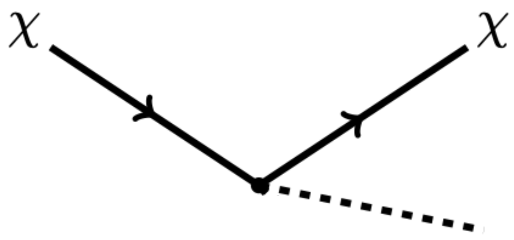
Multiphonon excitations in dark matter direct detection

Tongyan Lin
UCSD

September 9, 2022
GUINEAPIG workshop

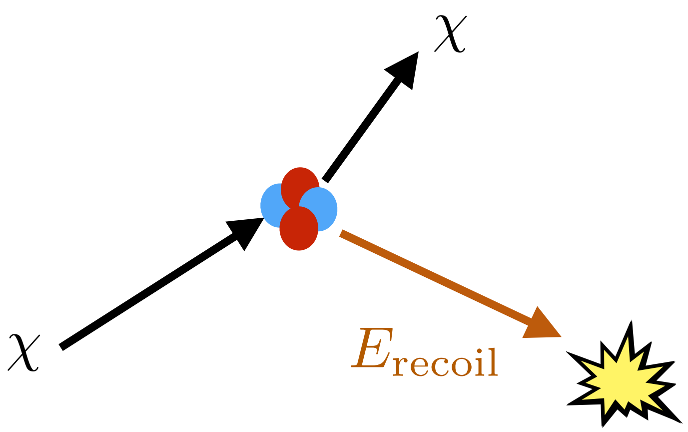
Dark matter mass

keV MeV GeV TeV

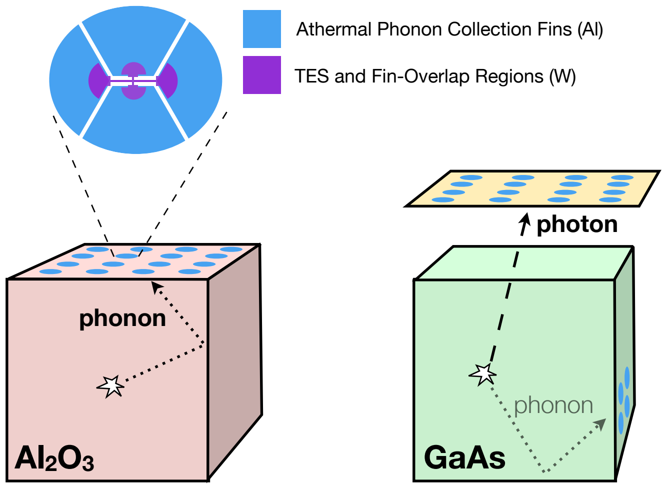


Single phonon excitation

?

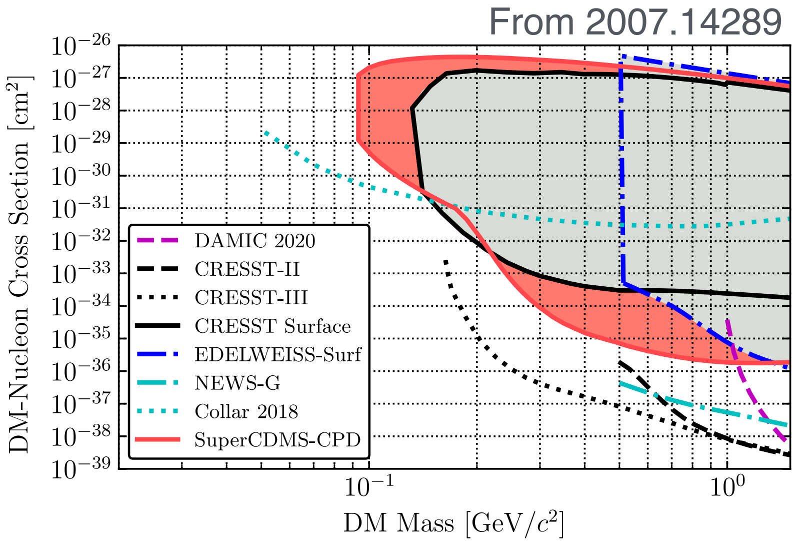


Nuclear recoils



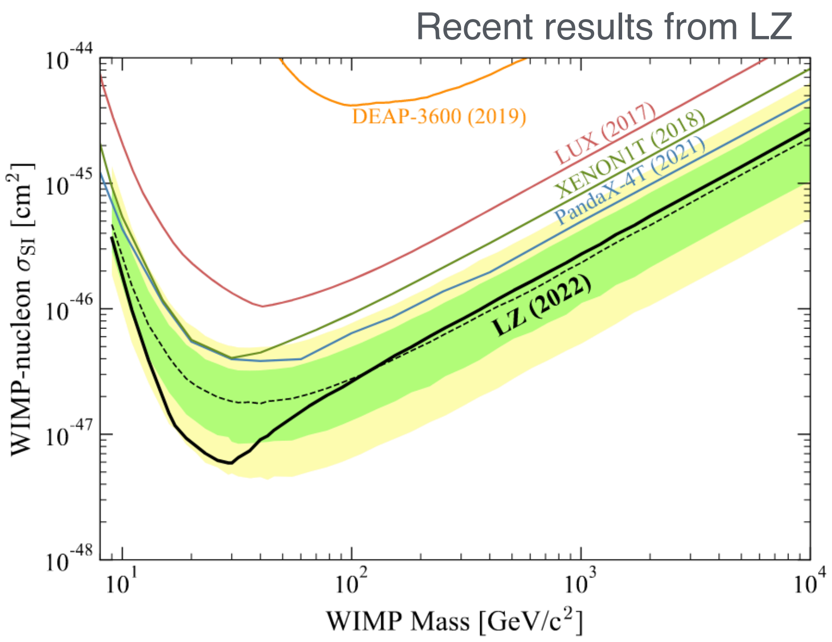
From TESSERACT white paper

Single phonons excitations with energy 1-100 meV (SPICE)



