Quantum simulations of models from high energy physics

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Quantum Optics Theory

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Postdoc position available

We want to understand:

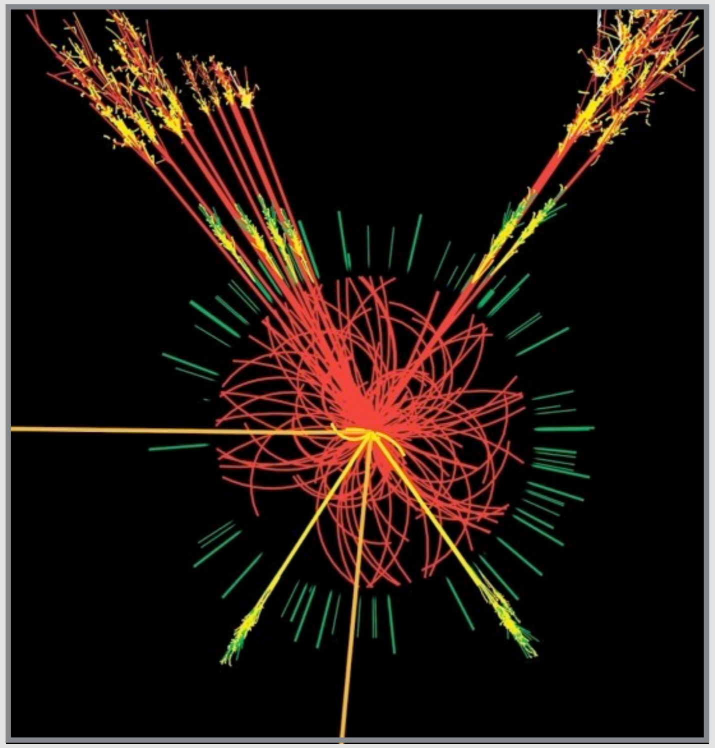
- Why is there more matter than antimatter in the universe?
- What happens inside neutron stars?
- What happened in the early universe?
- What happens in heavy ion collisions in particle accelerators?

Explore sign-problem free methods for lattice gauge theories

Dynamical problems:

What happens in heavy ion collisions

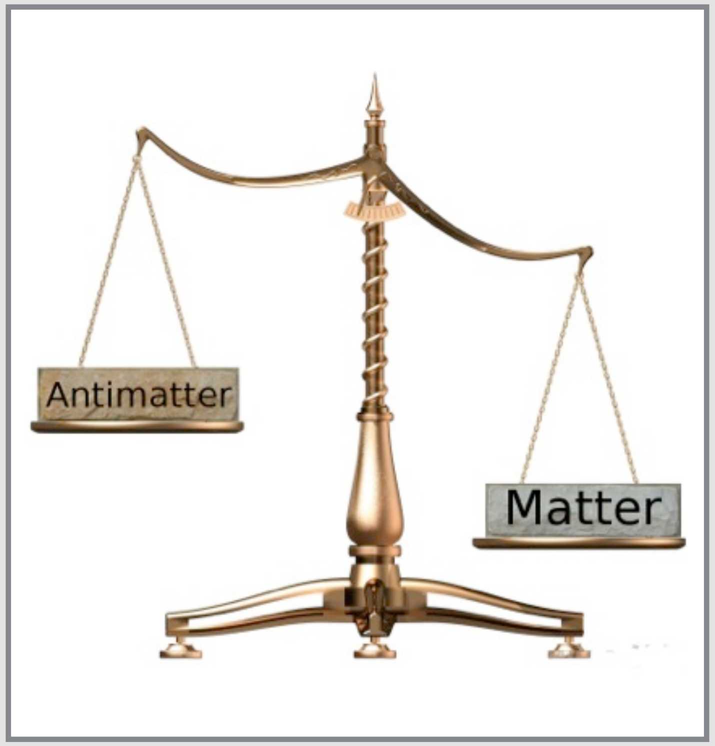
?



Topological terms:

How can we understand the large degree of CP violation in nature?

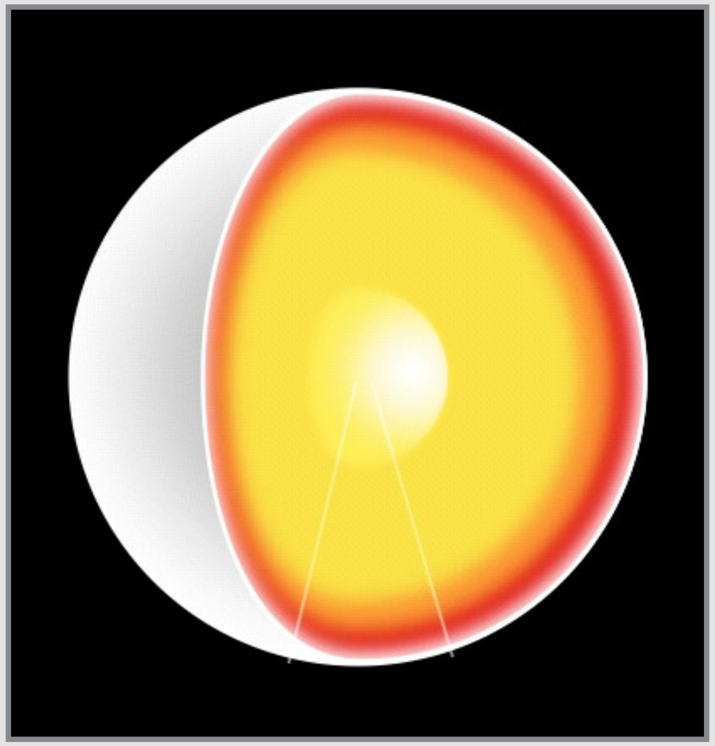
?



High baryon density:

What happens inside neutron stars

?



Gauge Theories:

Quest to find sign-problem free methods

- Quantum Simulations
- Numerical methods based on tensor network states

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Quest to find sign-problem free methods

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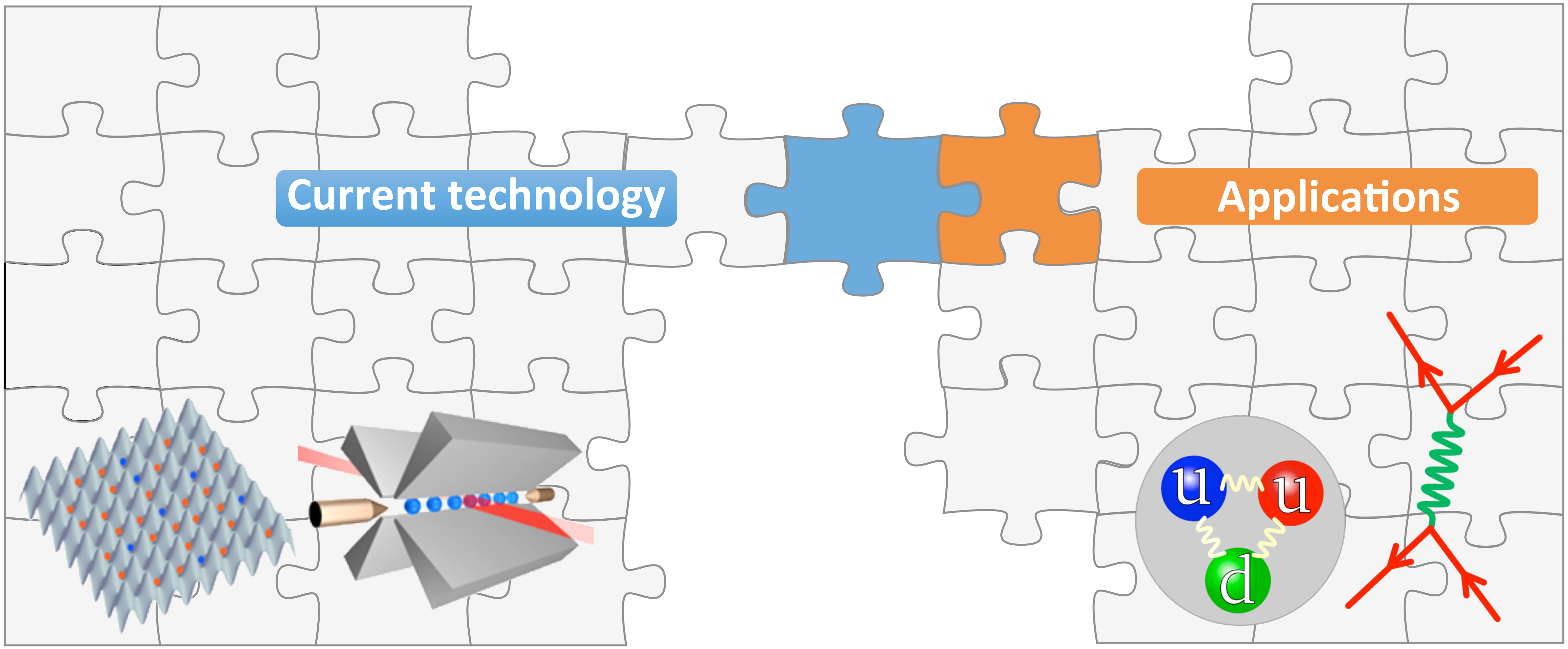
Two routes towards the same goal.
Both paths are actively explored.

This talk: Quantum simulations

Quantum information science

High energy physics



Current technology

Applications

Review Articles: Ann. Phys. 525, 777 (2013); Rep. Prog. Phys. 79, 014401 (2016); Contemporary Physics 57 388 (2016).

Short-term goal:

Develop a new type of
Quantum Simulator

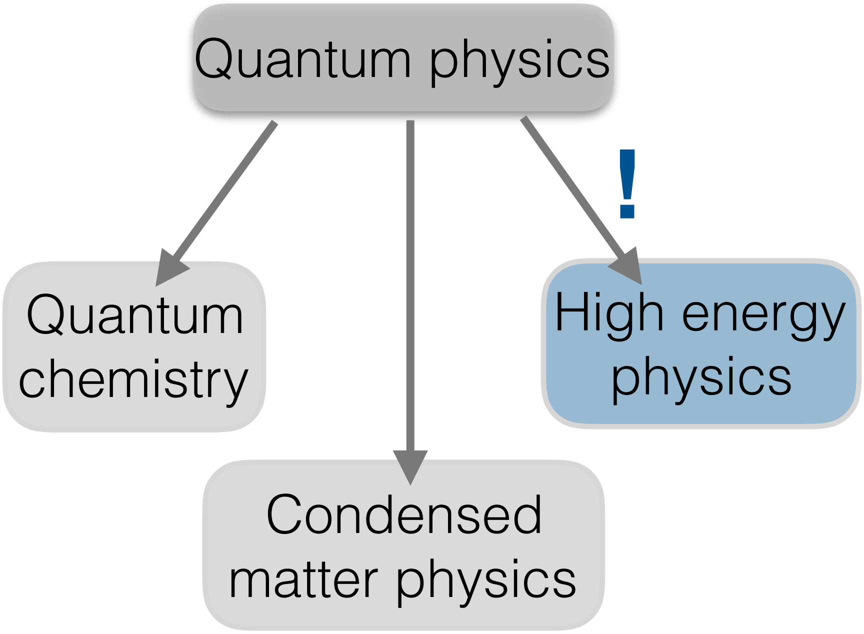
Perform proof-of-concept
Experiments

Long-term vision:

Simulate
Quantum Chromo Dynamics

Answer questions that
can not be tackled
numerically

Time



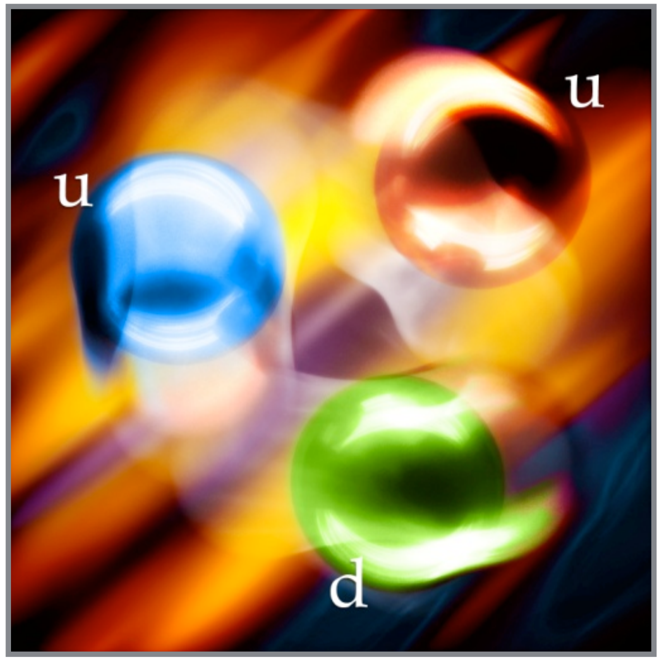
Quantum physics

Quantum chemistry

Condensed matter physics

!

High energy physics



Develop a new type of quantum simulator

Simulated states and dynamics must be gauge-invariant

Review Articles: Ann. Phys. 525, 777 (2013); Rep. Prog. Phys. 79, 014401 (2016); Contemporary Physics 57 388 (2016).

Develop a new type of quantum simulator

Simulated states and dynamics must be gauge-invariant

Difficulty for realizing quantum simulations of lattice gauge theories:
Implement a quantum many-body Hamiltonian
and a large set of local constraints (‘Gauss law’, in the case of QED: $\nabla E(r) = \rho(r)$)

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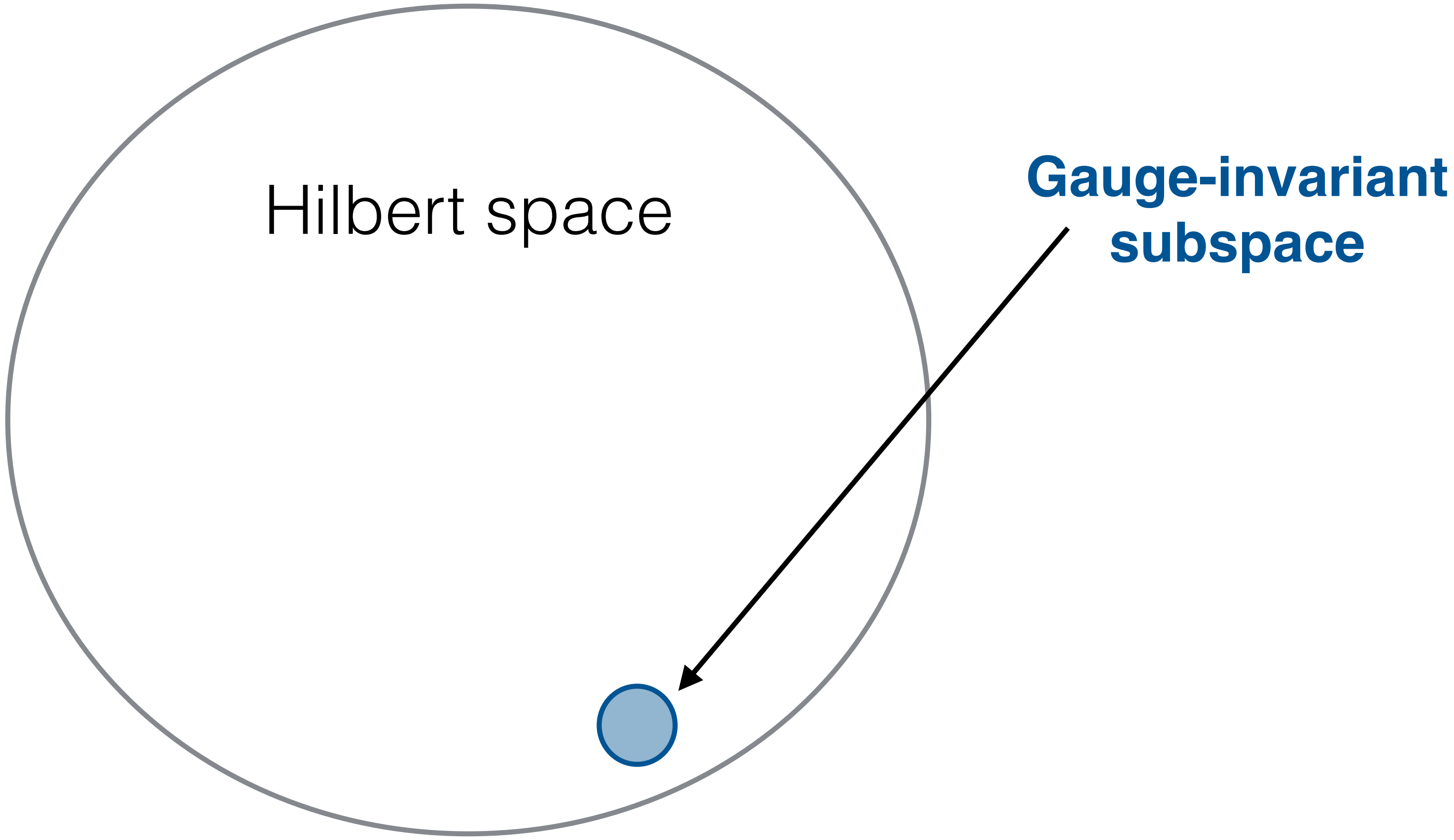

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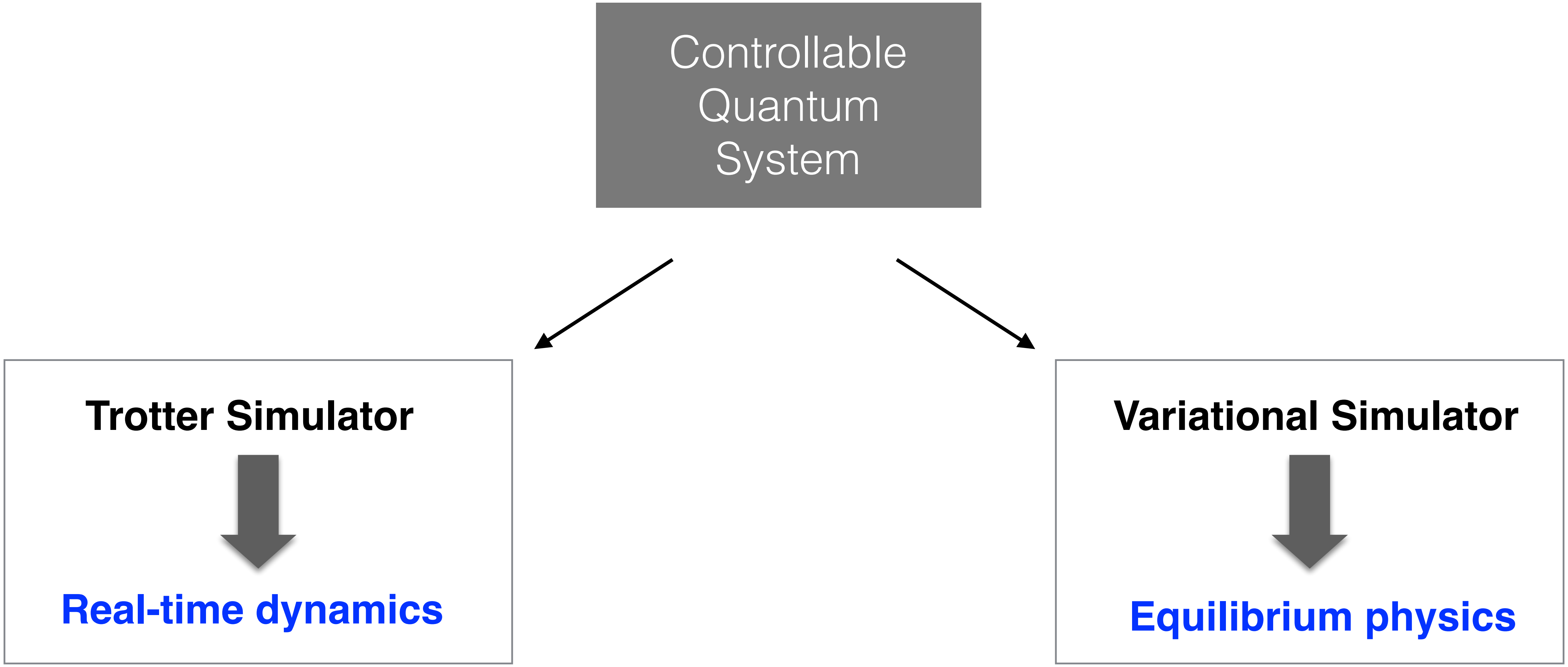

Controllable Quantum System

Trotter Simulator

Real-time dynamics

Variational Simulator

Equilibrium physics

Nature 534, 516 (2016).

1D-QED:
Pair creation

In preparation (2018).

1D-QED:
Parity breaking phase transiton

QED in (1+1) dimensions

Electromagnetic fields:

Vector potential: $A_0(x), A_1(x)$

Electric field: $E(x) = \partial_0 A_1(x)$

$[E(x), A_1(x')] = -i\delta(x - x')$

QED in (1+1) dimensions

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Matter fields:

$$\Psi(x) = \left(\begin{array}{c} \Psi_1(x) \\ \Psi_2(x) \end{array} \right)$$

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Matter fields:

$$\Psi(x) = \left(\begin{array}{c} \Psi_1(x) \\ \Psi_2(x) \end{array} \right)$$

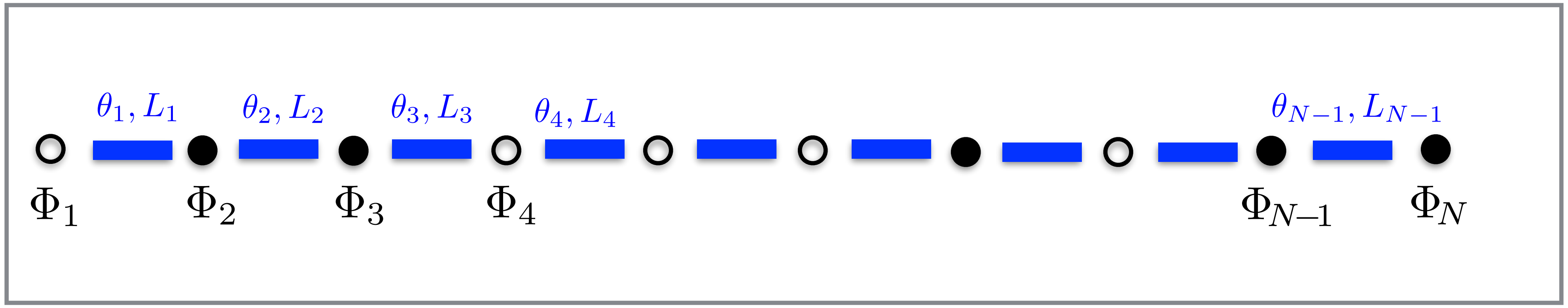
Hamiltonian:

$$H_{\mathrm{cont}} = \int dx \left[-i\Psi^\dagger(x)\gamma^1 \left(\delta_1 - igA_1\right)\Psi(x) + m\Psi^\dagger(x)\Psi(x) + \frac{1}{2}E^2(x) \right]$$

$\gamma_1 = -i\sigma_y$ coupling strength (charge) Fermion mass

The lattice Schwinger Model

The lattice Schwinger Model



Continuum

Vector potential $A_1(x)$

Electric field $E(x)$

$$[E(x), A_1(x')] = -i\delta(x - x')$$

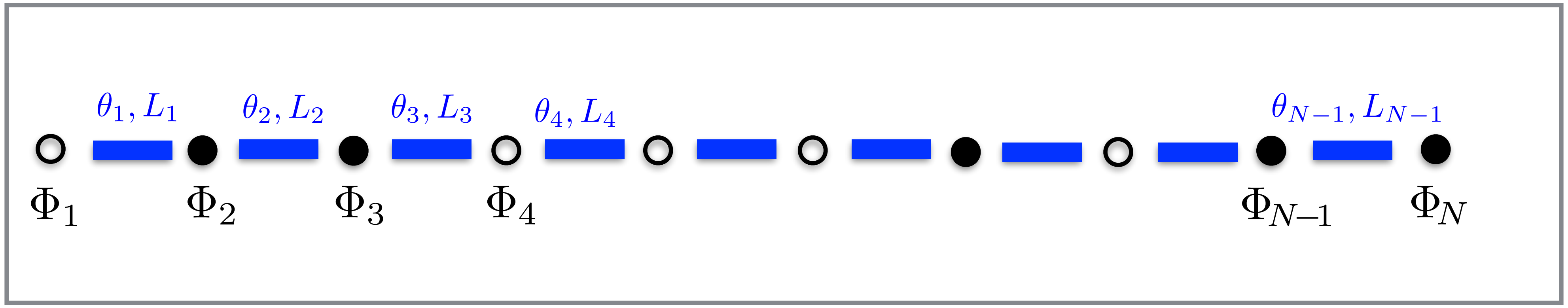
Lattice

$$\theta_n = agA_1(x_n)$$

$$L_n = \frac{1}{g}E(x_n)$$

$$[\theta_n, L_m] = i\delta_{n,m}$$

The lattice Schwinger Model



Continuum

Vector potential $A_1(x)$

Electric field $E(x)$

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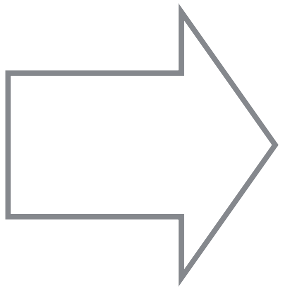
Dirac spinor

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$



Lattice

$$\theta_n = agA_1(x_n)$$

$$L_n = \frac{1}{g}E(x_n)$$

$$[\theta_n, L_m] = i\delta_{n,m}$$

odd lattice sites:

$$\Phi_n = \sqrt{a}\Psi_1(x_n)$$

even lattice sites:

$$\Phi_n = \sqrt{a}\Psi_2(x_n)$$

Hamiltonian formulation of the Schwinger model:

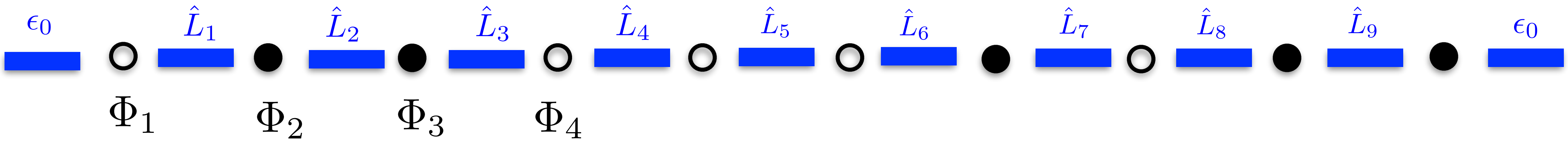
J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

$$\hat{H} = -iw \sum_{n=1}^{N-1} \left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{H.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n$$

The dynamics is constraint by the Gauss law:

In the continuum in 3D: $\nabla E = \rho$

Here: $\hat{L}_n - \hat{L}_{n-1} = \hat{\Phi}_n^\dagger \hat{\Phi} - \frac{1}{2} \left[1 - (-1)^n \right]$



One-dimensional QED

on a trapped ion quantum computer

We explore:

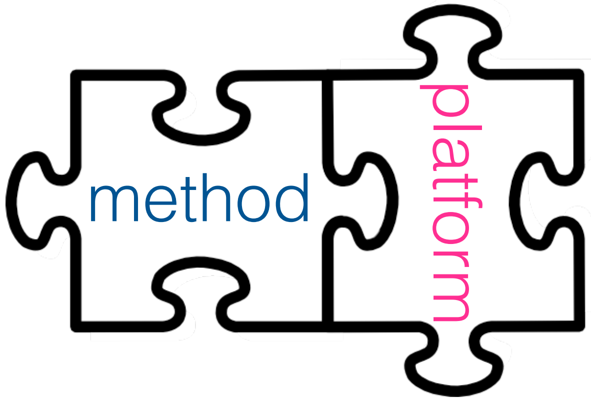
- Coherent real-time dynamics of particle-antiparticle creation
- Entanglement generation during pair creation

First experiment:

Real-time dynamics of lattice gauge theories on a few-qubit quantum computer
E. Martinez*, C. Muschik* et al, Nature 534, 516 (2016).

U(1) Wilson lattice gauge theories in digital quantum simulators
C. Muschik et al. New J. Phys. 19 103020 (2017).

Physics world: one of the top ten Breakthroughs in physics 2016



Efficient implementation

Our approach

Our scheme:

(1) Mapping of the Schwinger Hamiltonian to a pure spin model with long range interactions

(2) Realization of the required interactions with an efficient digital simulation scheme using “shaking methods”.

Important features of the scheme

- Exact gauge invariance at all energy scales (by construction)
- Very efficient use of resources

Elimination of the gauge fields → Pure spin model with long-range interactions

The gauge fields don't appear explicitly in the encoded description. Instead, they act in the form of a non-local interaction that corresponds to the Coulomb-interaction between the simulated charged particles.

The Schwinger model as exotic spin model

$$\hat{H}_S = w\sum_i \left(\hat{\sigma}_i^+\hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+\hat{\sigma}_i^-\right)$$

particle - antiparticle creation/annihilation

$$+\ J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z$$

long - range interaction

$$+\ m\sum_i c_i\hat{\sigma}_i^z + J\sum_i \tilde{c}_i\hat{\sigma}_i^z$$

effective particle masses

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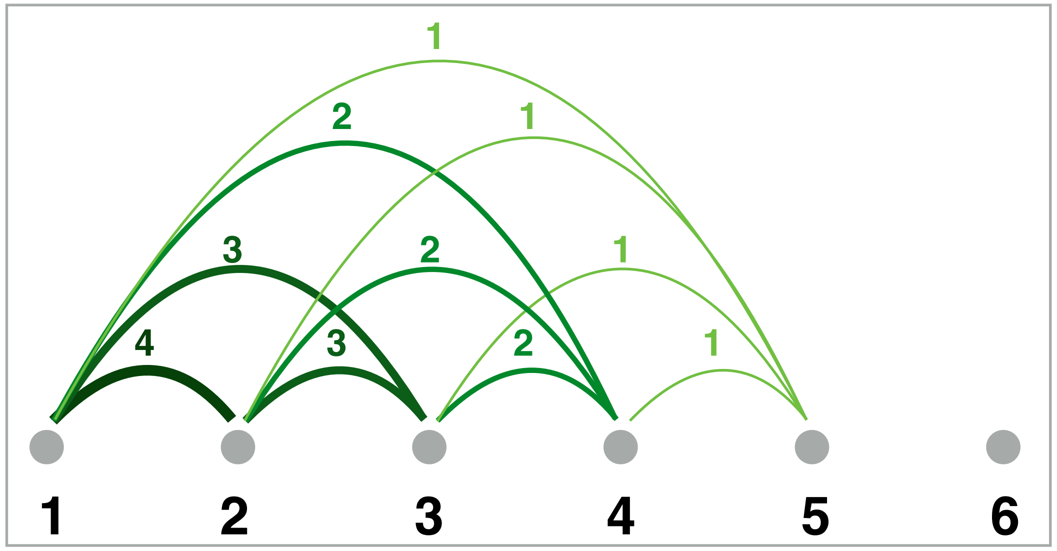

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$$+ \boxed{J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z}$$

long - range interaction

$$+ m\sum_i c_i\hat{\sigma}_i^z + J\sum_i \tilde{c}_i\hat{\sigma}_i^z$$

effective particle masses

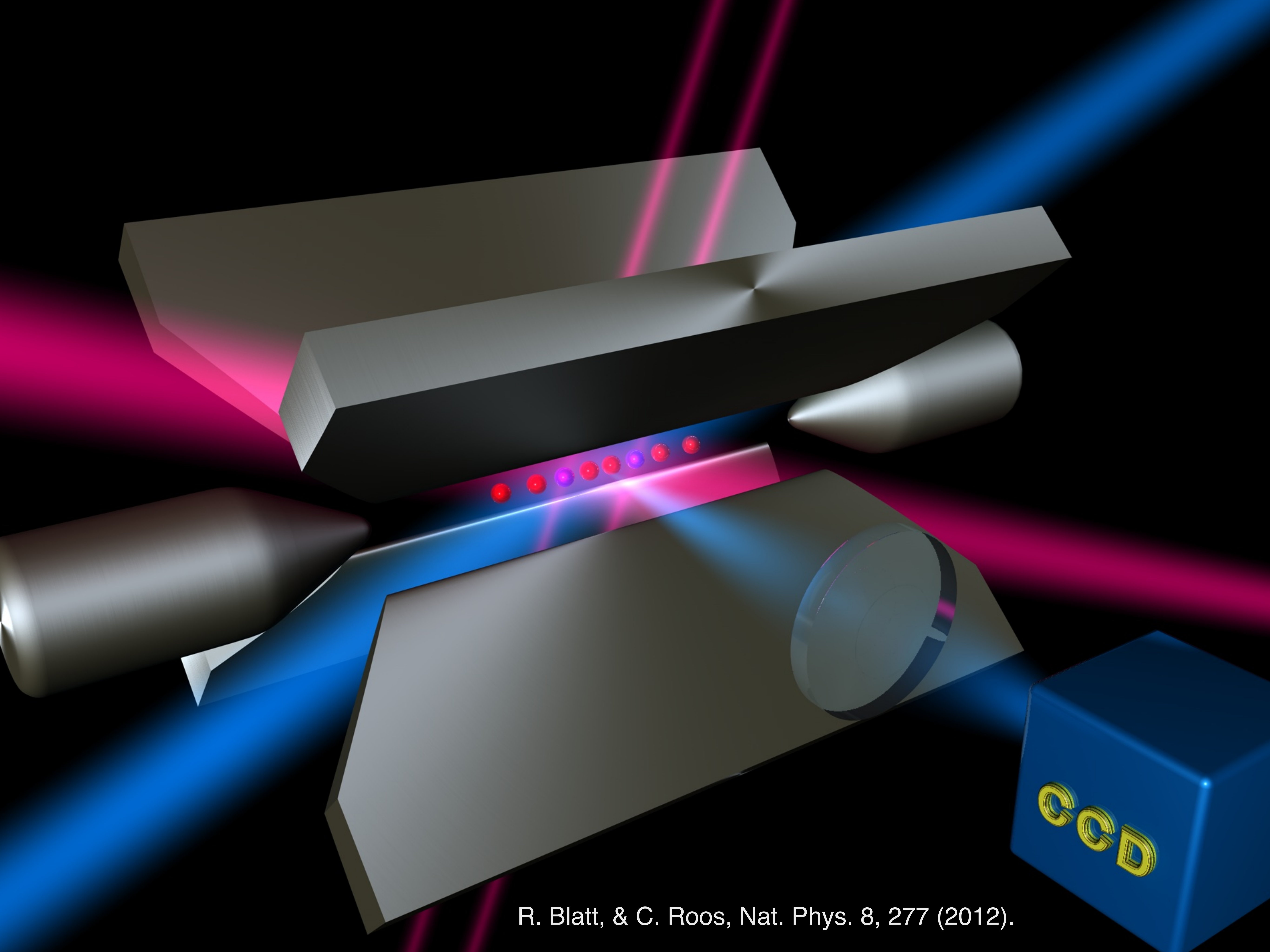
- Efficient implementation on an ion-quantum computer
- N spins simulate N matter fields and N-1 gauge fields
- Exotic spin interactions can be simulated efficiently: Digital scheme

Our toolbox

Ion trap quantum computers:

- Fast and accurate single qubit operations
- Entangling gates: Mølmer-Sørensen interaction

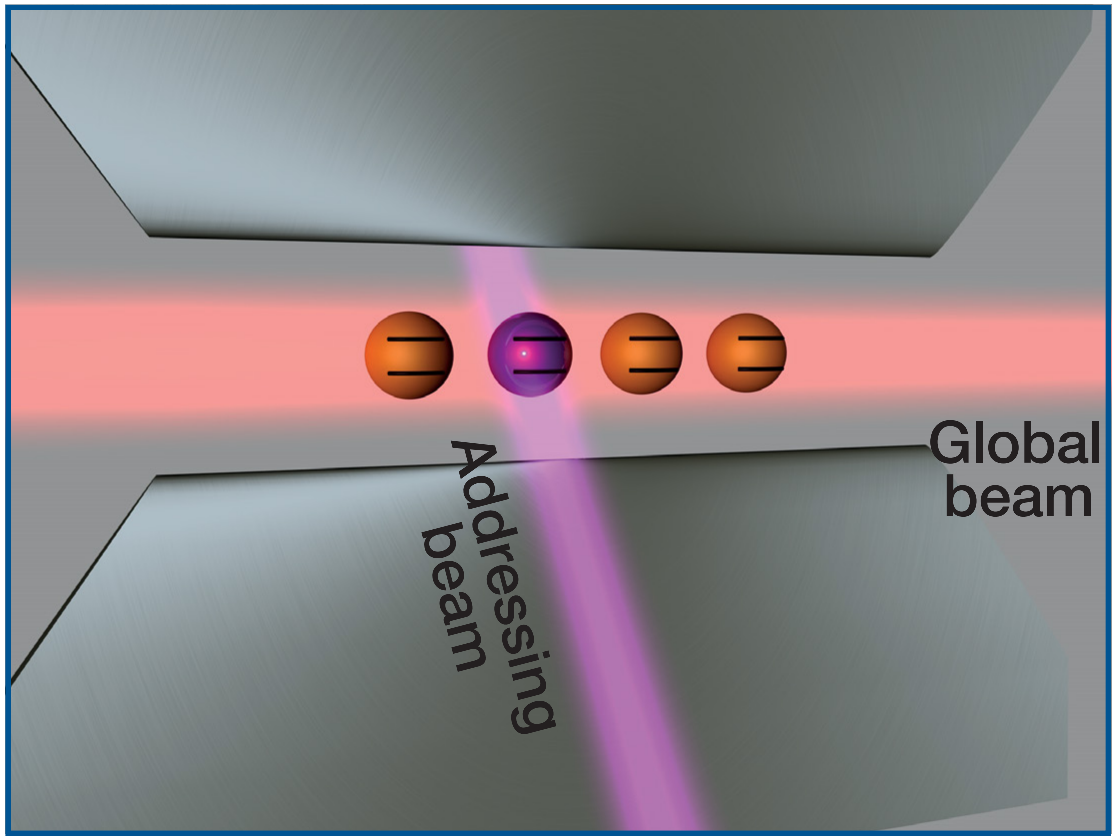
All-to-all 2-body interaction: $H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$



R. Blatt, & C. Roos, Nat. Phys. 8, 277 (2012).

Experiment

E. Martinez, P. Schindler, D. Nigg, A. Erhard, T. Monz, and R. Blatt



Tools for universal digital quantum simulation are available:

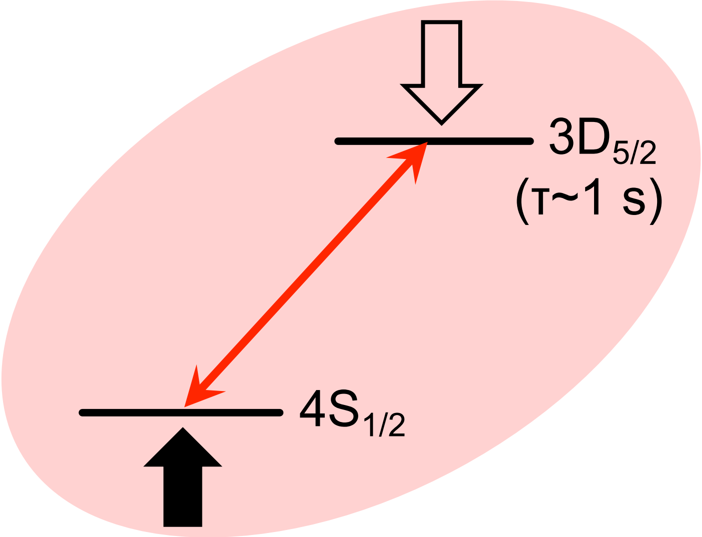
B. Lanyon, et al. Science 334, 57 (2011).

