TRISEP 2022

DARK MATTER THEORY DAY 2

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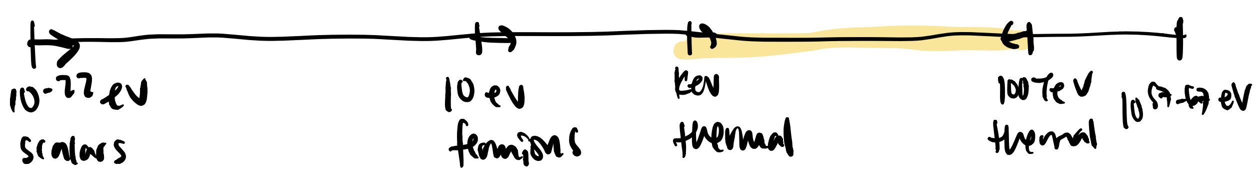
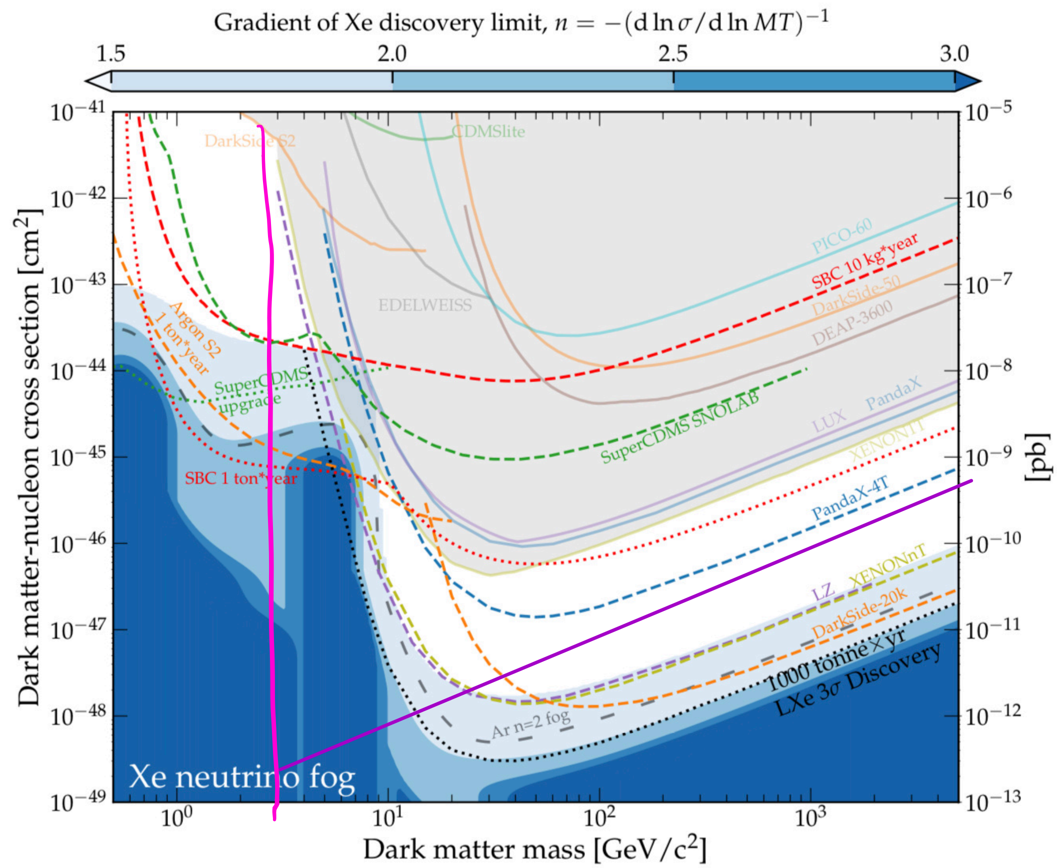
Yesterday:

- motivation for dark matter
- general properties of DM
  - ↳ constraints on masses
- introduction to thermal DM
  - ↳ how to populate relic abundance via thermal freeze-out
  - ↳ "WIMP" miracle
  - ↳ "Lee-Weinberg" bound motivates lighter mediators for sub-GeV masses
- prelude to direct detection



