TRISEP 2022

DARK MATTER THEORY DAY 1

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What is Dark Matter?

- galactic rotation curves
- Bullet cluster / galaxy cluster mergers
- large scale structure formation
- CMB

- these are all sensitive to the gravitational effects of DM → massive (CMB indicates it is a particle)
  - matter
  - stable w/ lifetime(s) long comparable to cosmological scales
  - weakly-interacting + minimal self-interactions
  - "dark" → EM neutral
  - needs to account for observed relic abundance
- now, let's look at galactic rotation curves

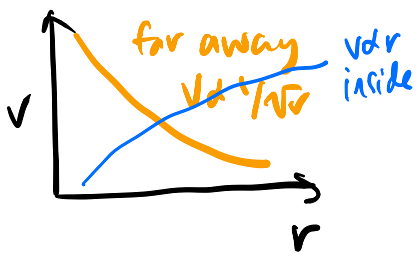
  Newton's 2nd Law: $v_c(r) = \sqrt{\frac{GM}{r}}$ circular velocity

  M = enclosed mass

  r = radial distance

  G = Newton's gravitational constant

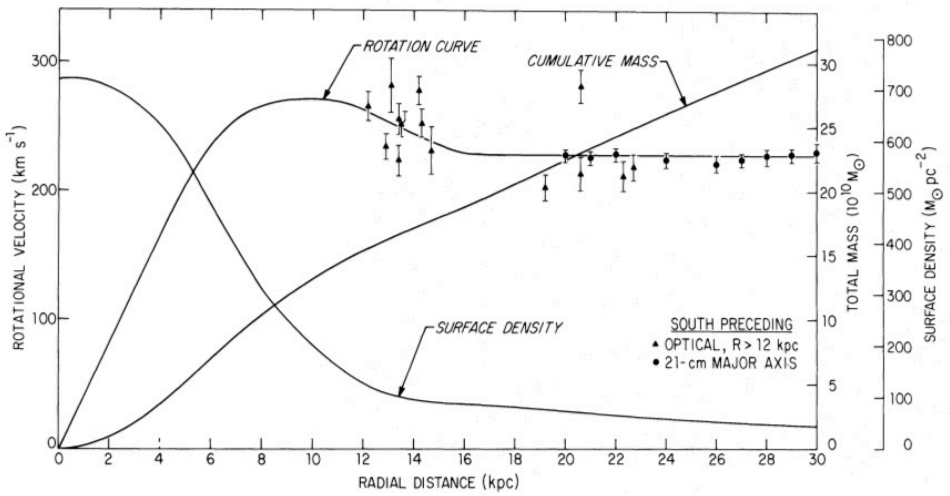
- Far away from galaxy, $M(r)$ = constant = mass of galaxy



inside: $M(r) = 4\pi \int_0^r dr' r'^2 \rho(r')$

assume $\rho(r')$ constant $\rightarrow v \propto r$

compare to observation: M31 Roberts & Whitehurst (1975)



Two things to note:

1. curve does not fall off like $1/\sqrt{r}$ at large r despite surface density going to zero

lessons:

1. matter distribution extends past visible disk
2. $M(r) \sim r \Rightarrow \rho(r) \propto 1/r^2$ assuming spherical sym

- from stellar kinematics, we can get the total mass of the MW halo and DM density

$M_{halo} \sim 10^{12} M_\odot \qquad \rho_{DM} \sim 0.4\ \text{GeV}/\text{cm}^3$

↳ $M_{halo} \sim 4\pi \int_0^{R_{halo}} dr\, r^2 \rho(r) \rightarrow R_{halo} \sim 100\,\text{kpc}$

- virial theorem tells us the average velocity

$$\langle v \rangle \sim \sqrt{\frac{G M_{halo}}{R_{halo}}} \sim 200\ \text{km/s} \sim 10^{-3}$$

- from this we learn that DM is non-relativistic
- we can deduce some general, model-independent properties about DM based on the fact that DM forms halos
- suppose we have an ultralight scalar candidate for DM
  - ↳ Bose statistics
  - ↳ occupation number is high → treat as a classical field
  - ↳ stability of the halo is set by uncertainty princ.

$$\Delta x \Delta p \sim 1 \qquad \Delta p \sim m_\chi v \qquad \Delta x \sim 2R_{halo}$$

