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New tool for sensitivity analysis in *r*-process nucleosynthesis studies — a case study in the rare-earth peak region

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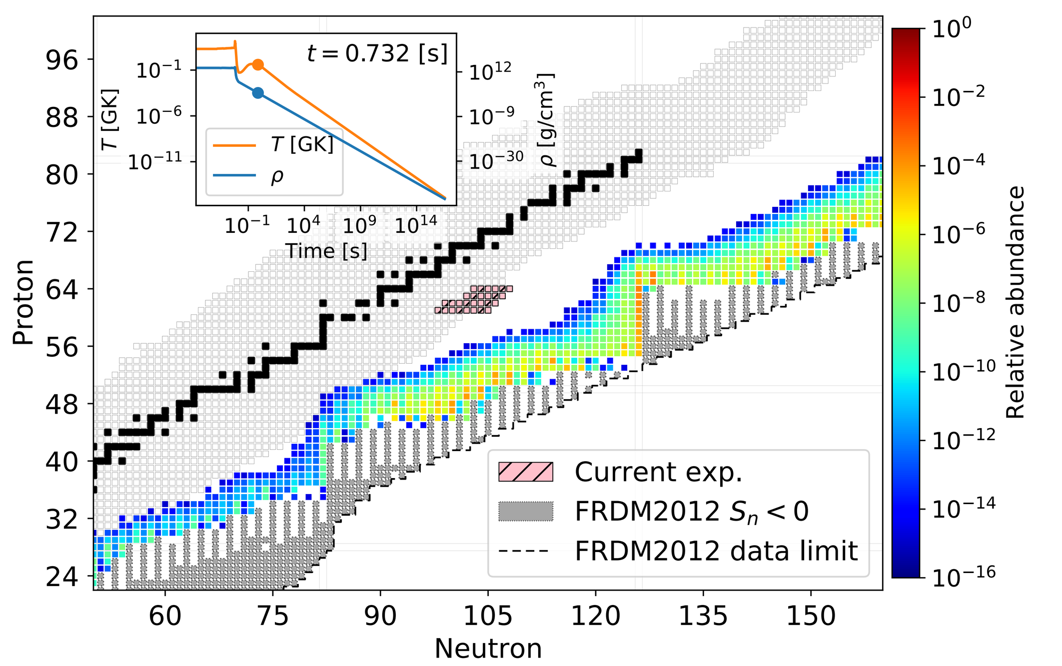
Heavy element nucleosynthesis and need for sensitivity analysis

Roughly half of the elements heavier than iron in the Universe are believed to be created in the rapid neutron capture process (the *r*-process). The *r*-process involve thousands of neutron-rich isotopes. While new radioactive beam facilities, such as ARIEL, FRIB, etc., will allow us to perform experiments with such exotic isotopes, it is crucial to understand what needs to be measured to efficiently reduce the nuclear physics uncertainty in our understanding of the r-process. Sensitivity analysis allows us to identify and quantify the sources of uncertainty. In this work, we introduce an improved sensitivity analysis method using a readily interpretable definition of sensitivity.



A snapshot of the path of r-process nucleosynthesis in neutron star merger ejecta, calculated with PRISM [1]. "Current exp" shows the nuclides of interest in this work.

The rare-earth peak (REP)

The rare-earth peak (REP) is a smaller peak located at A~165. This peak is formed in the late phase of the *r*-process, when the neutron-rich material decays back to stability (freeze-out). Interplay of neutron captures and β-decays during the freeze-out affects the shape of the REP, therefore understanding the impact of these nuclear processes allows us to probe the late time evolution of the *r*-process.