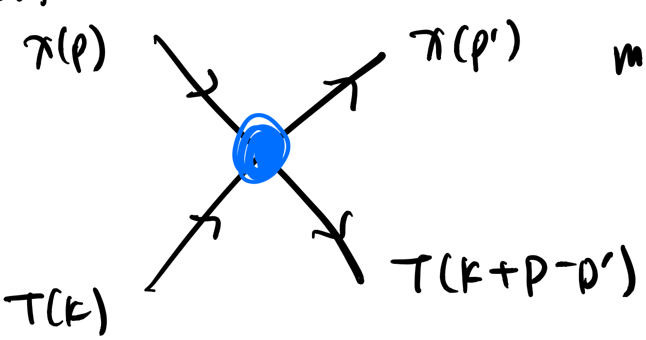


Snowmass WP1 from CF1

$$m_\chi \gtrsim \frac{2E_{min}}{v_{esc}^2} \qquad n_\chi = \frac{\rho_\chi}{m_\chi}$$

Direct Detection of Dark Matter

Basic idea



momentum transfer

$$q = p - p' = k - k'$$

- DM-nuclear scattering, treat nucleus as a particle at rest $k=0$

$$E_i = \frac{|\vec{p}|^2}{2m_\chi} \qquad \vec{p} = m_\chi \vec{v}_\chi$$ initial energy & momentum

$$E_f = \frac{|\vec{p}-\vec{q}|^2}{2m_\chi} + \underbrace{\frac{|\vec{q}|^2}{2m_N}}_{E_R}$$

E_R recoil energy of nucleus

Energy conservation:

$\cos\theta = \hat{p}\cdot\hat{q}$

$$\frac{|\vec{p}||\vec{q}|\cos\theta}{m_\chi} = \frac{q^2}{2\mu_{\chi N}} \rightarrow q_{max} = \frac{2\mu_{\chi N}\, p}{m_\chi} = 2\mu_{\chi N} v_\chi$$

$$\mu_{\chi N} \equiv \frac{m_\chi m_N}{m_\chi + m_N}$$

$$E_R^{max} = \frac{q_{max}^2}{2m_N} = \frac{2\mu_{\chi N}^2}{m_N} v^2 \simeq 1\,\text{eV} \times \left(\frac{m_\chi}{100\,\text{MeV}}\right)^2 \left(\frac{20\,\text{GeV}}{m_N}\right)$$

$m_\chi \sim 100$ GeV $m_N \sim 131$ GeV (xenon) $v \sim 10^{-3}$ $E_R \sim 49$ keV

$m_\chi \sim 100$ MeV $m_N \sim 131$ GeV $v \sim 10^{-3}$ $E_R \sim 0.1$ eV

<u>Example thresholds</u>

- 2019 CRESST-III: 30.1 eV → $m_\chi \sim 160$ MeV
- 2020 XENON1T: keV

↳ sub-GeV DM motivates looking at DM-electron scattering

- For DM-e scattering, we cannot take $k=0$ since e^- is moving

Deposited energy:

$$\Delta E_e = E_e(\vec{k}') - E_e(\vec{k}) = \frac{\vec{p}\cdot\vec{q}}{m_\chi} - \frac{q^2}{2m_\chi}$$

Energy conservation:

$$\Delta E_e = -\Delta E_\chi - \Delta E_N = -\frac{|m_\chi \vec{v} - \vec{q}|^2}{2m_\chi} + \frac{1}{2}m_\chi v^2 - \frac{q^2}{2m_N}$$

$$= \vec{q}\cdot\vec{v} - \frac{q^2}{2\mu_{\chi N}}$$

- In practice $\Delta E_N \ll 1$ which allows us to replace $\mu_{\chi N} \to m_\chi$

$$* \quad \boxed{\Delta E_e = \vec{q}\cdot\vec{v} - \frac{q^2}{2m_\chi}}$$

- arbitrary-sized momentum transfer is possible
  We can calculate ΔE_e^{max} by maximizing * wrt $\vec{q}$:

$$\Delta E_e^{max} = \frac{1}{2}\mu_{\chi N} v^2 \simeq \frac{1}{2}\,\text{eV}\left(\frac{m_\chi}{\text{MeV}}\right)$$

↳ all of the DM kinetic energy is available to excite the electron

Q: What are some typical values for q, ΔE_e?