- High fidelity local rotations ✓
- Entangling gates ✓

Mølmer-Sørensen interaction

$$H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$$

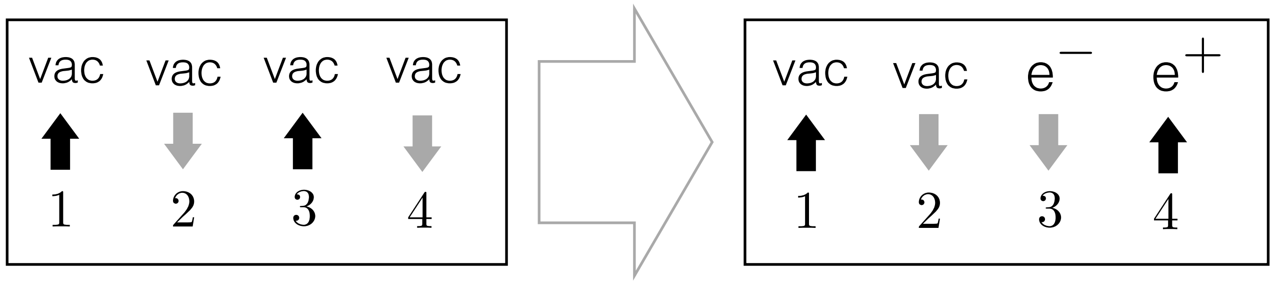
Qubit



Quantum Simulation of pair creation

Particle number density: $\nu(t) = \frac{1}{N} \sum_{n=1}^{N} \langle (-1)^n \sigma_n^z(t) + 1 \rangle$

Creation of a particle antiparticle pair:



$\nu = 0$ $\nu = 0.5$

Odd lattice sites:

$\underset{n}{\uparrow} \cong$ vac

$\underset{n}{\Downarrow} \cong$ e$^-$

Even lattice sites:

$\underset{n}{\uparrow} \cong$ e$^+$

$\Downarrow \cong$ vac

Schwinger Mechanism

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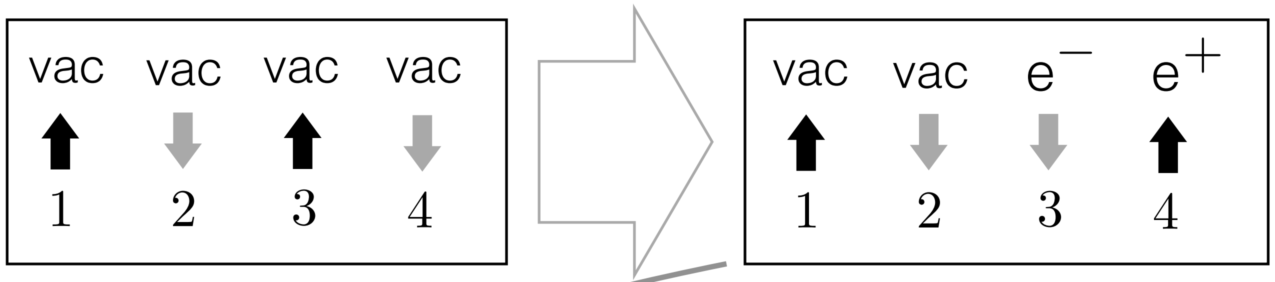
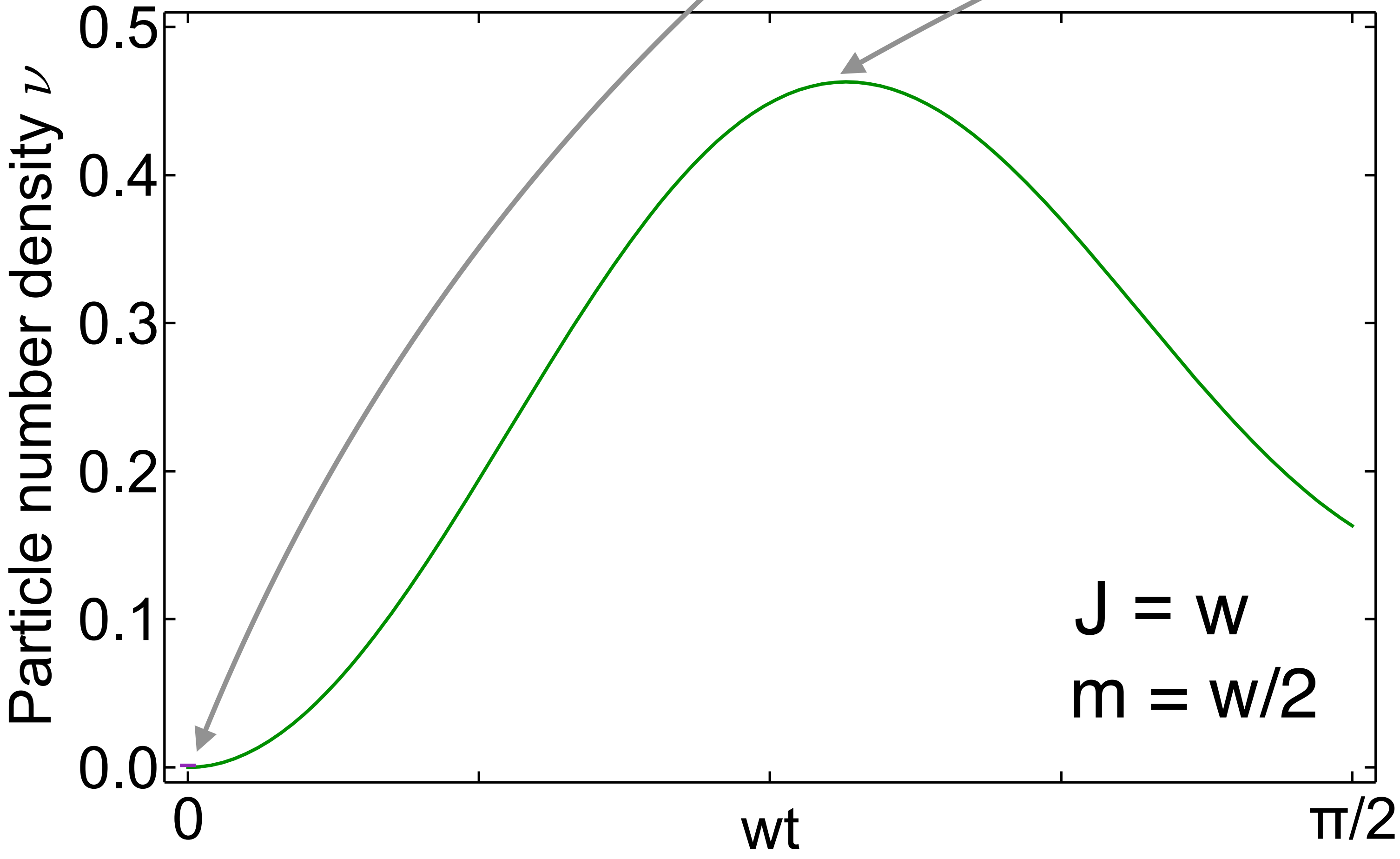


In the ideal case (N=4):



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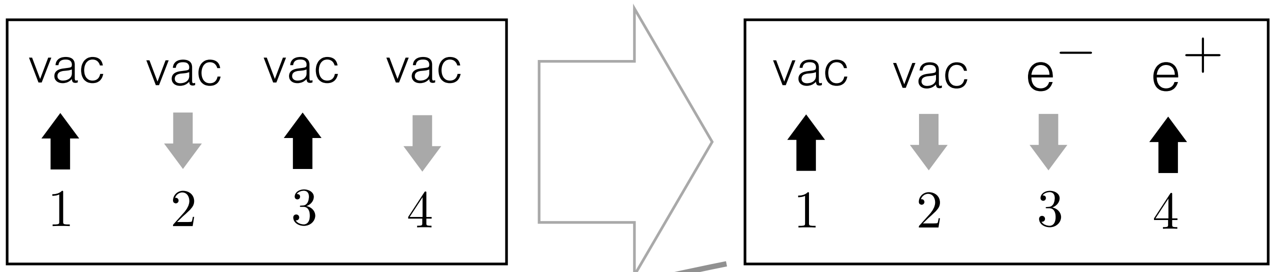

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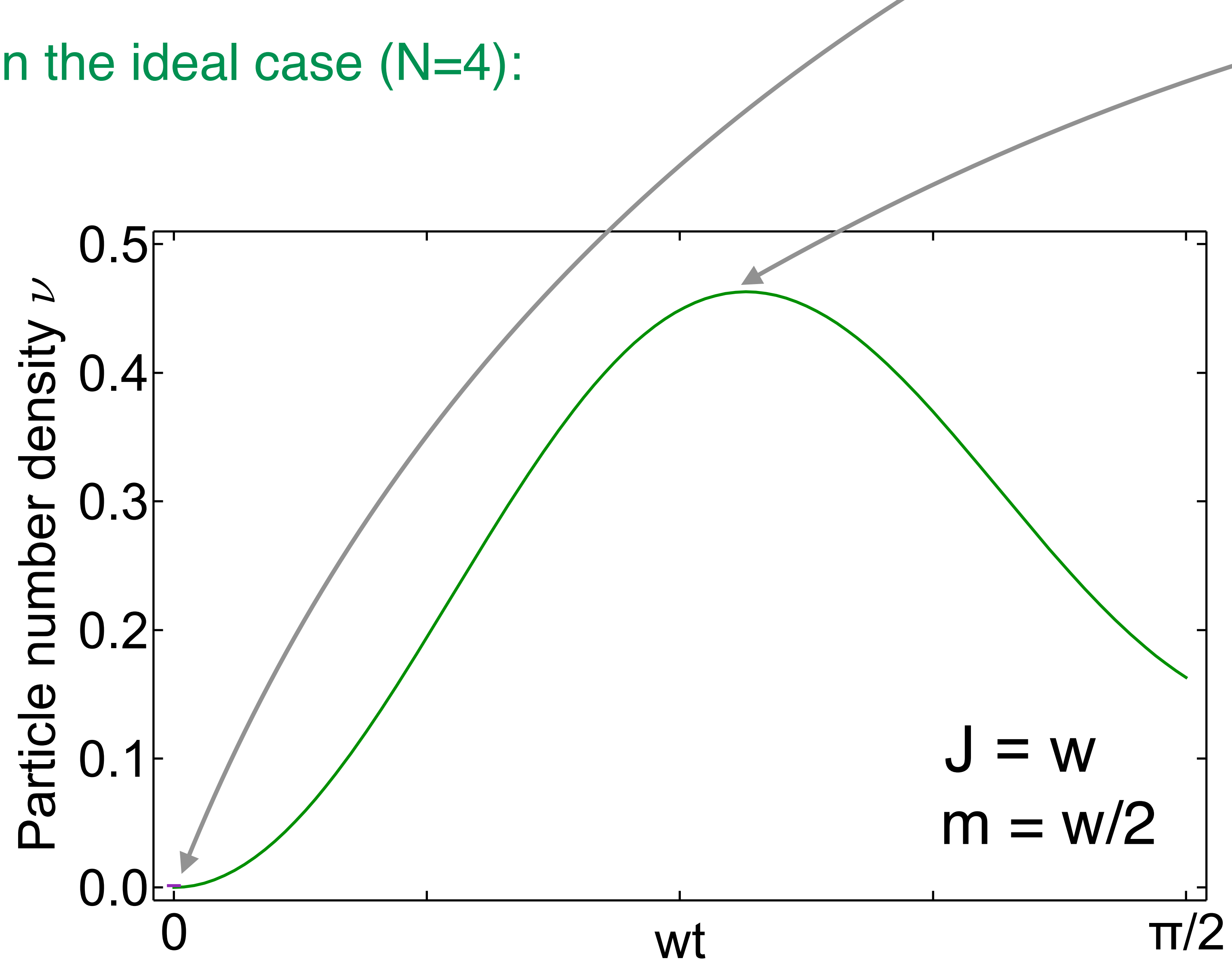

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$$+ J\sum_{i<j} c_{ij}\hat{\sigma}_i^z\hat{\sigma}_j^z$$

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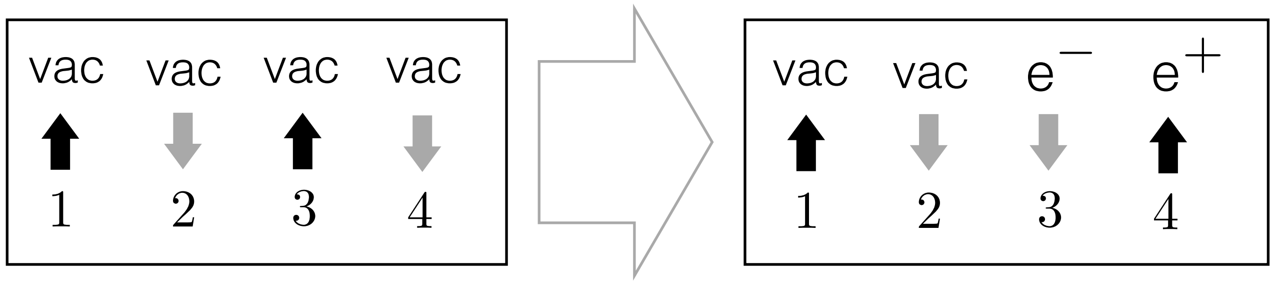


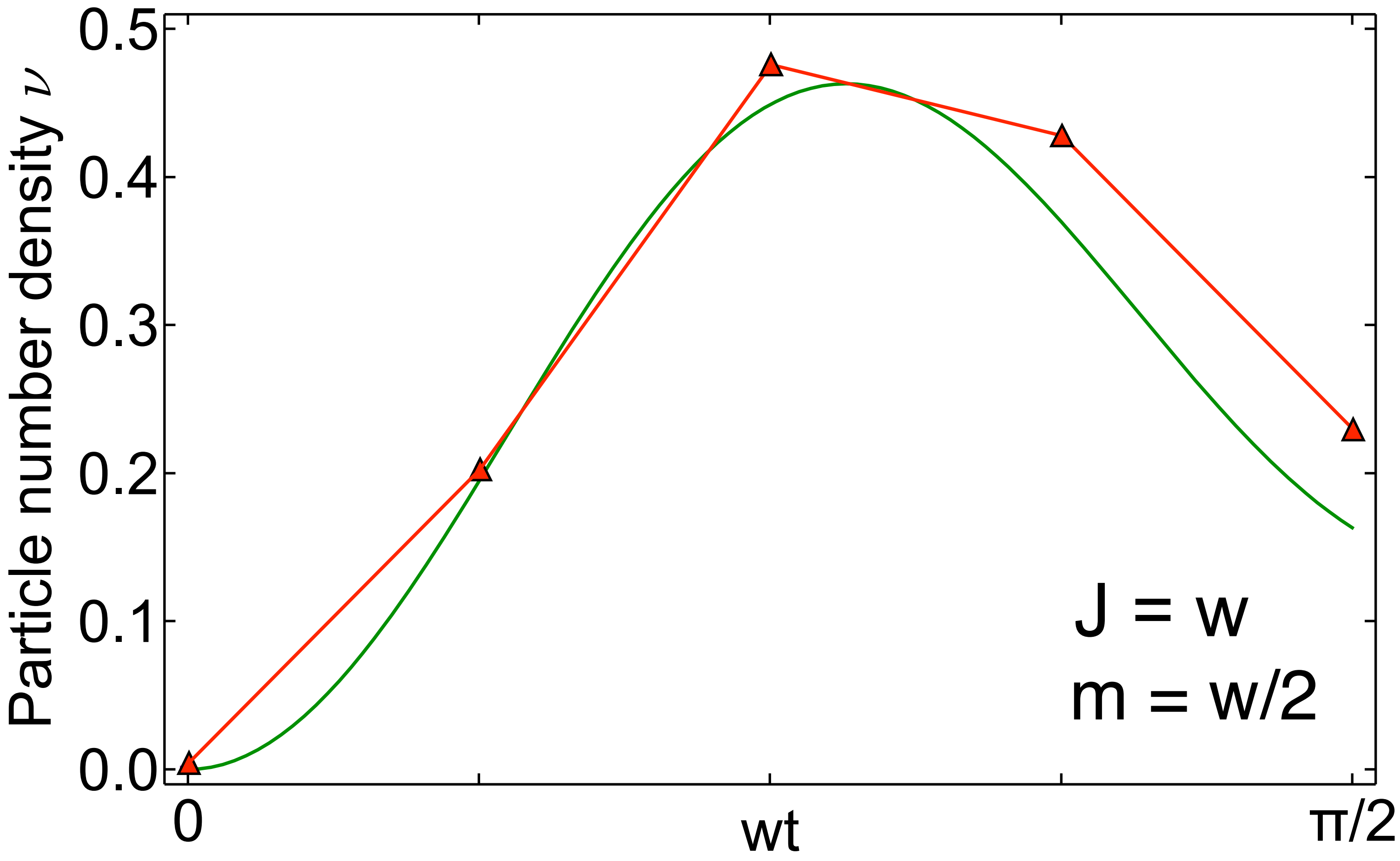
vac vac vac vac

1 2 3 4

vac vac e^- e^+

1 2 3 4

Including discretisation errors (N=4):



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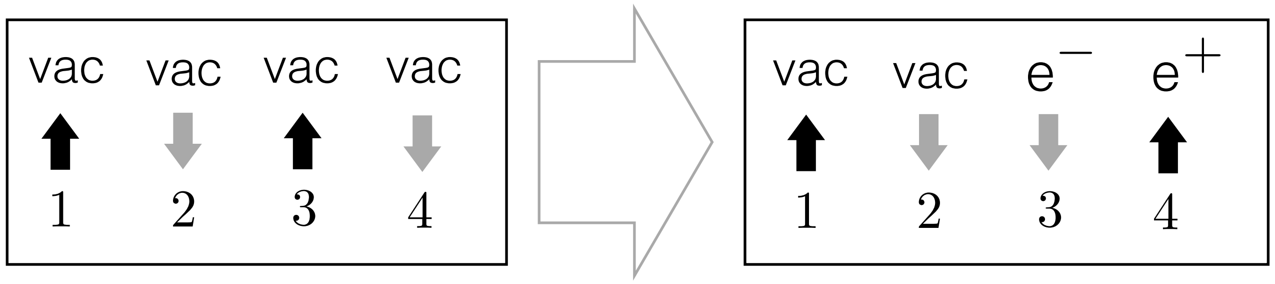
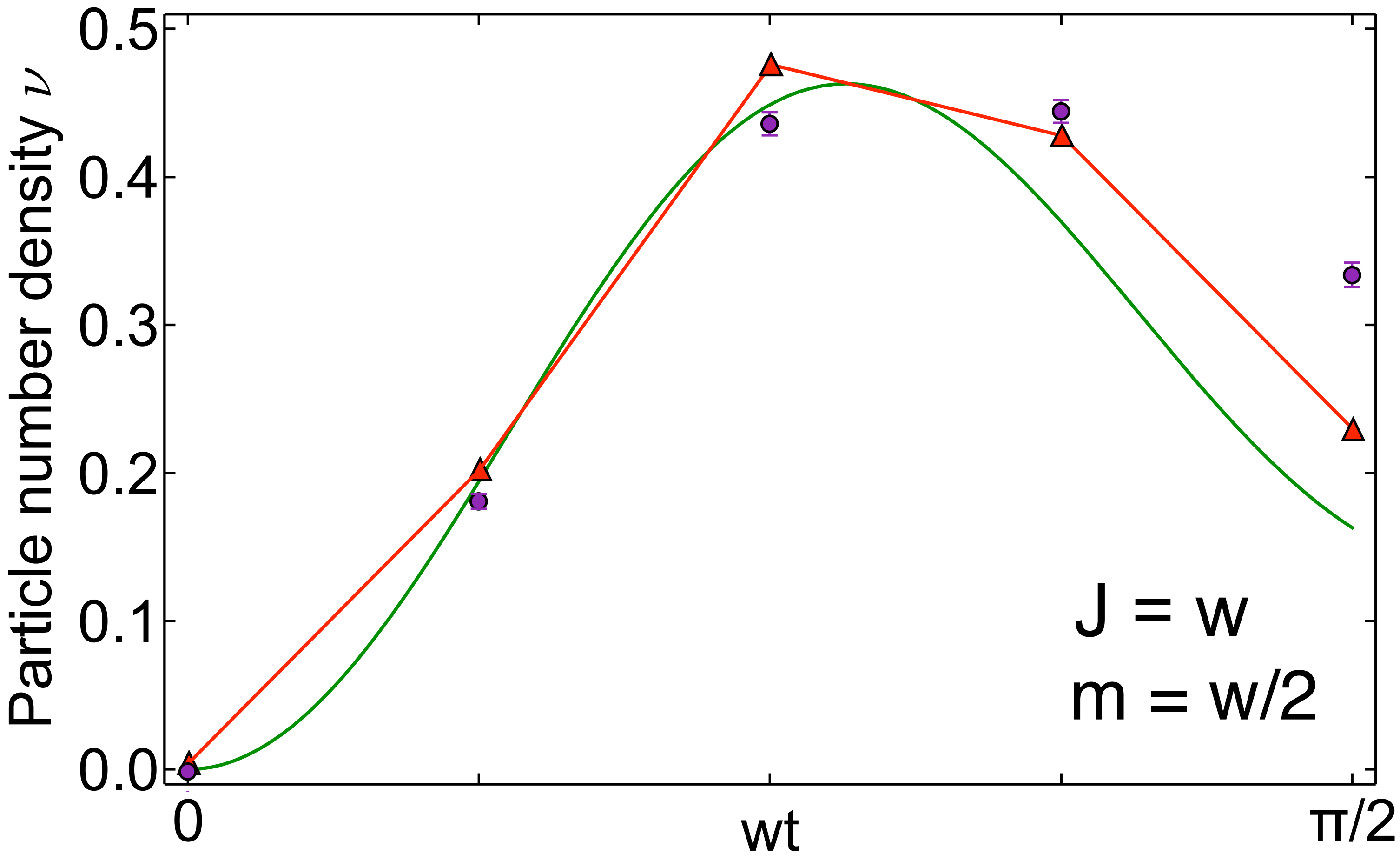


Experimental data (after postselection):



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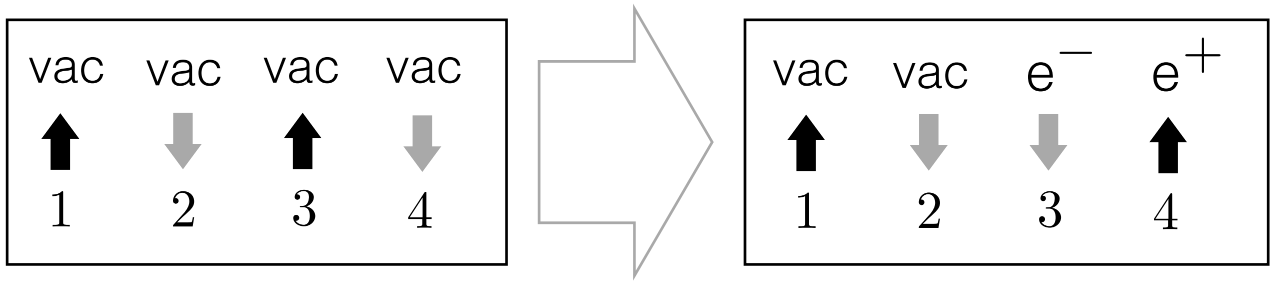
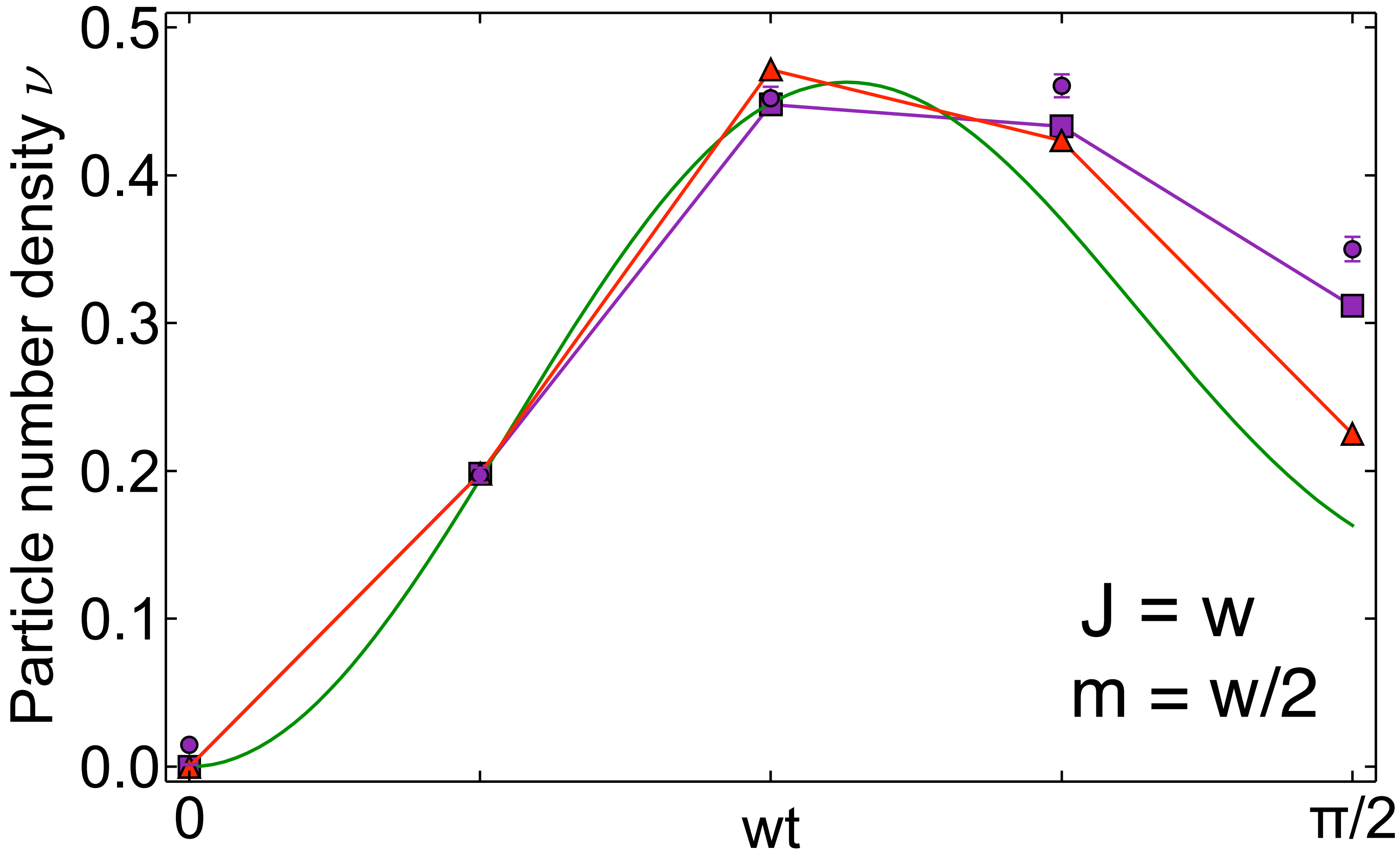
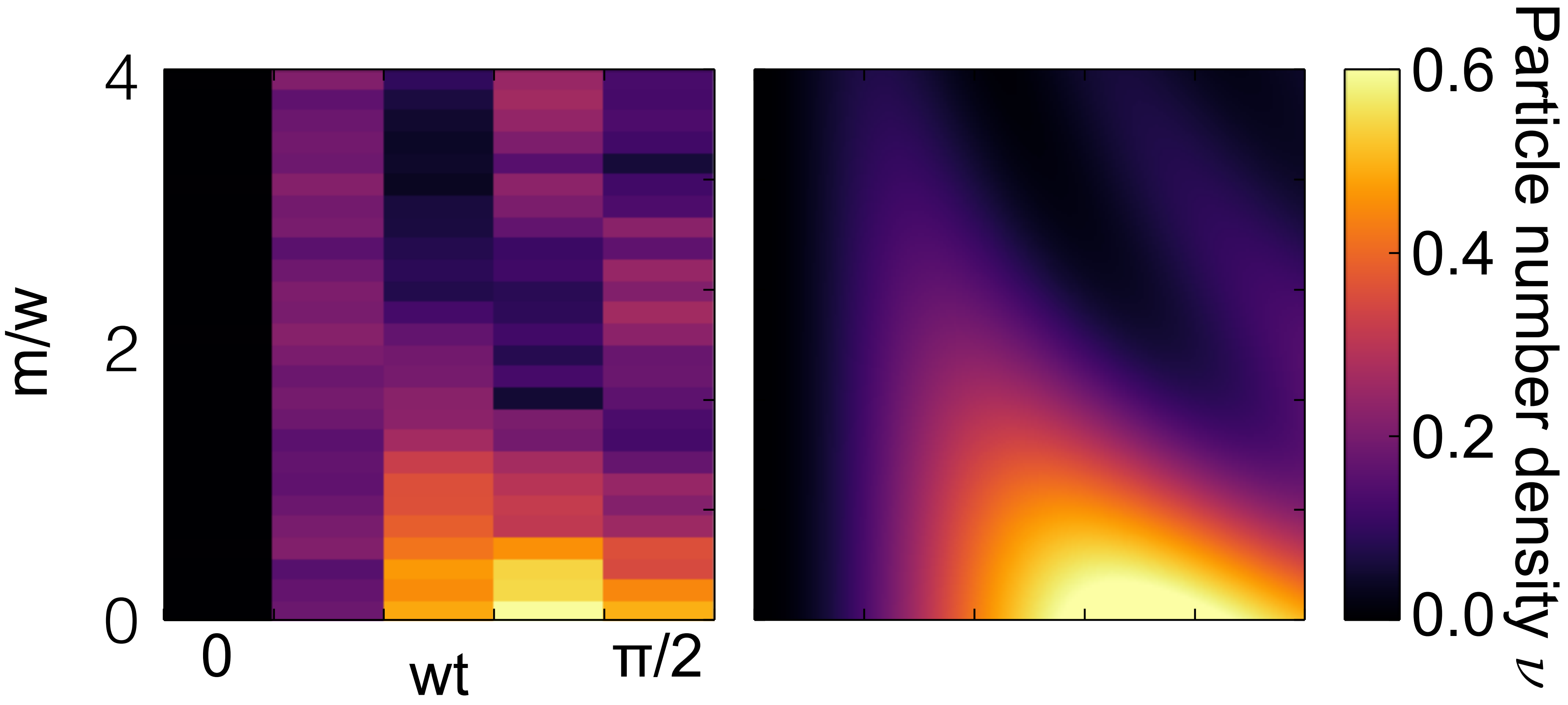


Simple error model (uncorrelated dephasing):



Schwinger Mechanism

Time evolution for different values of the particle mass m



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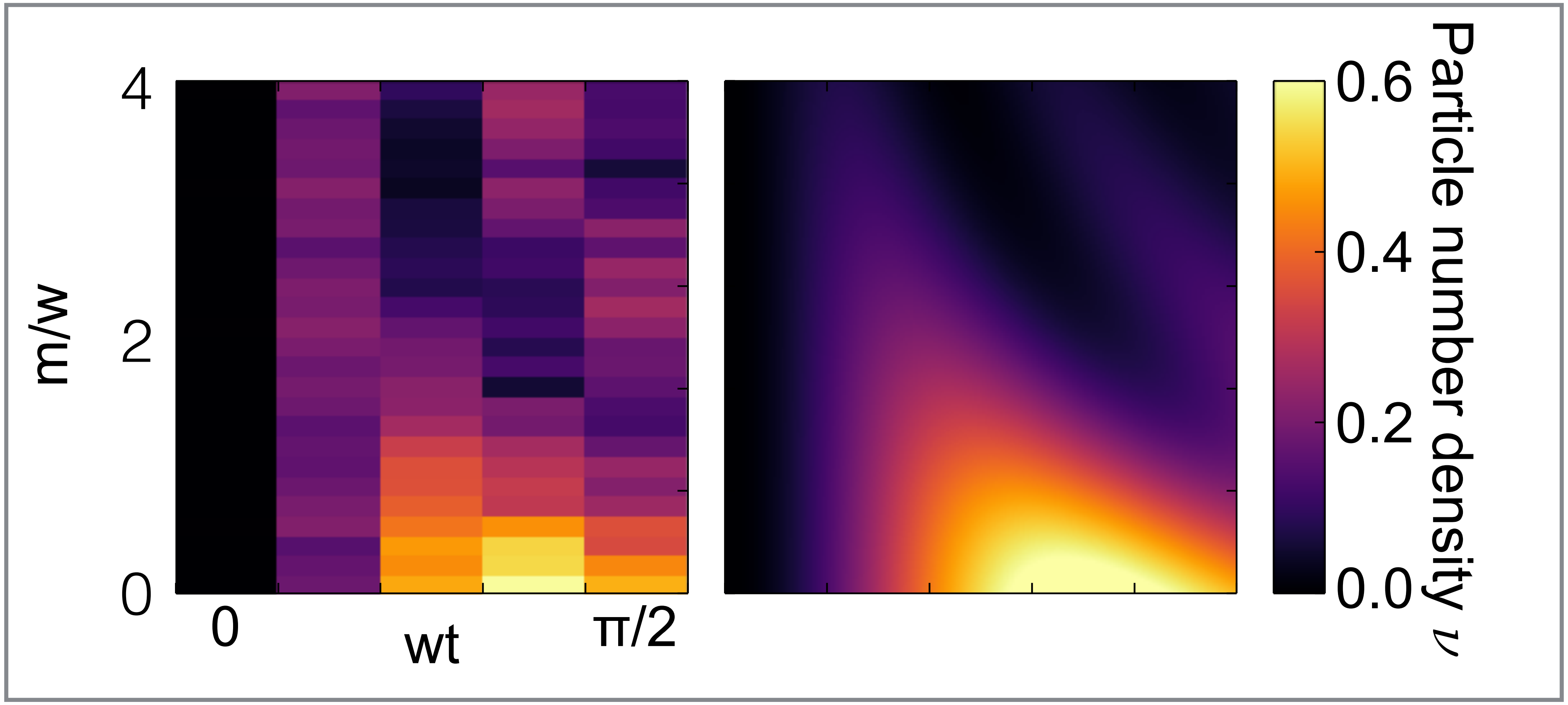
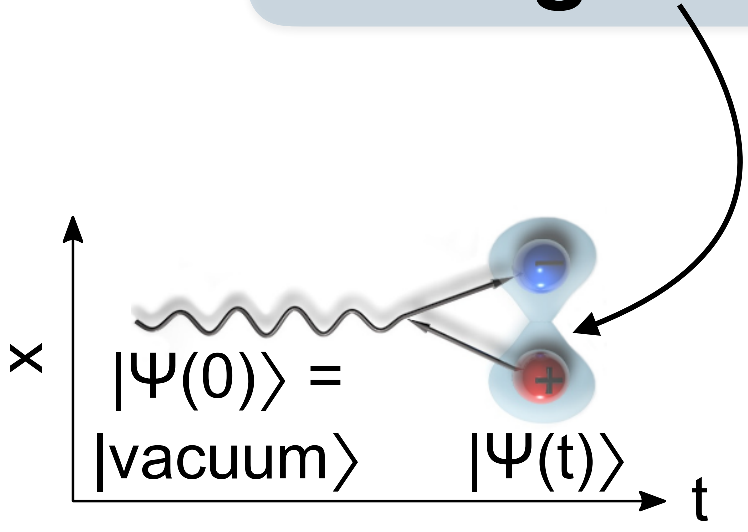


→ also: measurement of the vacuum persistence amplitude $|\langle \mathrm{vacuum}|\Psi(t)\rangle|^2$
see Nature 534, 516 (2016).