Variance-based sensitivity analysis and application to experimental data

Propagated uncertainty (variance) of calculated *r*-process abundance pattern can be obtained from nuclear reaction network calculations. Monte Carlo samples represent input uncertainties. Obtained variance can be decomposed:

$$V = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \cdots + V_{1,2,\ldots,k}$$

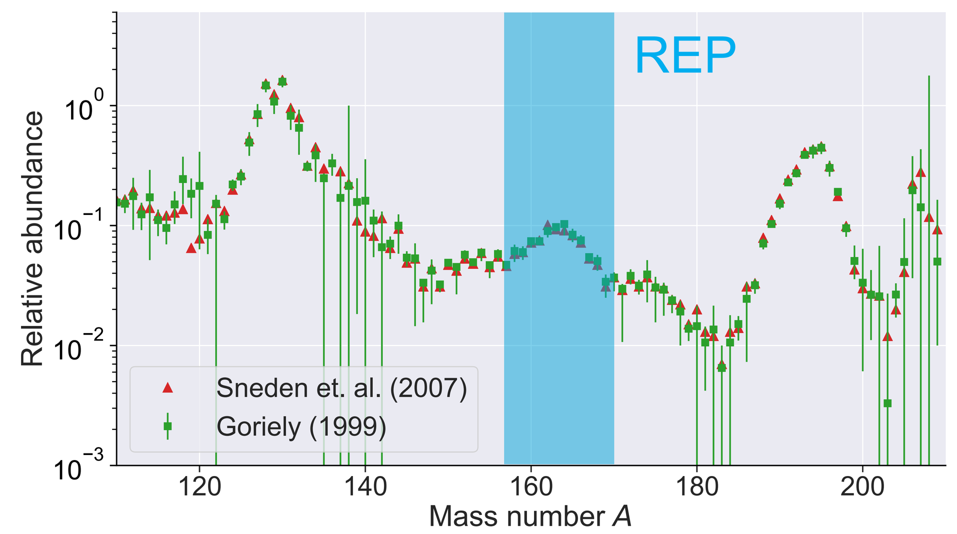
Dividing both sides with the total variance V

$$1 = \sum_i S_i^{(1)} + \sum_i \sum_{j>i} S_{ij}^{(2)} + \cdots + S_{1,2,\ldots,k}^{(k)}$$

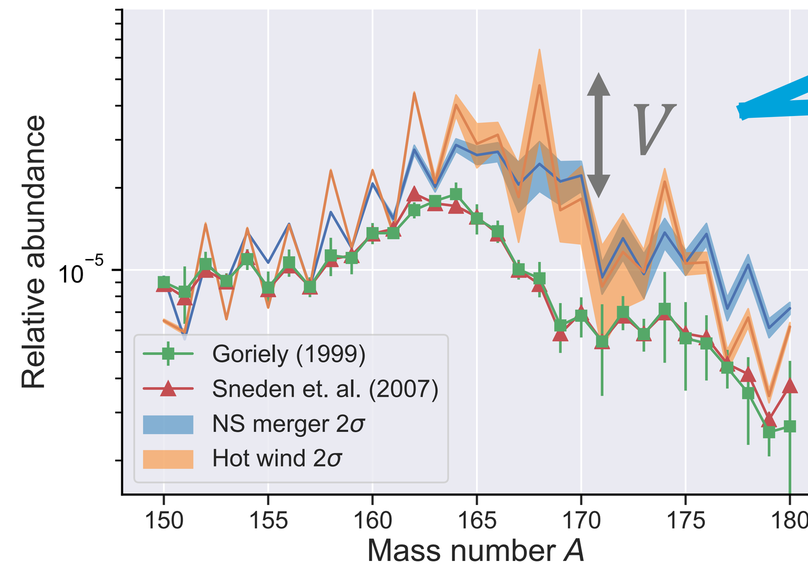
Where $S^{(k)}$ is called a k-th order **sensitivity index** [2].

We apply this to experimental β-decay half-lives and β-delayed one neutron emission probabilities of $^{159\text{-}166}$Pm, $^{161\text{-}168}$Sm, $^{165\text{-}170}$Eu, and $^{167\text{-}172}$Gd, newly obtained by the BRIKEN collaboration [3].

