Let's consider two limits. Because the Z boson, for example, is massive, we can boost to its rest frame. In that frame, there is complete symmetry between all three polarization states. So that longitudinal mode has to contribute exactly as much as the other polarizations. In the opposite limit, as the Z becomes more and more highly boosted, the longitudinal polarization must decrese in importance because if the energy of the Z boson were infinite, $E \to \infty$, it would effectively be massless. These considerations motivate the longitudinal mode contributes proportional to the mass of the Z boson; or, that the Z boson couples to the Higgs in proportion to its mass. This is excellent, because it also means that the massless photon is completely ignorant of the Higgs and stays massless.

Thus, we will assume that the Higgs couples to <u>all</u> particles in proportion to their mass, which can be justified further, but only need this observation for now. So, we would like to discover a spin-0 particle that couples in proportion to the mass of particles. We will just describe how to do this at a hadron collider like the LHC. The LHC collides protons on protons at energies of (now) up to nearly 7000 times the mass of the proton.

That is, the de Broglie wavelength of the protons at the LHC are 7000 times smaller than their compton wavelengths, so the substructure of the protons is very clearly resolved. Protons are bound states of quarks, so, like we observed in e^+e^- collisions, perhaps the Higgs can be produced as a resonance in $q\bar{q} \to H$ processes. That is, a schematic diagram is:



Let's estimate the cross section for this to occur, or, just the wavefunction overlap, $|\langle H|q\bar{q}\rangle|^2$. The Higgs couples proportional to the mass of the quarks and the probability is dimensionless so,

$$|\langle H|q\bar{q}\rangle|^2 \propto \frac{m_q^2}{m_H^2}.$$

What is the approximate size of this? The most massive quark in the proton with appreciable probability is the strange quark, which has a mass of $m_s \sim 100$ MeV. As of yet, we do not know the mass of the Higgs, but because we haven't seen it (in these lectures) it must be pretty massive, $\gtrsim 100$ GeV, but not too massive, otherwise it would form weak boson bound states or other wild things, $\lesssim 1$ TeV. For concreteness, let's assume $m_H \sim 100$ GeV, so that the probability is

$$|\langle H|q\bar{q}\rangle|^2 \propto \frac{m_q^2}{m_H^2} \sim \frac{m_s^2}{m_H^2} \sim 10^{-6},$$ which is very small.

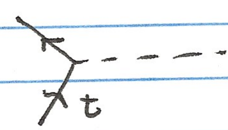
For context, let's estimate the probability that e^+e^-

annihilate into a photon. Electrons and positrons couple to photons proportional to their electric charge and so

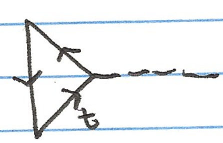
$$|\langle \gamma | e^+ e^- \rangle|^2 \sim e^2 = 4\pi\alpha \sim \frac{4\pi}{137} \sim 0.1.$$

So, electron-positron scattering occurs about 10^5 times more than $q\bar{q} \to H$. Hmm, this doesn't seem very promising if we need to collect hundreds of thousands times more data to get decent statistics.

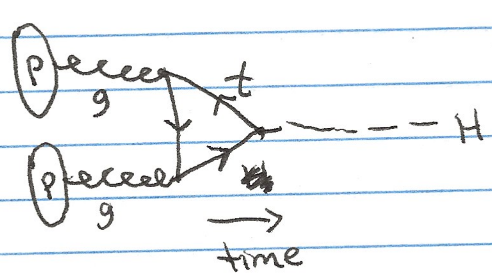
Let's re-think this in the opposite way, starting from the Higgs and then getting to protons. The Higgs couples to mass, so let's couple it to the most massive particle, the top quark:



We don't want to produce the top quark as well as the Higgs because we want to see a resonance. So let's make the top quarks a quantum fluctuation or loop in this diagram:



Finally, top quarks, while much too massive to be constituents of the proton, do couple strongly to gluons, which are copious. So lets pull out gluons from the protons:

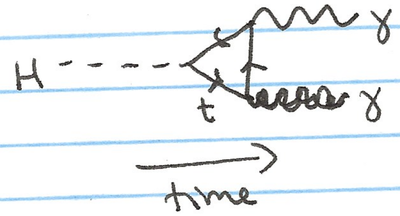


While we won't do it here, you can estimate the probability for this to occur and it is orders of magnitude larger than $q\bar{q} \to H$. So, we have found our production mechanism.

How do we actually observe the Higgs? Just like we did for the Z boson: through its decays. The Higgs can decay in manifold ways, so let's use some experimental constraints to determine the quickest path to discovery. First, we want to observe all decay products, so that means no neutrinos. Neutrinos are such low mass they don't couple to the Higgs directly but could be produced from secondary decay of W bosons. So, no $H \to W^+W^-$. Next, we want small backgrounds and clean signatures, so no jets or quarks from decay, like $H \to b\bar{b}$. The rate for producing two bottom quarks just in QCD is enormous, so we would have to sift through a mountain of data. These considerations leave two possibilities: $H \to \gamma\gamma$ and $H \to ZZ \to \ell^+\ell^-\ell'^+\ell'^-$. These final states are extremely clean and involve particles that interact electromagnetically. For the most robust observation, we only consider electrons and muons from Z decay (the possible leptons ℓ, ℓ' are only e, μ). Depending on the mass of the Higgs, the mass of the Z bosons might suppress the four-lepton channel, but the beauty of the final state greatly outweighs this mild ugliness. These two decays are referred to as the "golden channels" for these reasons.