From 2007.14289

O(10) eV thresholds

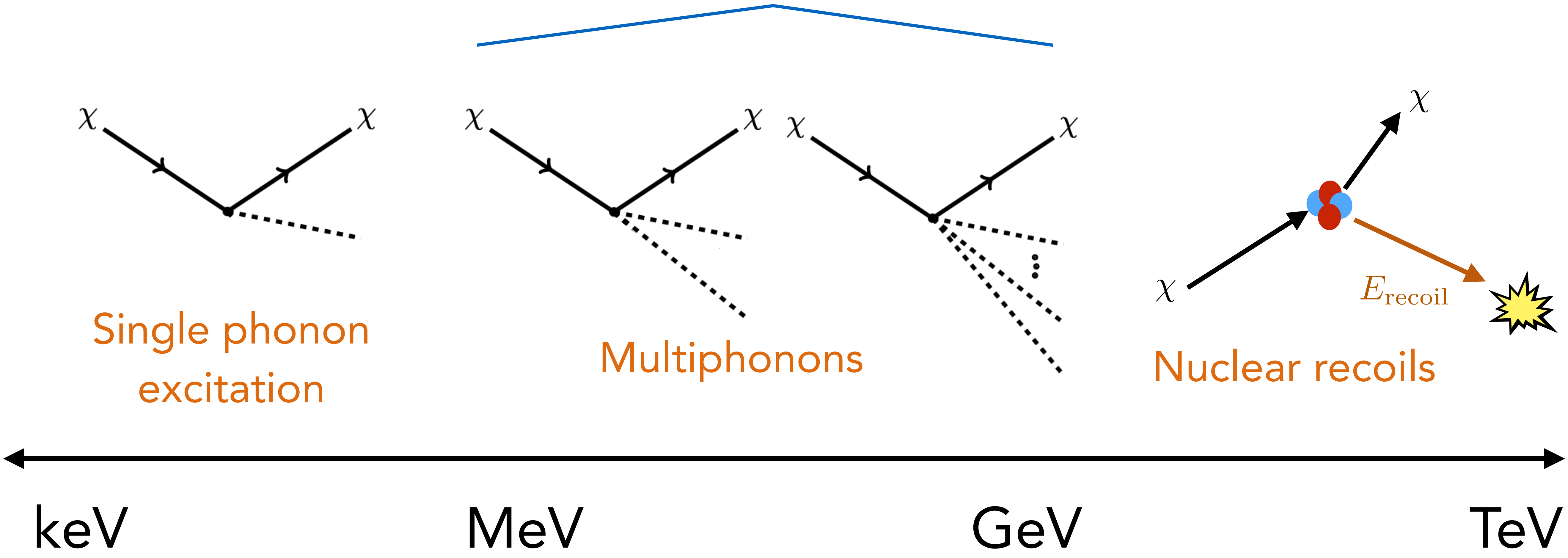


Recent results from LZ

O(keV) thresholds

DM-nucleus scattering in crystals

Applications also for the Migdal effect
and calculating backgrounds



Single phonon
excitation

Multiphonons

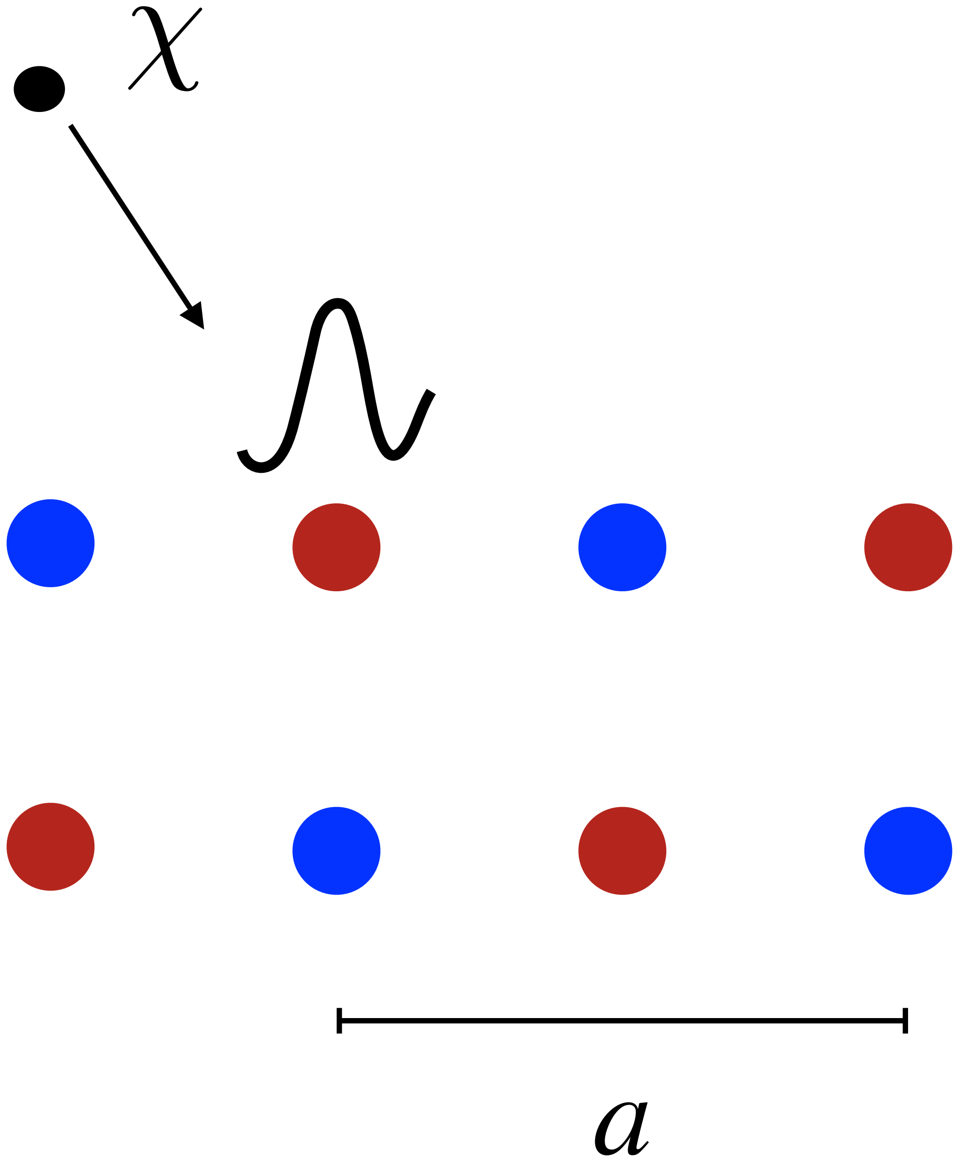
Nuclear recoils

keV MeV GeV TeV

Dark matter mass

Brian Campbell-Deem, Knapen, TL, Ethan Villarama 2205.02250
Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482
Knapen, Kozaczuk, TL 2011.09496

What does DM-nucleus scattering look like in a crystal?