Solar r-process abundance pattern



REP abundance uncertainty propagated from experimental $T_{1/2}$ and P_{1n}



Decomposition

$S^{(1)}$ for NS merger scenario

Nuclide	Variable	Max. relative uncertainty [%]	100×$S^{(1)}$ (95% C.I.) [%] $A = 168$	169	170	171	172	173
^{165}Pm	$T_{1/2}$	37.4	1.9 (± 1.1)	3.2 (± 1.5)	4.9 (± 1.9)	2.7 (± 1.5)	0.8 (± 0.9)	—
^{166}Pm	$T_{1/2}$	57.5	—	—	0.5 (± 0.6)	0.7 (± 0.7)	—	—
^{166}Sm	$T_{1/2}$	15.9	—	1.7 (± 1.2)	4.8 (± 1.9)	3.8 (± 1.7)	1.5 (± 1.0)	0.8 (± 0.7)
^{167}Sm	$T_{1/2}$	24.9	0.6 (± 0.6)	—	—	1.1 (± 0.9)	0.9 (± 0.8)	0.6 (± 0.7)
^{168}Sm	$T_{1/2}$	59.5	**60.9** (± 6.6)	**55.1** (± 7.1)	**14.6** (± 4.4)	**32.6** (± 5.0)	**43.5** (± 5.5)	**41.6** (± 5.6)
^{168}Eu	$T_{1/2}$	10.9	0.5 (± 0.7)	—	—	—	—	—
^{169}Eu	$T_{1/2}$	23.7	—	3.6 (± 1.4)	—	—	0.9 (± 0.8)	0.7 (± 0.7)
^{170}Eu	$T_{1/2}$	37.6	—	—	0.6 (± 0.9)	—	—	—
^{167}Gd	$T_{1/2}$	80.1	6.1 (± 2.5)	**26.6** (± 4.3)	**34.2** (± 6.2)	**14.6** (± 3.9)	3.5 (± 1.8)	1.2 (± 1.1)
^{168}Gd	$T_{1/2}$	15.8	**24.3** (± 4.6)	8.3 (± 2.7)	8.1 (± 2.8)	2.2 (± 1.5)	—	—
^{169}Gd	$T_{1/2}$	11.0	—	0.8 (± 0.8)	—	—	—	—
^{170}Gd	$T_{1/2}$	13.9	—	—	**25.2** (± 4.7)	1.4 (± 1.2)	2.6 (± 1.4)	3.5 (± 1.7)
^{171}Gd	$T_{1/2}$	37.0	—	—	—	**20.5** (± 4.1)	4.6 (± 2.0)	1.0 (± 1.1)
^{172}Gd	$T_{1/2}$	69.3	—	—	—	3.6 (± 2.1)	**35.7** (± 5.1)	**49.3** (± 5.9)
^{165}Pm	P_{1n}	47.0	—	0.6 (± 0.6)	0.7 (± 0.5)	—	—	—
^{168}Sm	P_{1n}	(100)	—	—	—	0.8 (± 0.8)	0.6 (± 0.6)	—
^{169}Eu	P_{1n}	39.8	5.4 (± 2.1)	—	3.7 (± 1.6)	3.6 (± 1.7)	1.3 (± 1.0)	0.6 (± 0.7)
^{170}Eu	P_{1n}	(100)	—	0.5 (± 0.6)	—	—	—	—
^{172}Gd	P_{1n}	(100)	—	—	—	5.5 (± 2.0)	3.2 (± 1.5)	0.6 (± 0.7)
	$S^{(1)}(T_{1/2})$ total:		94.9 (± 8.6)	100.1 (± 9.2)	93.9 (± 9.9)	84.0 (± 8.5)	95.1 (± 8.3)	99.7 (± 8.6)
	$S^{(1)}(P_{1n})$ total:		5.9 (± 2.3)	1.1 (± 1.1)	5.6 (± 2.0)	11.0 (± 2.9)	5.7 (± 2.0)	2.0 (± 1.1)
	$S^{(1)}$ total:		100.9 (± 8.9)	101.3 (± 9.2)	99.5 (± 10.1)	95.0 (± 9.0)	100.7 (± 8.6)	101.6 (± 8.6)