Entanglement in the Schwinger mechanism



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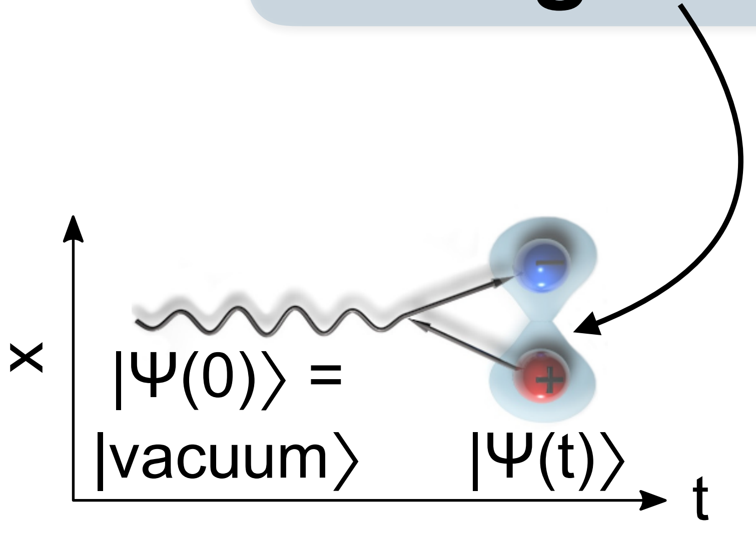
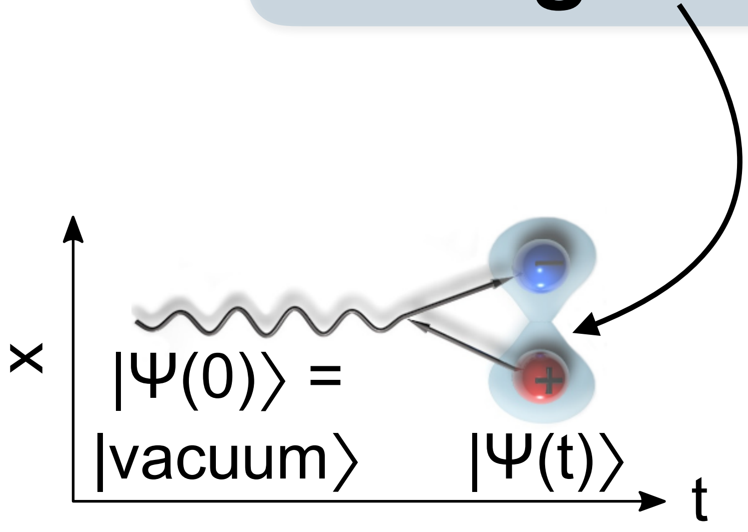


State tomography:
access to the full density matrix

Entanglement in the Schwinger mechanism



State tomography:
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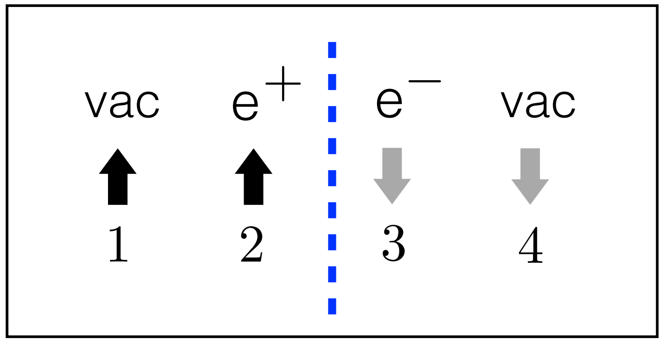
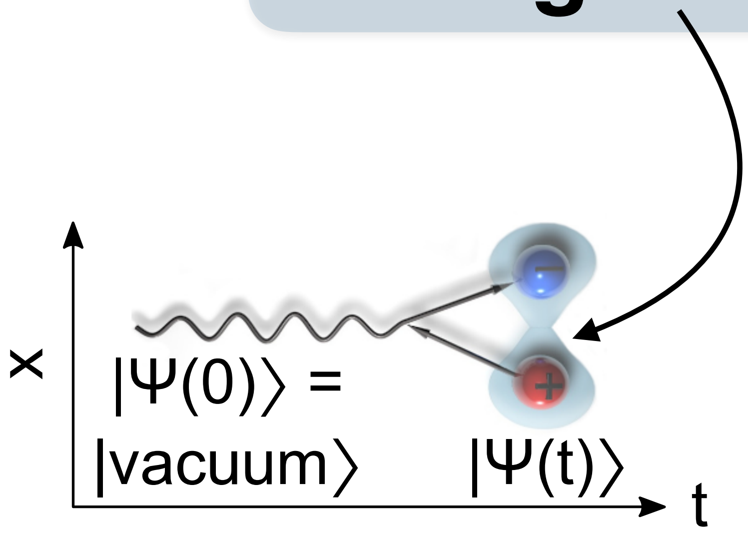
E_n : logarithmic negativity
evaluated with respect to this bipartition:



Entanglement between the two halves of the system.

Entanglement in the Schwinger mechanism



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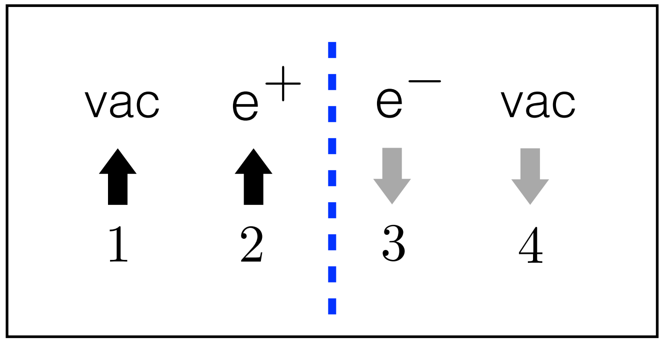

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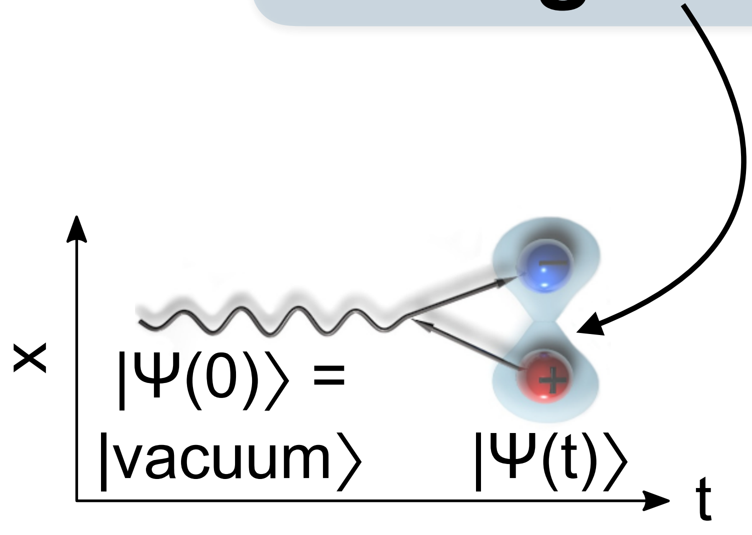

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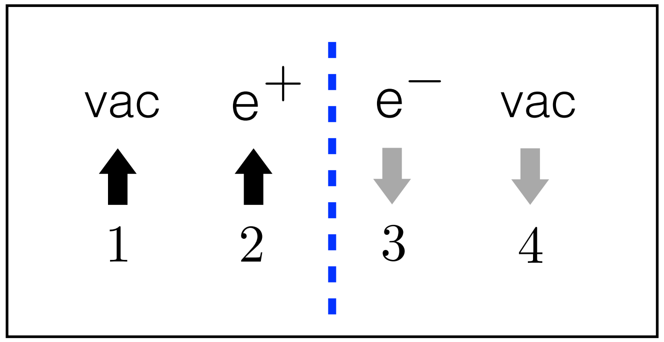

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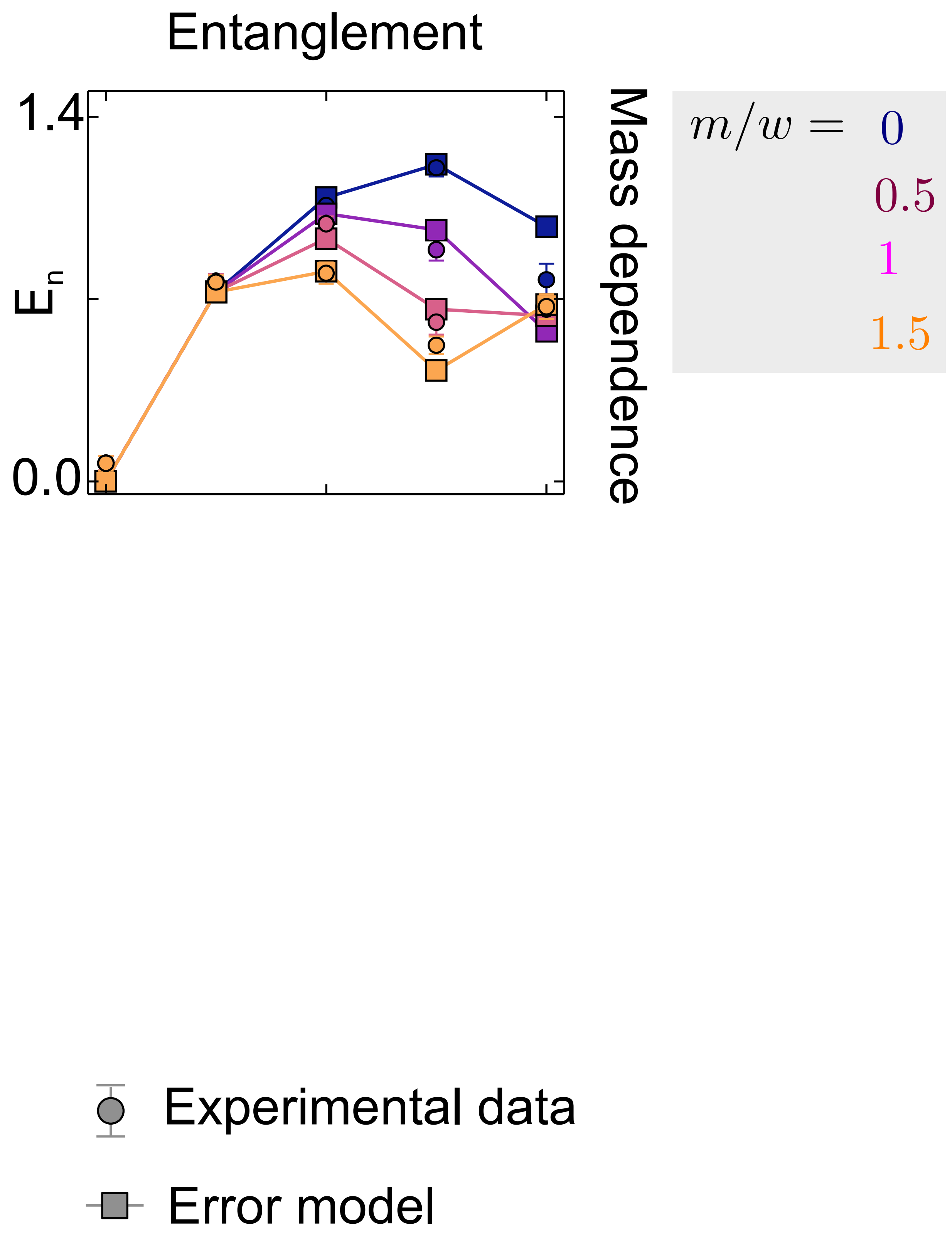

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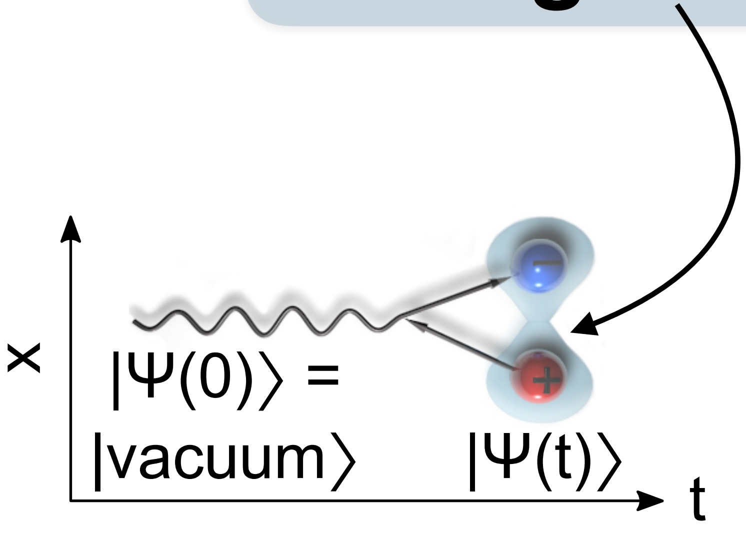

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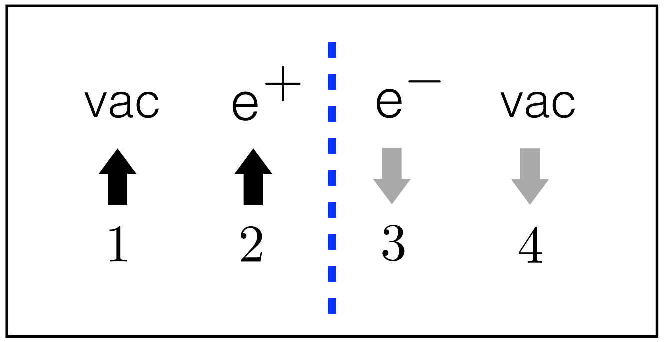
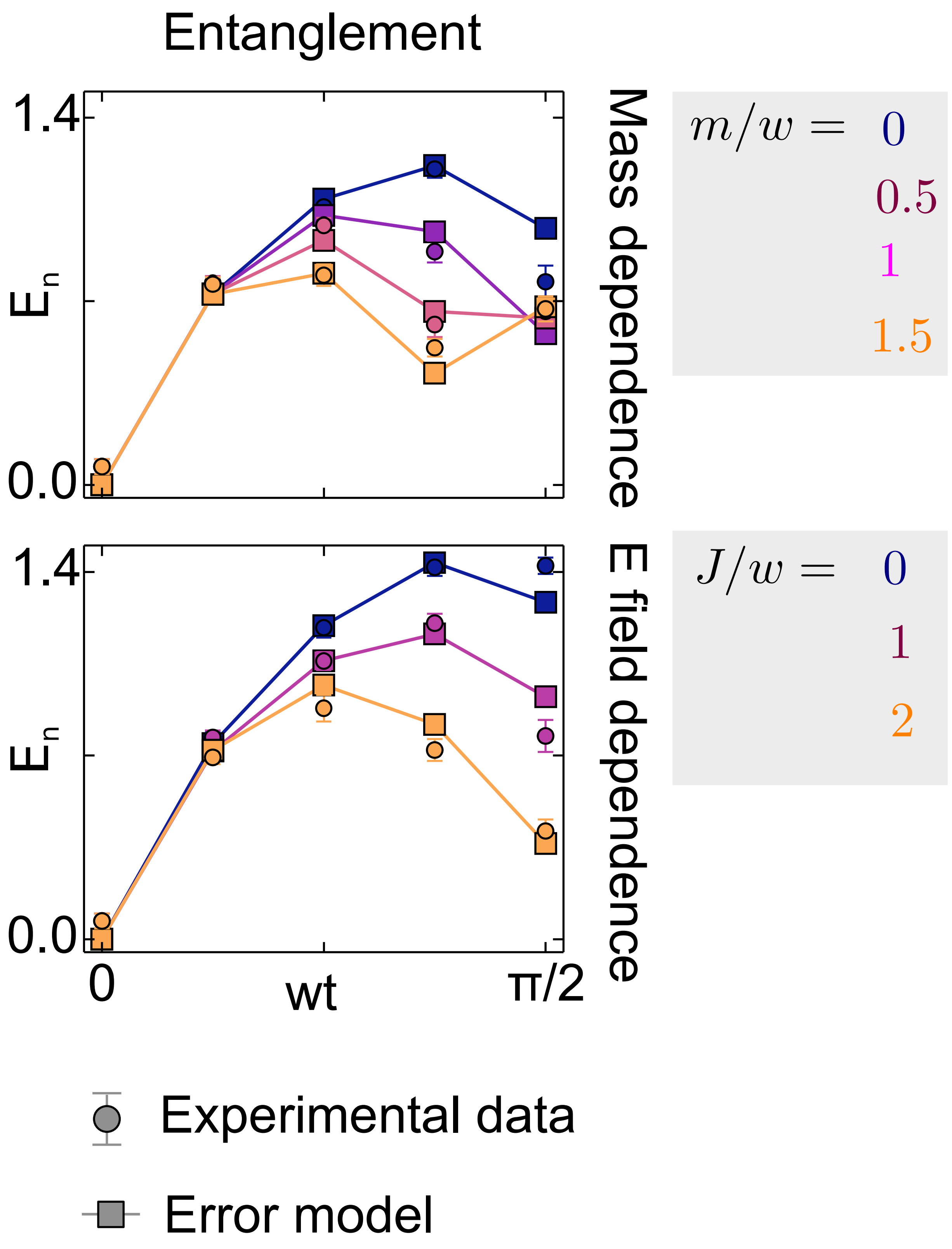
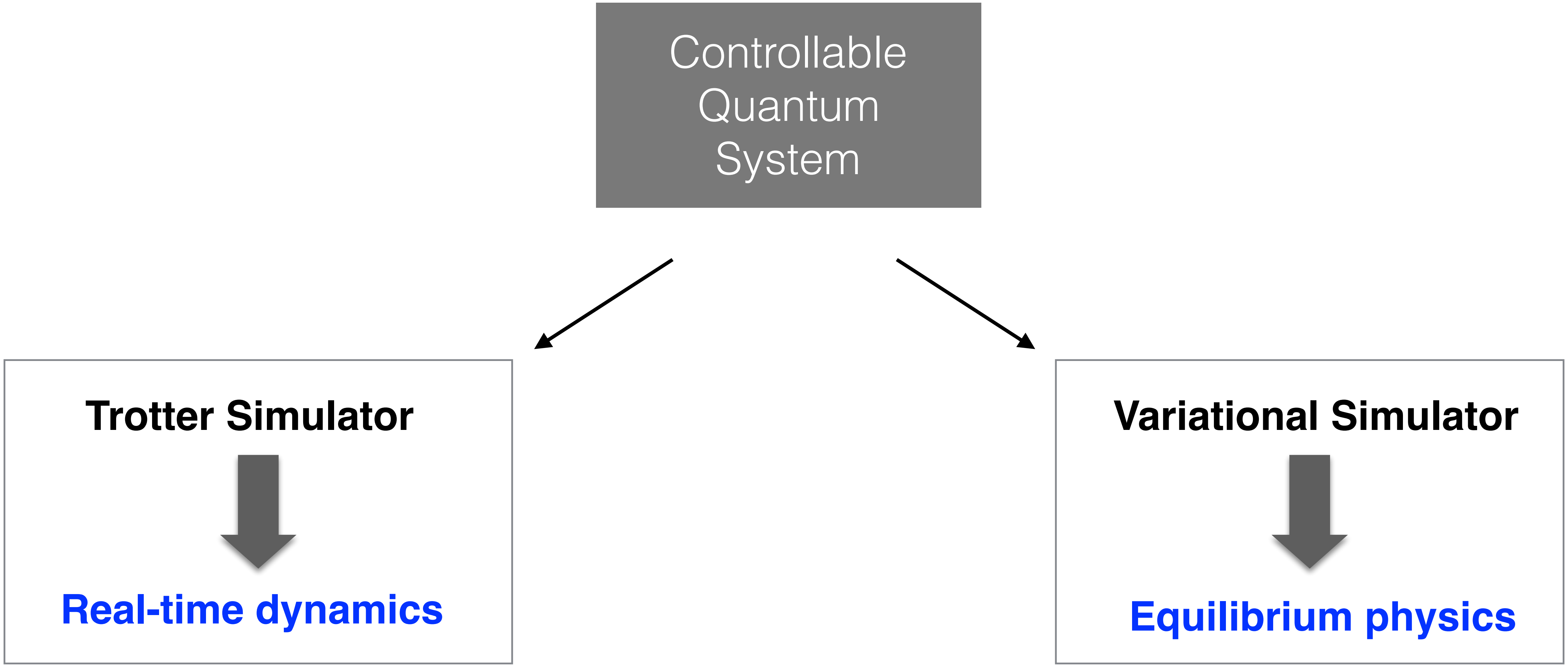


Entanglement between the two halves of the system.





Controllable Quantum System

Trotter Simulator

Real-time dynamics

Variational Simulator

Equilibrium physics

Nature 534, 516 (2016).

1D-QED:
Pair creation

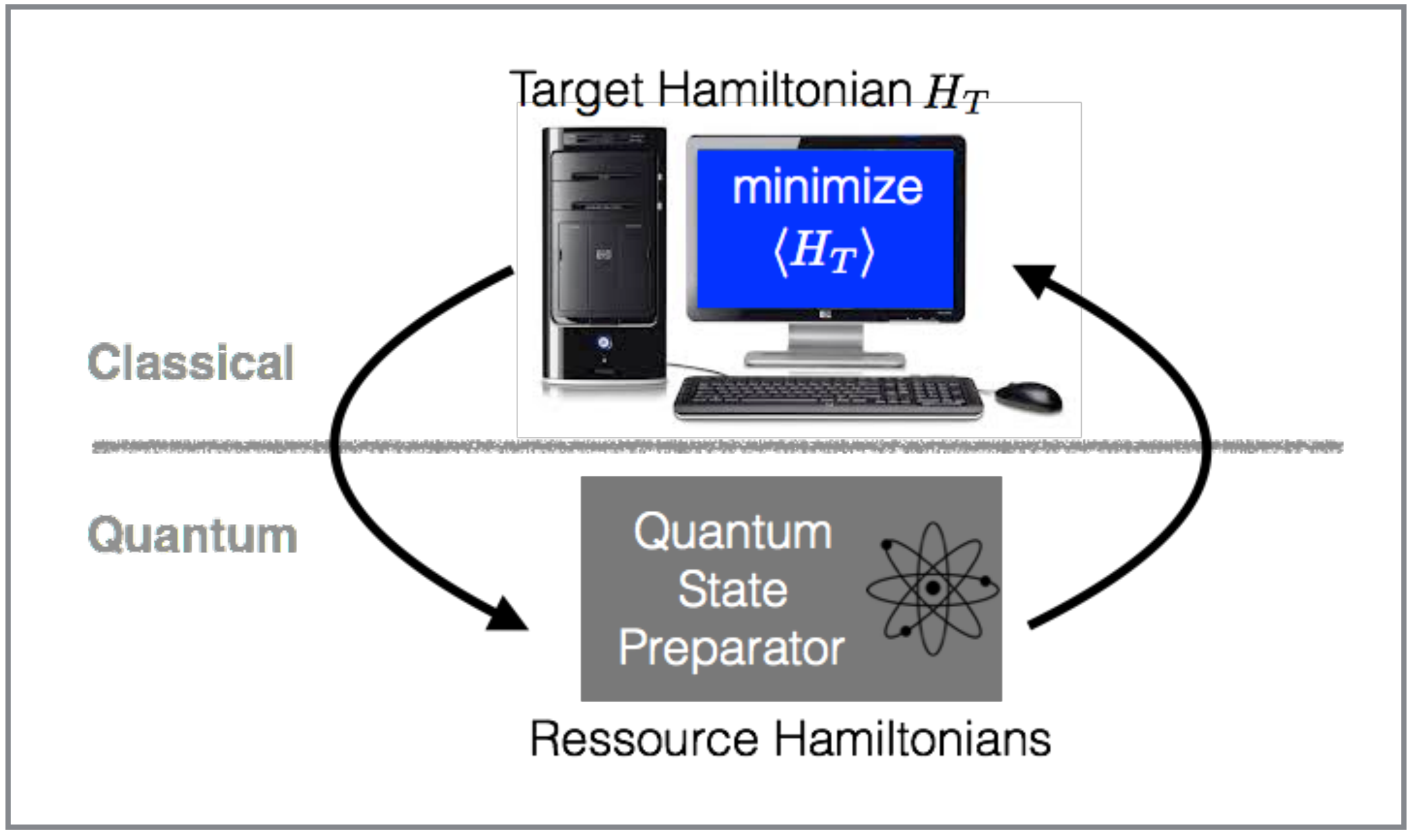
In preparation (2018).

1D-QED:
Coleman’s phase transiton

Variational Quantum Simulation

in preparation

IQI & IQC Institute for Quantum Computing



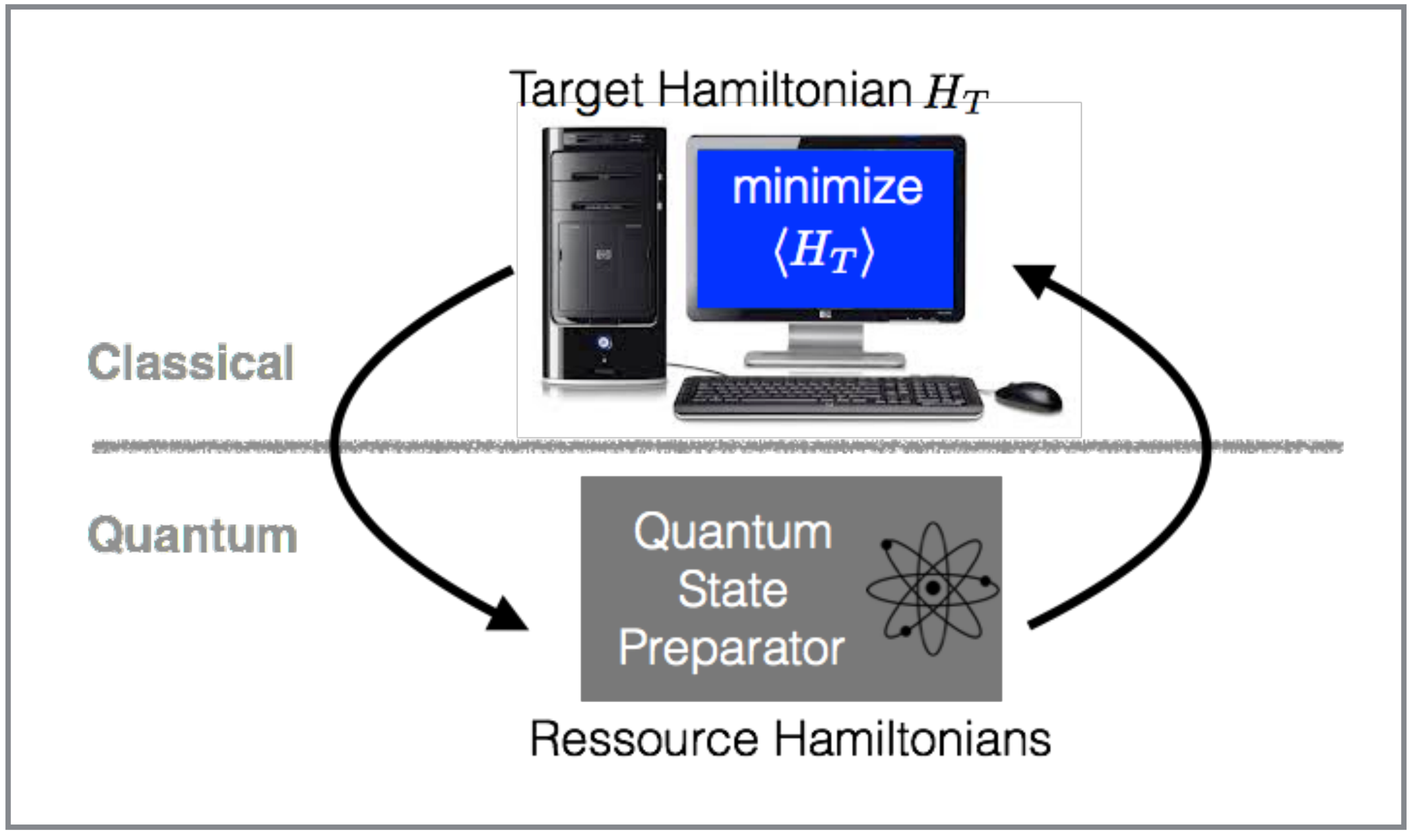
Inspiration: E. Farhi, J. Goldstone, S. Gutmann, H. Neven; MIT-CTP/4893 (2017)

Variational Quantum Simulation

in preparation

P. Zoller

IQI & IQC Institute for Quantum Computing



Target Hamiltonian H_T

minimize $\langle H_T \rangle$

Classical

Quantum

Quantum State Preparator

Ressource Hamiltonians

Inspiration: E. Farhi, J. Goldstone, S. Gutmann, H. Neven; MIT-CTP/4893 (2017)

Variational Quantum Simulation

➡ Target Hamiltonian: H_T (contains e.g. 3-body terms or long-range interactions)

Variational Quantum Simulation

- Target Hamiltonian: H_T (contains e.g. 3-body terms or long-range interactions)
- Experimentally available resource Hamiltonians: $\{\ldots, H_{\mathrm{res}}^{(j)}, H_{\mathrm{res}}^{(j+1)}, \ldots\}$

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Variational Quantum Simulation

- Target Hamiltonian: H_T (contains e.g. 3-body terms or long-range interactions)
- Experimentally available resource Hamiltonians: $\{\ldots, H_{\text{res}}^{(j)}, H_{\text{res}}^{(j+1)}, \ldots\}$
- Create variational state: $|\psi(\boldsymbol{\Theta})\rangle = \cdots e^{i\Theta_j H_{\text{res}}^{(j)}} e^{i\Theta_{j+1} H_{\text{res}}^{(j+1)}} \cdots |\psi_{\text{init}}\rangle$

Can be highly entangled,
yet parametrised with few parameters

Variational Quantum Simulation

- Target Hamiltonian: H_T (contains e.g. 3-body terms or long-range interactions)
- Experimentally available resource Hamiltonians: $\{\dots, H_{\text{res}}^{(j)}, H_{\text{res}}^{(j+1)}, \dots\}$
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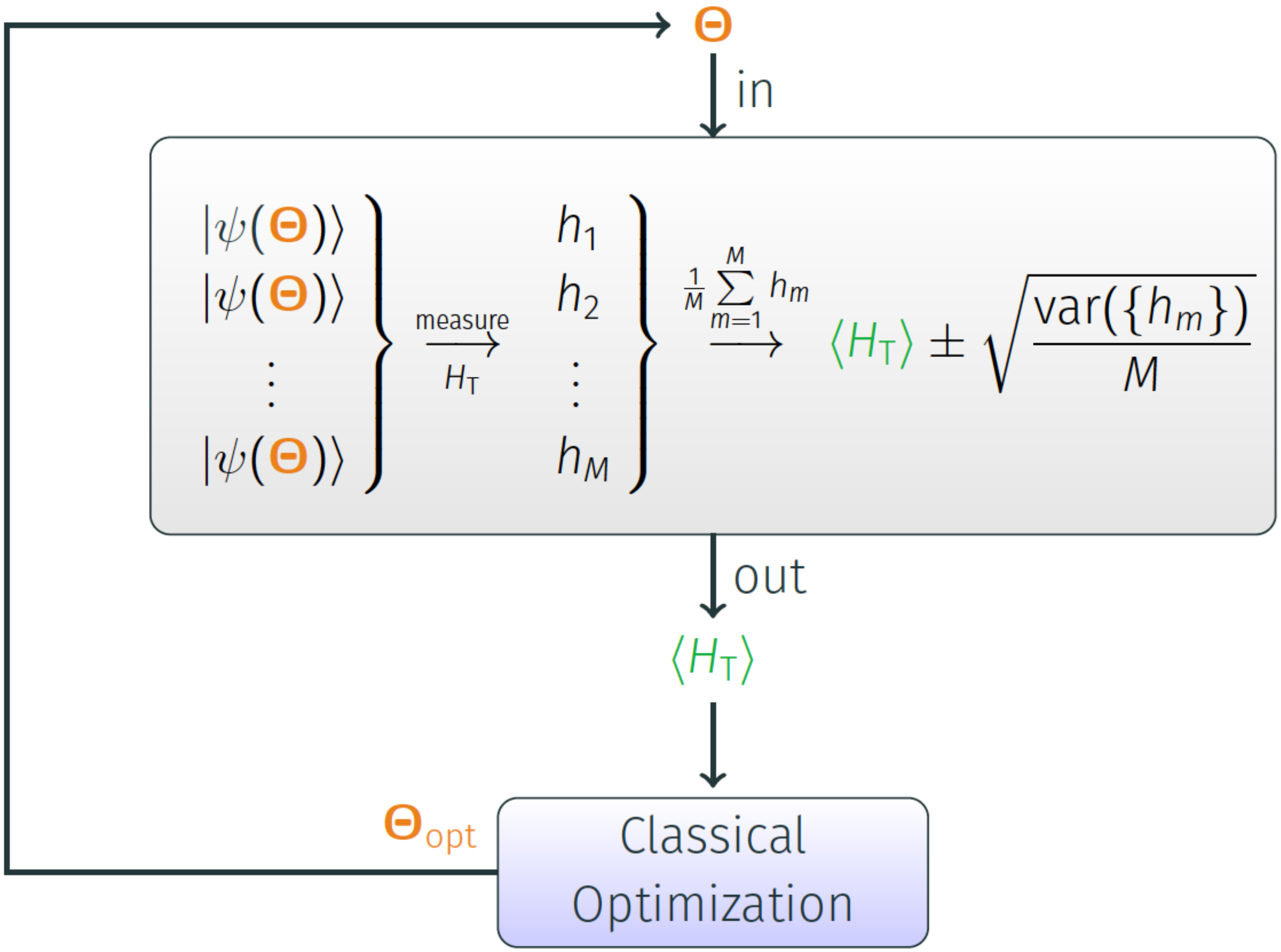
- The parameters Θ are varied such that $|\Psi(\Theta)\rangle$ becomes the ground state of a target Hamiltonian H_T:

$$\min_{\boldsymbol{\Theta}} \frac{\langle\psi(\boldsymbol{\Theta})|\, H_{\text{T}}\, |\psi(\boldsymbol{\Theta})\rangle}{\langle\psi(\boldsymbol{\Theta})|\psi(\boldsymbol{\Theta})\rangle}$$

Can be highly entangled,
yet parametrised with few parameters

Variational Quantum Simulation



Ground state preparation for 1D-QED with trapped ions

Resource Gates:

Spin-Spin Interaction Hamiltonian

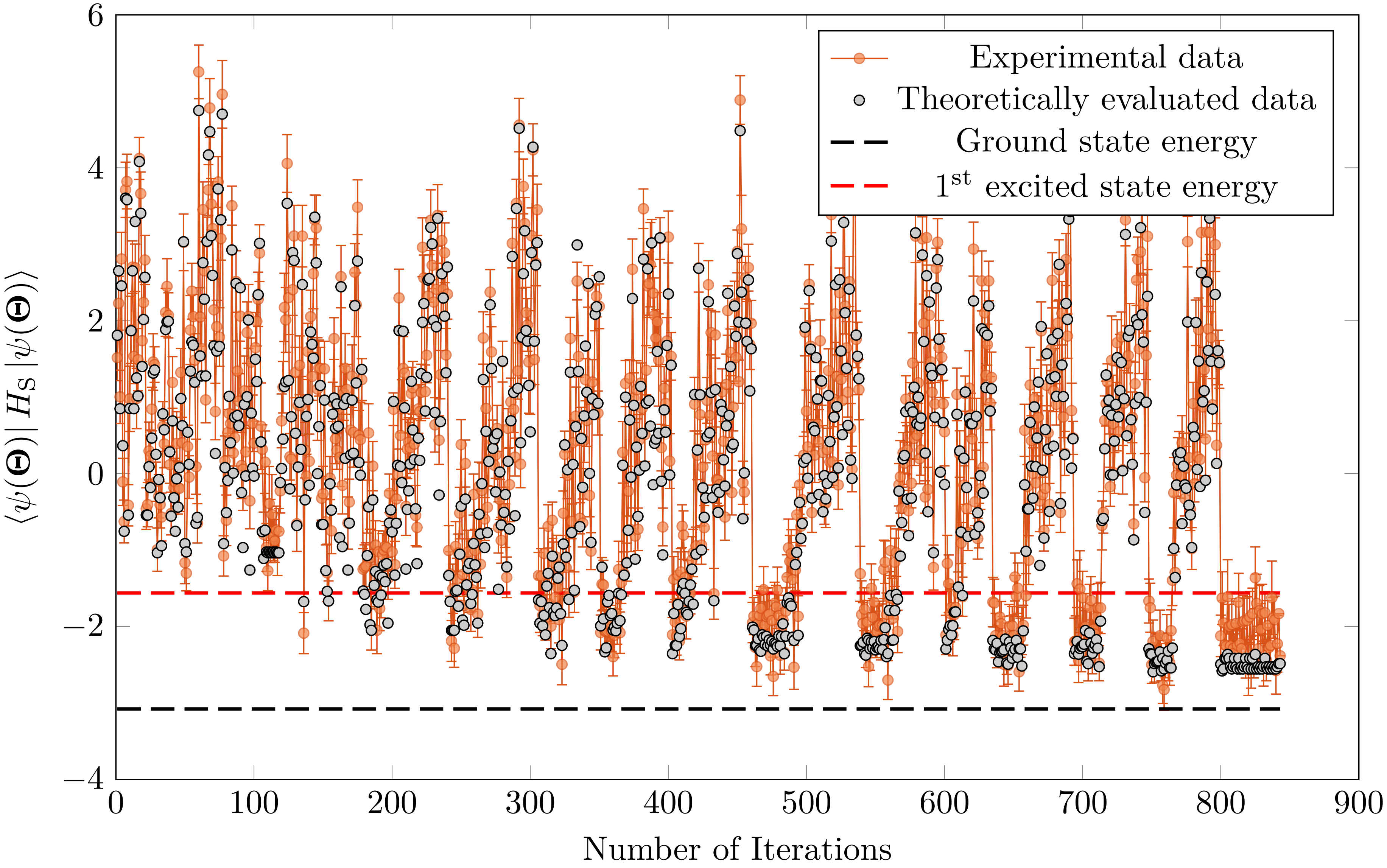
$$H_{\text{spin-spin}} = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} J_{ij} \sigma_i^x \sigma_j^x + \sum_i B_i(t) \sigma_i^z$$

Single Qubit rotations on the Bloch sphere

$$R(\mathbf{\Theta}) = \exp\left(-i \frac{\mathbf{\Theta}}{2} \cdot \boldsymbol{\sigma}\right)$$