A: electron is both the lightest & fastest particle

$v_e \sim Z_{eff}\alpha \sim 10^{-2}$

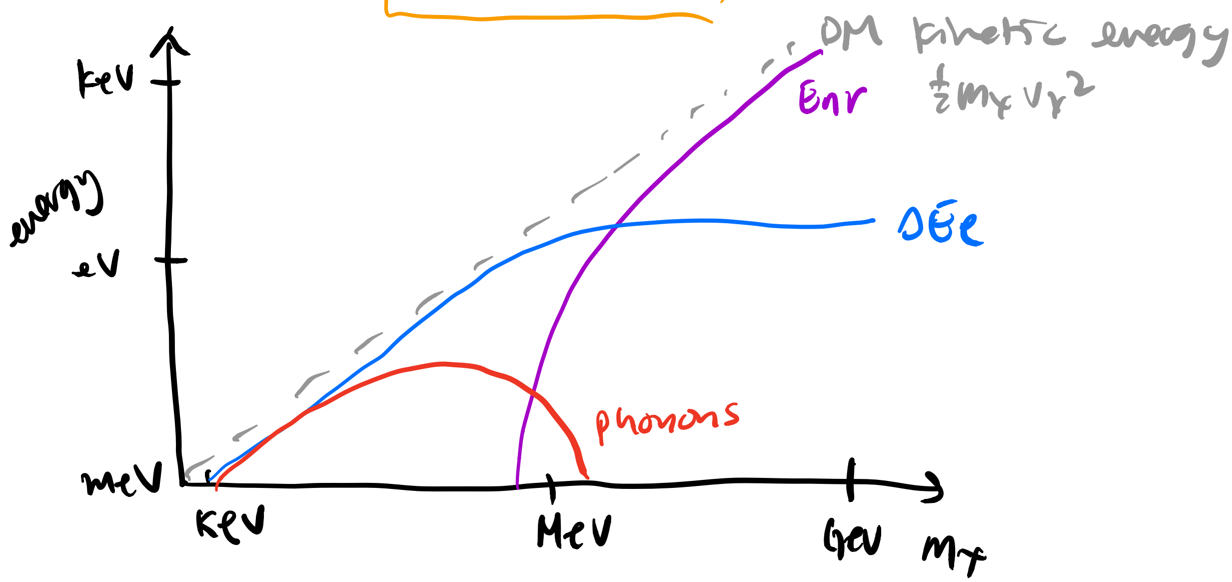
Then $q_{typ} \simeq \mu_{\chi e} v_{rel} \simeq m_e v_e \sim Z_{eff}\alpha m_e$

$$\boxed{q_{typ} \simeq Z_{eff} \cdot 4\text{ keV}}$$

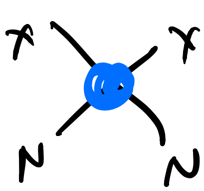
Away from threshold, 1st term of γ dominates

So $q \gtrsim \frac{\Delta E_e}{v} \simeq \frac{\Delta E_e}{4 Z_{eff}\, eV} \cdot q_{typ}$

$$\Rightarrow \boxed{\Delta E_e^{typ} \sim eV}$$



DM-nuclear scattering rates



DM interactions are given at the microscopic level, i.e. w/ quarks & gluons

- fermionic DM mediated by weak-scale mediators

dim. 6 ops: $\frac{c_V}{m_Z^2}\bar{\chi}\gamma^\mu\chi\,\bar{q}\gamma_\mu q + \frac{c_A}{m_Z^2}\bar{\chi}\gamma^\mu\gamma^5\chi\,\bar{q}\gamma_\mu\gamma^5 q$

$+ \frac{c_S m_q}{m_Z^2 v}\bar{\chi}\chi\,q\bar{q} + \ldots$

- next, we need to match the quark-level operators to nucleon-level operators

$$\bar{q}\gamma_\mu q \sim \bar{n}\gamma_\mu n \quad \text{etc...}$$

for example, vector current:

$$\langle n(k')|\bar{q}\gamma^\mu q|n(k)\rangle = \bar{u}_n(k')\left[F_1^{q,n}(q^2)\gamma^\mu + \frac{i}{2m_n}F_2^{q,n}(q^2)\sigma^{\mu\nu}q_\nu\right] \times u_n(k)$$

DM-n scattering, we are non-relativistic so $q^2 \to 0$

In this limit, F_1 captures the quark content

$$F_1^{u,p}(0) = 2 \quad F_1^{d,p}(0) = 1 \quad F_1^{s,p}(0) = 0$$

$$F_1^{u,n}(0) = 1 \quad F_1^{d,n}(0) = 2 \quad F_1^{s,n}(0) = 0$$

DM-nucleus scattering cross-section:

$$\frac{d\sigma}{dE_{nr}} = \frac{m_N}{2v^2\mu_{\chi N}^2}\left[\sigma_{SI}F_{SI}^2(q^2) + \sigma_{SD}F_{SD}^2(q^2)\right]$$