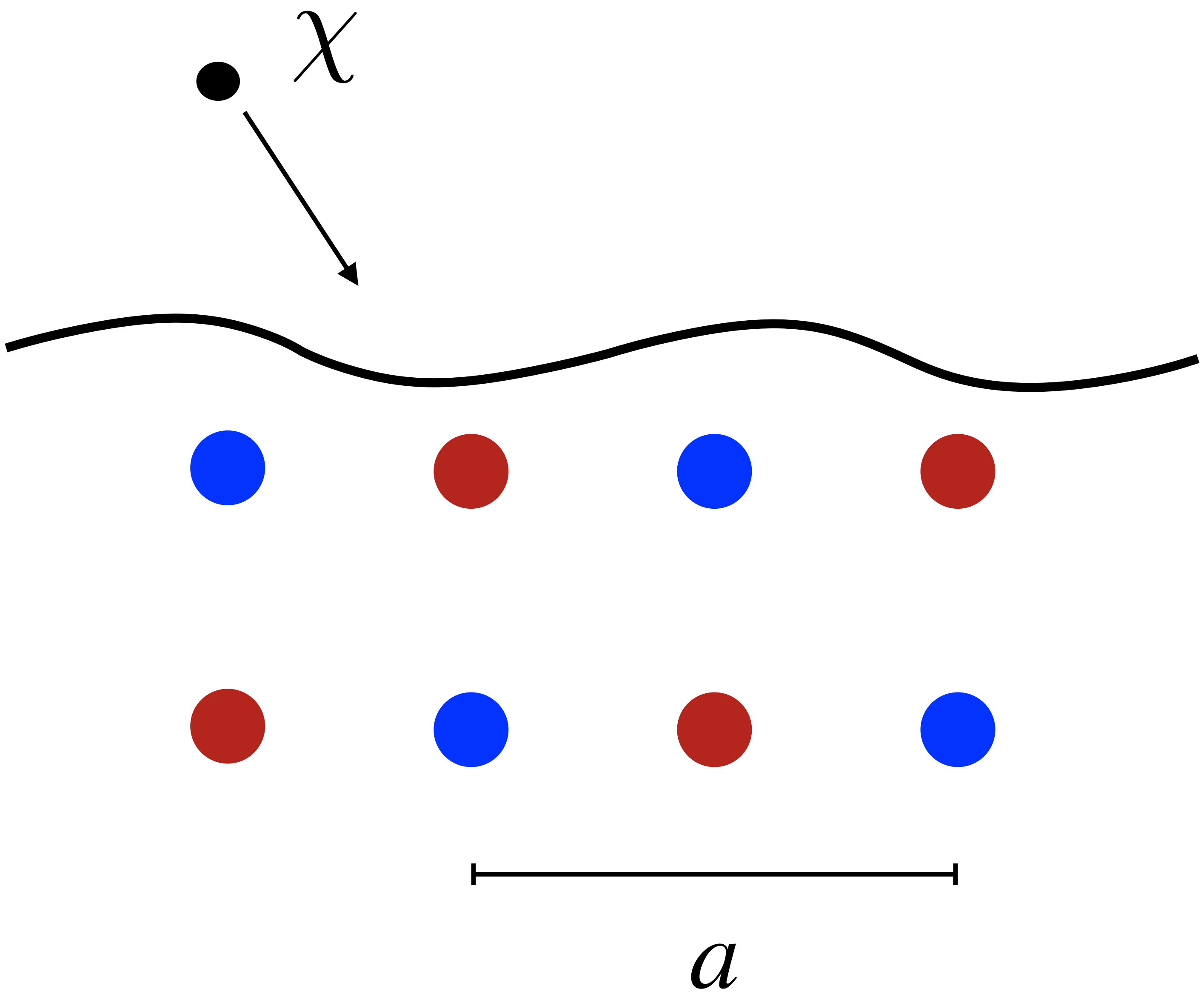
When momentum transfer

$$q \gg q_{\rm BZ} = \frac{2\pi}{a} \sim \text{few keV}$$

and $\omega \gg \bar{\omega}_{\rm phonon} \sim 10\text{-}100\ \text{meV}$

DM scatters off an individual nucleus

What does DM-nucleus scattering look like in a crystal?