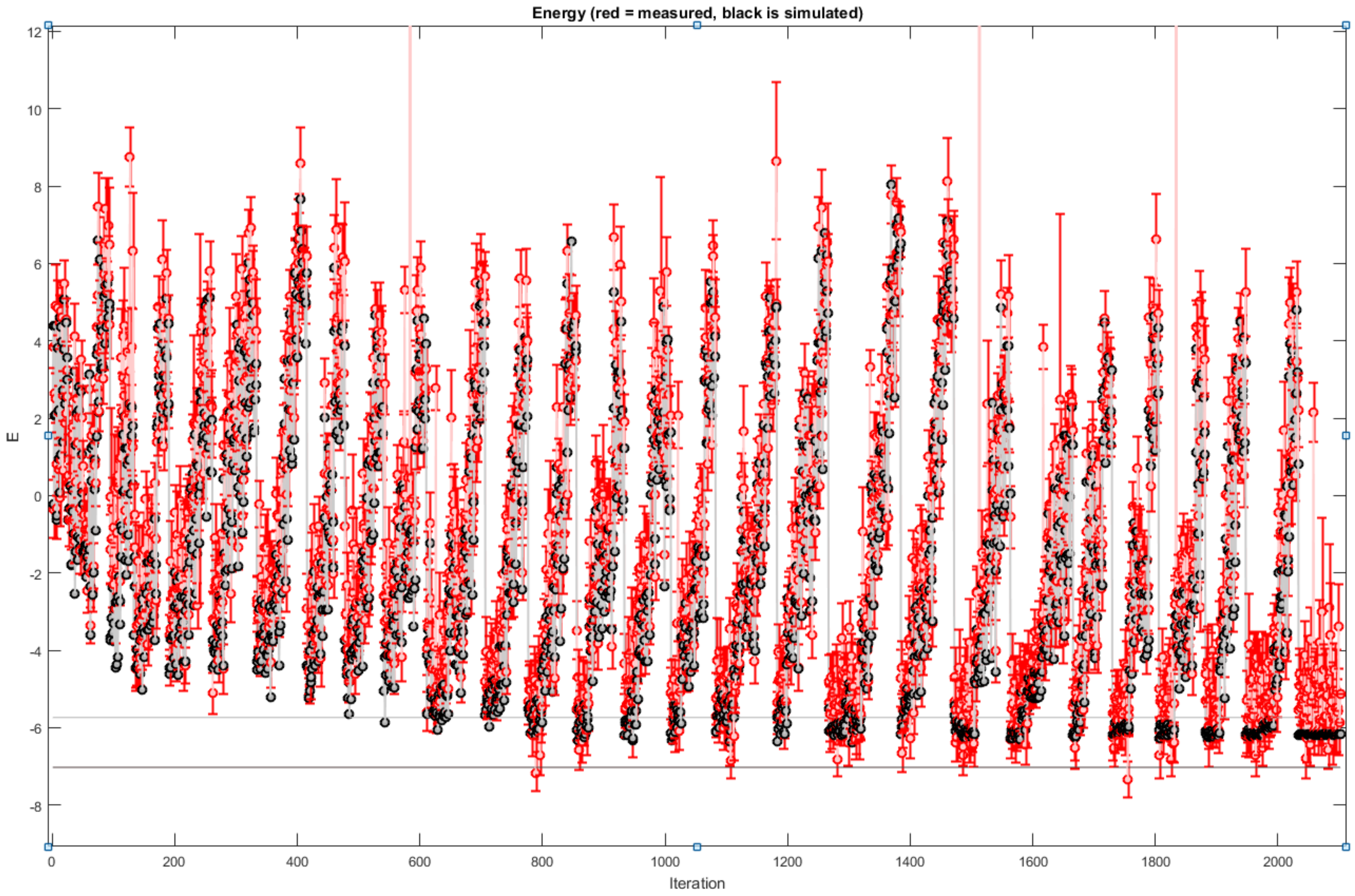
Ground state preparation for 1D-QED with trapped ions

8 qubits:

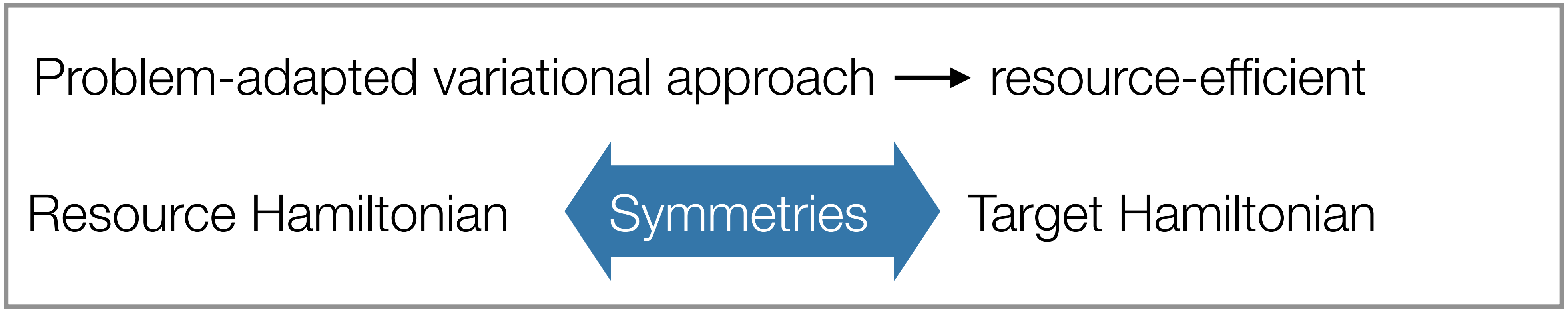


Ground state preparation for 1D-QED with trapped ions

an experimental run for 12 qubits:



Variational Quantum Simulation with trapped ions



Problem-adapted variational approach → resource-efficient

Resource Hamiltonian

Target Hamiltonian

New variational features:

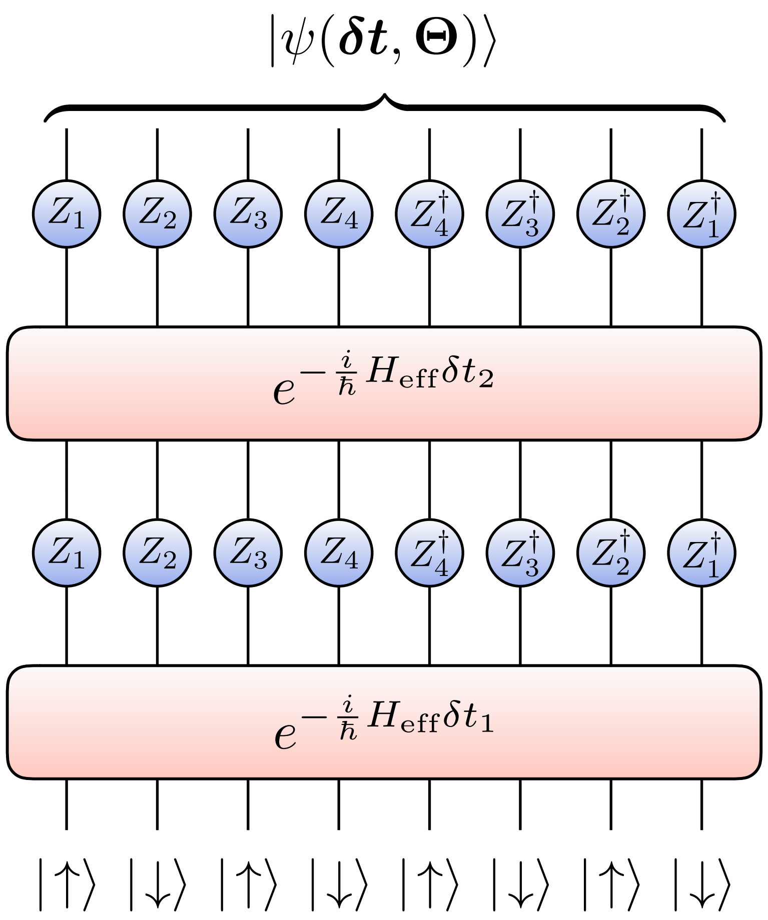
- self-validation
- access excited states

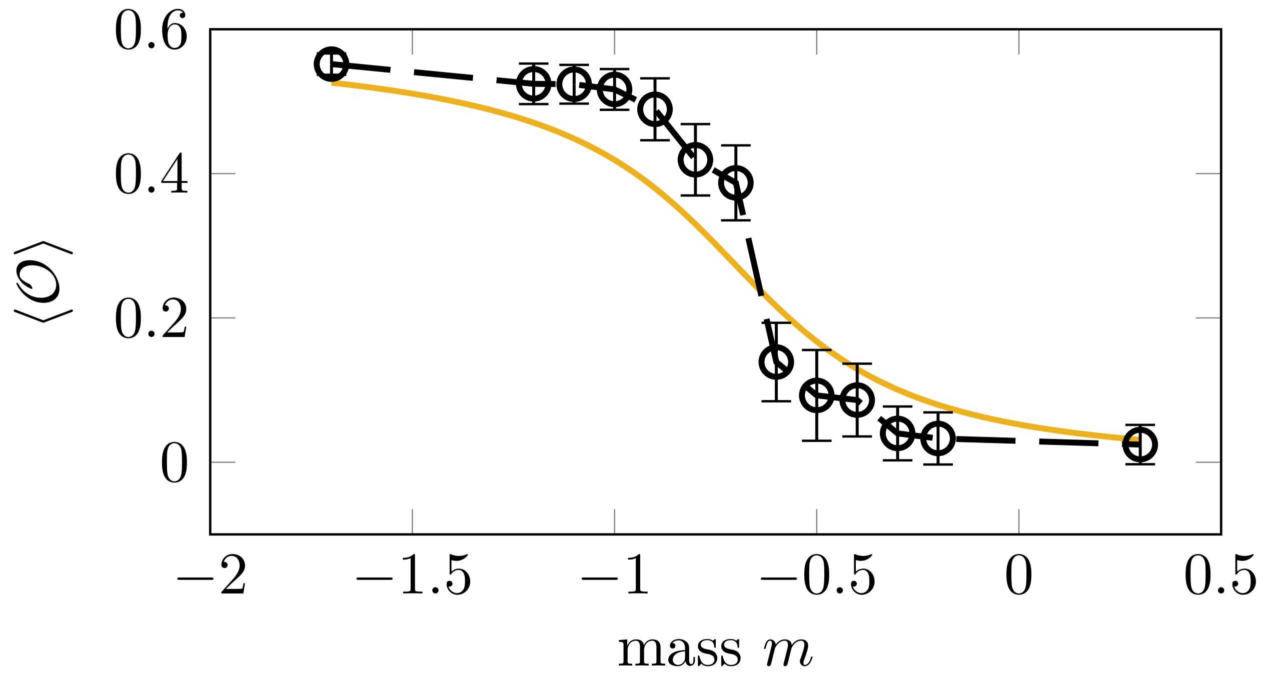
C. Kokail, R.van Bijnen, P. Silvi, P. Zoller, P. Jurcevic, E. Martinez, P. Monz, P. Schindler, R. Blatt

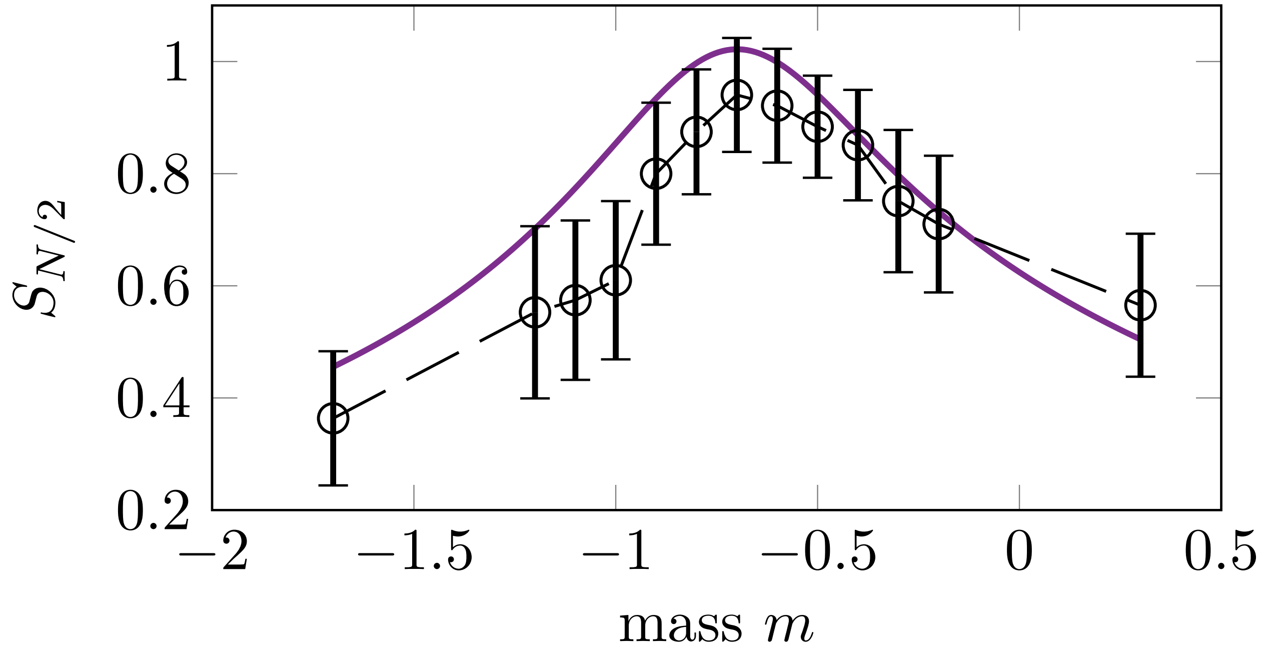
Parity Phase Transition via Variational Quantum Simulation

Numerical Results for 8 Ions

Quantum Circuit used for the numerical simulation of the experiment:







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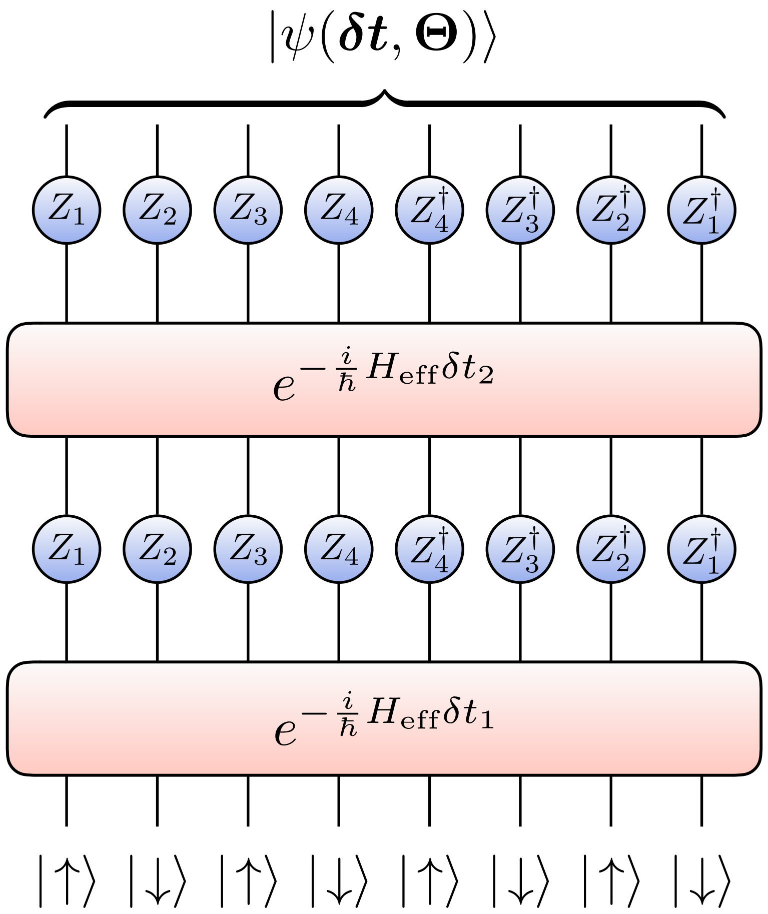

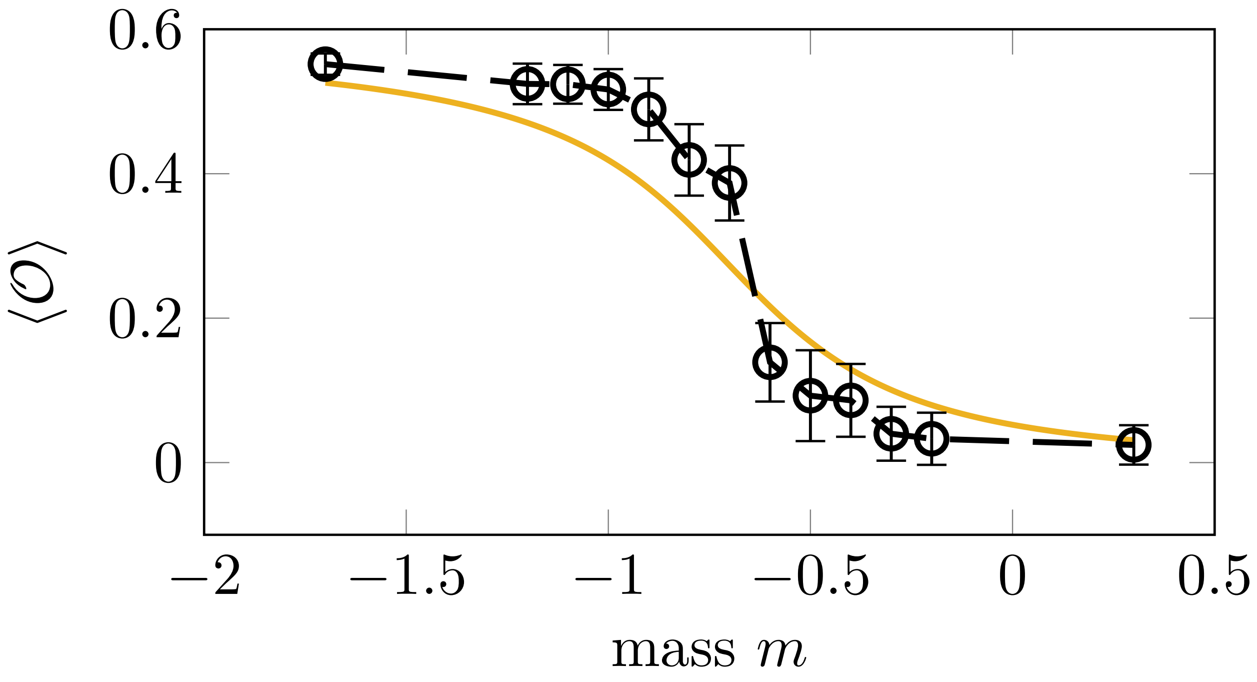

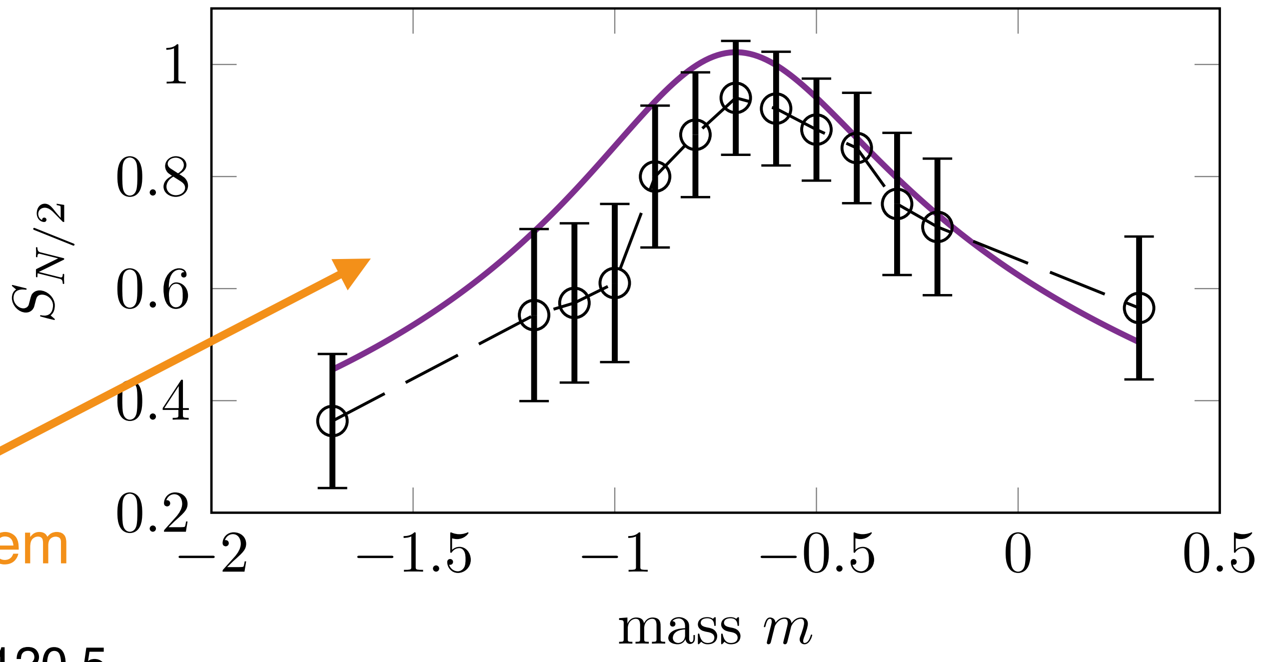

Renyi entropy measurements for determining entanglement in the system

Elben, Andreas, et al. *Physical review letters* 120.5 (2018): 050406.

Parity Phase Transition via Variational Quantum Simulation

Numerical Results for 8 Ions

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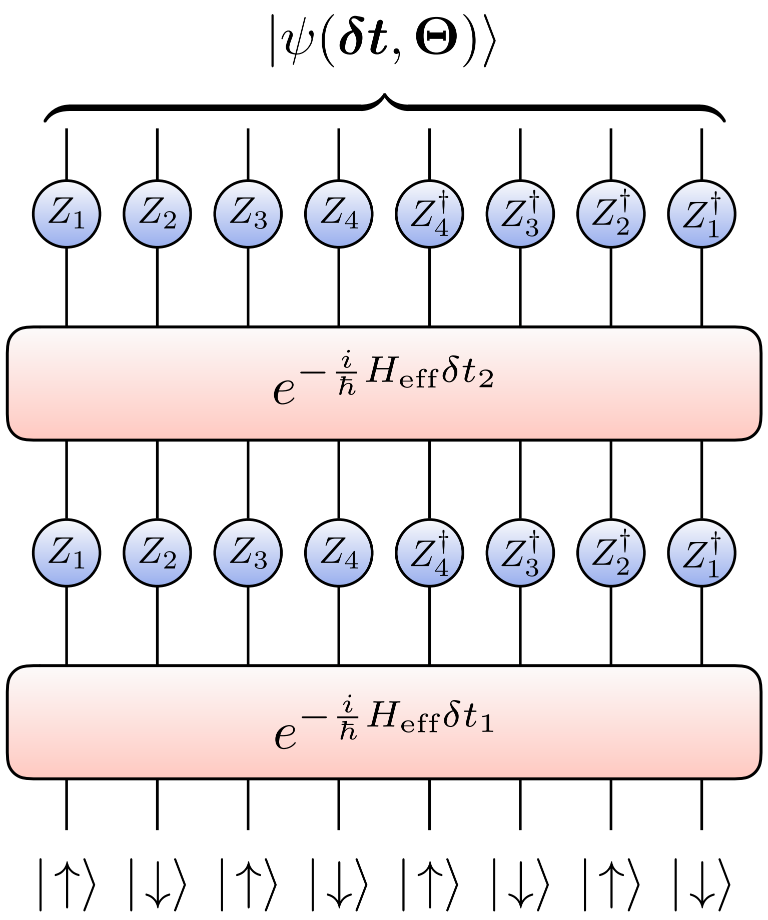
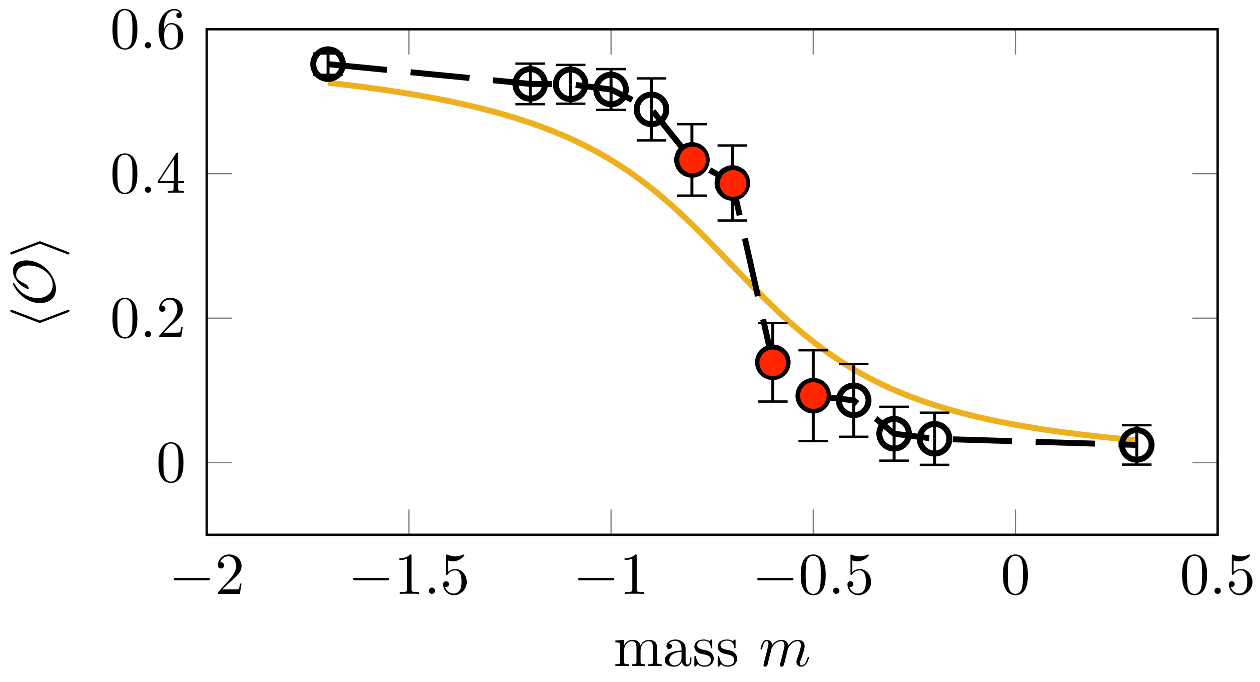
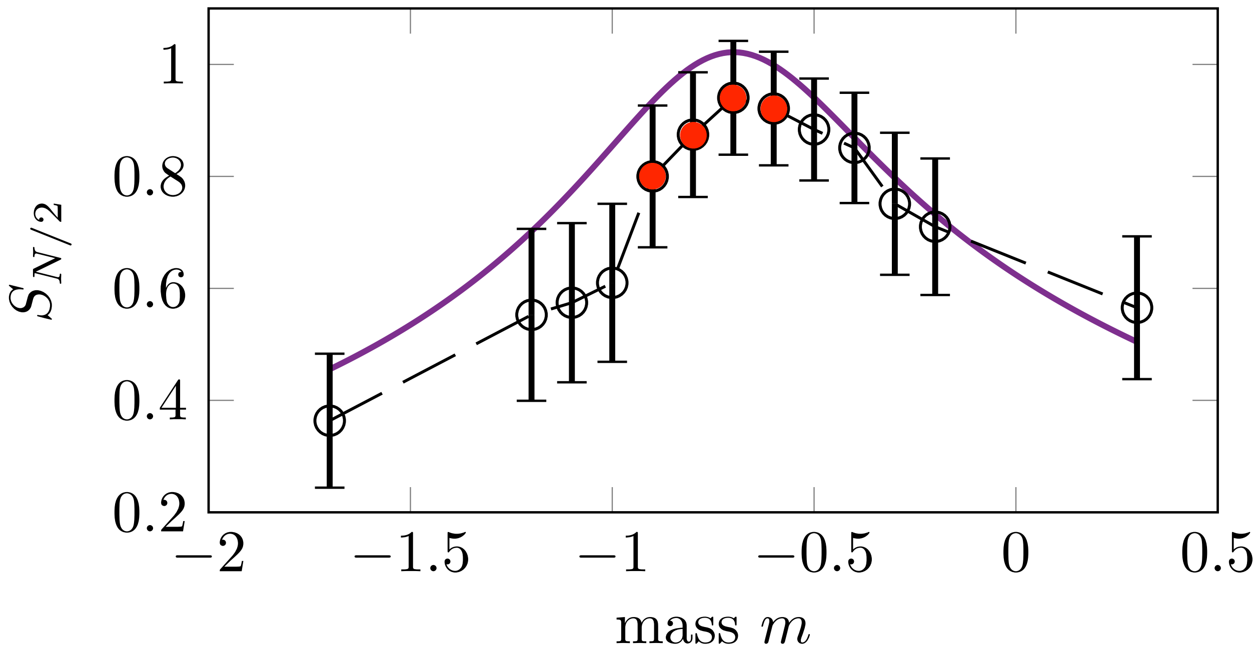






● measurement budget: $5 \cdot 10^4 \, \frac{\text{shots}}{\text{basis}}$

○ measurement budget: $2 \cdot 10^4 \, \frac{\text{shots}}{\text{basis}}$

Some related demonstrations:

Rigetti, IBM: Deuteron ⟶ 2,3 qubit variational simulation
IBM: Schwinger Model ⟶ 2,3 qubits variational simulation

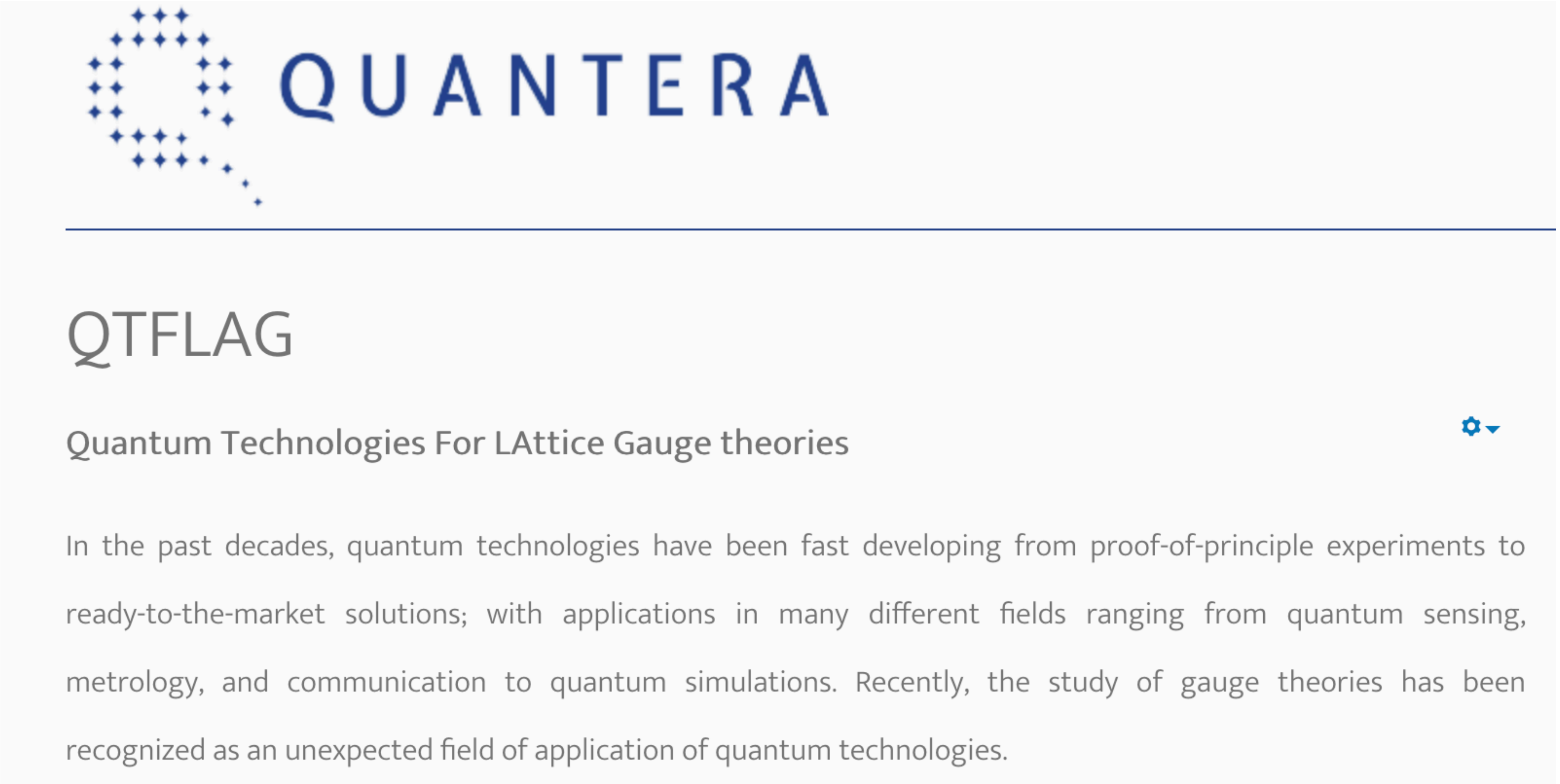
Ongoing: Chris Wilson (Waterloo) ⟶ 1D-QED with superconducting circuits

Ongoing: Markus Oberthaler (Heidelberg) ⟶ 1D-QED with cold atoms

Ongoing: Chris Monroe (JQI) ⟶ Deuteron with trapped ions

Planned: Misha Lukin (Harvard) ⟶ Rydberg atoms

See also:
Experimental quantum simulation of fermion-antifermion scattering via boson exchange in a trapped ion
Nature Commun. **9**, 195 (2018).

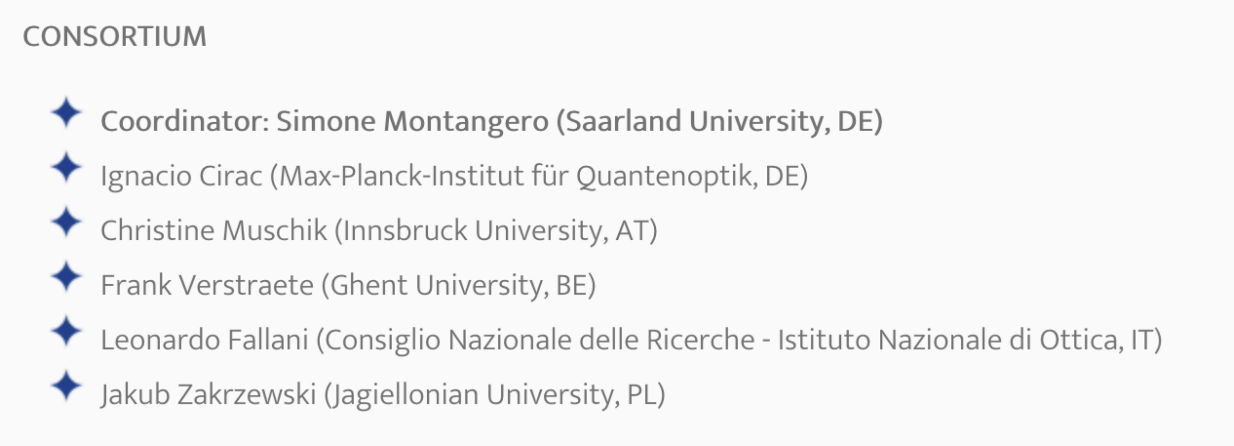


QUANTERA

QTFLAG

Quantum Technologies For LAttice Gauge theories

In the past decades, quantum technologies have been fast developing from proof-of-principle experiments to ready-to-the-market solutions; with applications in many different fields ranging from quantum sensing, metrology, and communication to quantum simulations. Recently, the study of gauge theories has been recognized as an unexpected field of application of quantum technologies.



CONSORTIUM

- **Coordinator: Simone Montangero (Saarland University, DE)**
- Ignacio Cirac (Max-Planck-Institut für Quantenoptik, DE)
- Christine Muschik (Innsbruck University, AT)
- Frank Verstraete (Ghent University, BE)
- Leonardo Fallani (Consiglio Nazionale delle Ricerche - Istituto Nazionale di Ottica, IT)
- Jakub Zakrzewski (Jagiellonian University, PL)

Next challenges:

- ➡ Realisation of 2D models
- ➡ Simulate increasingly complex dynamics
- ➡ Realisation of non-Abelian theories
- ➡ …

UNIVERSITY OF WATERLOO

IQC Institute for Quantum Computing

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CANADA FIRST
RESEARCH EXCELLENCE FUND

ARL

Thank you very much
for your attention!

Local (gauge) symmetries

Local symmetry generators: $\{G_n\}$

The Hamiltonian is invariant under gauge transformations of the form:

$$H' = \left(\Pi_n e^{i\alpha_n G_n}\right)\ H\ \left(\Pi_n e^{-i\alpha_n G_n}\right) \qquad [H, G_n] = 0$$

For 1D QED: $G_n = L_n - L_{n-1} - \Phi^\dagger \Phi - \frac{1}{2}\left[1 - (-1)^n\right]$

The Hamiltonian does not mix eigenstates of G_n with different eigenvalues λ_n.