For spin-independent, we can approximate nuclear FF w/ Helm FF: $F(x) = \frac{3j_1(x)}{x}\exp\left(\frac{-(xs)^2}{2R_N}\right)$

$x \equiv qR_N$

$R_N \simeq 1.2A^{1/3}$

$s \simeq 0.5\,\mathrm{fm}$

$$\sigma_{SI} = \sigma_n \frac{\mu_{\chi N}^2}{\mu_{\chi n}^2}\frac{[f_p Z + f_n(A-Z)]^2}{f_n^2}$$

	A
Xe	131
Ar	39
Na	22
I	127

isospin-conserving: $\sigma_{SI} \sim A^2$

$$\frac{d\sigma_{SD}}{dE_{nr}} = \frac{8 G_F^2}{\pi v^2}\left[a_p \langle S_p\rangle + a_n \langle S_n\rangle\right] \frac{J+1}{J} \frac{S(|q|)}{S(0)}$$

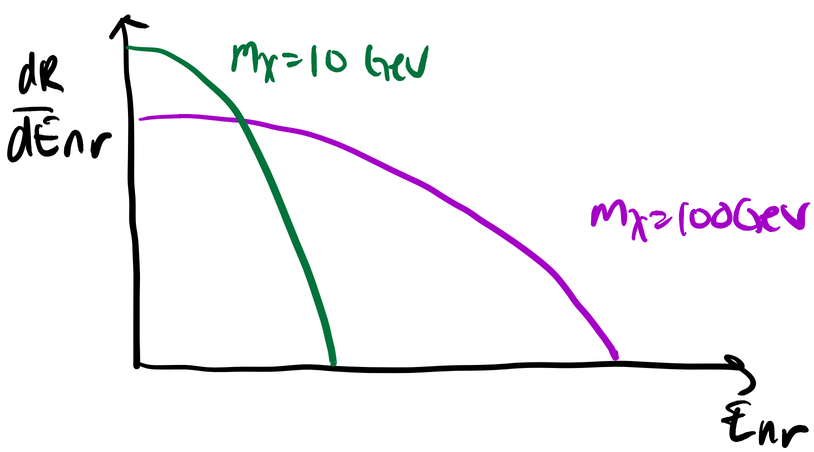
	J	
F	1/2	protons
Xe	1/2, 3/2	neutrons
Ge	9/2	neutrons

Total DM scattering rate comes from convolving the cross-section w/ the DM velocity distribution

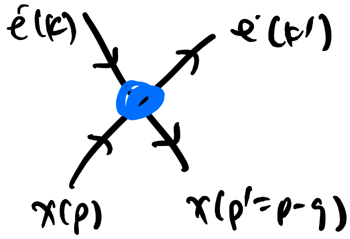
$$\frac{dR}{dE_{nr}} = \frac{M_T}{m_N}\frac{\rho_\chi}{m_\chi}\int d^3v\, v\, g(\vec{v})\, \frac{d\sigma}{dE_{nr}}$$ ← DM velocity distribution

let's define $\eta(v_{min}) \equiv \int_{v_{min}} d^3v\, \frac{g(\vec{v})}{v}$

$$\frac{dR}{dE_{nr}} = \frac{\rho_\chi}{m_\chi}\frac{M_T}{m_N}\frac{\sigma_n m_N}{2\mu_{\chi n}^2}\left[\frac{f_p Z + f_n(A-Z)}{f_n}\right]^2 F^2(q^2) \times \eta(v_{min})$$ ← for spin-independent scattering



DM-electron scattering



$$\sigma v_{free} = \frac{1}{4E_\chi' E_e'} \int \frac{d^3q}{(2\pi)^3} \frac{d^3k'}{(2\pi)^3} \frac{1}{4E_\chi E_e} (2\pi)^4 \delta(E_i - E_f) \times \delta^{(3)}(\vec{k}+\vec{q}-\vec{k}') \overline{|M_{free}(\vec{q})|^2}$$

If the electrons are unbound, non-relativistic scattering amplitude:

$$\langle \chi_{\vec{p}-\vec{q}}, e_{\vec{k}'} | H_{int} | \chi_{\vec{p}}, e_{\vec{k}} \rangle = C\, M_{free}(\vec{q}) \times (2\pi)^3 \delta^{(3)}(..)$$