Hot neutrino-driven wind scenario

Nuclide	Variable	Max. relative uncertainty [%]	100×$S^{(1)}$ (95% C.I.) [%] $A = 168$	169	170	171	172	173
^{165}Pm	$T_{1/2}$	37.4	—	0.5 (± 0.6)	—	—	—	—
^{168}Sm	$T_{1/2}$	59.5	**96.1** (± 14.1)	**71.4** (± 7.0)	**95.2** (± 8.2)	**56.8** (± 7.1)	**44.6** (± 7.2)	**80.7** (± 13.3)
^{169}Eu	$T_{1/2}$	23.7	—	2.6 (± 1.4)	0.5 (± 0.6)	—	—	—
^{167}Gd	$T_{1/2}$	80.1	—	0.6 (± 0.6)	—	—	—	—
^{168}Gd	$T_{1/2}$	15.8	—	2.8 (± 1.5)	—	—	—	—
^{170}Gd	$T_{1/2}$	13.9	—	—	1.1 (± 0.9)	0.7 (± 0.8)	—	—
^{171}Gd	$T_{1/2}$	37.0	—	—	—	6.9 (± 2.6)	0.5 (± 0.7)	1.8 (± 1.2)
^{172}Gd	$T_{1/2}$	69.3	—	—	—	9.9 (± 3.2)	**53.3** (± 7.6)	**11.1** (± 3.3)
^{168}Sm	P_{1n}	(100)	2.0 (± 1.5)	3.5 (± 1.7)	0.5 (± 0.6)	—	—	—
^{169}Eu	P_{1n}	39.8	1.0 (± 0.9)	**10.8** (± 2.9)	0.5 (± 0.7)	—	—	—
^{170}Eu	P_{1n}	(100)	—	6.7 (± 2.3)	2.1 (± 1.2)	—	—	—
^{172}Gd	P_{1n}	(100)	—	—	—	**25.2** (± 4.6)	2.6 (± 1.7)	5.5 (± 2.1)
	$S^{(1)}(T_{1/2})$ total:		97.0 (± 14.1)	78.9 (± 7.4)	97.4 (± 8.3)	74.6 (± 8.2)	98.6 (± 10.5)	93.8 (± 13.7)
	$S^{(1)}(P_{1n})$ total:		3.0 (± 1.8)	21.5 (± 4.1)	3.7 (± 1.6)	25.9 (± 4.7)	2.8 (± 1.7)	5.6 (± 2.1)
	$S^{(1)}$ total:		100.0 (± 14.3)	100.5 (± 8.5)	101.1 (± 8.4)	100.5 (± 9.5)	101.3 (± 10.7)	99.4 (± 13.9)

Pros and cons of the method

Pros:

- Normalized and interpretable sensitivity
- Can be applied to nonlinear (nonmonotonic) output response
- Generated samples provides insight into flows of nuclear reactions

Cons:

- Computation rather costly (scales with *k*)
- Input uncertainty must be defined

SUMMARY / OUTLOOK

Sensitivity analysis in the r-process abundance prediction is important for guiding future experimental efforts. In this work, we have introduced variance-based sensitivity analysis method. This provides detailed sensitivity information and a new tool for investigating the impact of nuclear physics inputs.

In future work, more isotopes will be included, and theoretical uncertainty with correlated inputs will also be employed to draw more general conclusions.

References

[1] Mumpower, M., Surman, R., McLaughlin, G., & Aprahamian, A. 2016, Progress in Particle and Nuclear Physics, 86, 86

[2] Saltelli, A., Annoni, P., Azzini, I., et al. 2010, Computer Physics Communications, 181, 259

[3] Kiss, G. G., Vitéz-Sveiczer, A., Saito, Y., et al. 2022, to be submitted to the Astrophysical Journal

Discovery, accelerated