  - ↳ from dwarf galaxies : $m_s \gtrsim 10^{-22}\ \text{eV}$
- Now, let's consider fermionic DM
  - ↳ fermi statistics : Pauli-exclusion principle

$$M_{halo} = m_f V \int f(p)\, d^3p \lesssim m_f V \int d^3p \sim m_f R_{halo}^3 (m_f v)^3$$

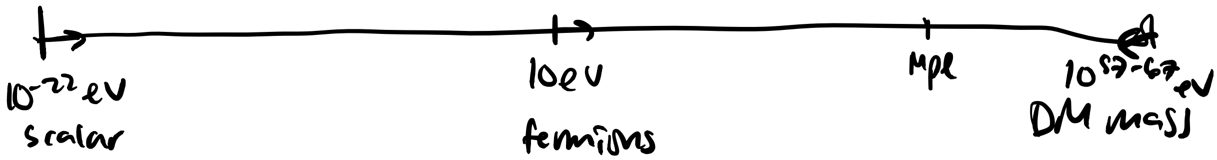
$V = \frac{4}{3}\pi R^3$

let v = virial velocity → $m_f \gtrsim (G^3 M_{halo} R_{halo}^3)^{-1/8}$

$\geq \mathcal{O}(10)\,\text{eV}$

Tremaine-Gunn bound : Fornax dwarf → $m_f \gtrsim 70\,\text{eV}$

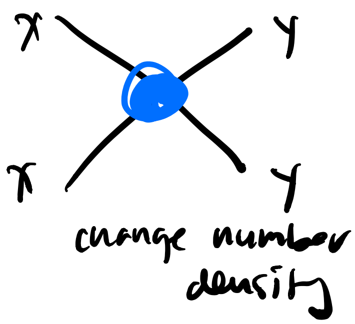
- generic upper limit on DM mass from searches for MACHOs



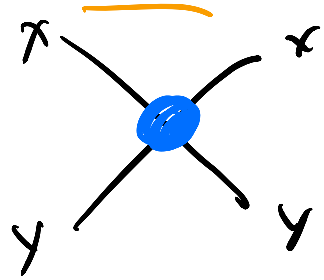
Thermal DM

- DM is in thermal equilibrium in the early Universe due to its interactions with the SM
  ↳ attractive: economical and predictive
- two types of interactions are occurring:

inelastic



elastic



- as the Universe expands $\chi\chi \to YY$ shuts off
  "freeze-out" $\Gamma_{inelastic} = n_\chi \langle \sigma v \rangle \sim H$
- cold DM is non-relativistic at freeze-out and
  $n_\chi \sim T^{3/2} e^{-m_\chi/T}$
- hot DM is relativistic at freeze-out and $n_\chi \sim T^3$
- after freeze-out, DM is no longer in chemical equilibrium but remains in thermal equilibrium w/ surrounding plasma through $\Gamma_{elastic}$.
- the elastic rate will also decouple
  "kinetic-decoupling": $\Gamma_{elastic} = n_Y \langle \sigma v \rangle$ $\sim T^3$ b/c Y are relativistic

- note, for CDM, kinetic-decoupling after DM has fallen out of chemical equilibrium - earlier for hotter

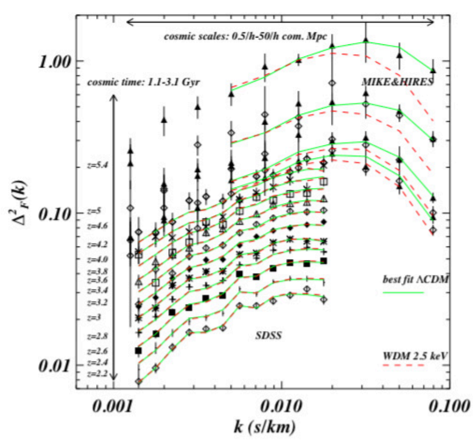
↳ DM is "free-streaming"

- the time at which DM becomes free-streaming sets a cut-off scale for the DM power spectrum

↳ suppression of perturbation spectrum below some characteristic wave number (or above some free-streaming length): hotter the DM the longer the free-streaming length → lower cutoff

- by studying the Ly-α power spectra @ high-z quasars we can exclude $m_{thermal} \lesssim 3.3\,\text{keV}$

EROS-2 collaboration astro-ph/0607207



- these results constrain how <u>warm</u> DM can be
- punchline: WDM predicts less structure on small scales
  - rules out neutrinos as <u>all</u> of the DM

Thermal freeze-out

goal: calculate the abundance of DM today

$$* \quad \frac{1}{a^3}\frac{d}{dt}(na^3) = \frac{dn}{dt} + 3Hn \overset{= \text{ collision term}}{} \qquad \text{where } H \equiv \dot{a}/a$$

Boltzmann eqn.