However, for bound electrons, we must take into account their wavefunctions

$$\langle e_2 | H_{int} | e_1 \rangle = \left[\int \frac{\sqrt{V}\, d^3k'}{(2\pi)^3} \tilde{\psi}_2^*(\vec{k}') \langle \chi_{p}, e_{\vec{k}'} | \right] H_{int} \left[\int \frac{\sqrt{V}\, d^3k}{(2\pi)^3} \tilde{\psi}_1(\vec{k}) | e_{\vec{k}}, \chi_{\vec{p}} \rangle \right]$$

$$= C\, M_{free}(\vec{q}) \times \int \frac{V\, d^3k}{(2\pi)^3} \tilde{\psi}_2^*(\vec{k}+\vec{q}) \tilde{\psi}_1(\vec{k})$$

bound-state scattering amplitude:

$$V (2\pi)^3 \delta^{(3)}(\vec{k}-\vec{q}-\vec{k}') |M_{free}|^2 \longrightarrow$$

$$|M_{free}|^2 \cdot V^2 |f_{1\to 2}(\vec{q})|^2$$

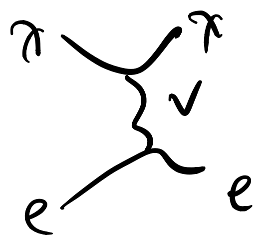
$$\boxed{f_{1\to 2}(\vec{q}) \equiv \int \frac{d^3k}{(2\pi)^3} \tilde{\psi}_2^*(\vec{k}+\vec{q}) \tilde{\psi}_1(\vec{k})}$$

calculating the ψ is primary challenge of DM-e scattering

$$\sigma v_{1\to 2} = \frac{1}{4E_\chi E_e'} \int \frac{d^3q}{(2\pi)^3} \frac{1}{4E_\chi E_e} 2\pi\, \delta(E_1 - E_2)\, \overline{|M_{free}|^2} \times |f_{1\to 2}(\vec{q})|^2$$

We can parameterize the microphysics that governs the DM-e interaction via

$$\overline{|M_{free}(\vec{q})|^2} \equiv \overline{|M_{free}(\alpha m_e)|^2} \times |F_{DM}(q)|^2$$



$m_V \gg \alpha m_e \qquad F_{DM}(q) \sim 1$

$m_V \ll \alpha m_e \qquad F_{DM}(q) \sim \frac{1}{q^2}$

$$\bar{\sigma}_e \equiv \frac{\mu_{\chi e}^2\, \overline{|M_{free}(\alpha m_e)|^2}}{16\pi m_\chi^2 m_e^2}$$

← DM velocity distribution

← momentum dependence of DM-e interaction

$$R_{1\to 2} = \rho_T \frac{\rho_\chi}{m_\chi} \frac{\bar{\sigma}_e}{8\pi \mu_{\chi e}^2} \int d^3q\, \frac{1}{q}\, \underbrace{\eta(v_{min})}_{\text{astrophysics}}\, \underbrace{|F_{DM}(q)|^2}_{\text{particle physics}}\, |f_{1\to 2}(q)|^2$$