In the following, we restrict ourselves to the zero-charge subsector: $\lambda_{G_n} = 0,\ \forall n$
(# of particles = # of antiparticles).

$$G_n |\Psi_{\text{physical}}\rangle = 0 \qquad \forall n$$

QED in (1+1) dimensions

Electromagnetic fields:

Vector potential: $A_0(x), A_1(x)$

Electric field: $E(x) = \partial_0 A_1(x)$

$[E(x), A_1(x')] = -i\delta(x - x')$

Matter fields:

$$\Psi(x) = \left(\begin{array}{c} \Psi_1(x) \\ \Psi_2(x) \end{array} \right)$$

Hamiltonian:

$$H_{\text{cont}} = \int dx \left[-i\Psi^\dagger(x)\gamma^1 \left(\delta_1 - igA_1\right)\Psi(x) + m\Psi^\dagger(x)\Psi(x) + \frac{1}{2}E^2(x) \right]$$

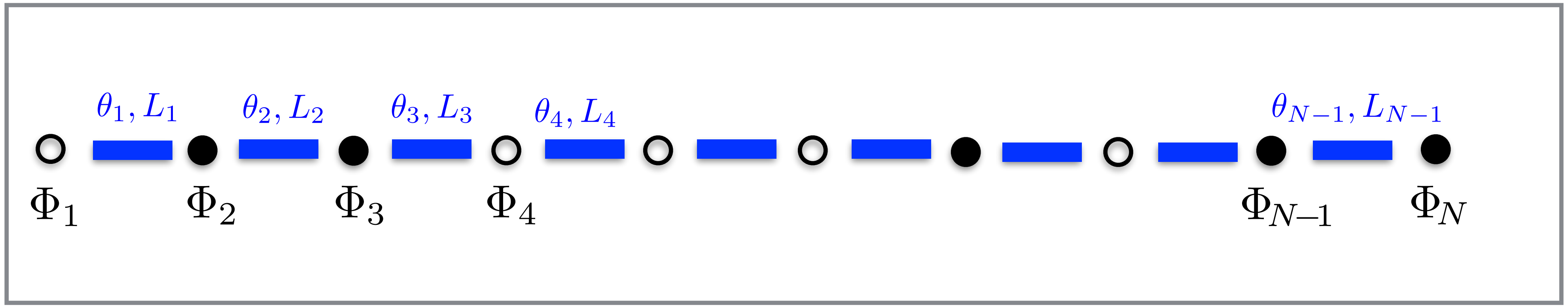
$\gamma_1 = -i\sigma_y$

coupling strength (charge)

Fermion mass

The lattice Schwinger Model

The lattice Schwinger Model



Continuum

Vector potential $A_1(x)$

Electric field $E(x)$

$$[E(x), A_1(x')] = -i\delta(x - x')$$

Dirac spinor

$$\Psi(x) = \begin{pmatrix} \Psi_1(x) \\ \Psi_2(x) \end{pmatrix}$$

Lattice

$$\theta_n = agA_1(x_n)$$

$$L_n = \frac{1}{g}E(x_n)$$

$$[\theta_n, L_m] = i\delta_{n,m}$$

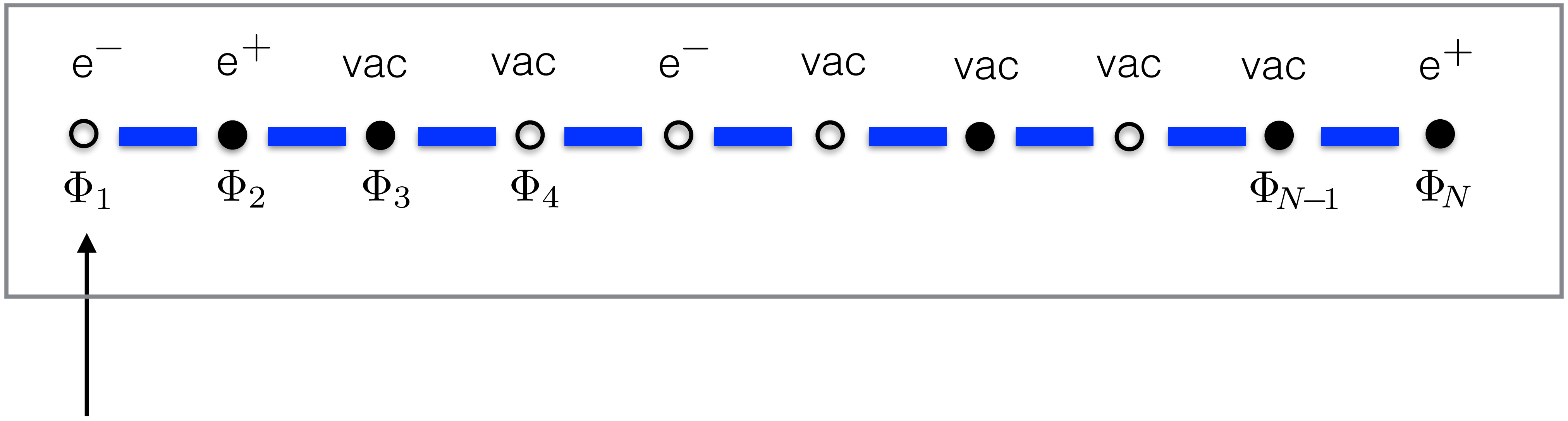
odd lattice sites:

$$\Phi_n = \sqrt{a}\Psi_1(x_n)$$

even lattice sites:

$$\Phi_n = \sqrt{a}\Psi_2(x_n)$$

Wilson’s staggered Fermions



e^- e^+ vac vac e^- vac vac vac vac e^+

Φ_1 Φ_2 Φ_3 Φ_4 Φ_{N-1} Φ_N

one-component fermion fields

odd sites:

● $\cong$ vac

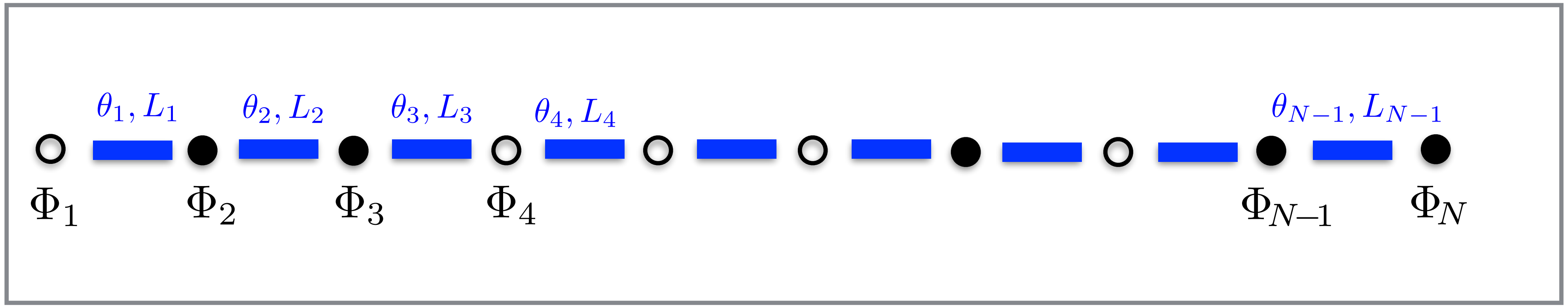
○ $\cong$ e^-

even sites:

● $\cong$ e^+

○ $\cong$ vac

The lattice Schwinger Model



Continuum

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Lattice

$$H_{\text{lat}} = \quad \underset{\overset{\uparrow}{w = \frac{1}{2a}}}{-iw} \sum_{n=1}^{N-1} \left[\Phi_n^\dagger e^{i\theta_n} \Phi_{n+1} - H.C.\right] + m \sum_{n=1}^{N} (-1)^n \Phi_n^\dagger \Phi_n + \underset{\overset{\uparrow}{J = \frac{g^2 a}{2}}}{J} \sum_{n=1}^{N-1} L_n^2$$

J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

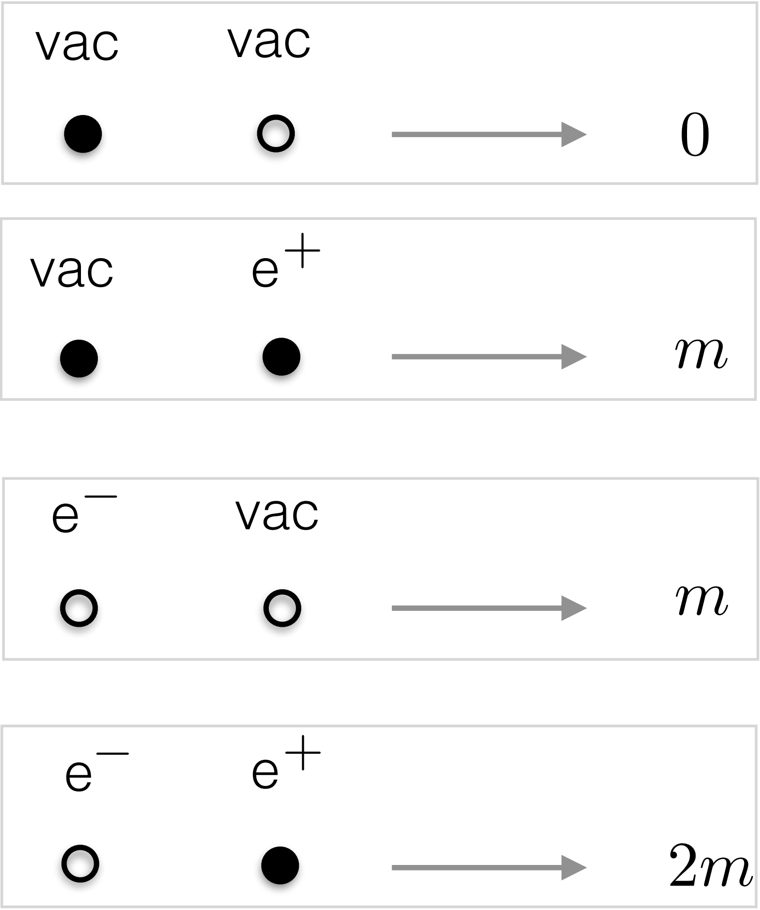
The Schwinger model

Hamiltonian formulation of the Schwinger model:

J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

$$\hat{H} = -iw \sum_{n=1}^{N-1} \left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n} \hat{\Phi}_{n+1} - \text{H.c.}\right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} \left[(-1)^n \hat{\Phi}_n^\dagger \hat{\Phi}_n + 0.5\right]$$

Fermion rest mass



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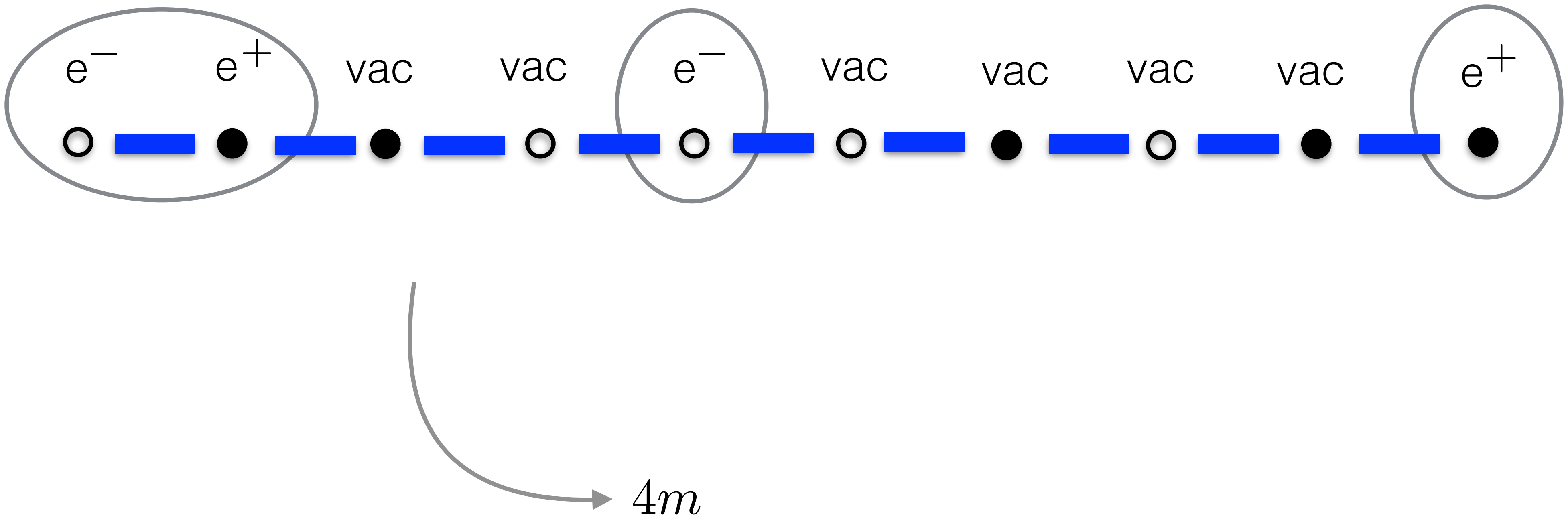

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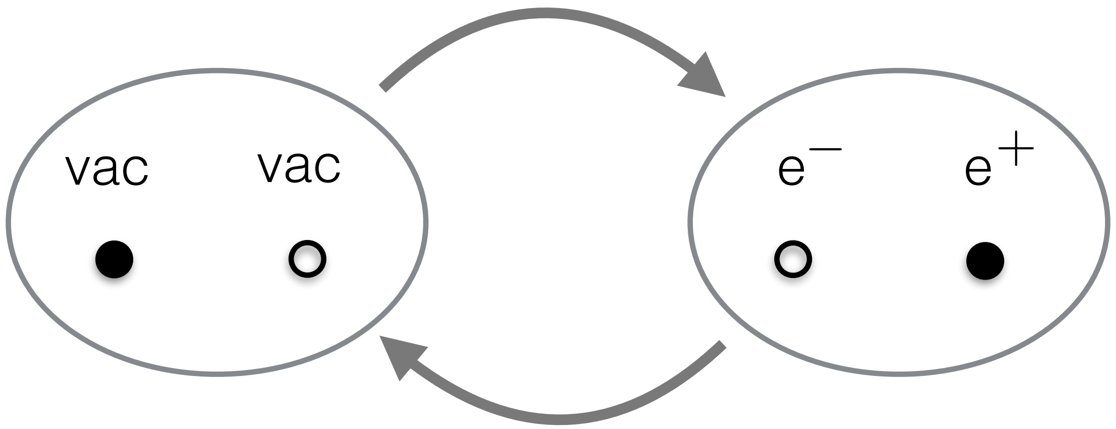
Pair creation and annihilation

Particle masses

$$w = \frac{1}{2a}$$

(a =lattice spacing)



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Pair creation and annihilation

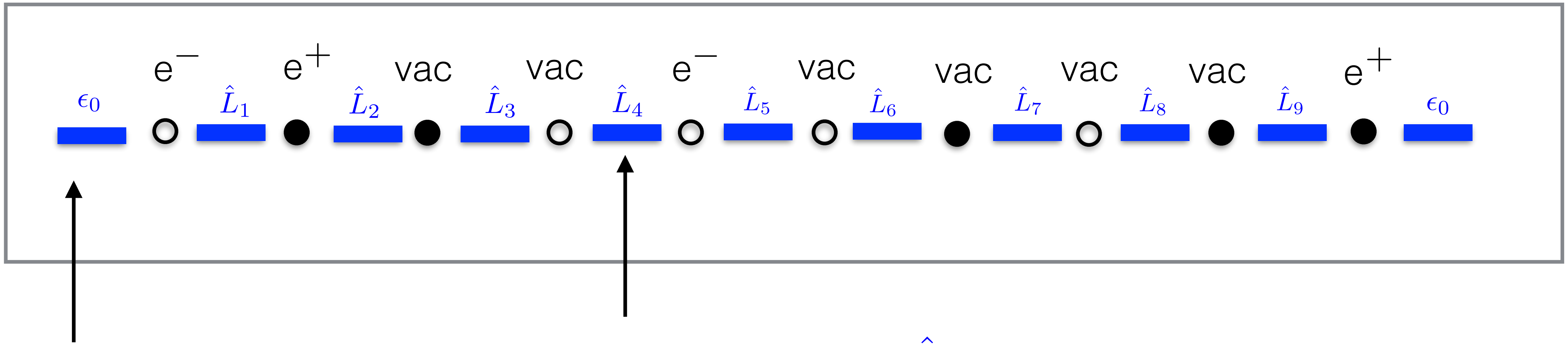
E-field energy

Particle masses

$$J = \frac{g^2 a}{2}$$

a =lattice spacing

g = light-matter coupling



background field

The operators $\hat{L}_n$ represent the electric fields on the links.
They take eigenvalues $\hat{L}_n = 0, \pm 1, \pm 2, \pm 3...$

Hamiltonian formulation of the Schwinger model:

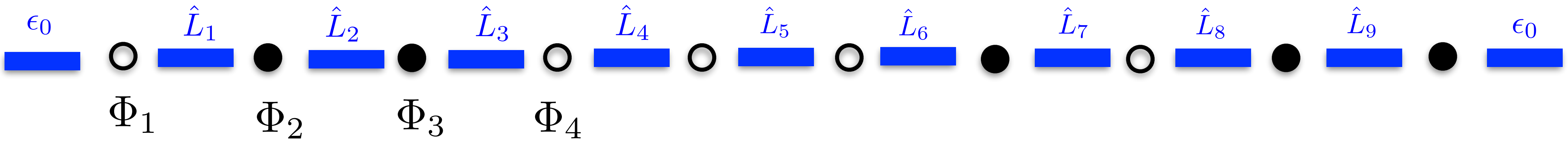
J. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).

$$\boxed{\hat{H} = -iw\sum_{n=1}^{N-1}\left[\hat{\Phi}_n^\dagger e^{i\hat{\theta}_n}\hat{\Phi}_{n+1} - \text{H.c.}\right] + J\sum_{n=1}^{N-1}\hat{L}_n^2 + m\sum_{n=1}^{N}(-1)^n\hat{\Phi}_n^\dagger\hat{\Phi}_n}$$

The dynamics is constraint by the Gauss law:

In the continuum in 3D: $\nabla E = \rho$

Here: $\hat{L}_n - \hat{L}_{n-1} = \hat{\Phi}_n^\dagger\hat{\Phi} - \frac{1}{2}\left[1 - (-1)^n\right]$



Our approach

Quantum simulation of a Wilson model → Include the whole infinite dimensional Hilbert space of the gauge fields

Our scheme:

(1) Mapping of the Schwinger Hamiltonian to a pure spin model with long range interactions

(2) Realization of the required interactions with an efficient digital simulation scheme using “shaking methods”.

Important features of the scheme

- Exact gauge invariance at all energy scales (by construction)
- Very efficient use of resources

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- Very efficient use of resources

Two simple transformations:

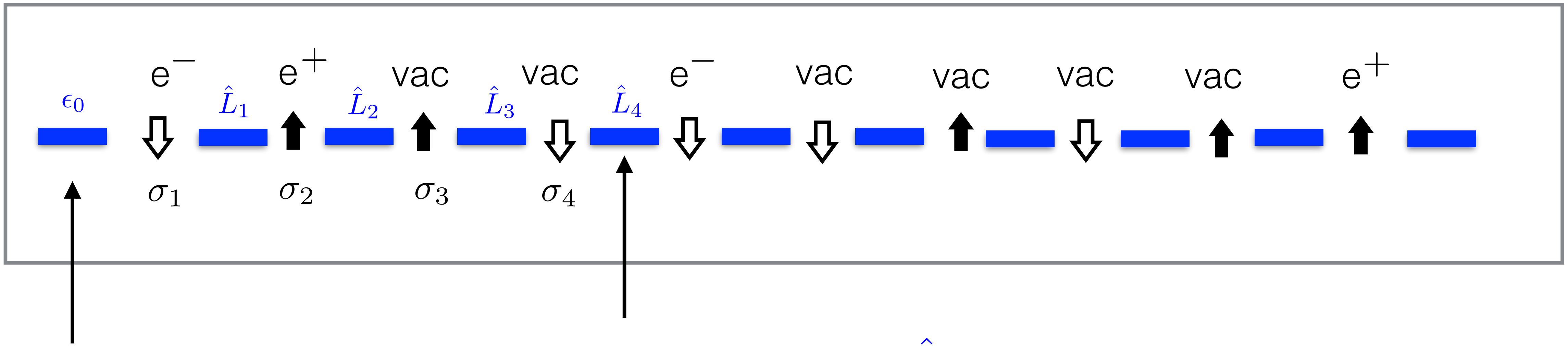
(1) Fermions —> spins $\Phi_n = \prod_{l<n} [i\sigma_l^z]\, \sigma_n^-$

(2) Elimination of $\hat{\theta}_n$ $\hat{\sigma}_n^- \rightarrow \prod_{l<n} \left[e^{-i\hat{\theta}_l}\right] \hat{\sigma}_n^-$

Hamiltonian in terms of spins and electric fields

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \mathrm{H.c.} \right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$



background field

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Odd lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ vac $\quad L_n = L_{n-1}$

$\underset{n}{\circ} \cong \underset{n}{\Downarrow} \cong$ e$^-$ $\quad L_n = L_{n-1} - 1$

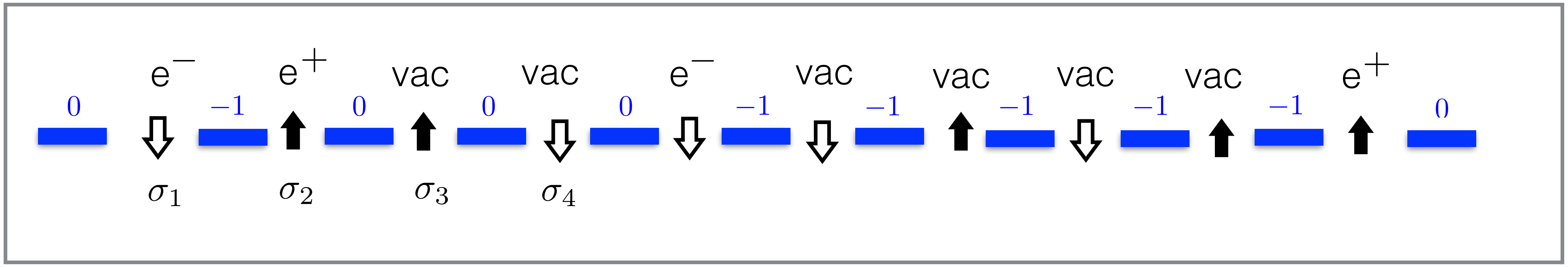
Even lattice sites:

$\underset{n}{\bullet} \cong \underset{n}{\uparrow} \cong$ e$^+$ $\quad L_n = L_{n-1} + 1$

$\underset{n}{\circ} \cong \underset{n}{\Downarrow} \cong$ vac $\quad L_n = L_{n-1}$

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \text{H.c.}\right] + J \sum_{n=1}^{N-1} \hat{L}_n^2 + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$



A given configuration of spins and choice of background field completely determines the gauge degrees of freedom.

Odd lattice sites:

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Transformed Gauss law:

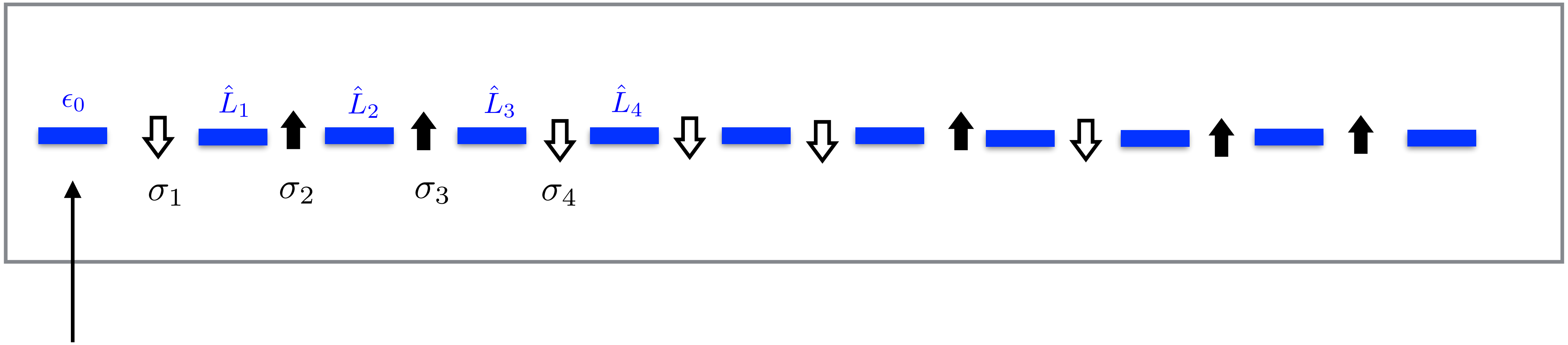
$$\hat{L}_n - \hat{L}_{n-1} = \frac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$

Transformed Hamiltonian:

$$\hat{H} = w \sum_{n=1}^{N-1} \left[\hat{\sigma}_n^+ \hat{\sigma}_{n+1}^- + \text{H.c.}\right] + \boxed{J \sum_{n=1}^{N-1} \hat{L}_n^2} + m \sum_{n=1}^{N} (-1)^n \hat{\sigma}_n^z$$

$$\boxed{+J \sum_{n=1}^{N-1} \left[\epsilon_0 + \frac{1}{2} \sum_{m=1}^{n} \left[\hat{\sigma}_m^z + (-1)^m\right]\right]^2}$$

$$\epsilon_0 = 0 \qquad \hat{L}_n - \hat{L}_{n-1} = \frac{1}{2}\left[\hat{\sigma}_n^z + (-1)^n\right]$$



background field

Elimination of the gauge fields → **Pure spin model with long-range interactions**

The gauge fields don't appear explicitly in the encoded description. Instead, they act in the form of a non-local interaction that corresponds to the Coulomb-interaction between the simulated charged particles.

The Schwinger model as exotic spin model

$$\hat{H}_S = w \sum_i (\hat{\sigma}_i^+ \hat{\sigma}_{i+1}^- + \hat{\sigma}_{i+1}^+ \hat{\sigma}_i^-)$$

particle - antiparticle creation/annihilation

$$+ J \sum_{i<j} c_{ij} \hat{\sigma}_i^z \hat{\sigma}_j^z$$

long - range interaction

$$+ m \sum_i c_i \hat{\sigma}_i^z + J \sum_i \tilde{c}_i \hat{\sigma}_i^z$$

effective particle masses

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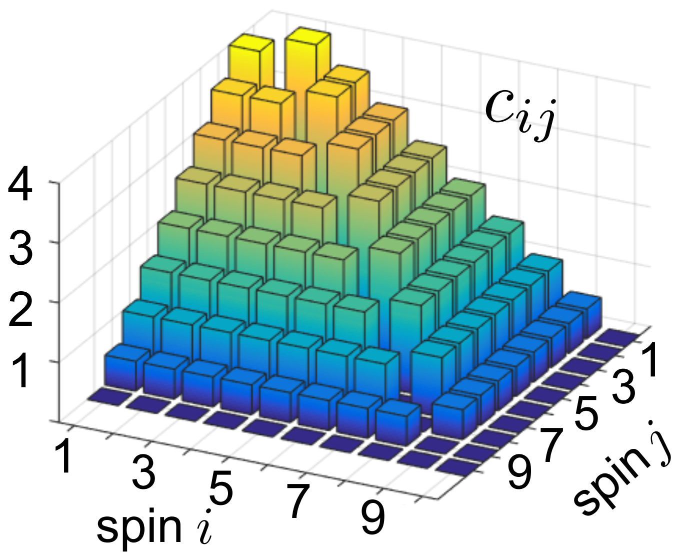

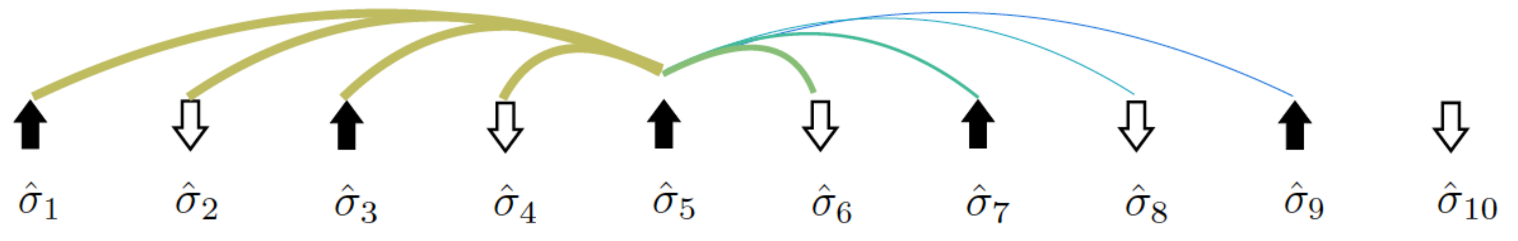

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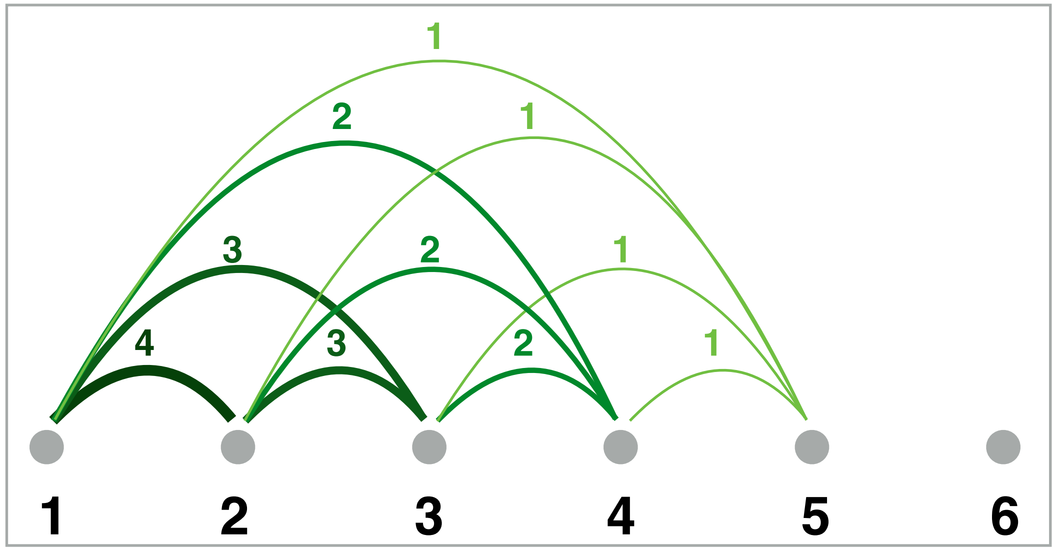

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effective particle masses

- Efficient implementation on an ion-quantum computer
- N spins simulate N matter fields and N-1 gauge fields
- Exotic spin interactions can be simulated efficiently:
  Digital scheme

Digital quantum simulation

Approximate time evolution by a stroboscopic sequence of gates

The evolution under a desired Hamiltonian is realised on a coarse-grained time scale

$$H = H_1 + H_2$$

$$U(t) \equiv e^{-iHt/\hbar} = e^{-iH\Delta t_n/\hbar} \ldots^{-iH\Delta t_1/\hbar}$$

↑

Trotter expansion:

$$e^{-iH\Delta t/\hbar} \simeq \underbrace{e^{-iH_1\Delta t/\hbar}}_{\text{first term}} \underbrace{e^{-iH_2\Delta t/\hbar}}_{\text{second term}} \underbrace{e^{\frac{1}{2}\frac{(\Delta t)^2}{\hbar^2}[H_1,H_2]}}_{\substack{\text{Trotter errors for} \\ \text{non-commuting terms}}}$$

S. Lloyd, Science 273, 1073 (1996).

Digital quantum simulation

Approximate time evolution by a stroboscopic sequence of gates

The evolution under a desired Hamiltonian is realised on a coarse-grained time scale

$$U_{\mathrm{S}} \;=\; e^{-i\hat{H}_{\mathrm{S}}t}$$

$$U_{\mathrm{sim}} = \left(e^{-iH_1 t/n}...e^{-iH_n t/n}\right)^n$$

Operations that can be performed straightforwardly

Trotter error: $U_{\mathrm{S}} - U_{\mathrm{sim}} = \frac{t^2}{2n}\sum_{i,j}[H_i, H_j] + \epsilon$

This scheme: Trotter errors do not violate gauge invariance

Our toolbox

Ion trap quantum computers:

➡ Fast and accurate single qubit operations

➡ Entangling gates: Mølmer-Sørensen interaction

All-to-all 2-body interaction: $H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$

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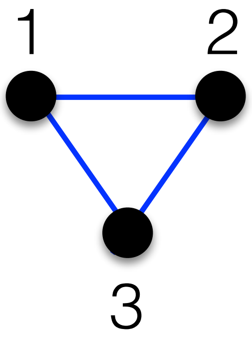
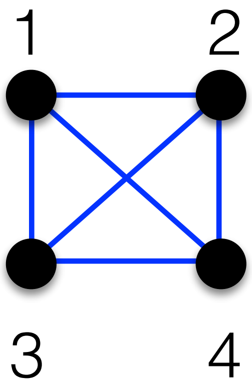
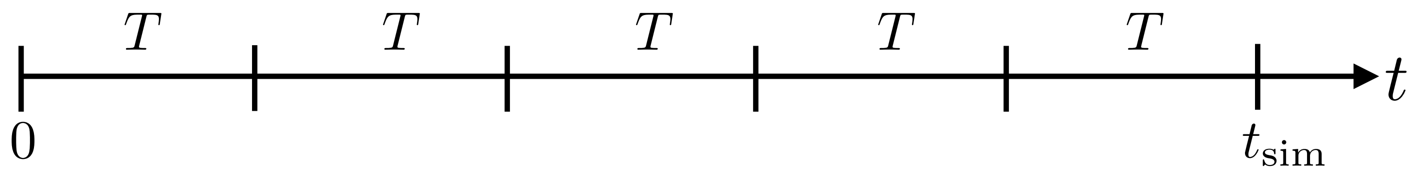
$\sigma_1^x \sigma_2^x$



$\sigma_1^x \sigma_2^x + \sigma_2^x \sigma_3^x + \sigma_1^x \sigma_3^x$



$\sigma_1^x \sigma_2^x + \sigma_1^x \sigma_3^x + \sigma_1^x \sigma_4^x + \sigma_2^x \sigma_3^x + \sigma_2^x \sigma_4^x + \sigma_3^x \sigma_4^x$



Time coarse graining

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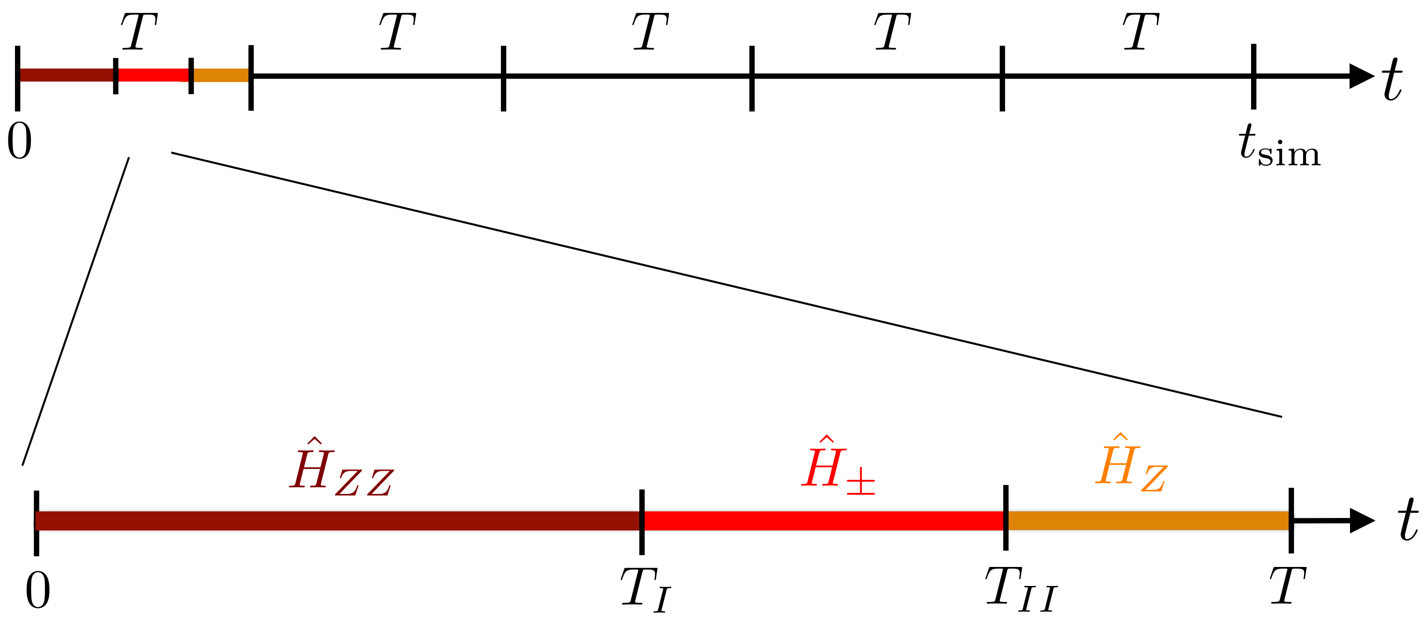

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long - range interaction

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particle - antiparticle creation/annihilation

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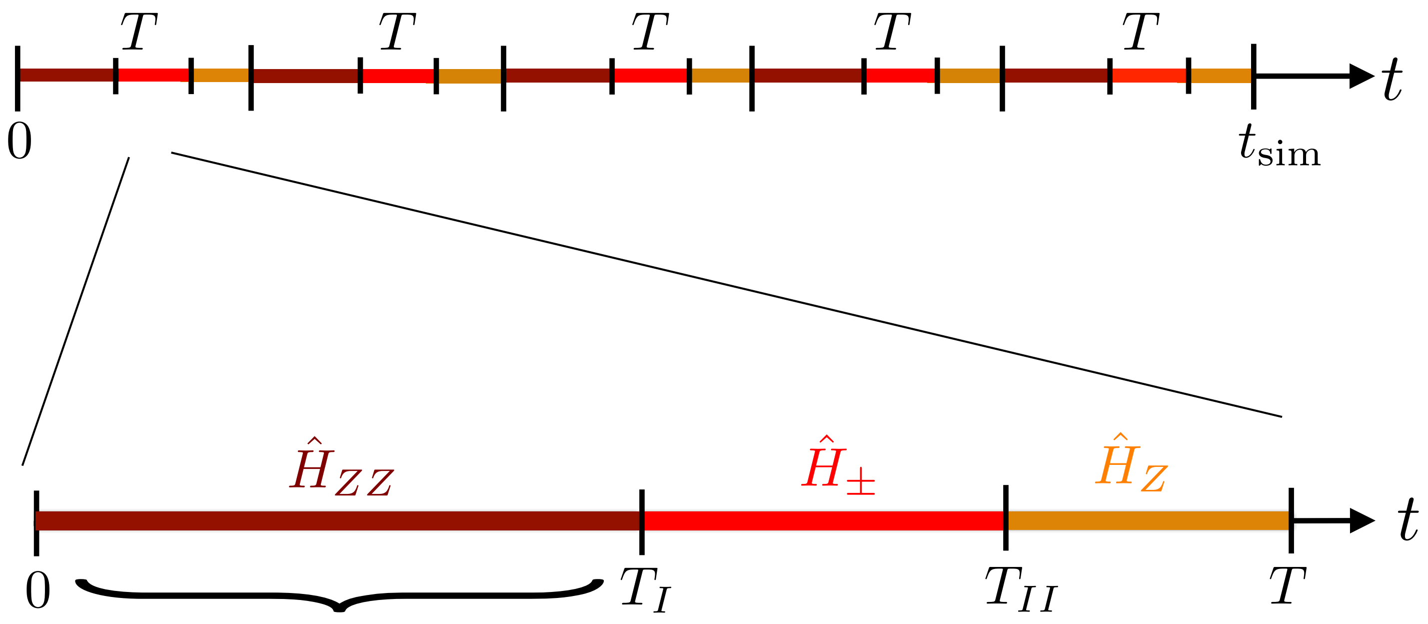
effective particle masses