When momentum transfer

$$q \ll q_{\rm BZ} = \frac{2\pi}{a}$$

and $\omega \sim \bar{\omega}_{\rm phonon}$

DM excites collective excitations = phonons

DM scattering rate

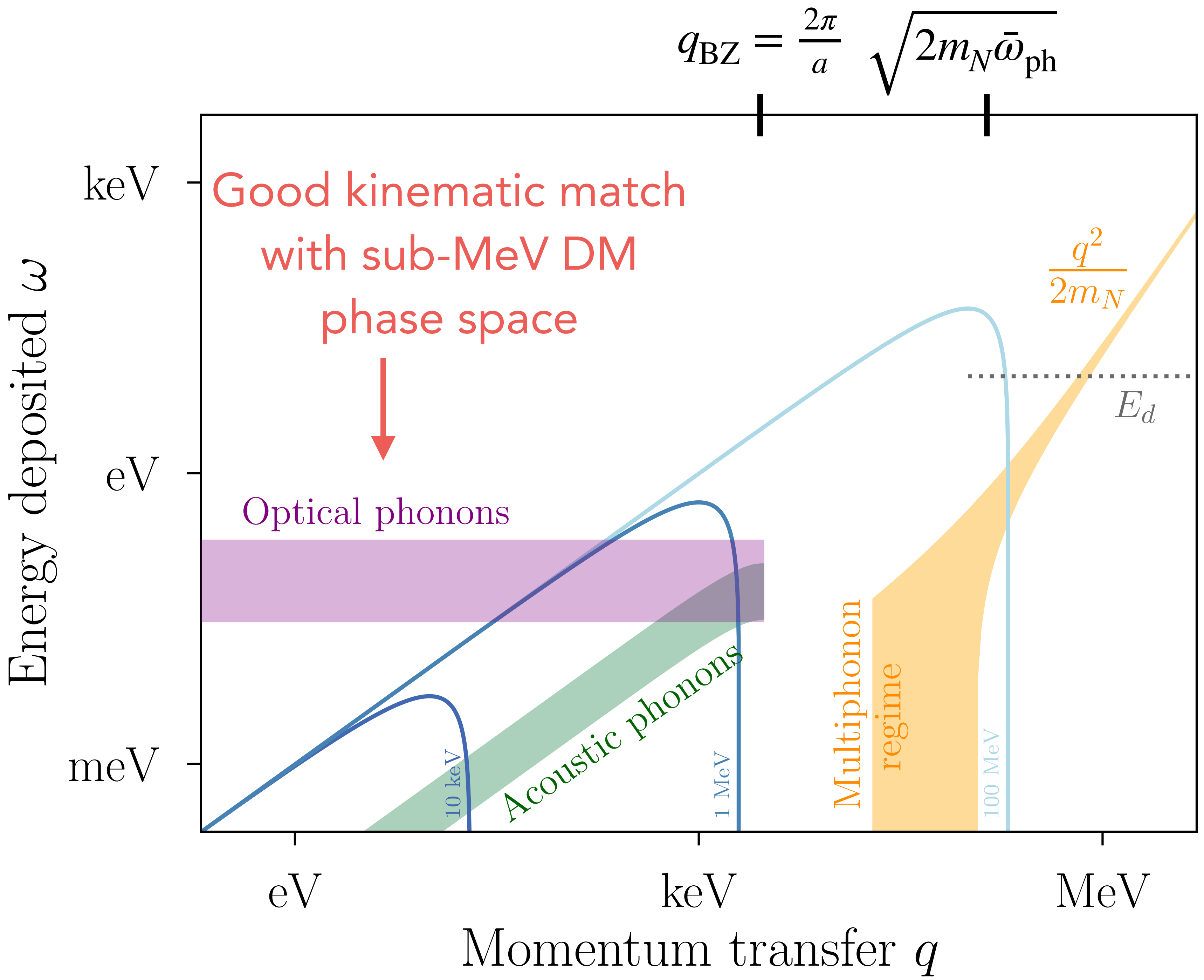
$$\frac{d\sigma}{d^3\mathbf{q}\,d\omega} \propto \sigma_{\chi p}\,|\overbrace{\tilde{F}_{\rm med}(q)}^{\text{DM-mediator form factor}}|^2\,\underbrace{S(\mathbf{q},\omega)}_{\text{Dynamic structure factor captures response of target}}\,\delta(\omega - \mathbf{q}\cdot\mathbf{v} + \tfrac{q^2}{2m_\chi})$$

For free nuclei and spin-independent interactions:

$$S(\mathbf{q},\omega) \propto A_N^2\,\delta\left(\omega - \tfrac{q^2}{2m_N}\right)$$

Goal: understand $S(\mathbf{q},\omega)$ from the single phonon to the nuclear recoil regime