But wait, How does the Higgs decay to photons? We just said that it was great that the Higgs doesn't couple to photons, so what gives?

Quantum mechanics is what giveth! Just as we produced the Higgs via a loop of top quarks, it can decay through a loop of top quarks:



The top quark is electrically charged ($q_t = \frac{2}{3}e$), and so it couples to photons. Can we estimate the rate at which it decays to photons, $\Gamma_{H\to\gamma\gamma}$? This decay rate can be expressed as:

$$\Gamma_{H\to\gamma\gamma} \propto m_H |\langle\gamma\gamma|H\rangle|^2.$$

The decay rate has units of inverse time, or, in natural units, energy. The only characteristic energy for Higgs decay is its mass. Now, let's estimate the wavefunction overlap, as quantified by the diagram. First, the top quarks couple to photons in proportion to their electric charge:

$$\langle\gamma\gamma|H\rangle \propto e^2 Q_t^2, \text{ where } Q_t = 2/3.$$

Next, the Higgs couples to the top in proportion to its mass, m_t. This proportionality is called the top Yukawa coupling y_t, and so

$$\langle\gamma\gamma|H\rangle \propto y_t e^2 Q_t^2.$$

Finally, the overlap can depend on the Higgs or top mass in principle, so let's include them:

$$\langle \gamma\gamma | H \rangle \propto y_t e^2 Q_t^2 \, m_H^a m_t^b .$$

This overlap must be dimensionless and so $b=-a$. Also, if $m_t \to \infty$ with fixed m_H, then it cannot exist as a quantum fluctuation and the rate of decay to photons must vanish, $b<0$, or $a>0$. Finally, if the $m_t \to \infty$ limit is to be controlled within the mass mechanism of the Standard Model, $b=-1$ so that the dependence on the Yukawa coupling (implicitly) drops out. Then,

$$\langle \gamma\gamma | H \rangle \propto y_t e^2 Q_t^2 \frac{m_H}{m_t} .$$

There's one more thing we can include. Loops in Feynman diagrams always come with a factor of $1/(4\pi)^2$ essentially from Fourier transforming to momentum space. This diagram has one loop so we can include it:

$$\langle \gamma\gamma | H \rangle \propto \frac{y_t e^2 Q_t^2}{(4\pi)^2} \frac{m_H}{m_t} = \frac{y_t \alpha Q_t^2}{4\pi} \frac{m_H}{m_t} .$$

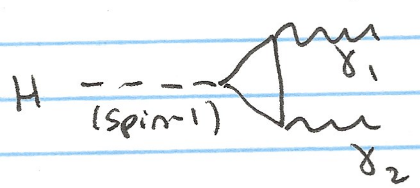
This $1/(4\pi)^2$ prescription is called naive dimensional analysis, or NDA. Then, our approximation for the decay rate to photons is:

$$\Gamma_{H\to\gamma\gamma} \propto \frac{y_t^2 \alpha^2 Q_t^4}{(4\pi)^2} \frac{m_H^3}{m_t^2} .$$

By far, the smallest factor in this rate is α, so this decay is dominated by electromagnetic effects, but then suppressed further by that pesky loop factor, $1/(4\pi)^2 \sim 1/100$. Nevertheless, these factors do not conspire to be excessively small, so there is hope of discovery. Let's look at some plots!

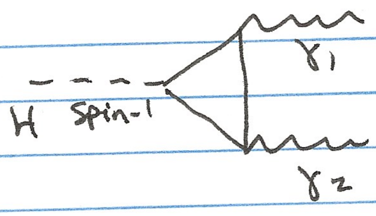
Finally, I just want say some brief words on establishing the two properties of the Higgs we started from: coupling proportional to mass and spin-0. Establishing that the Higgs couples proportional to mass is very challenging because only a few particles have large enough masses. Light quarks and leptons might forever be hopeless, but for now the story is very consistent, especially for particles with masses above about a GeV.

Finally, the spin of the Higgs. We observe the Higgs decay to two photons, which are identical, spin-1 particles. Therefore, the rules of angular momentum addition in quantum mechanics states that the Higgs must be spin-0, spin-1, or spin-2. Spin-0 requires that the spins of the photons are anti-aligned, while spin-2 requires that they are aligned. Spin-1 is subtle, and ultimately impossible due to a result called the Landau-Yang theorem, of which I can motivate here. Spin-1 Higgs means that all of the Higgs and two (identical) photons have some polarization vectors. This combination of three spin-1 polarization vectors must be rotationally-invariant because we assume full Lorentz invariance. The only combination of three-vectors that is rotationally invariant is the scalar triple product. So,



$$\propto \vec{\varepsilon}_H \cdot \left(\vec{\varepsilon}_{\gamma_1} \times \vec{\varepsilon}_{\gamma_2} \right).$$

This seems all well and good, but the photons are <u>identical</u>, so their wavefunction must be invariant under permutation:



$$\propto \vec{\varepsilon}_H \cdot \left(\vec{\varepsilon}_{\gamma_1} \times \vec{\varepsilon}_{\gamma_2}\right) + \vec{\varepsilon}_H \cdot \left(\vec{\varepsilon}_{\gamma_2} \times \vec{\varepsilon}_{\gamma_1}\right)$$

$$= 0,$$

by the antisymmetry of the cross-product. Thus the Higgs boson <u>cannot</u> be spin-1.

Actually establishing the spin with high confidence is very subtle, and so the PDG has made no official declaration. Nevertheless, all signs point to spin-0, as we would expect.

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