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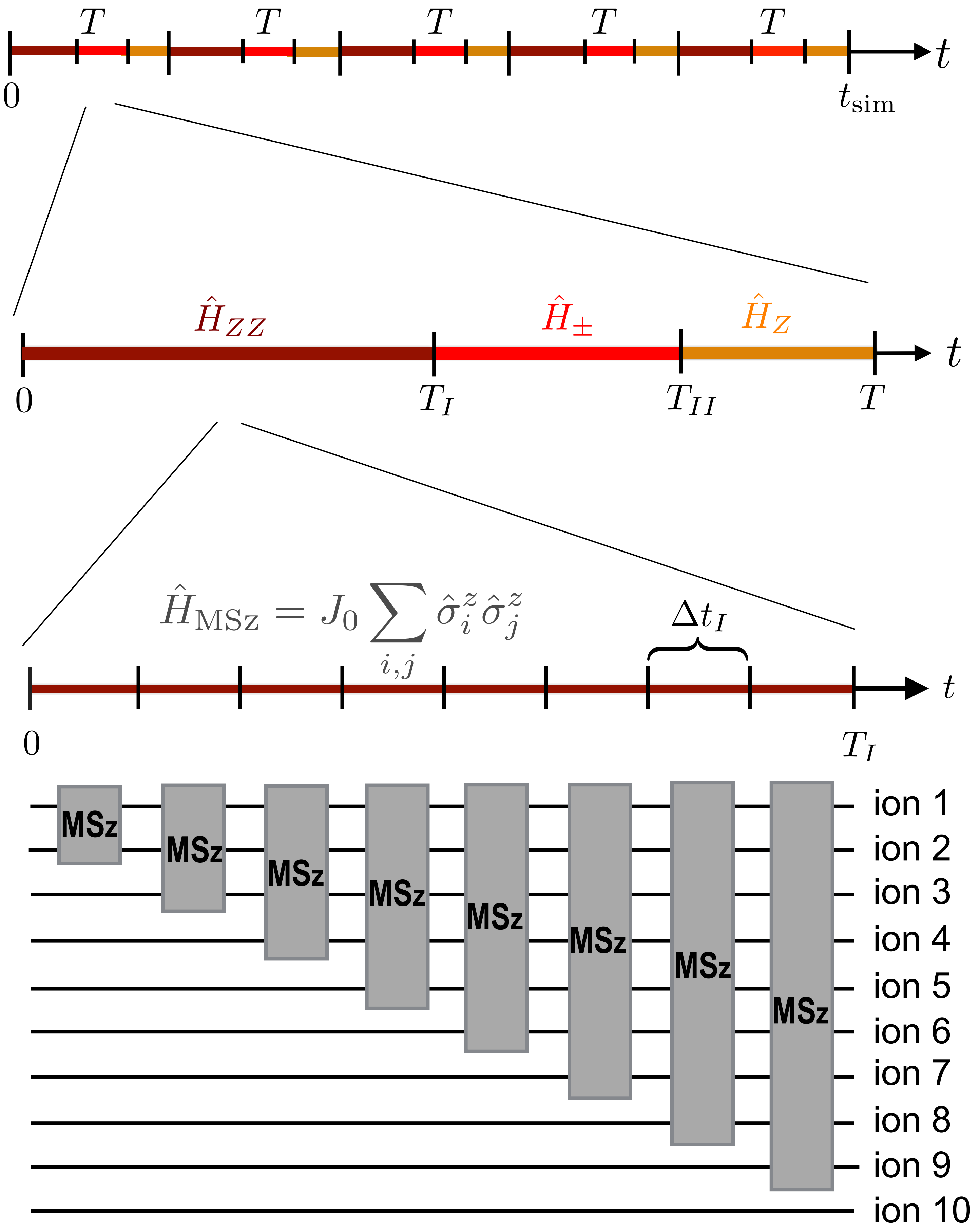

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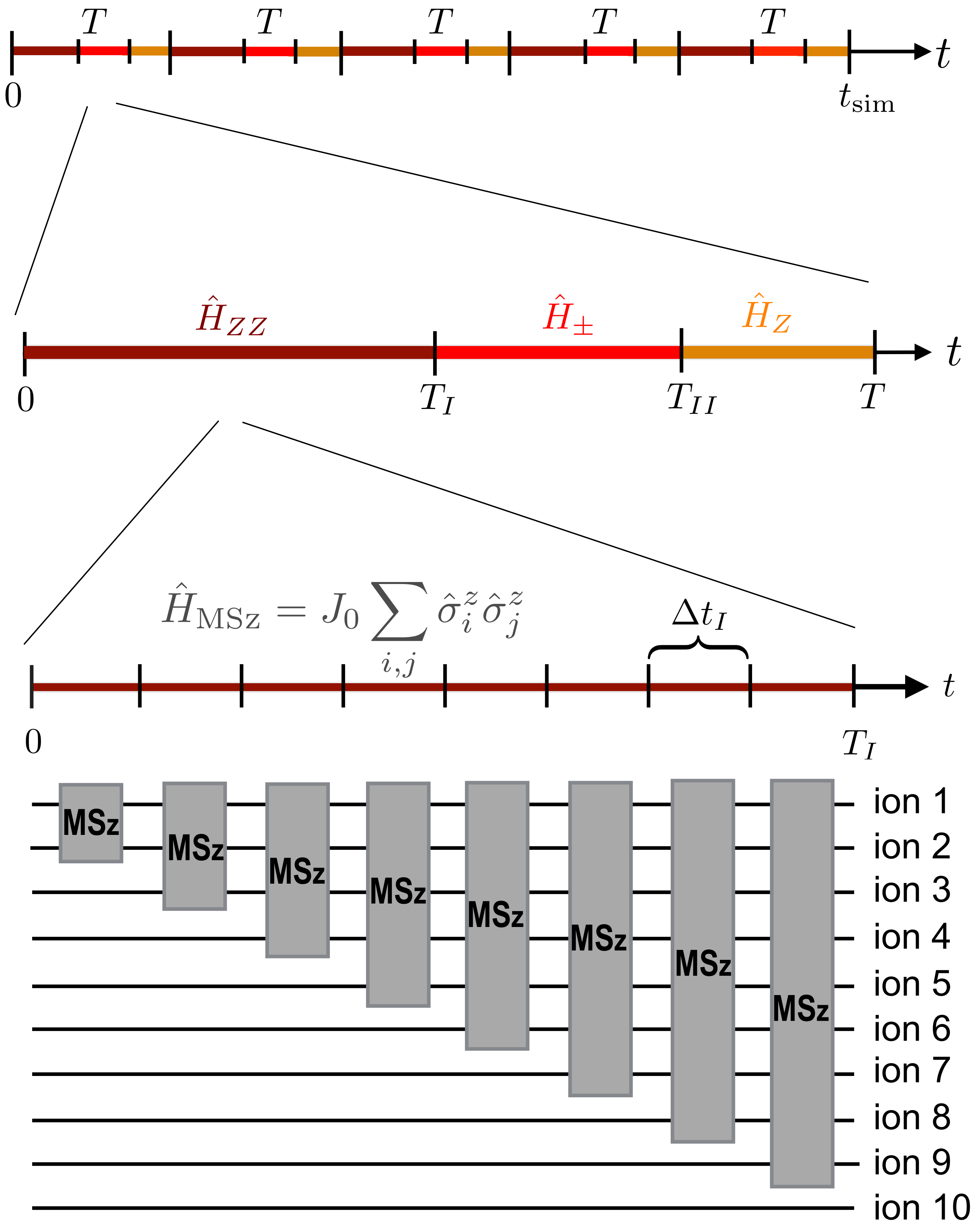

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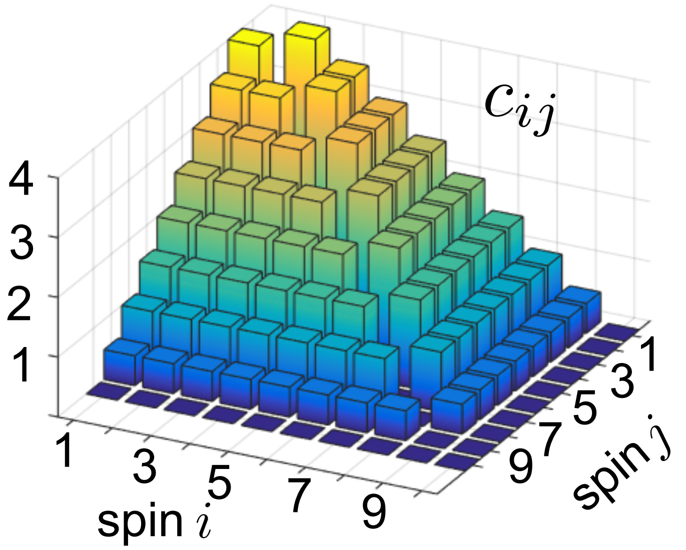
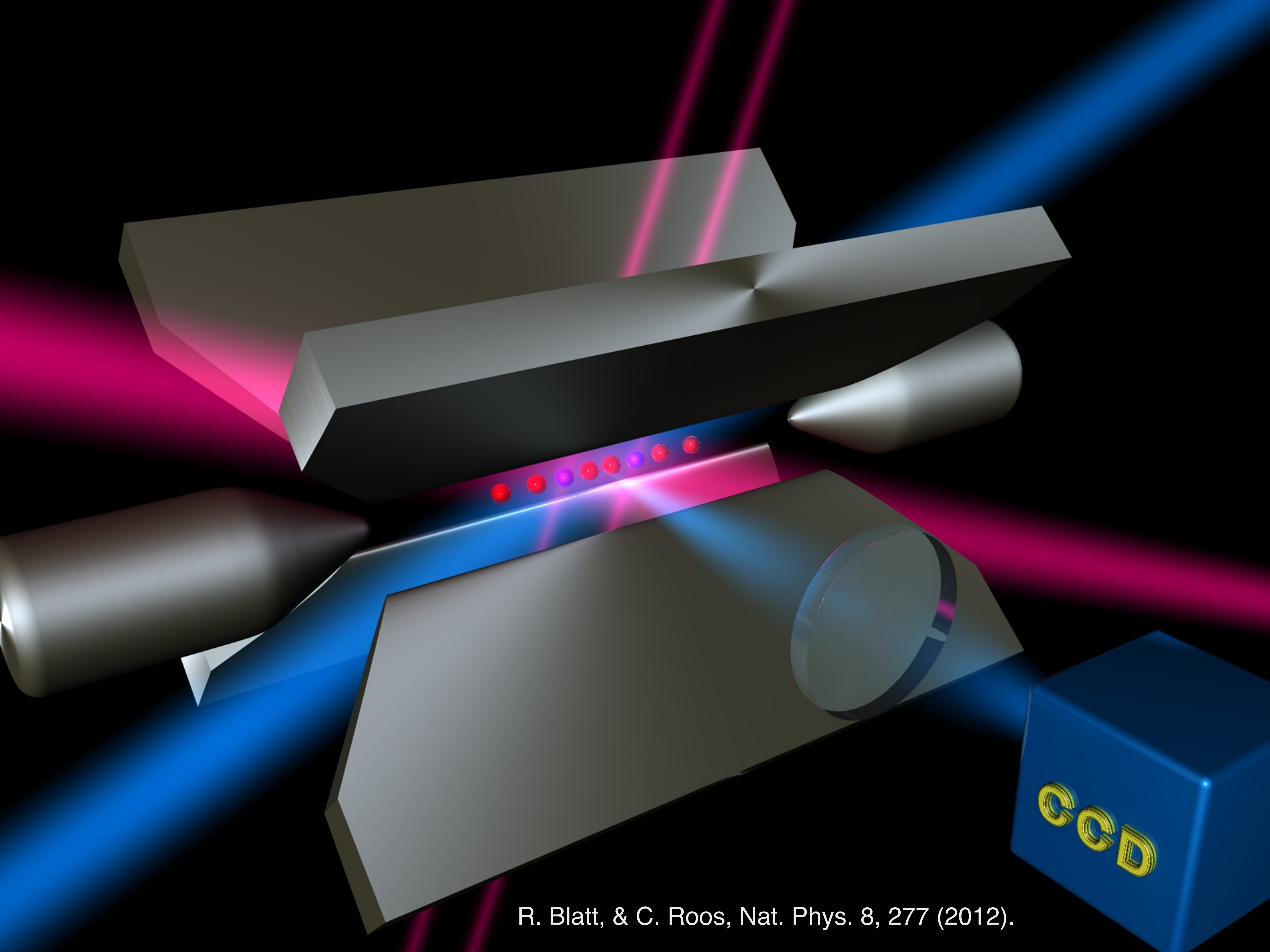
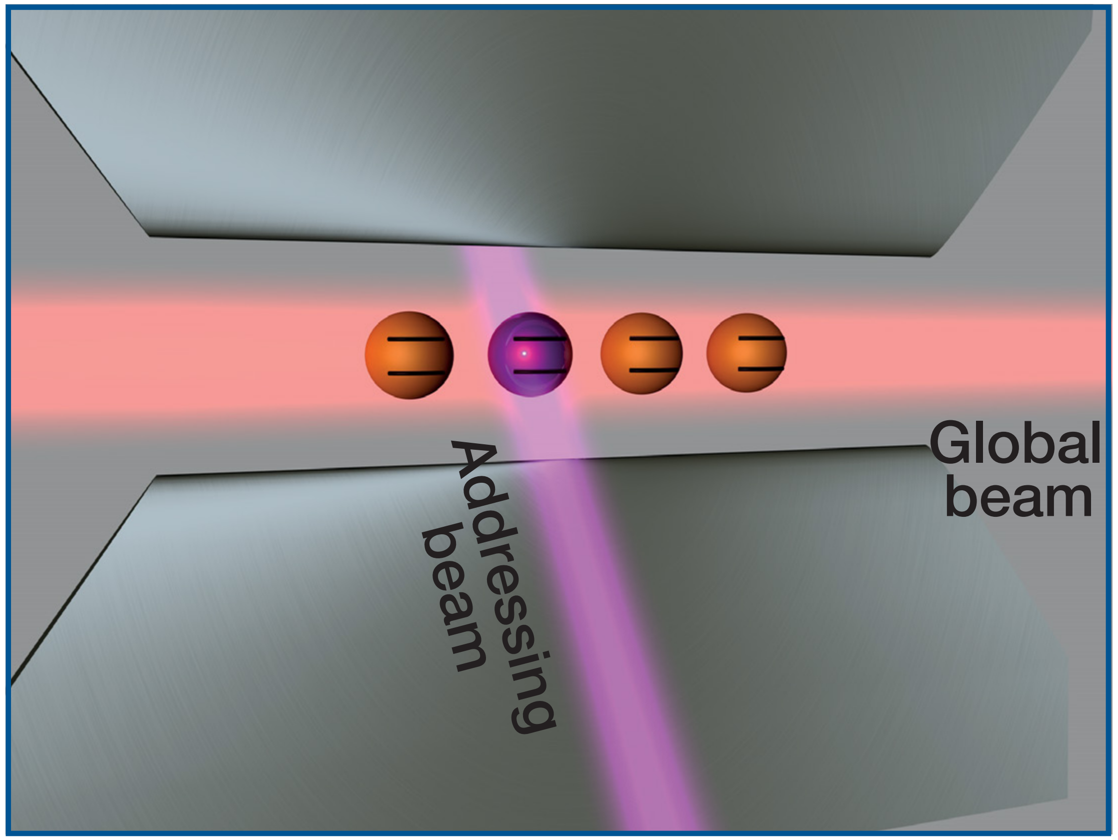




R. Blatt, & C. Roos, Nat. Phys. 8, 277 (2012).

Experiment

E. Martinez, P. Schindler, D. Nigg, A. Erhard, T. Monz, and R. Blatt



Tools for universal digital quantum simulation are available:

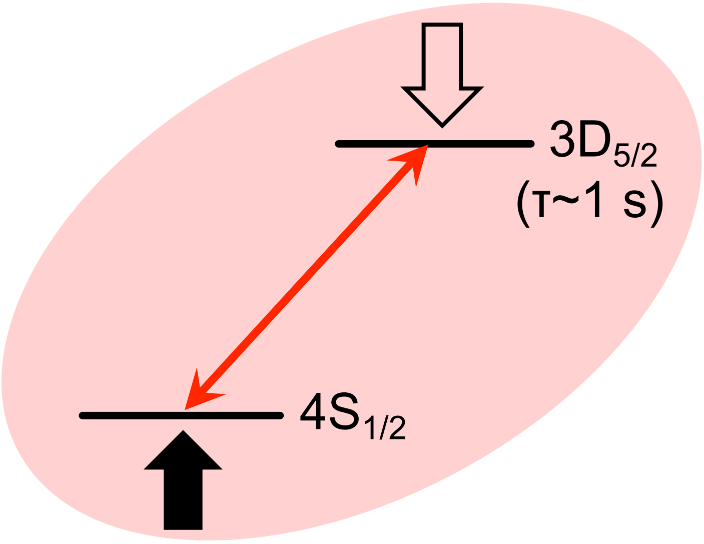
B. Lanyon, et al. Science 334, 57 (2011).

- High fidelity local rotations ✓
- Entangling gates ✓

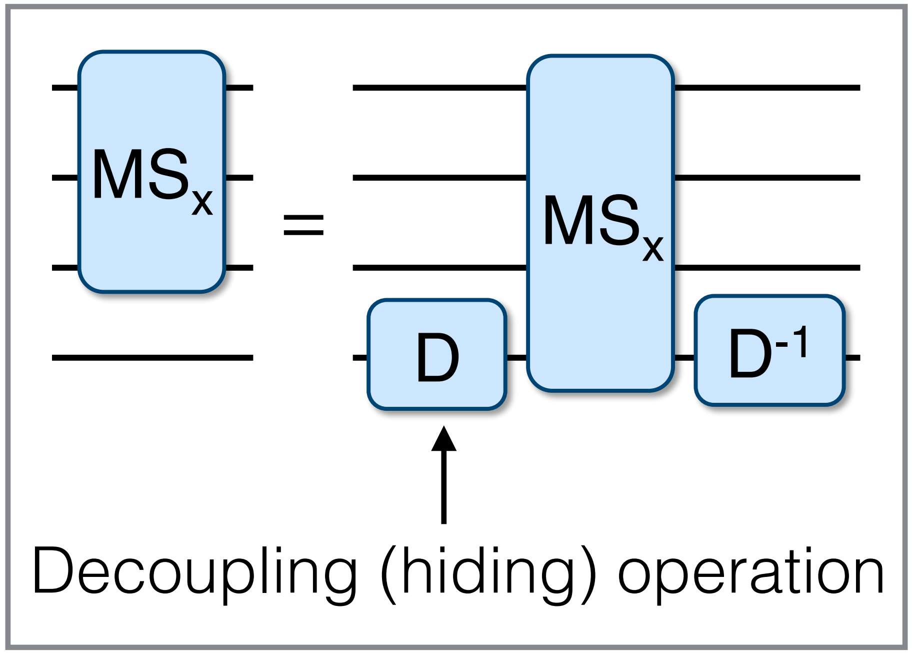
Mølmer-Sørensen interaction

$$H_0 = J_0 \sum_{i,j} \sigma_i^x \sigma_j^x$$

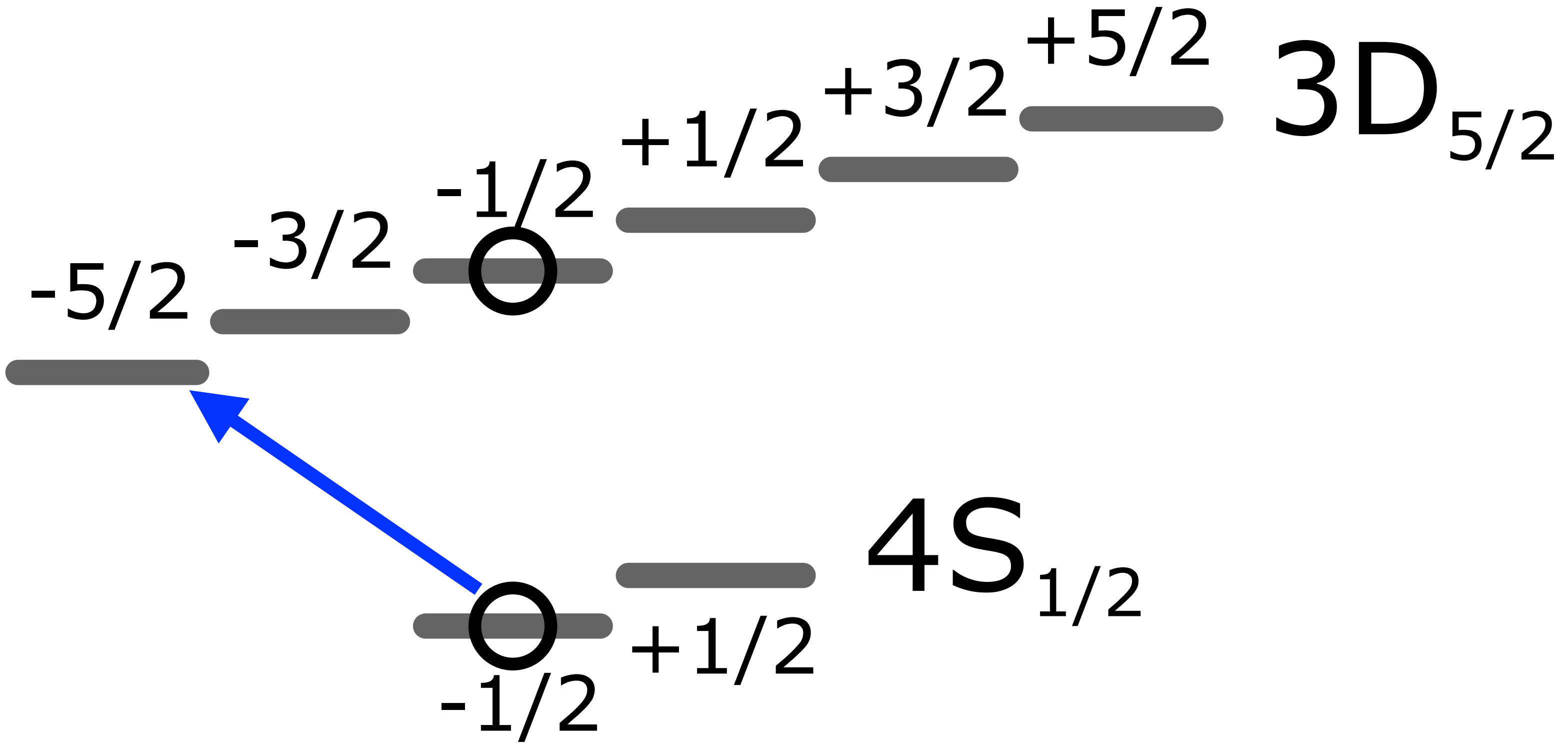
Qubit



Decoupling of individual ions



Ions are selectively decoupled from the MS interaction by transferring their population to off-resonant Zeeman levels:



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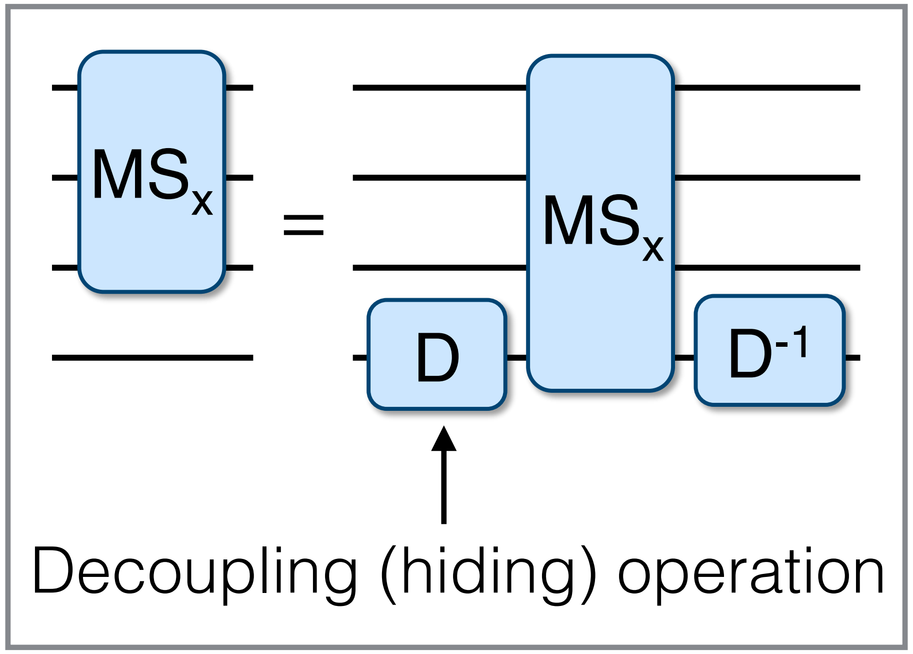

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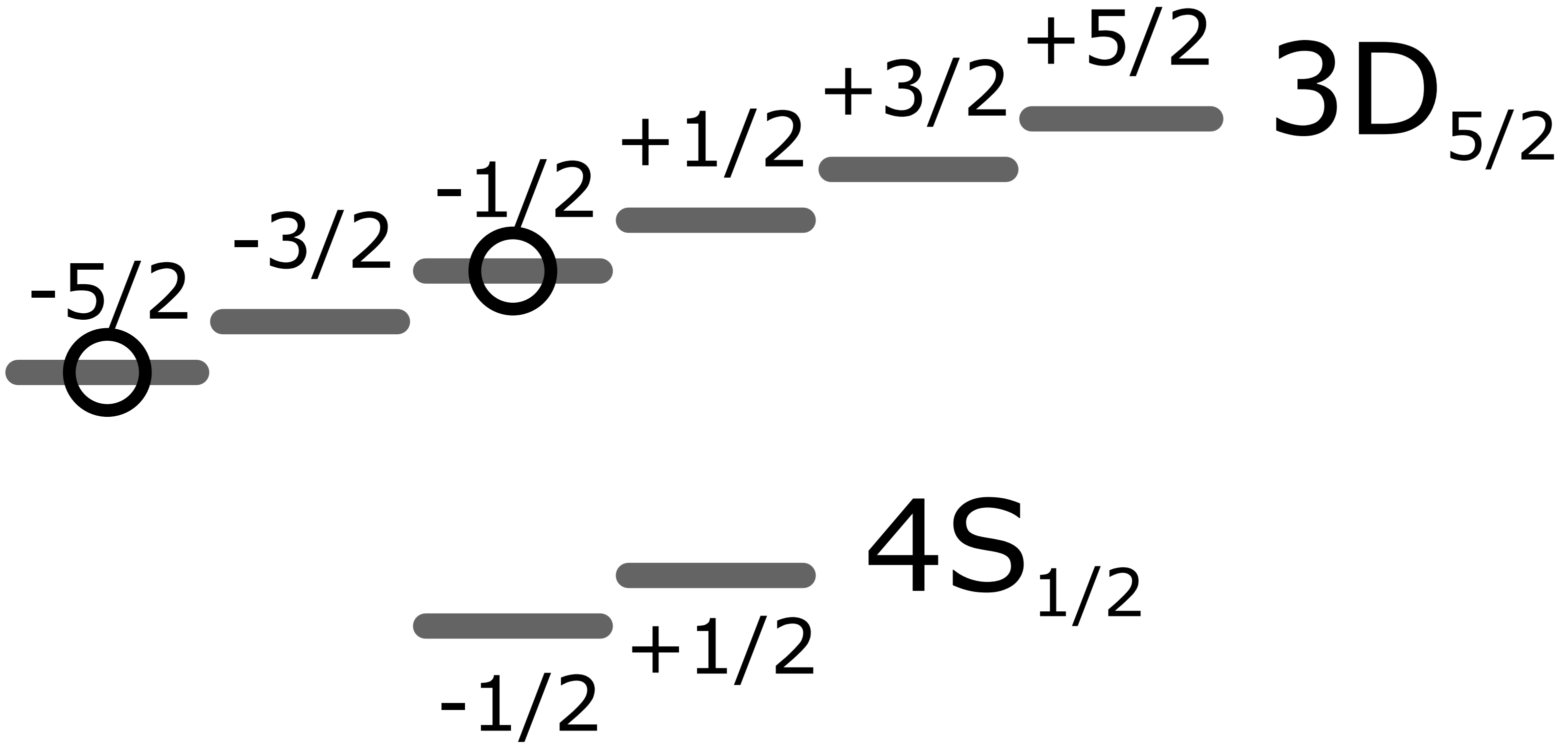

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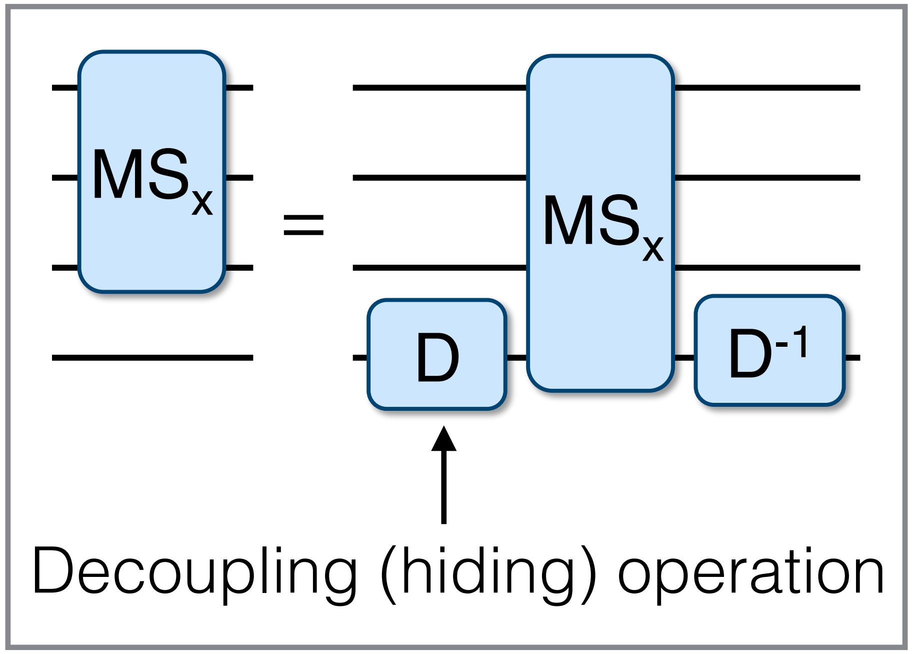

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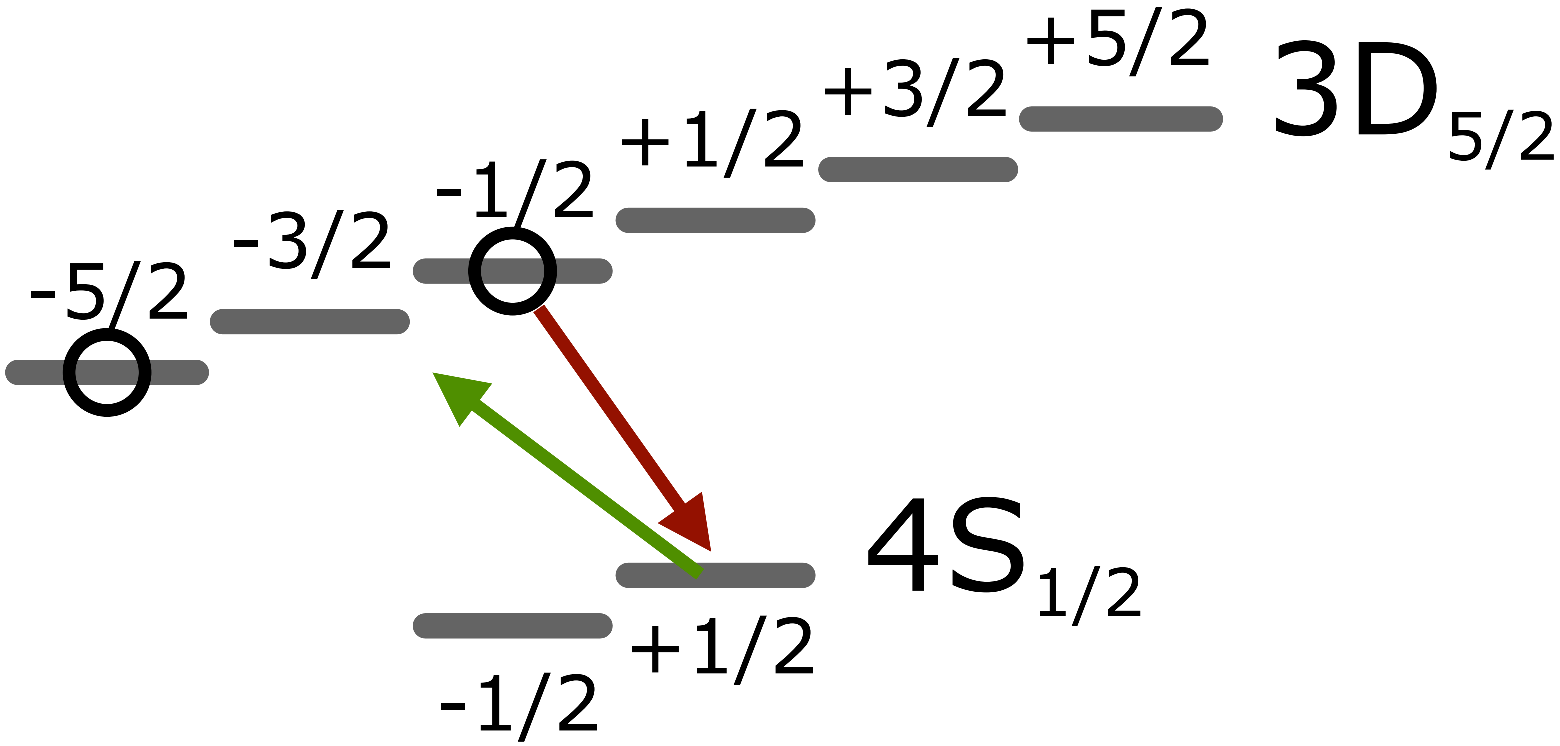

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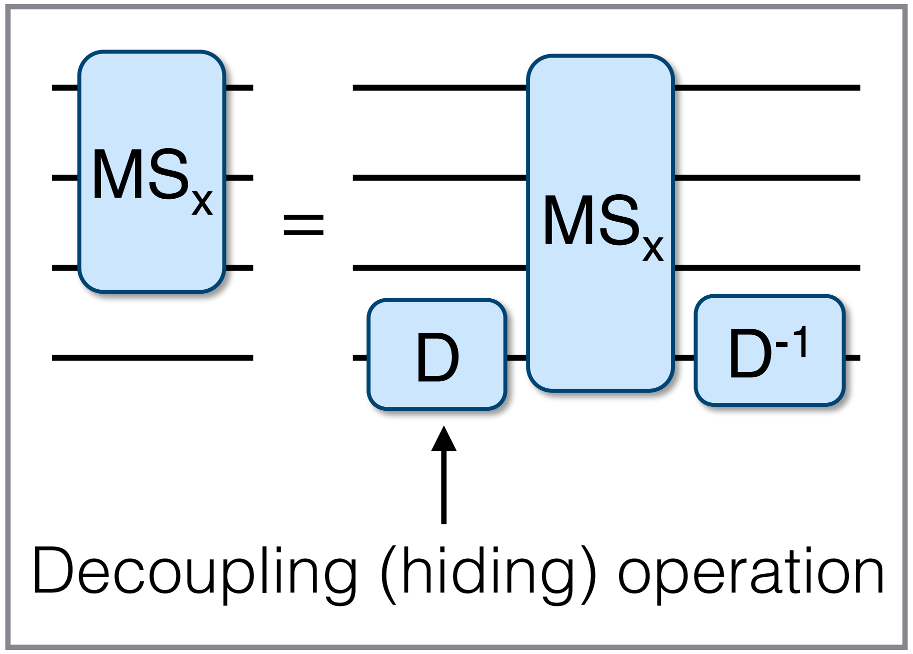

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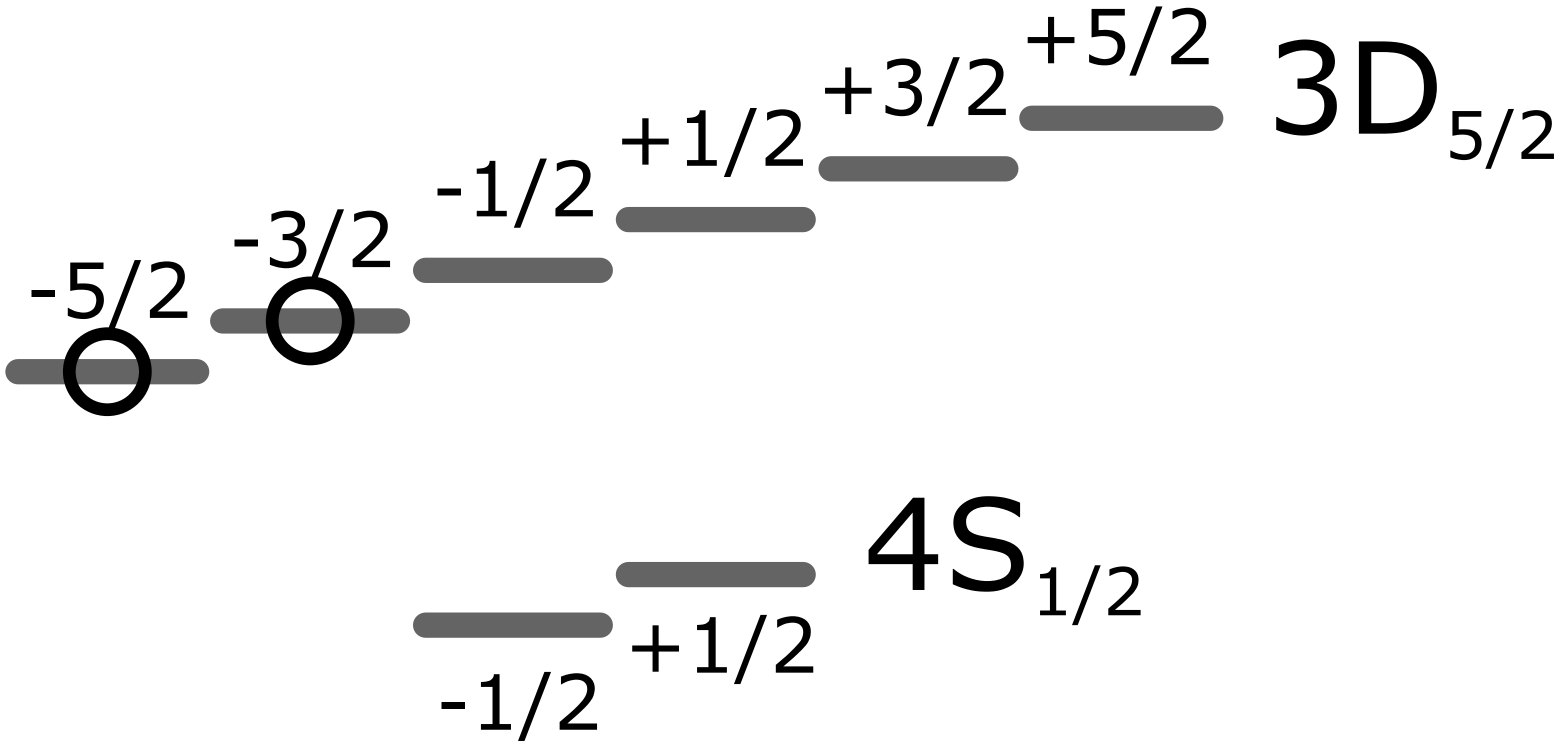