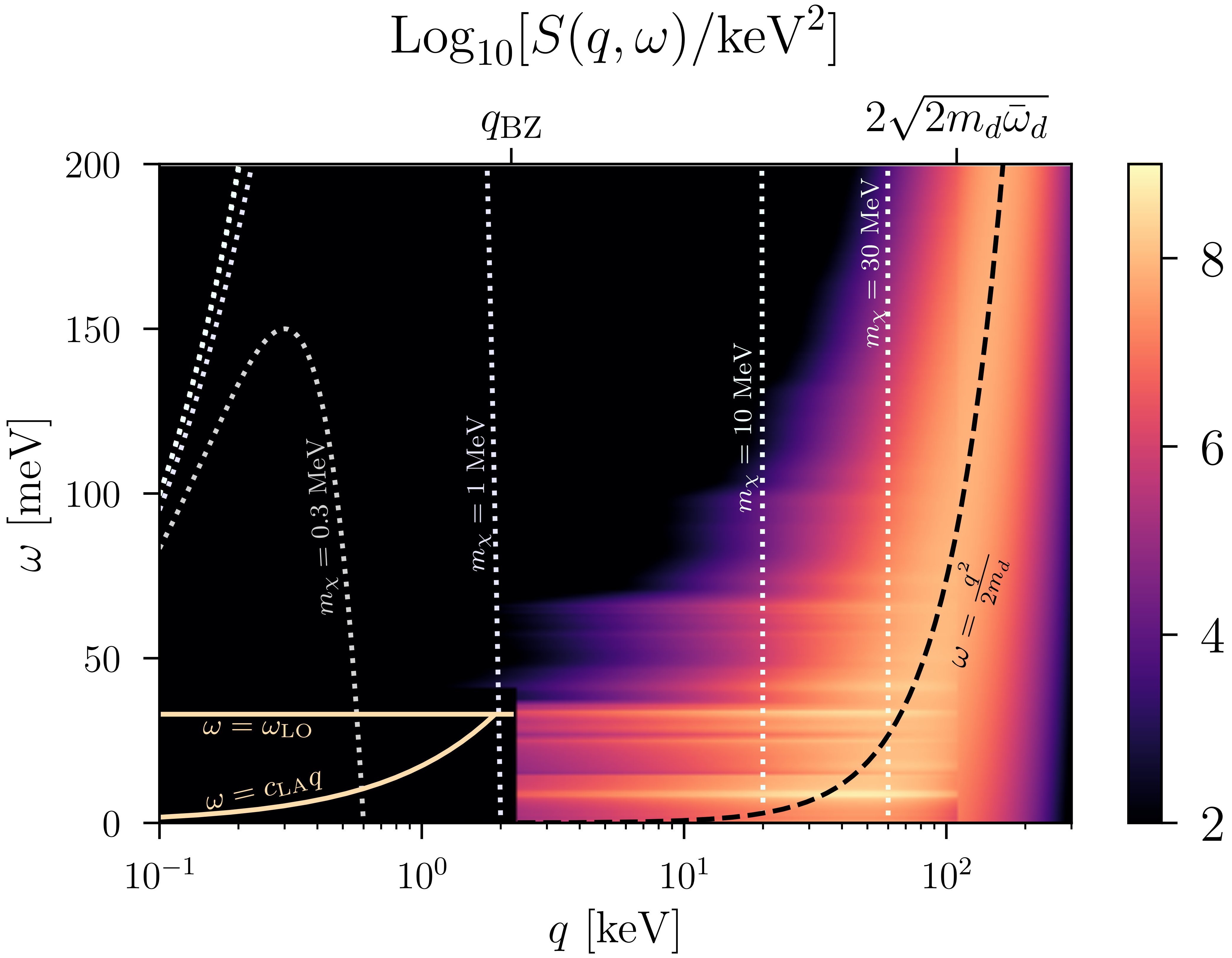
DM-nucleus scattering in a crystal



Good kinematic match with sub-MeV DM phase space

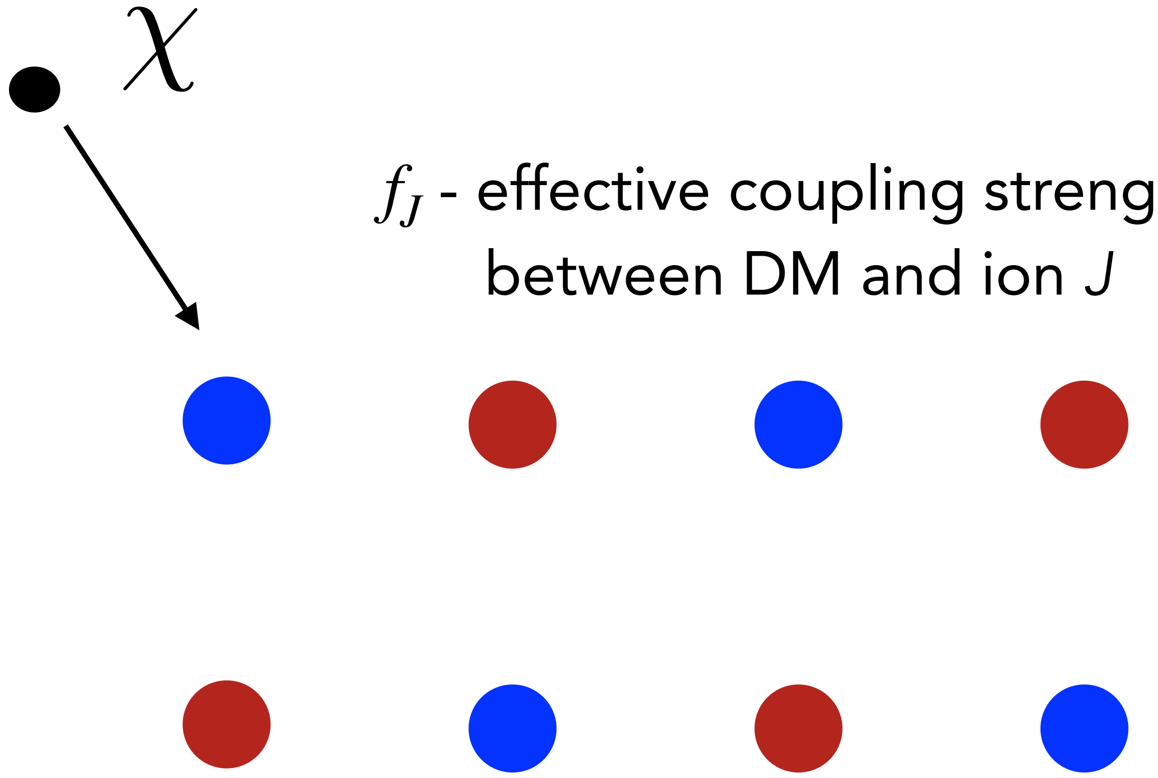
Kahn, TL 2108.03239
Trickle, Zhang, Zurek, Inzani, Griffin 1910.08092

Structure factor for GaAs



Campbell-Deem, Knapen, TL, Villarama 2205.02250

DM-nucleus interaction



f_J - effective coupling strength between DM and ion J

Short range SI interaction

$$\sigma_{\chi p} = 4\pi b_p^2$$

Scattering potential in Fourier space

$$V(\mathbf{q}) \propto b_p \sum_J f_J \, e^{i\mathbf{q}\cdot\mathbf{r}_J}$$

$$S(\mathbf{q},\omega) \equiv \frac{2\pi}{V} \sum_f \left| \sum_J \langle \Phi_f | f_J e^{i\mathbf{q}\cdot\mathbf{r}_J} | 0 \rangle \right|^2 \delta\left(E_f - \omega\right)$$

$$= \frac{1}{V} \sum_{J,\, J'}^{N} f_J f_{J'}^{*} \int\limits_{-\infty}^{\infty} dt \, \langle e^{-i\mathbf{q}\cdot\mathbf{r}_{J'}(0)} e^{i\mathbf{q}\cdot\mathbf{r}_J(t)} \rangle e^{-i\omega t}$$

Contains interference terms between different atoms → single phonon excitations

Theory of neutron scattering: Squires 1996, Schober 2014

Dynamic structure factor

Phonons appear through positions of ions:

$$\mathbf{r}_J(t) = \mathbf{r}_J^0 + \underset{\uparrow}{\mathbf{u}_J(t)}$$

Quantized phonon field given in terms of phonon dispersions $\omega_{\mathbf{q}}$ and eigenvectors $\mathbf{e}_{\mathbf{q}}$

Single phonon contribution has been studied extensively in literature, with $\omega_{\mathbf{q}}$, $\mathbf{e}_{\mathbf{q}}$ calculated from first principles approaches

$$S^{n=1}(\mathbf{q}, \omega) \sim \sum_{J,J'} f_J f_{J'} \int dt \, \langle \mathbf{q} \cdot \mathbf{u}_J(0) \; \mathbf{q} \cdot \mathbf{u}_{J'}(t) \rangle e^{-i\omega t}$$

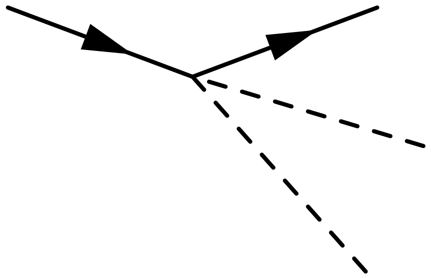
Griffin, Knapen, TL, Zurek 1807.10291; Griffin, Inzani, Trickle, Zhang, Zurek 1910.10716
Griffin, Hochberg, Inzani, Kurinsky, TL, Yu 2020; Coskuner, Tickle, Zhang, Zurek 2102.09567