- if there are no number-changing interactions then $* = 0 \Rightarrow na^3$ is constant in time.
- let's now add in interactions $1 + 2 \leftrightarrow 3 + 4$
- the collision term for particle 1:

$$** \quad g_1 \int \frac{d^3p_1}{(2\pi)^3} C[f_1] = -\sum_{\text{spins}} \int \Big[f_1 f_2 (1 \pm f_3)(1 \pm f_4) |M_{12\to34}|^2 - f_3 f_4 (1 \pm f_1)(1 \pm f_2) |M_{34\to12}|^2 \Big] \times (2\pi)^4 \delta^{(4)}(p_1 + p_2 - p_3 - p_4)\, d\Pi_1\, d\Pi_2\, d\Pi_3\, d\Pi_4$$

$$d\Pi_1 = \frac{d^3p_1}{(2\pi)^3 E_1} \quad \text{phase-space factor}$$

- simplifying assumptions:
  1. kinetic equilibrium: phase-space distributions take Fermi-Dirac or Bose-Einstein forms
  2. temperature of each species satisfies $T_i \ll E_i - \mu_i$
     ↳ follows Maxwell-Boltzmann dist. $(1 \pm f) \sim 1$
  3. SM particles involved are in thermal equilibrium w/ photon bath

- derive expression for cross-section

$$\sum_{\text{spins}} \int |\mathcal{M}_{ij \to k\ell}|^2 (2\pi)^4 \delta^{(4)}(p_i + p_j - p_k - p_\ell)\, d\Pi_k\, d\Pi_\ell$$

$$= 4 g_i g_j\, \sigma_{ij} \sqrt{(p_i \cdot p_j)^2 - (m_i m_j)^2}$$

s.t. [illegible] $= -\int \{ (\sigma v_{M\phi l})_{12}\, dn_1\, dn_2 - (\sigma v_{M\phi l})_{34}\, dn_3\, dn_4 \}$

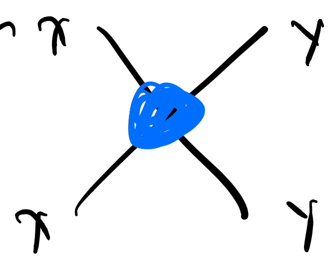
where $(v_{M\phi l})_{ij} \equiv \dfrac{\sqrt{(p_i \cdot p_j)^2 - (m_i m_j)^2}}{E_i E_j}$

$(\sigma v_{M\phi l})_{ij} \sim$ constant

$\hookrightarrow$ [illegible] $= -\langle \sigma v_{M\phi l} \rangle_{12}\, n_1 n_2 + \langle \sigma v_{M\phi l} \rangle_{34}\, n_3 n_4$

$$= \frac{dn_1}{dt} + 3Hn_1$$

identical w/ $n_1 = n_2 = n$



in thermal equilibrium w/ photon bath $n_3 = n_3^{eq}$, $n_4 = n_4^{eq}$

when DM is in equilibrium w/ SM final states:

$$\langle \sigma v \rangle_{12}\, n_{eq}^2 = \langle \sigma v \rangle_{34}\, n_3^{eq}\, n_4^{eq}$$

$$\Rightarrow \quad \frac{dn}{dt} + 3Hn = \langle \sigma v \rangle (n_{eq}^2 - n^2)$$

- since the Universe is expanding, it's useful to recast in terms of $Y \equiv n/s$ $\propto$ total entropy density of Universe