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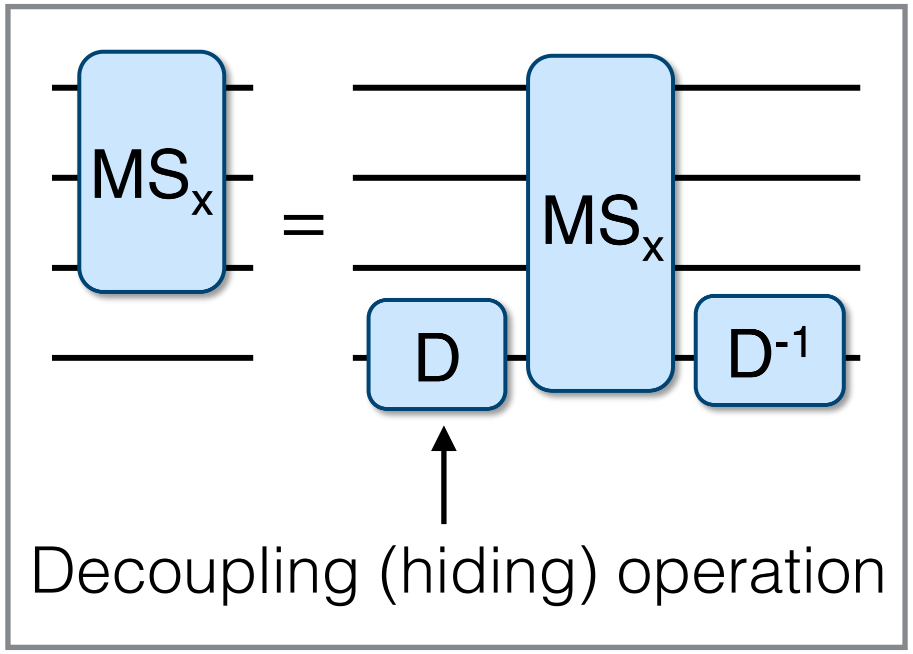

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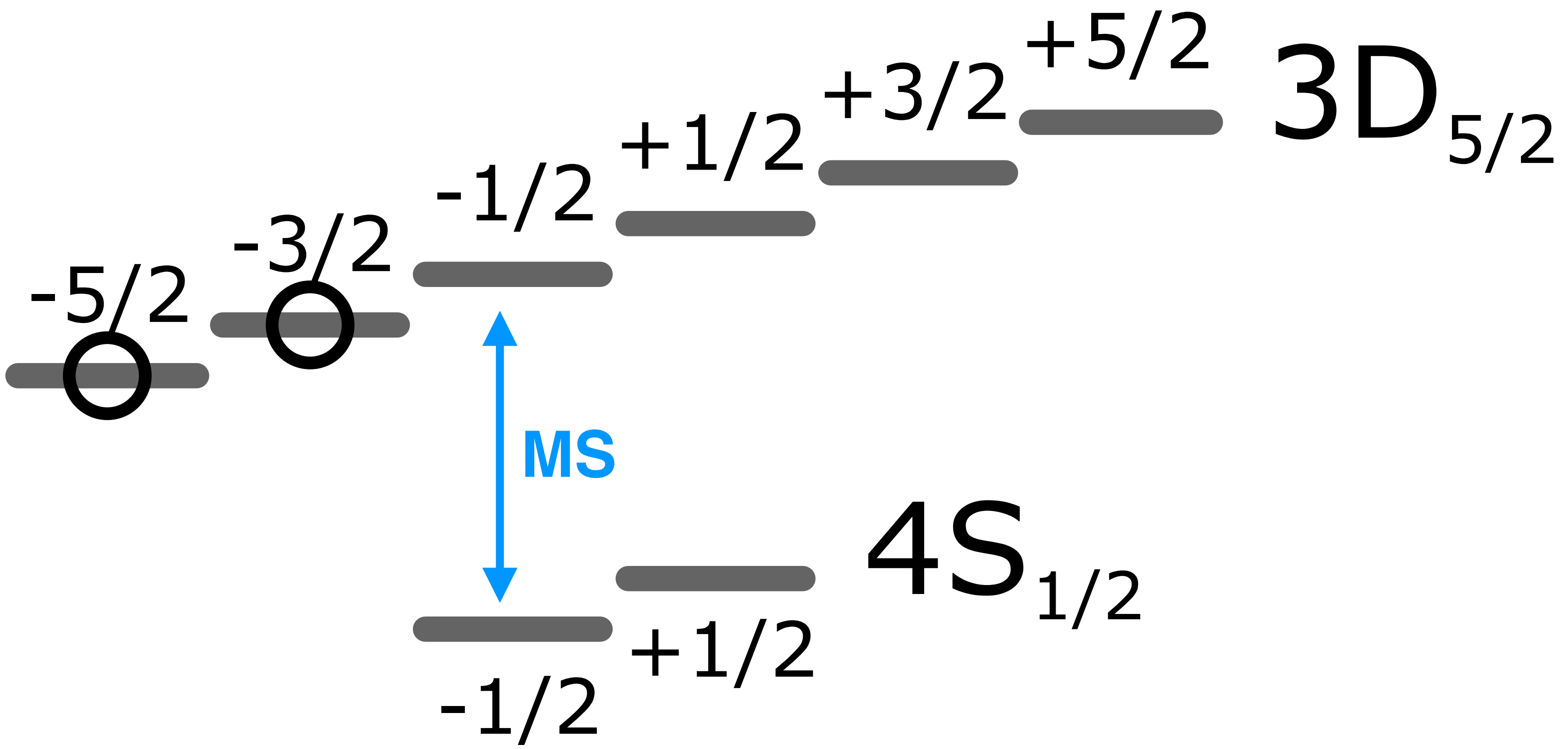
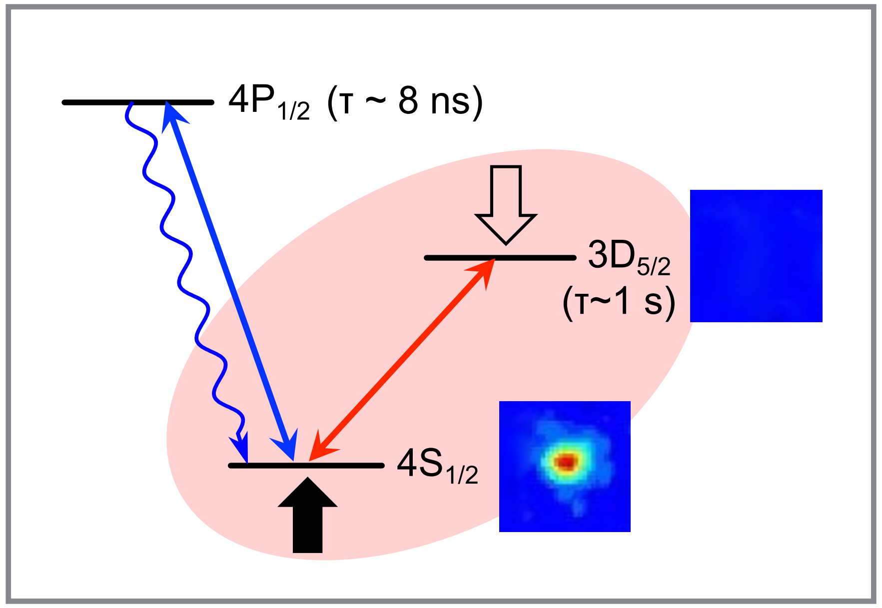
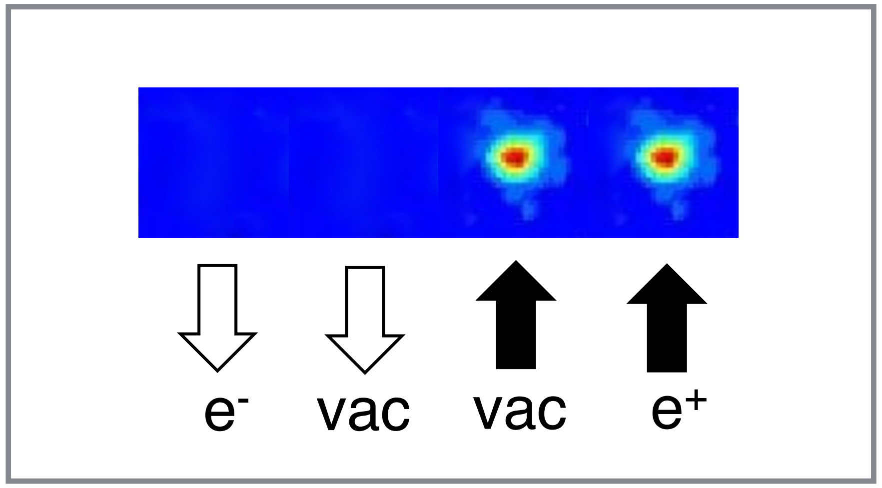


Measurements

- Electron shelving technique (projective measurement in the z-basis)



- Imaging of the whole ion chain on a CCD camera

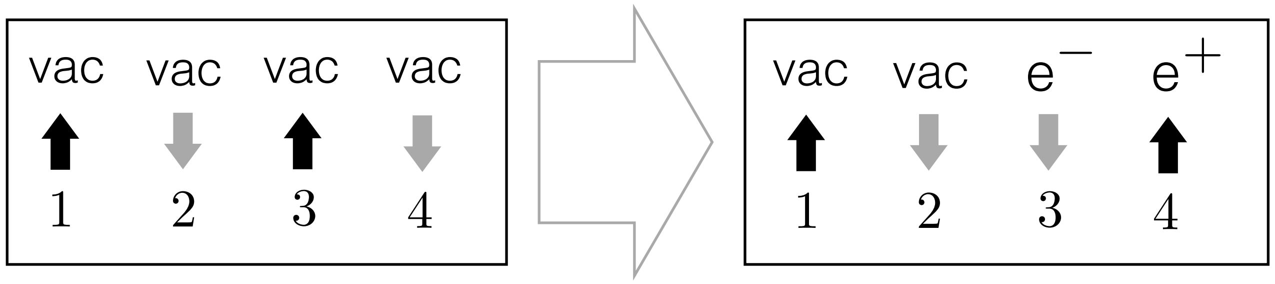


- Change of the measurement basis: full state tomography

Quantum Simulation of pair creation

Particle number density: $\nu(t) = \frac{1}{N}\sum_{n=1}^{N}\langle(-1)^n\sigma_n^z(t) + 1\rangle$

Creation of a particle antiparticle pair:



$\nu = 0$

$\nu = 0.5$

Schwinger Mechanism

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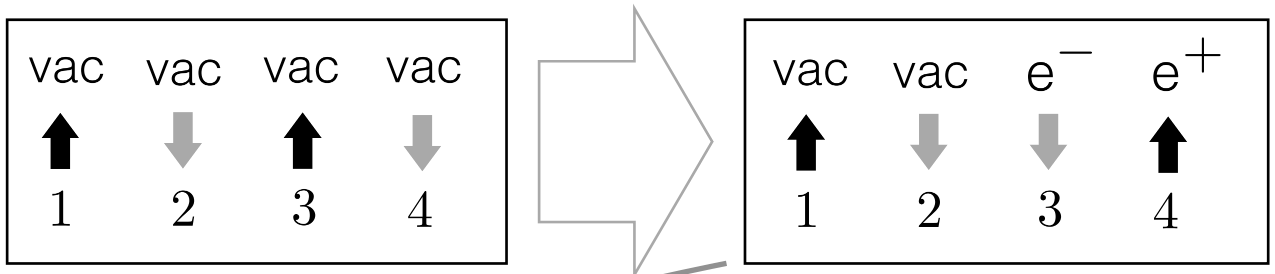
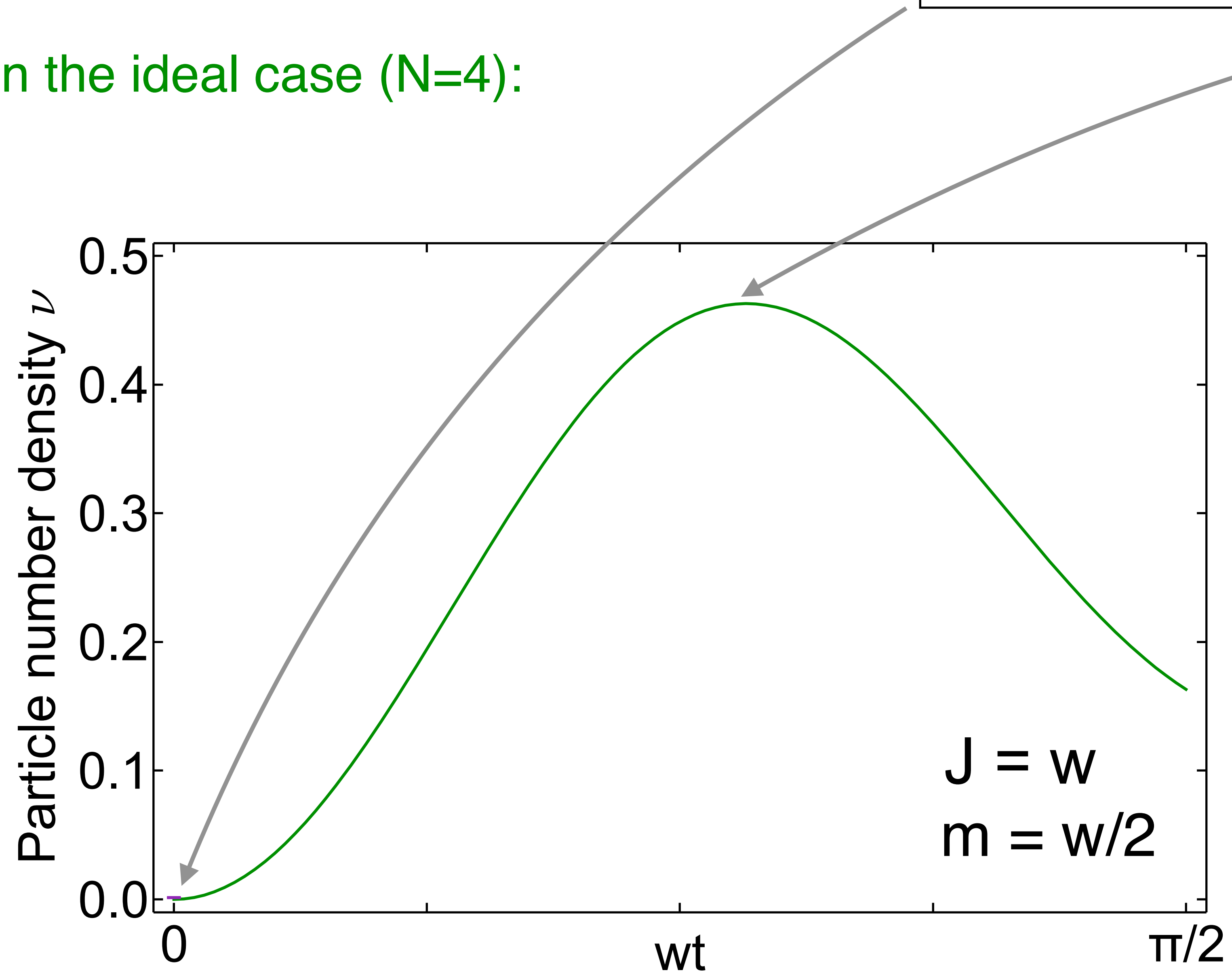


In the ideal case (N=4):



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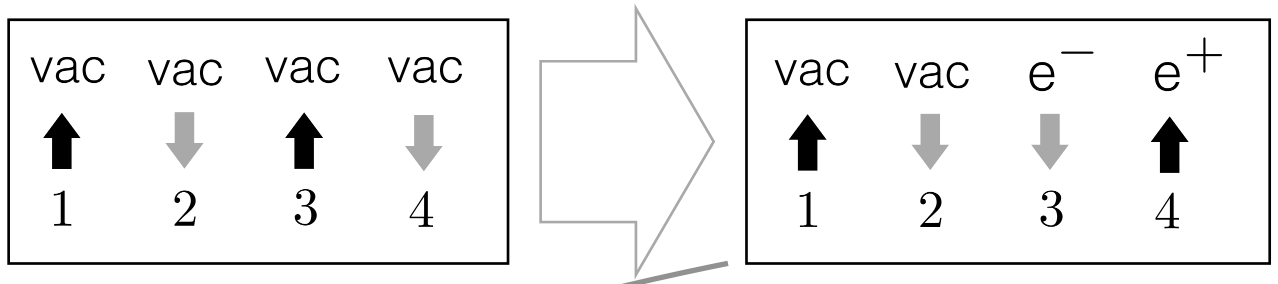

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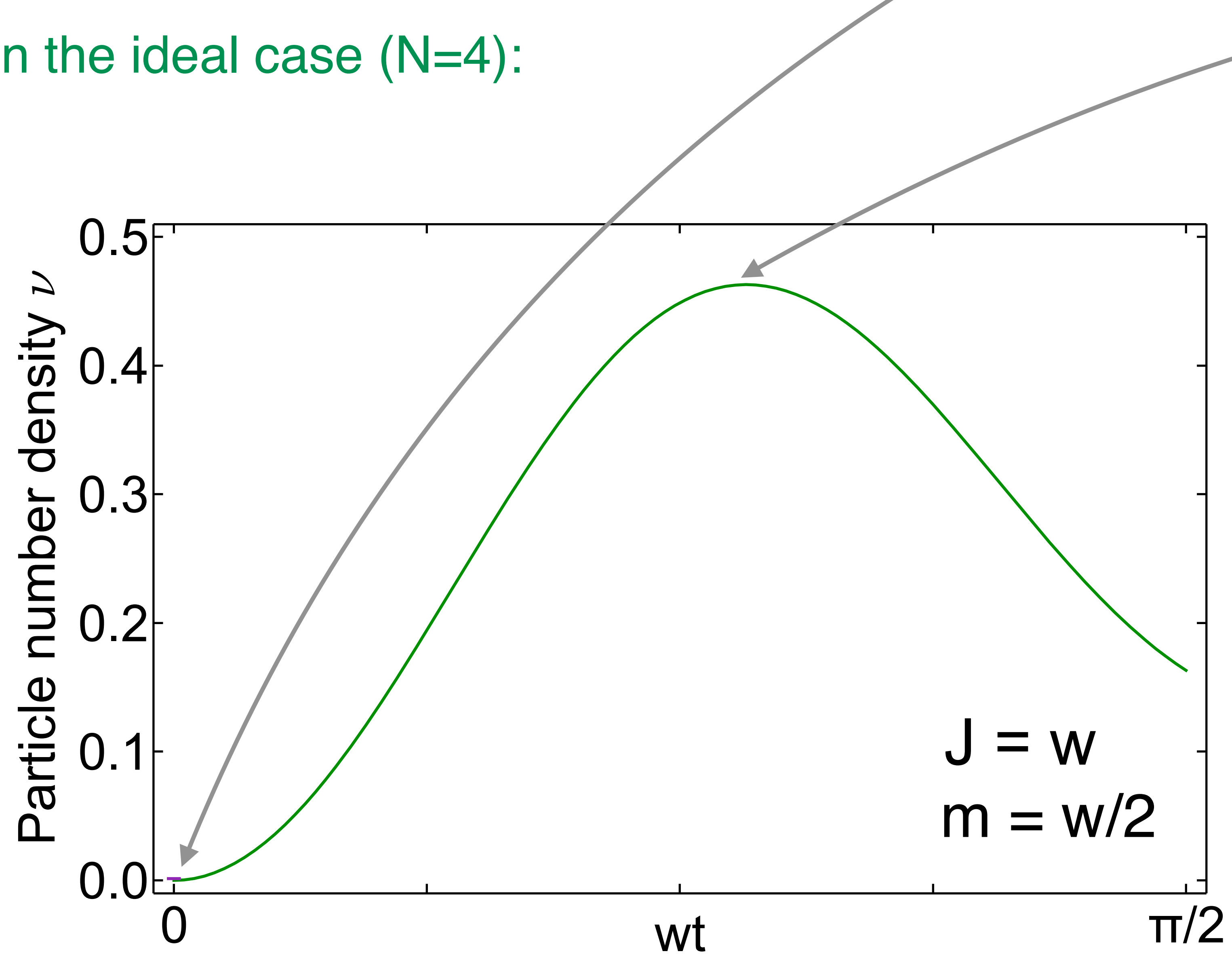

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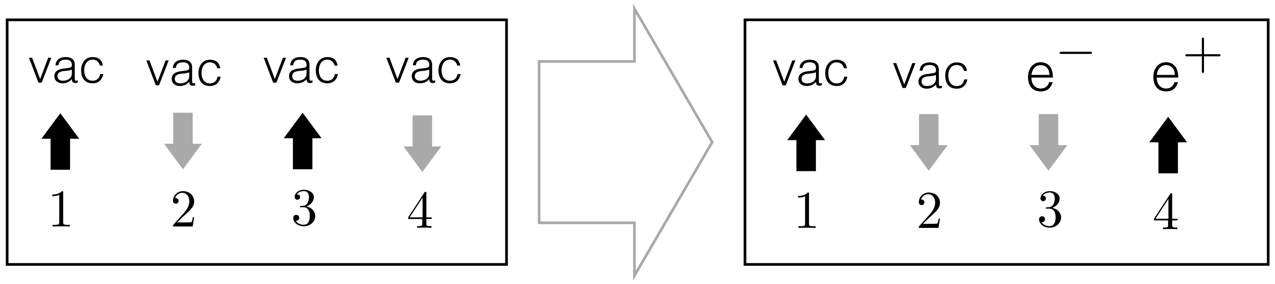
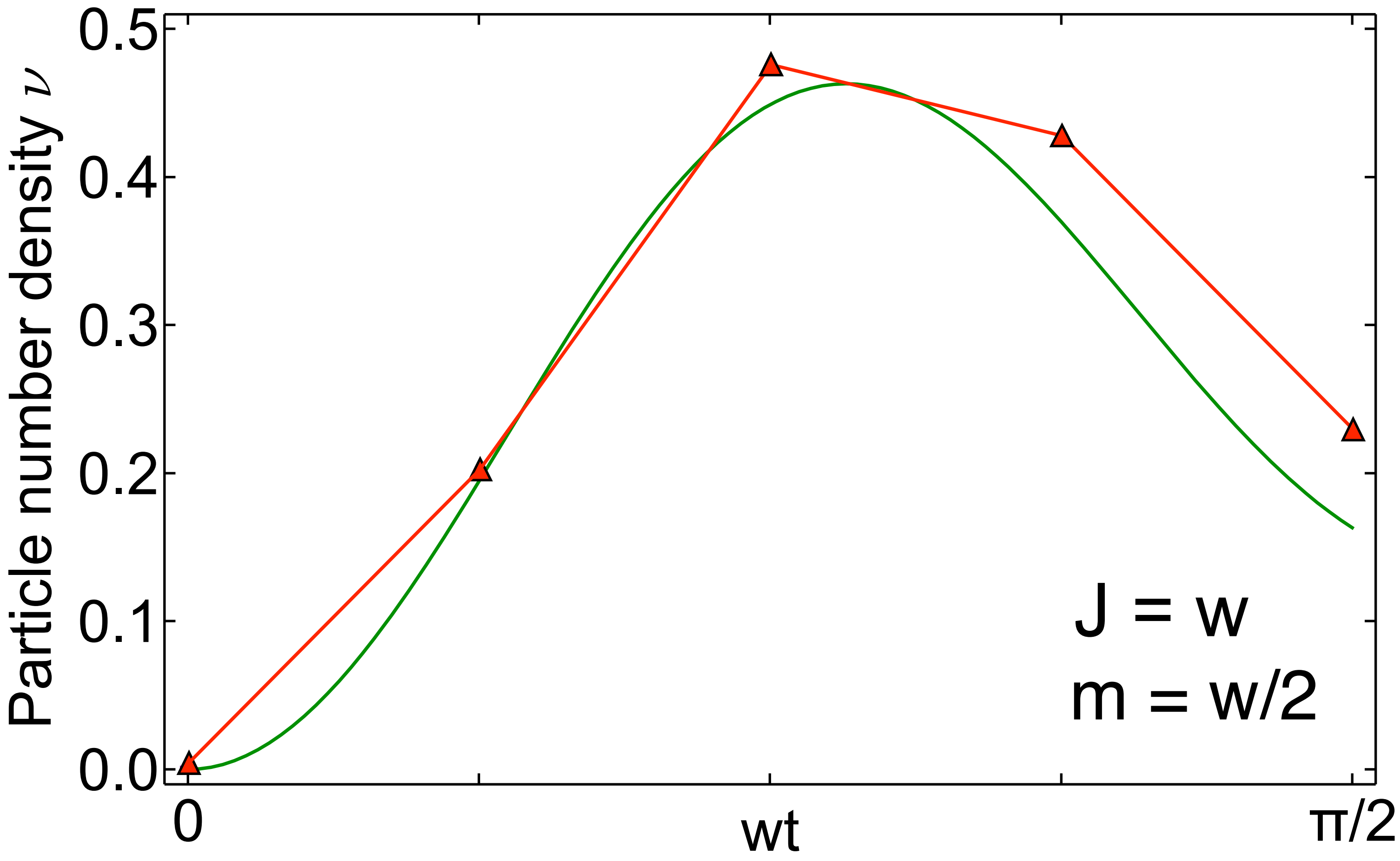


Including discretisation errors (N=4):



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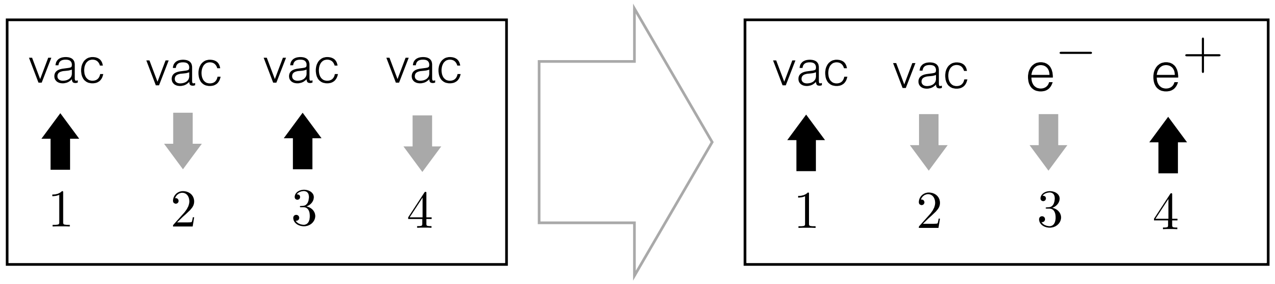
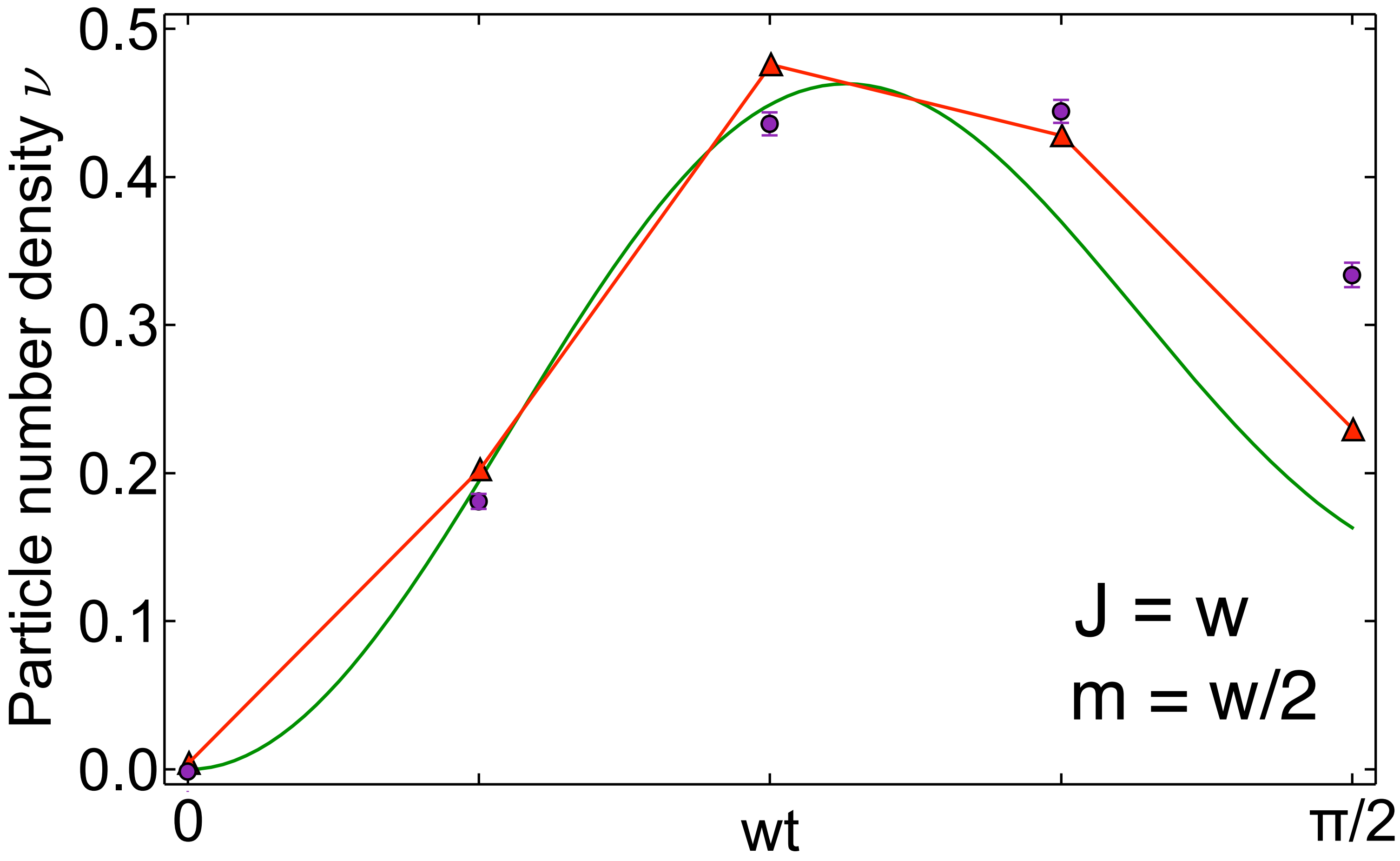
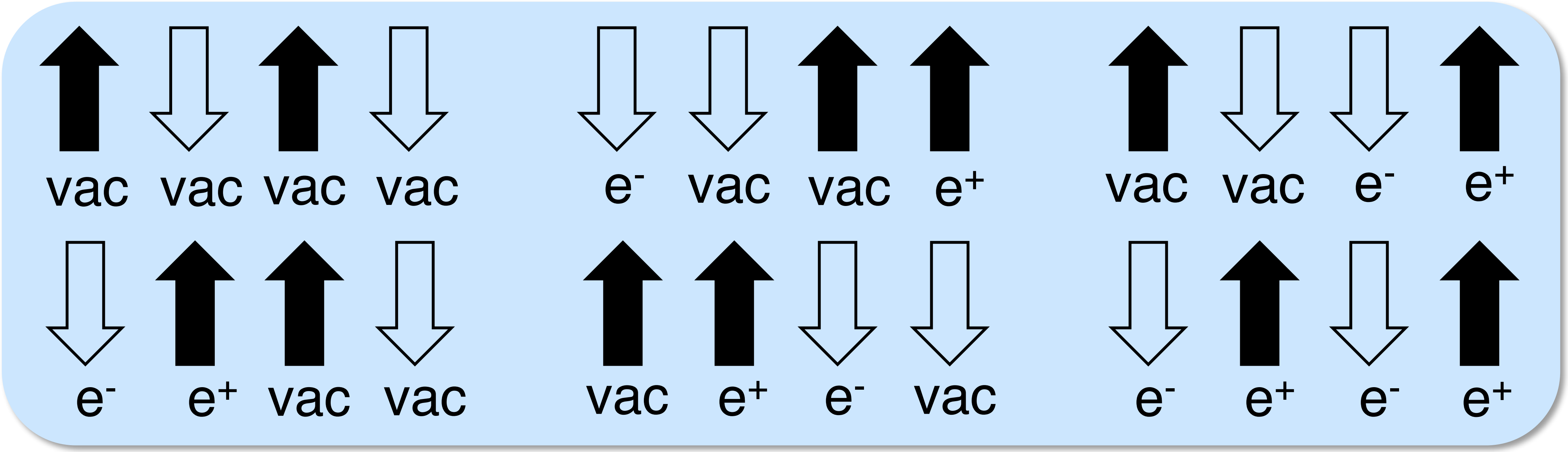


Experimental data (after postselection):



Postselection



Schwinger Model: zero charge subspace
Spin model: zero magnetization subspace

The desired dynamics preserve gauge invariance
Only implementation errors lead to states outside of this subspace