Dynamic structure factor

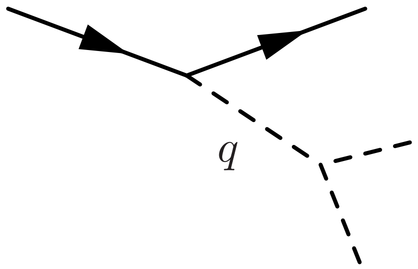
Expansion in $q^2/(M_N\omega)$ (and anharmonic interactions):

$$S(\mathbf{q},\omega) = \text{(0-phonon)} + \text{(1-phonon)} + \text{(2-phonon)} + \cdots$$

Harmonic



Anharmonic



Quickly becomes more complicated to evaluate for more than 1 phonon

Our approach: use harmonic & incoherent approximations

Incoherent approximation for $q > q_{\mathrm{BZ}}$ or $n > 1$ phonons

Neglect interference terms entirely:

$$S(\mathbf{q},\omega) \approx \frac{1}{V}\sum_J^N (f_J)^2 \int_{-\infty}^{\infty} dt\, \langle e^{-i\mathbf{q}\cdot\mathbf{u}_J(0)} e^{i\mathbf{q}\cdot\mathbf{u}_J(t)}\rangle e^{-i\omega t}$$

Given in terms of auto-correlation function

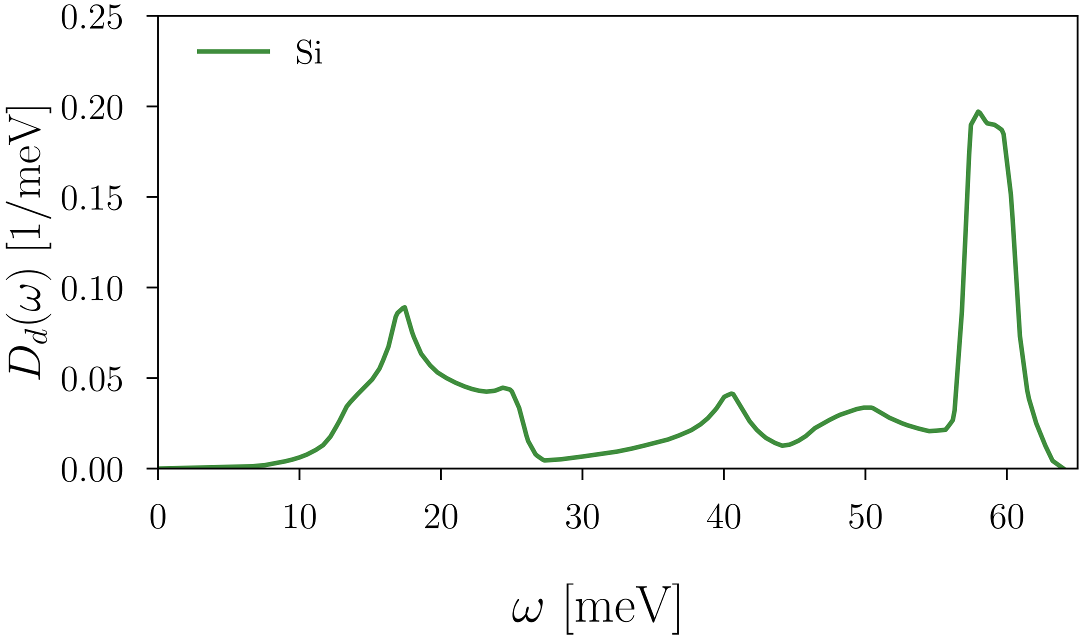
Motivation for $q > q_{\mathrm{BZ}}$: scatter off individual nuclei at large q

Motivation for $n > 1$: momentum gets distributed over multiple phonons, and the motions of individual atoms will be less correlated.

Auto-correlation can be approximated using the phonon density of states

$$\langle \mathbf{q}\cdot\mathbf{u}_J(0)\mathbf{q}\cdot\mathbf{u}_J(t)\rangle \approx \frac{q^2}{2m_N}\int d\omega' \frac{D(\omega')}{\omega'} e^{i\omega' t}$$