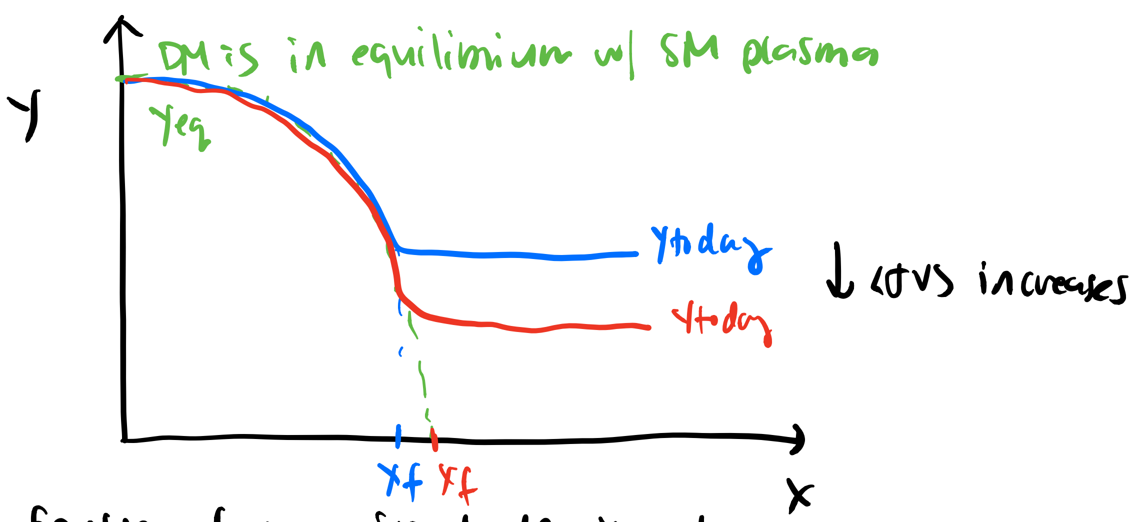
- $sa^3 =$ constant

$$\boxed{\frac{dY}{dt} = \langle\sigma v\rangle\left(Y_{eq}^2 - Y^2\right)} \rightarrow \frac{dY}{dx} = -\frac{x\, s\langle\sigma v\rangle}{H(m)}\left(Y^2 - Y_{eq}^2\right)$$

$$x \equiv m_{DM}/T$$

What have we learned?

- we have an expression for evolution of Y as the Universe cools
- Y is the rescaled DM number density
  - ↳ removes effects of Universe's expansion
- any changes in Y are from interactions of DM w/ states that are in thermal equilibrium w/ photon bath
- governed by velocity averaged cross-section $\langle\sigma v\rangle$

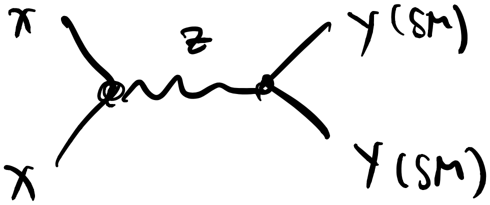


- fraction of the critical density due to DM

$$\Omega_\chi = \frac{m_\chi\, s_{today}\, Y_{today}}{\rho_c}$$

$$\Rightarrow \Omega_\chi h^2 \sim \frac{10^{-26}\ \text{cm}^3/\text{s}}{\langle\sigma v\rangle} \simeq \underbrace{0.1\left(\frac{0.01}{\alpha}\right)^2\left(\frac{m_\chi}{100\,\text{GeV}}\right)^2}_{\text{"WIMP miracle"}}$$

where $x_f \sim 10$ $\quad \langle \sigma v \rangle \sim \frac{\alpha^2}{m_{DM}^2}$



$\sigma v \lesssim \frac{m_\chi^2}{m_Z^2}$ perturbative

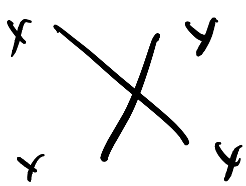
$m_\chi \gtrsim \frac{m_Z^2}{(T_{eq} M_{pl})^{1/2}} \sim \text{GeV}$

↳ sub-GeV DM requires lighter-mediators
"Lee-Weinberg" bound.

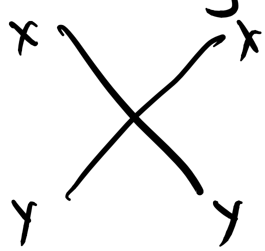
- unitarity bounds for thermal DM
  ↳ $m_\chi \lesssim 100$ TeV

- can evade those bounds by considering non-thermal relics
  ↳ axions
  ↳ super-heavy DM

- Now that we have populated our DM, we can start to search for and study it.
- So far we have been looking at $\chi\chi \to YY$



rotate



- this allows for direct detection