Schwinger Mechanism

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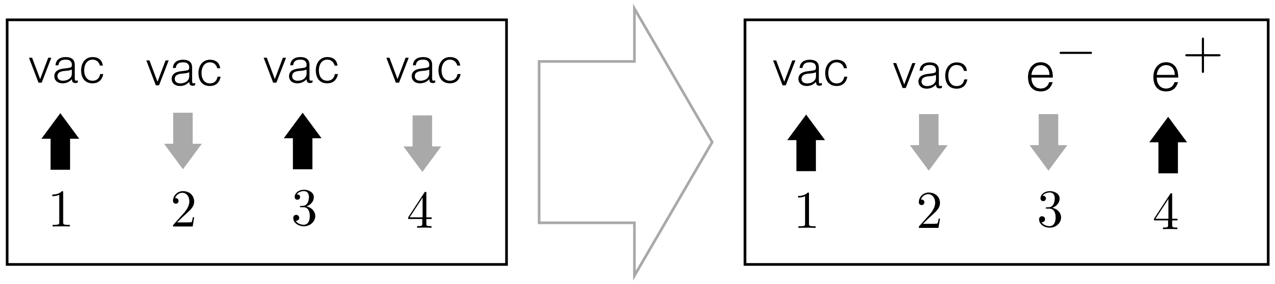
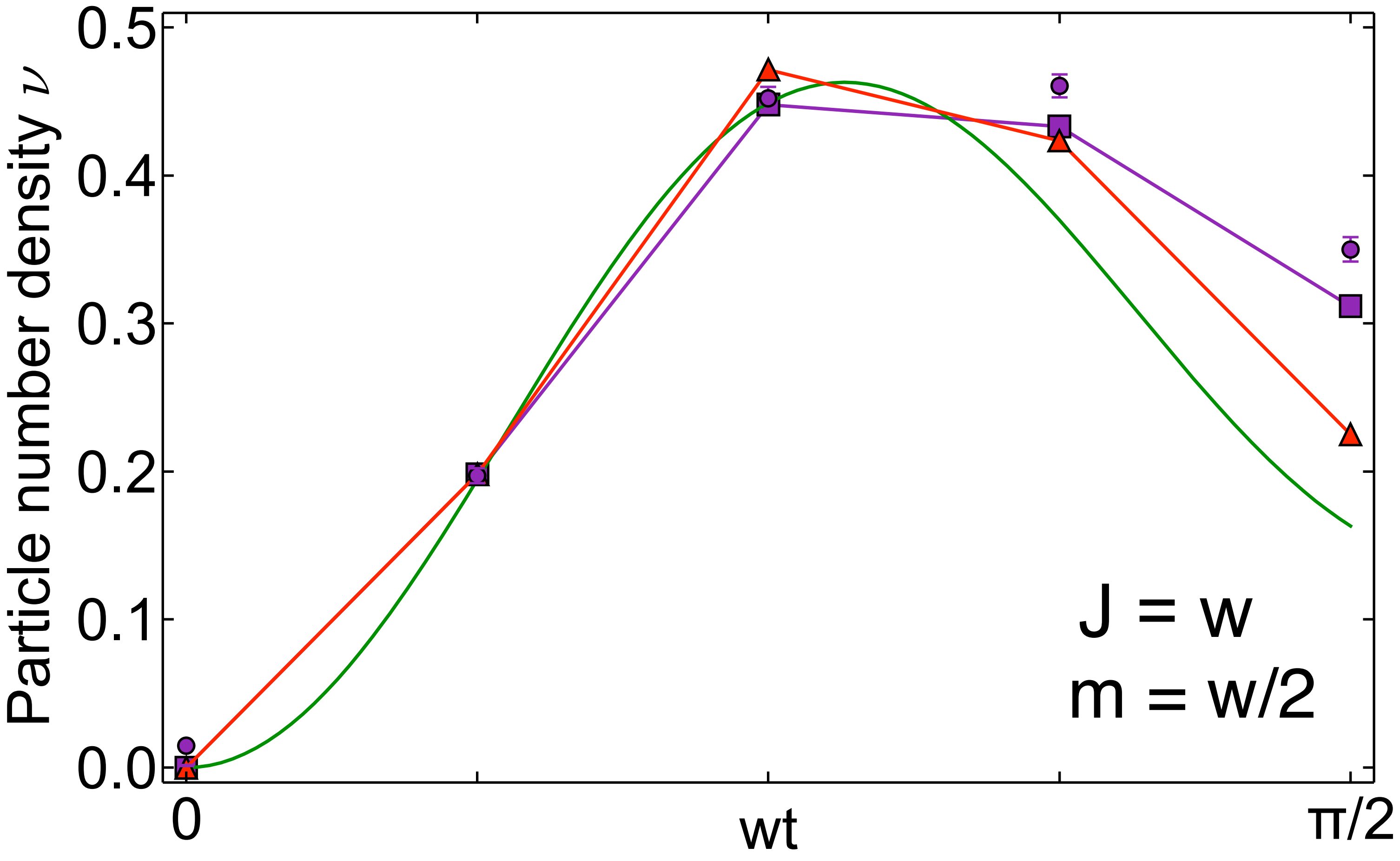
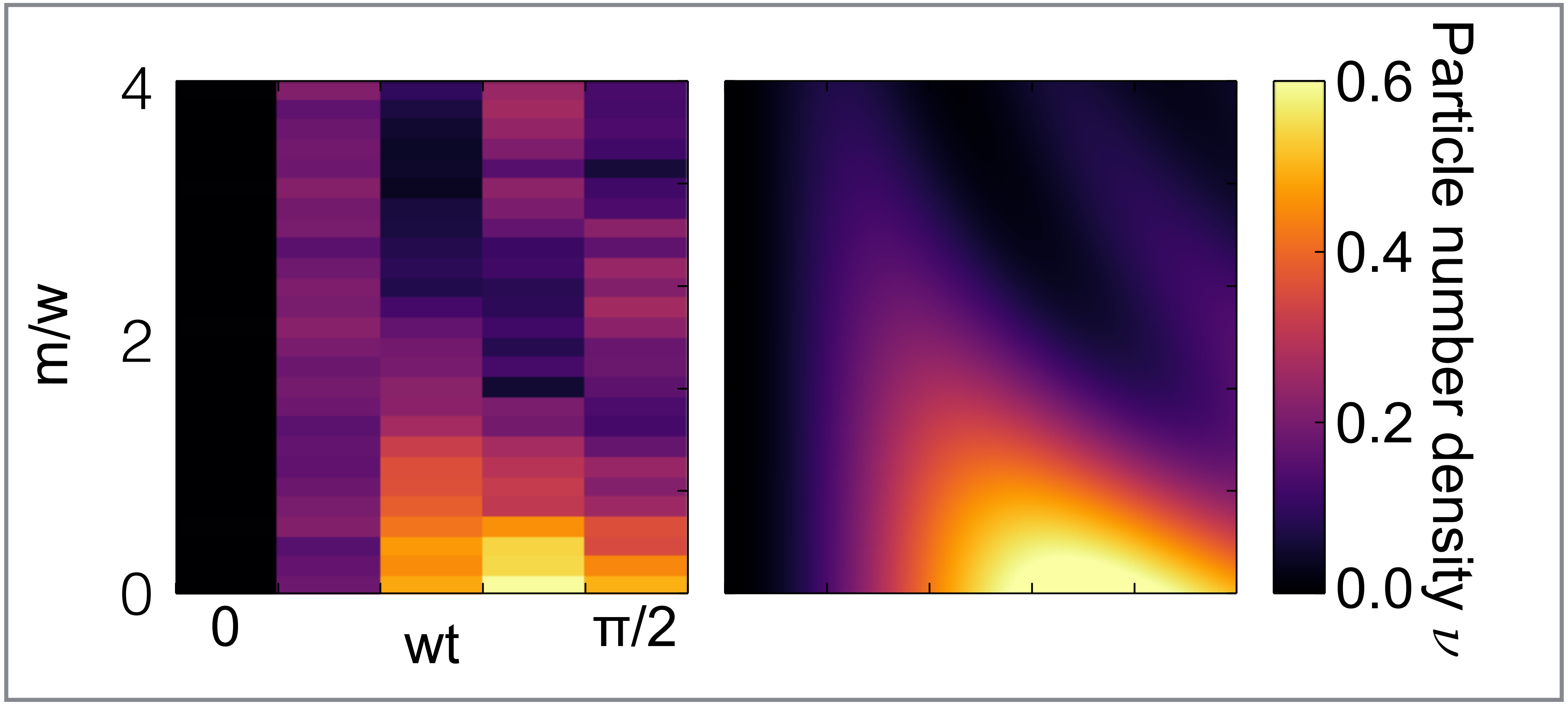


Simple error model (uncorrelated dephasing):



Schwinger Mechanism

Time evolution for different values of the particle mass m



Schwinger Mechanism

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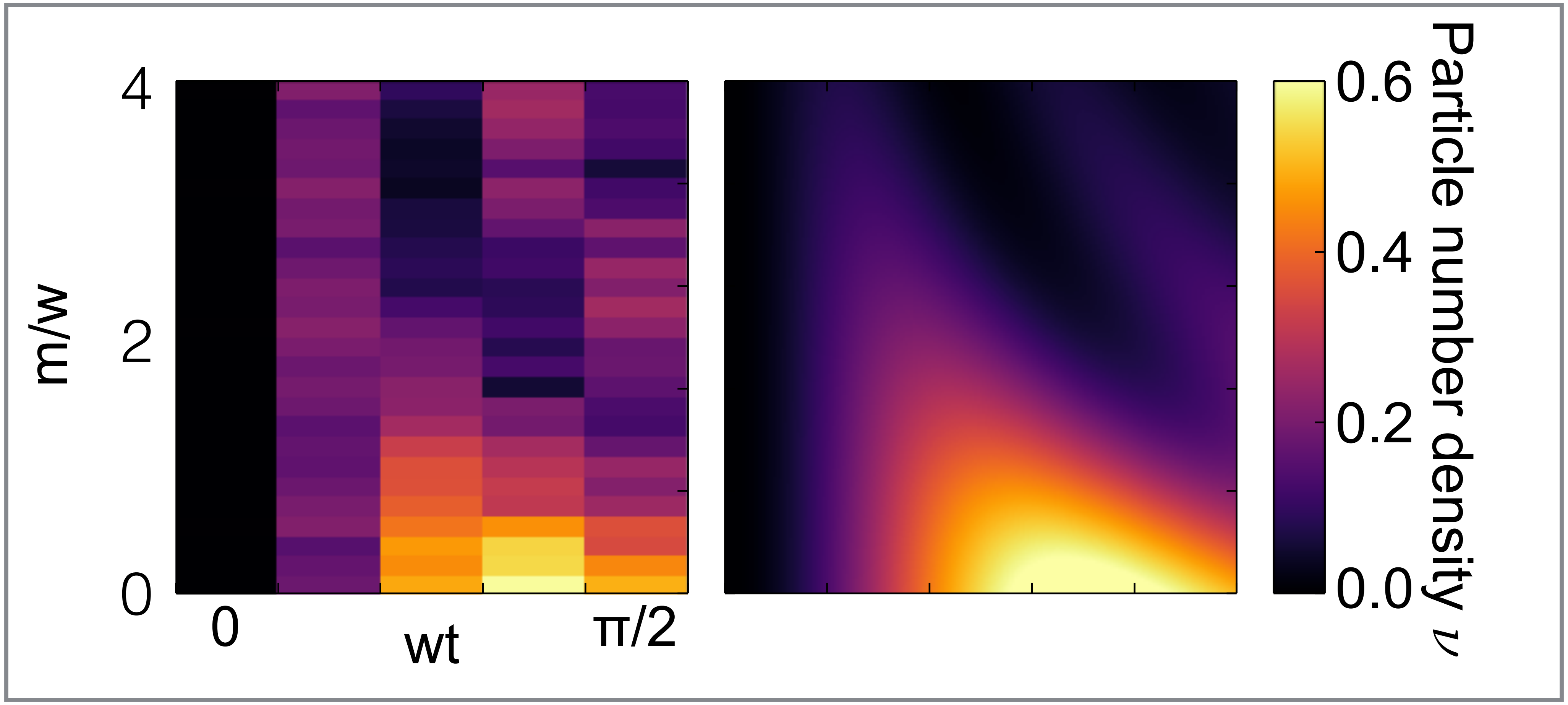
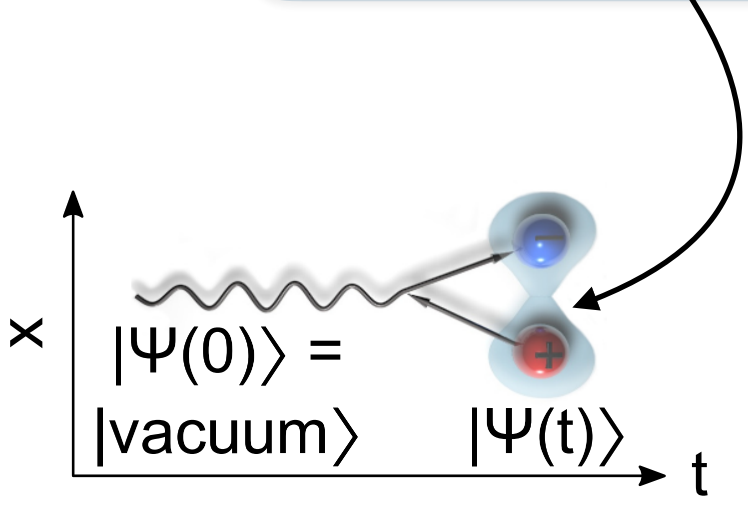


→ also: measurement of the vacuum persistence amplitude $|\langle \mathrm{vacuum}|\Psi(t)\rangle|^2$
see Nature 534, 516 (2016).

Entanglement in the Schwinger mechanism



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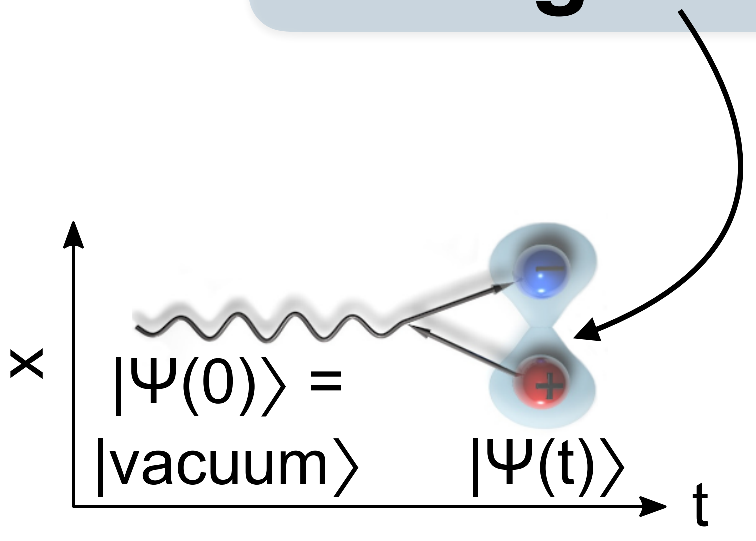
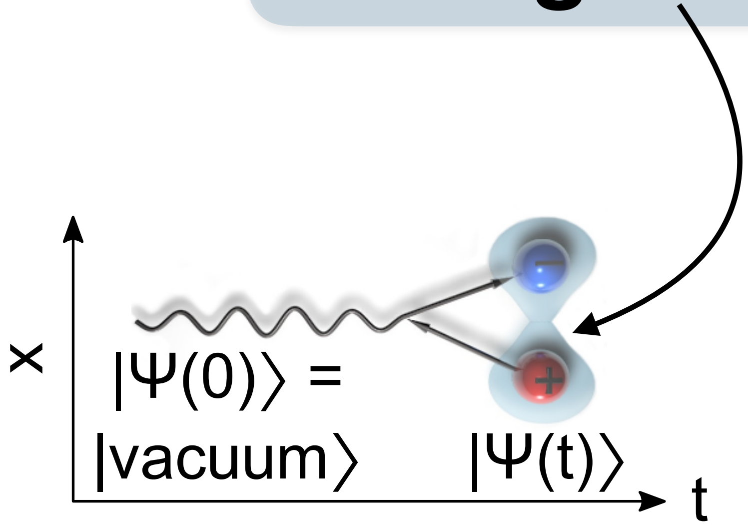


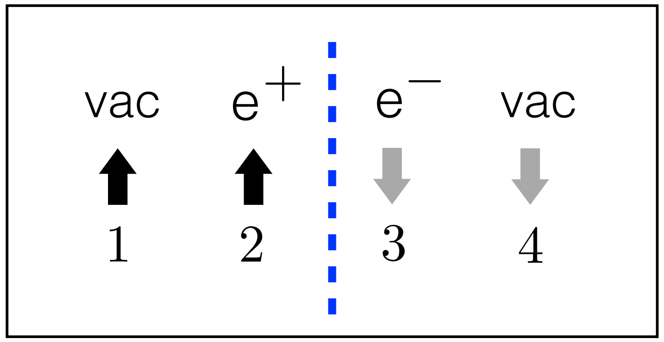
State tomography:
access to the full density matrix

Entanglement in the Schwinger mechanism



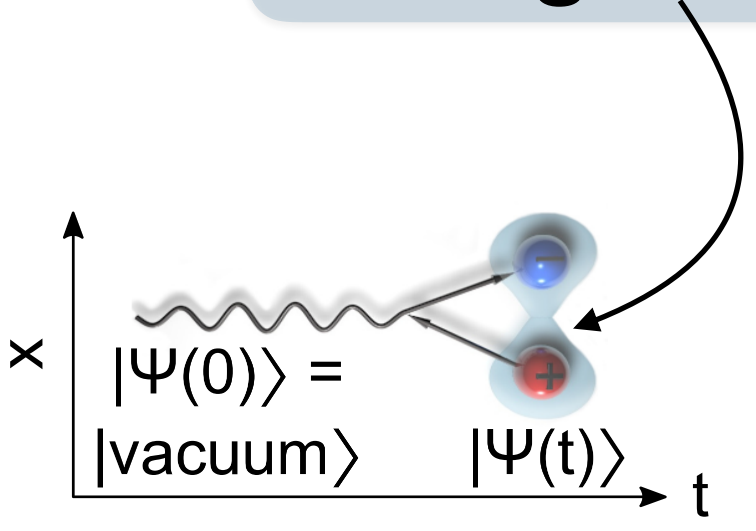
State tomography:
access to the full density matrix

E_n : logarithmic negativity
evaluated with respect to this bipartition:



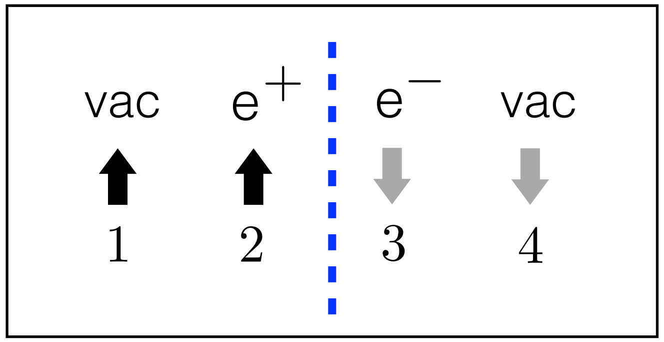
Entanglement between the two
halves of the system.

Entanglement in the Schwinger mechanism



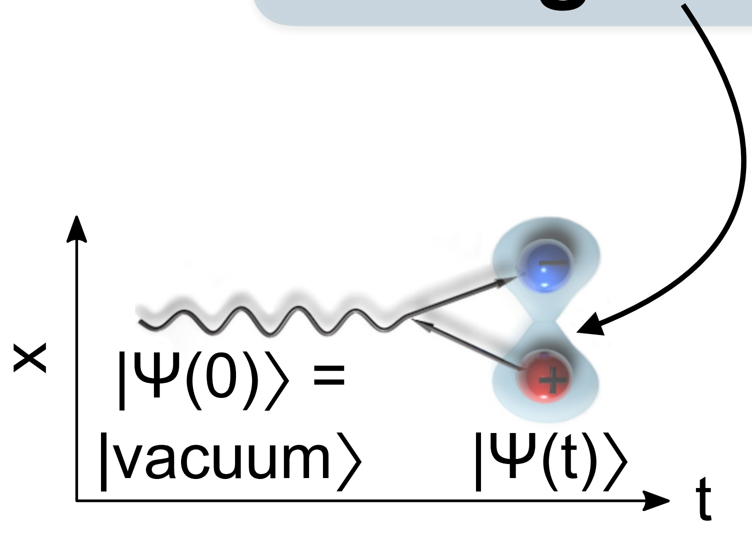
State tomography:
access to the full density matrix

E_n : logarithmic negativity
evaluated with respect to this bipartition:



Entanglement between the two
halves of the system.

Entanglement in the Schwinger mechanism



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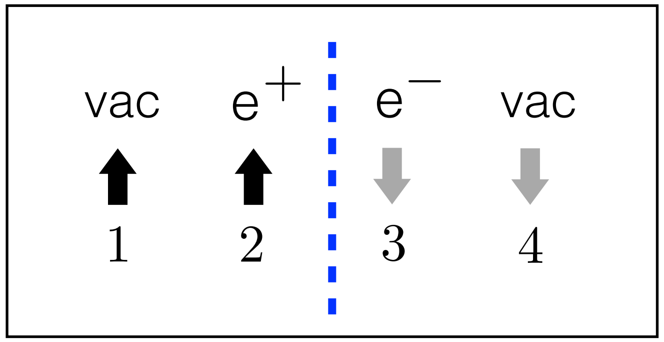
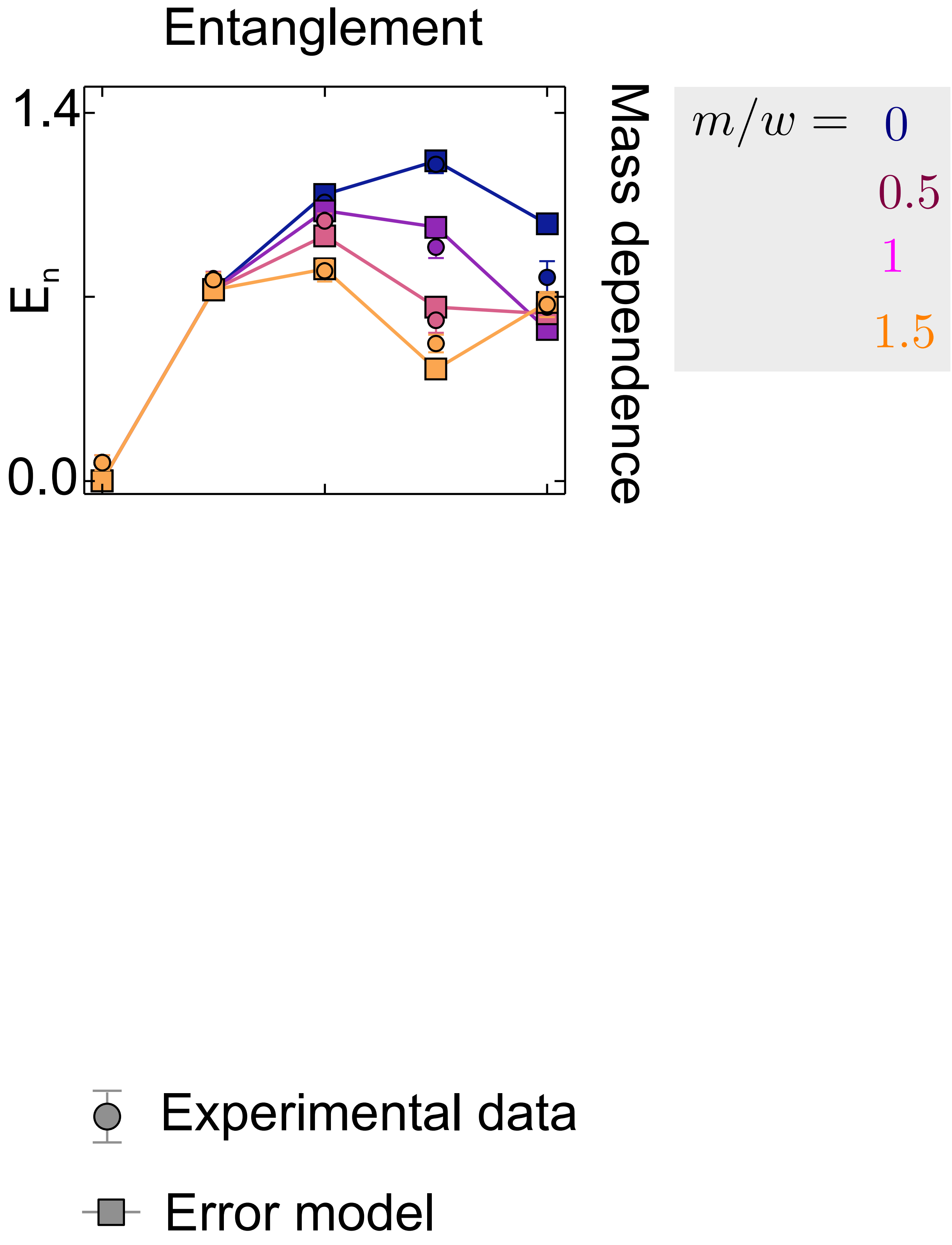
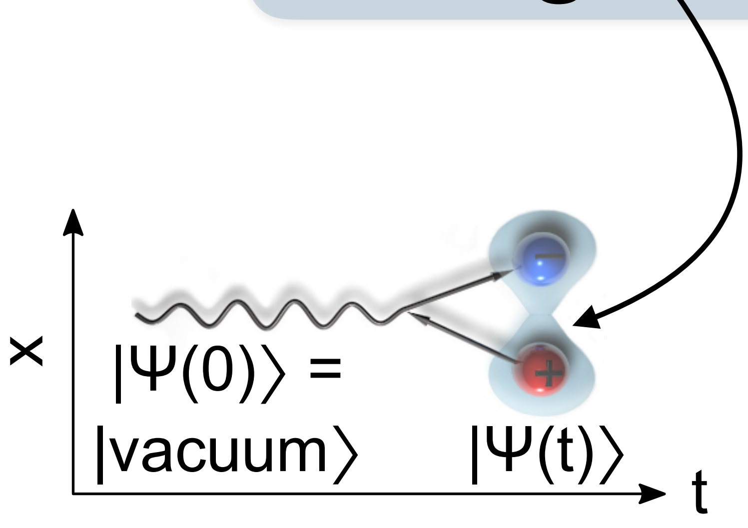


Entanglement between the two halves of the system.



Entanglement in the Schwinger mechanism



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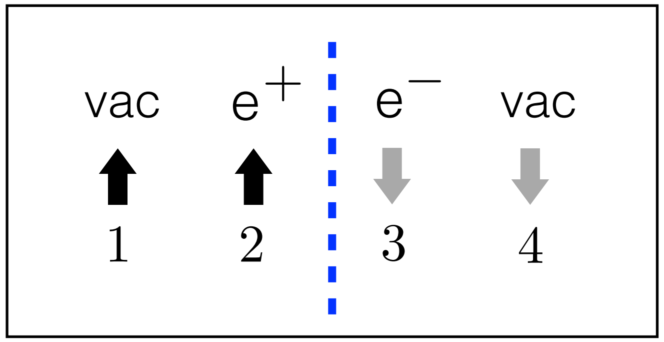

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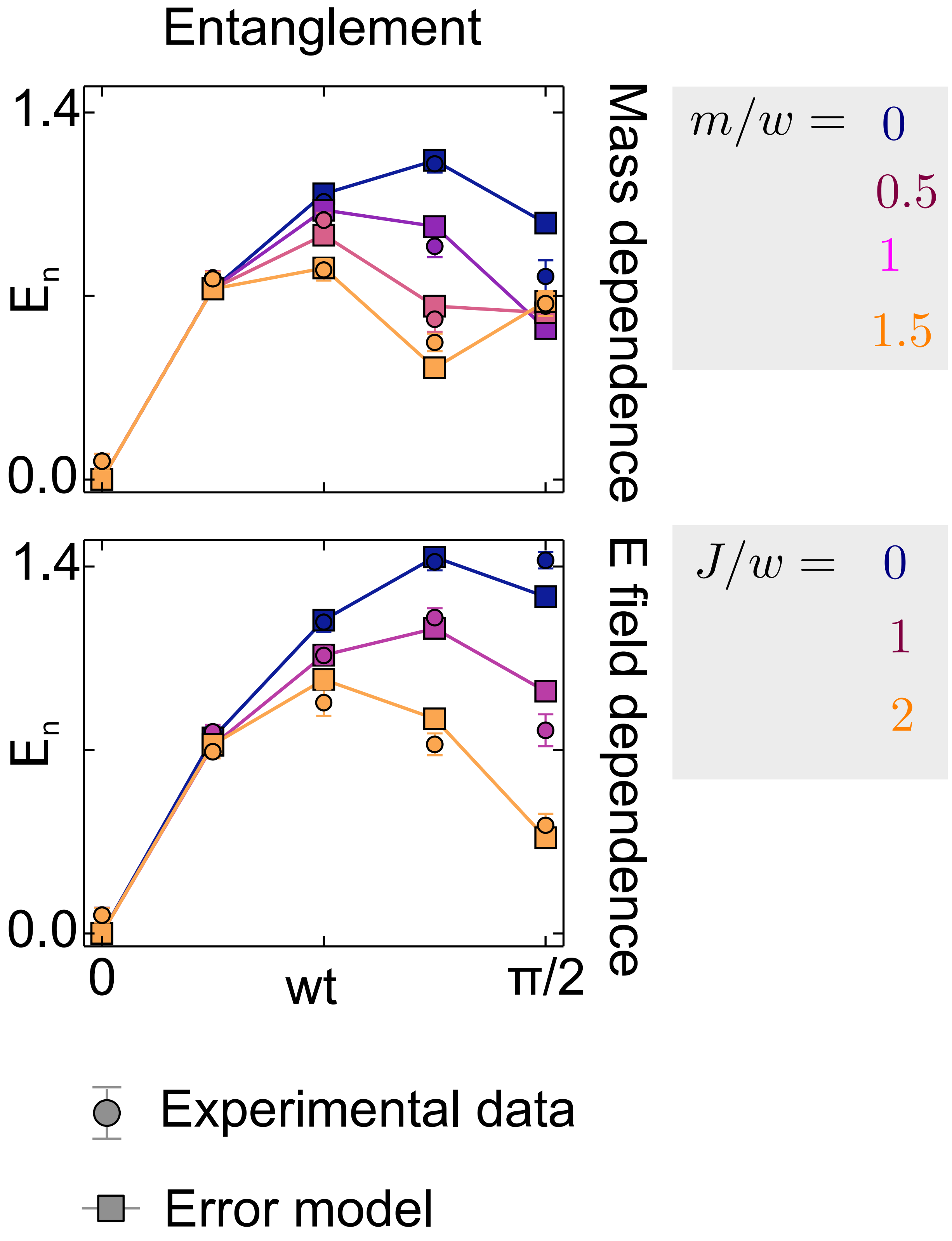
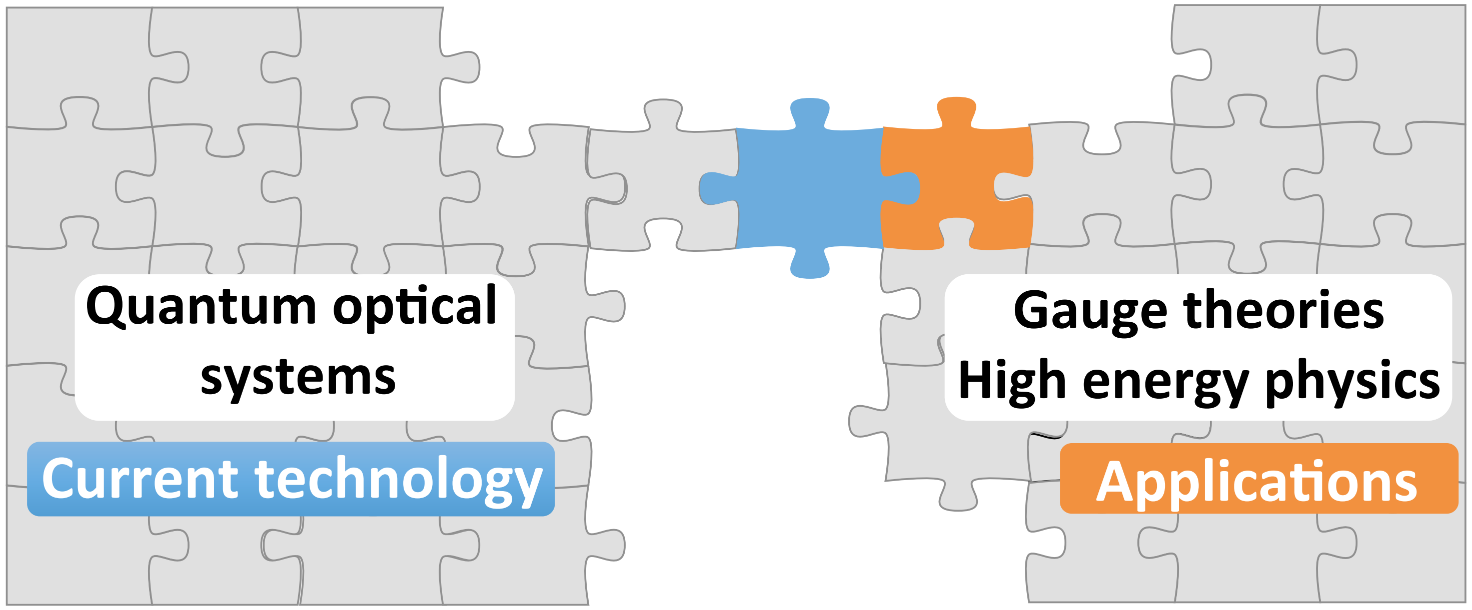


Conclusions

1.) Digital quantum simulation of a the Schwinger model
→ real-time dynamics

2.)



Quantum optical systems

Current technology

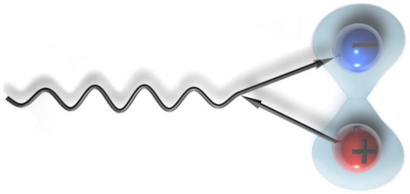
Gauge theories
High energy physics

Applications

3.) Our approach:

- Very efficient use of resources.
- Gauge invariance by construction.

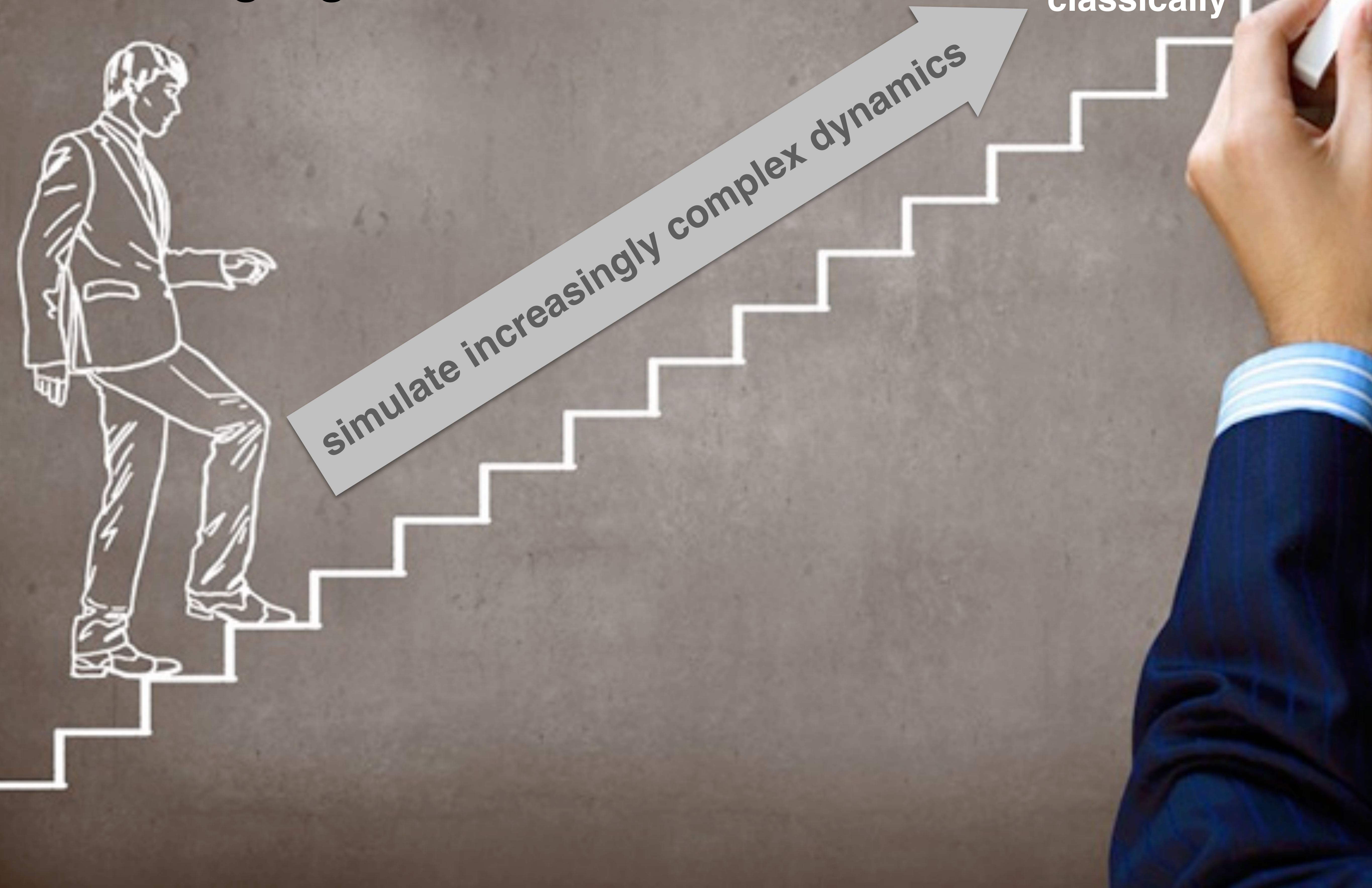
Explore new features like entanglement.



Quantum simulation of lattice gauge theories

simulate increasingly complex dynamics

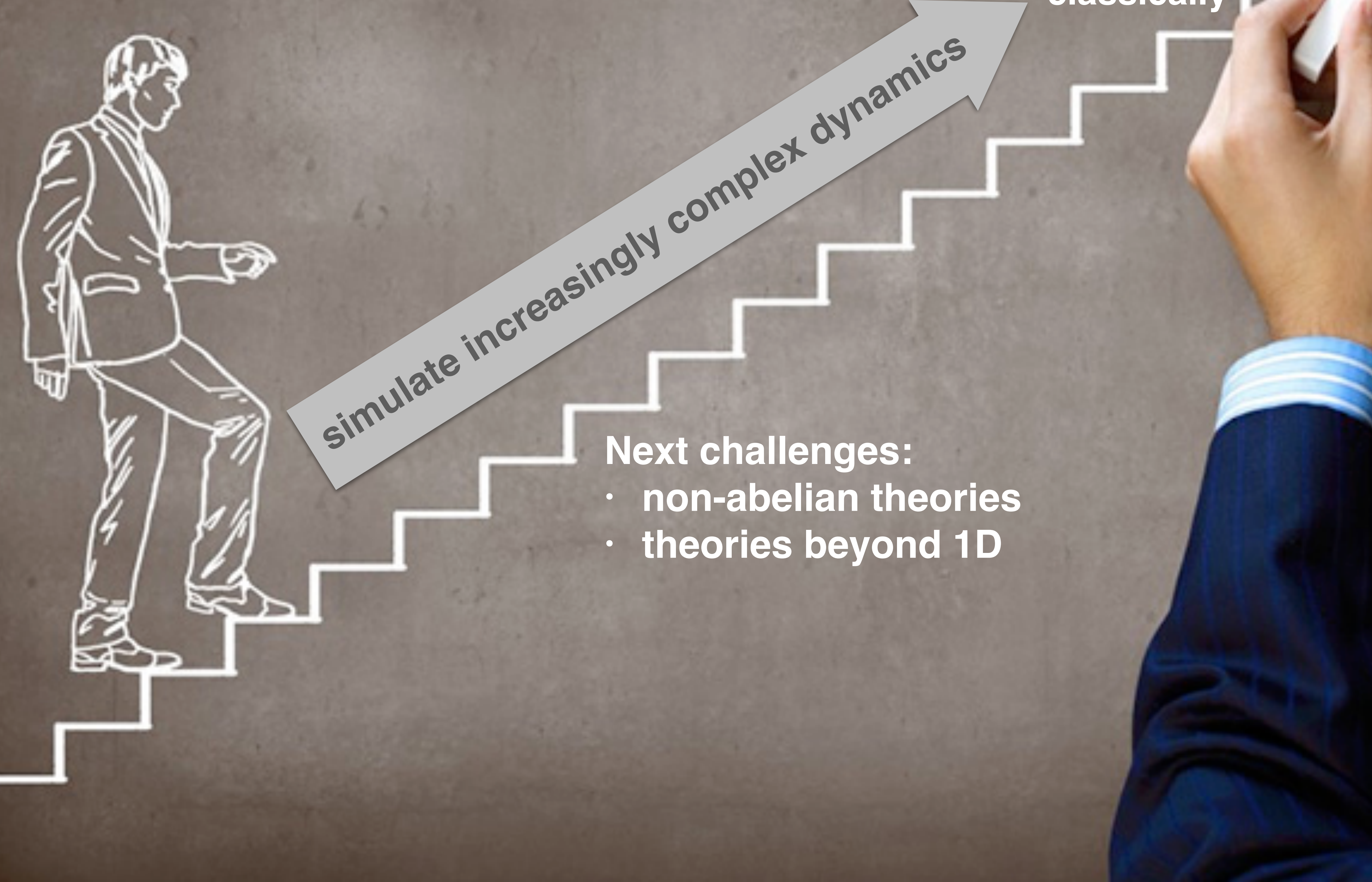
Quantum simulation of lattice gauge theories



simulate increasingly complex dynamics

solve problems that cannot be solved classically

Quantum simulation of lattice gauge theories



simulate increasingly complex dynamics

solve problems that cannot be solved classically

Next challenges:

- non-abelian theories
- theories beyond 1D