In the harmonic, isotropic limit

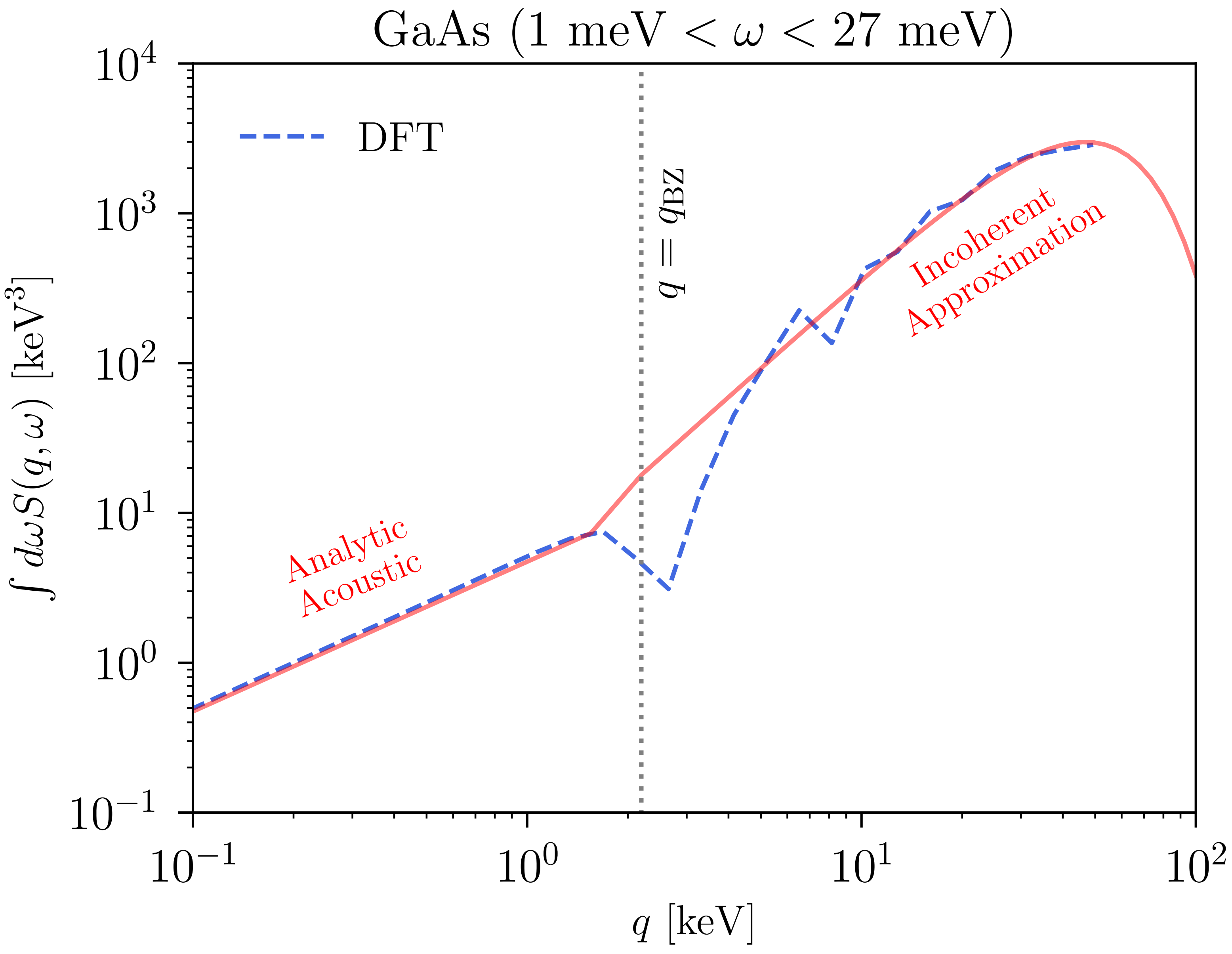


Dynamic structure factor with incoherent approximation:

$$S(q,\omega) \propto \sum_J e^{-2W_J(q)}(f_J)^2 \sum_n \frac{1}{n!} \underbrace{\left(\frac{q^2}{2m_N}\right)^n \left(\prod_{i=1}^{n}\int d\omega_i \frac{D(\omega_i)}{\omega_i}\right)}_{\sim \left(\frac{q^2}{2m_N\bar{\omega}_{\text{ph}}}\right)^n} \delta\left(\sum_j \omega_j - \omega\right)$$

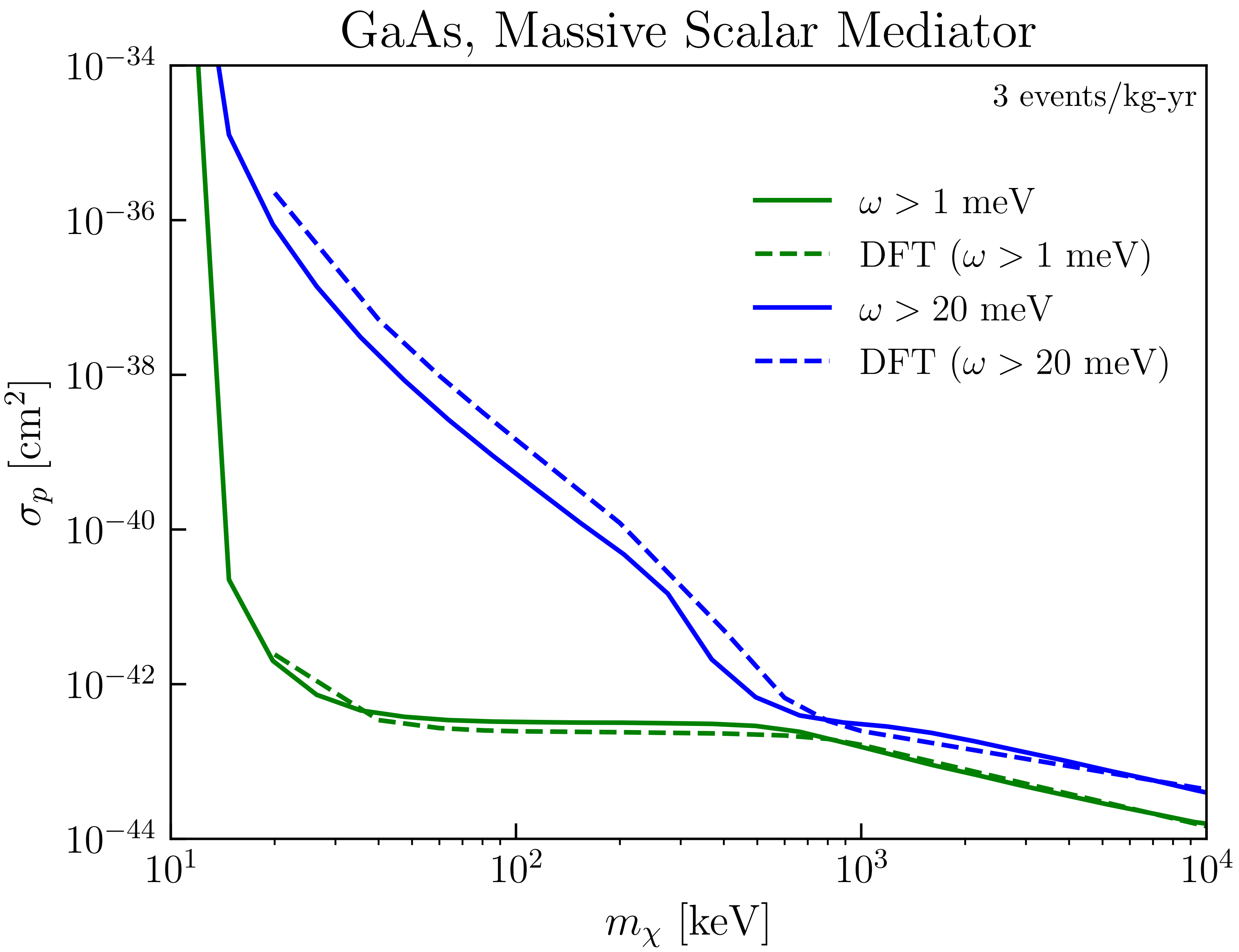
$q \approx \sqrt{2m_N\bar{\omega}_{\text{ph}}}$ for many phonons to contribute