↑ electron wavefunctions

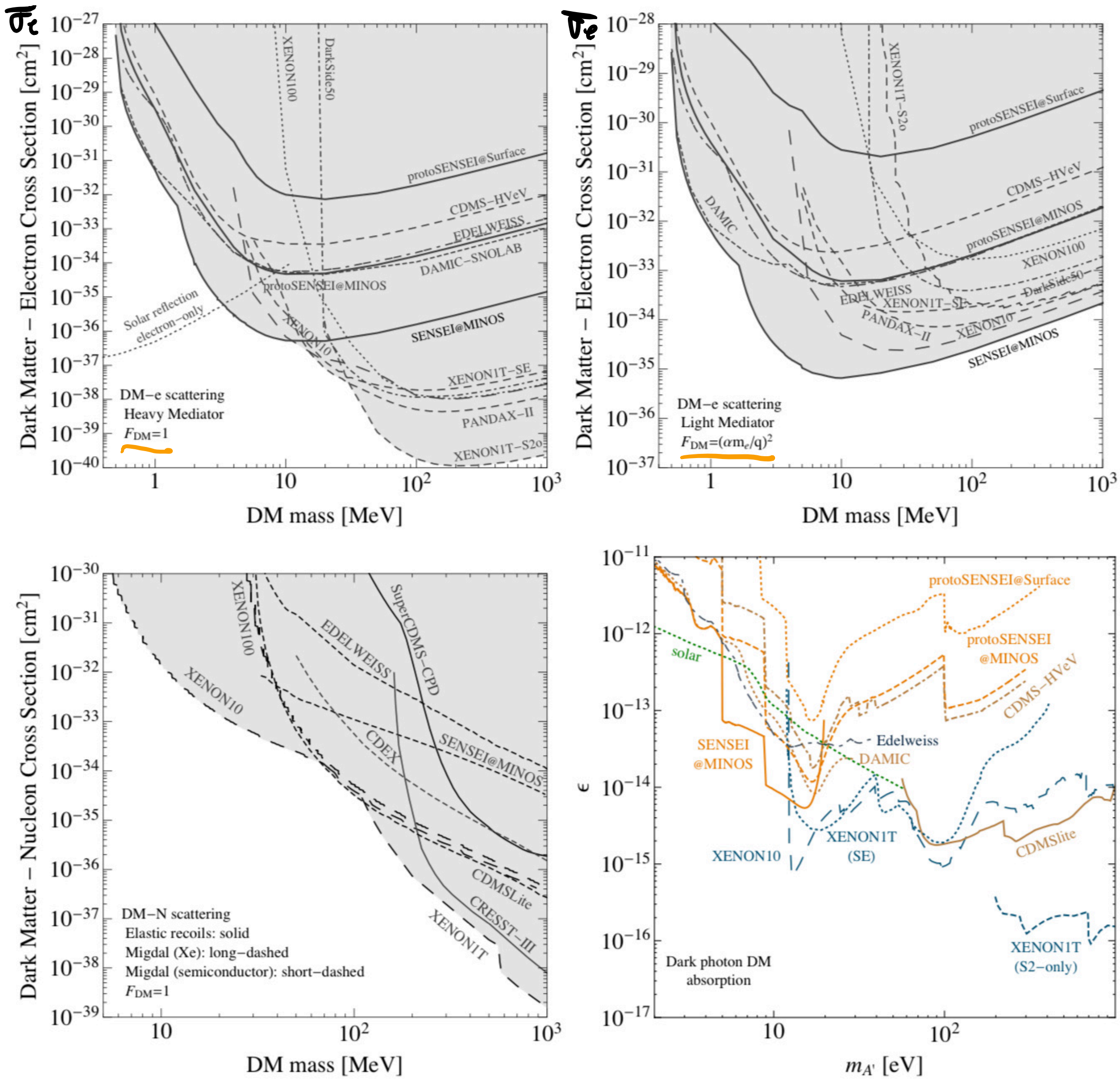
atomic physics

$m_\chi = 100$ MeV

$R \sim N_T \frac{\rho_\chi}{m_\chi} \bar{\sigma}_e v_0$

silicon: $N_T \sim 10^{26}$/kg

$\Big\}$ ~ 50-100 events/kg/day



σe
Dark Matter – Electron Cross Section [cm²]
DM mass [MeV]
XENON100
DarkSide50
protoSENSEI@Surface
CDMS-HVeV
EDELWEISS
DAMIC-SNOLAB
protoSENSEI@MINOS
Solar reflection electron-only
SENSEI@MINOS
XENON10
XENON1T-SE
PANDAX-II
XENON1T-S2o
DM–e scattering
Heavy Mediator
F_DM=1
σe
Dark Matter – Electron Cross Section [cm²]
DM mass [MeV]
XENON1T-S2o
protoSENSEI@Surface
CDMS-HVeV
DAMIC
protoSENSEI@MINOS
XENON100
DarkSide50
EDELWEISS
XENON1T-SE
PANDAX-II
XENON10
SENSEI@MINOS
DM–e scattering
Light Mediator
F_DM=(αm_e/q)²
Dark Matter – Nucleon Cross Section [cm²]
DM mass [MeV]
XENON100
EDELWEISS
SuperCDMS-CPD
XENON10
CDEX
SENSEI@MINOS
CDMSLite
CRESST-III
XENON1T
DM–N scattering
Elastic recoils: solid
Migdal (Xe): long–dashed
Migdal (semiconductor): short–dashed
F_DM=1
ϵ
m_A' [eV]
protoSENSEI@Surface
solar
protoSENSEI
@MINOS
CDMS-HVeV
SENSEI
@MINOS
Edelweiss
DAMIC
XENON10
XENON1T
(SE)
CDMSlite
XENON1T
(S2-only)
Dark photon DM
absorption