Comparison with full (DFT) calculation for n=1 phonon

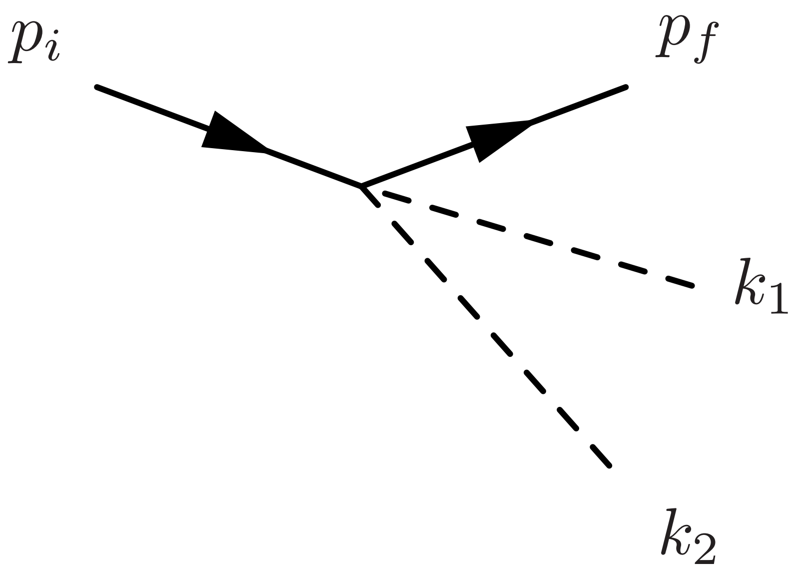


Incoherent approximation captures integrated structure factor

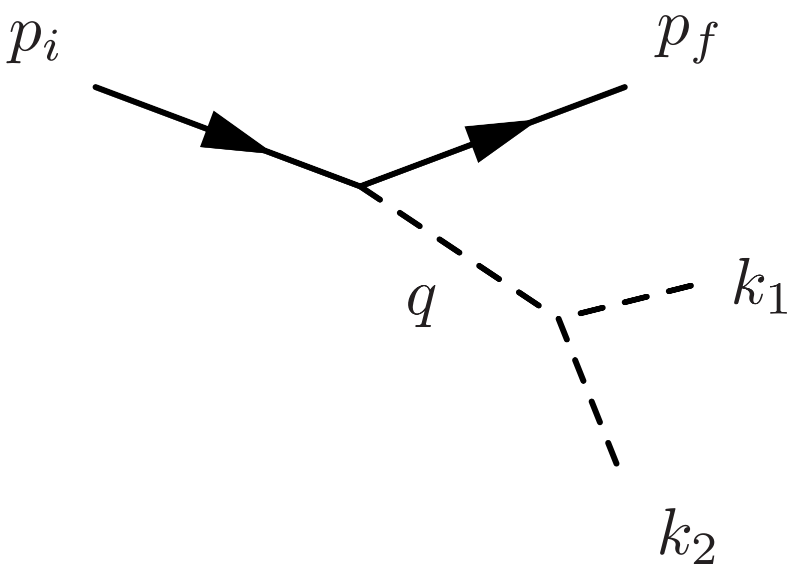
Comparison with full (DFT) calculation for n=1 phonon



2 phonons



Harmonic



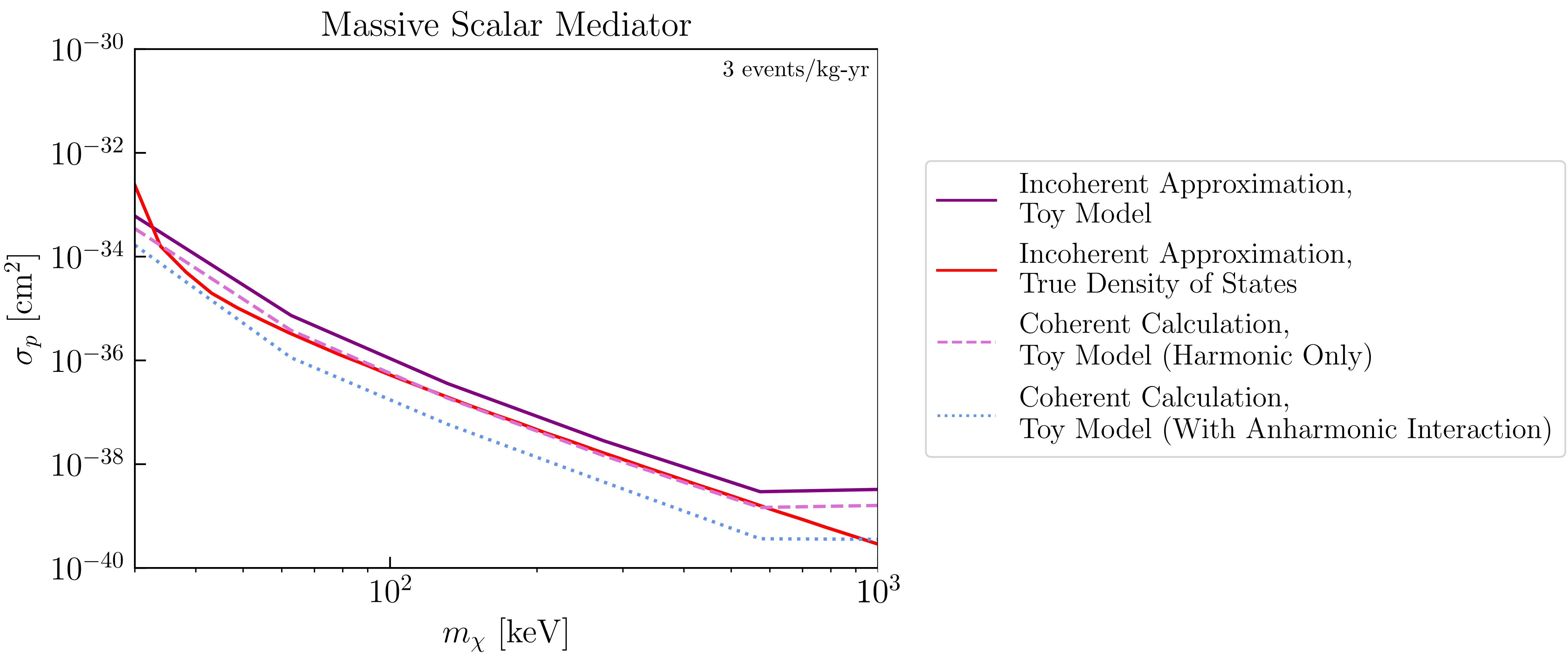
Anharmonic

Calculated in long-wavelength ($q \ll q_{\rm BZ}$) limit in crystals

Campbell-Deem, Cox, Knapen, TL, Melia 1911.03482

Calculated in superfluid He:
Schutz and Zurek 1604.08206
Knapen, TL, Zurek 1611.06228
Acanfora, Esposito, Polosa 1902.02361

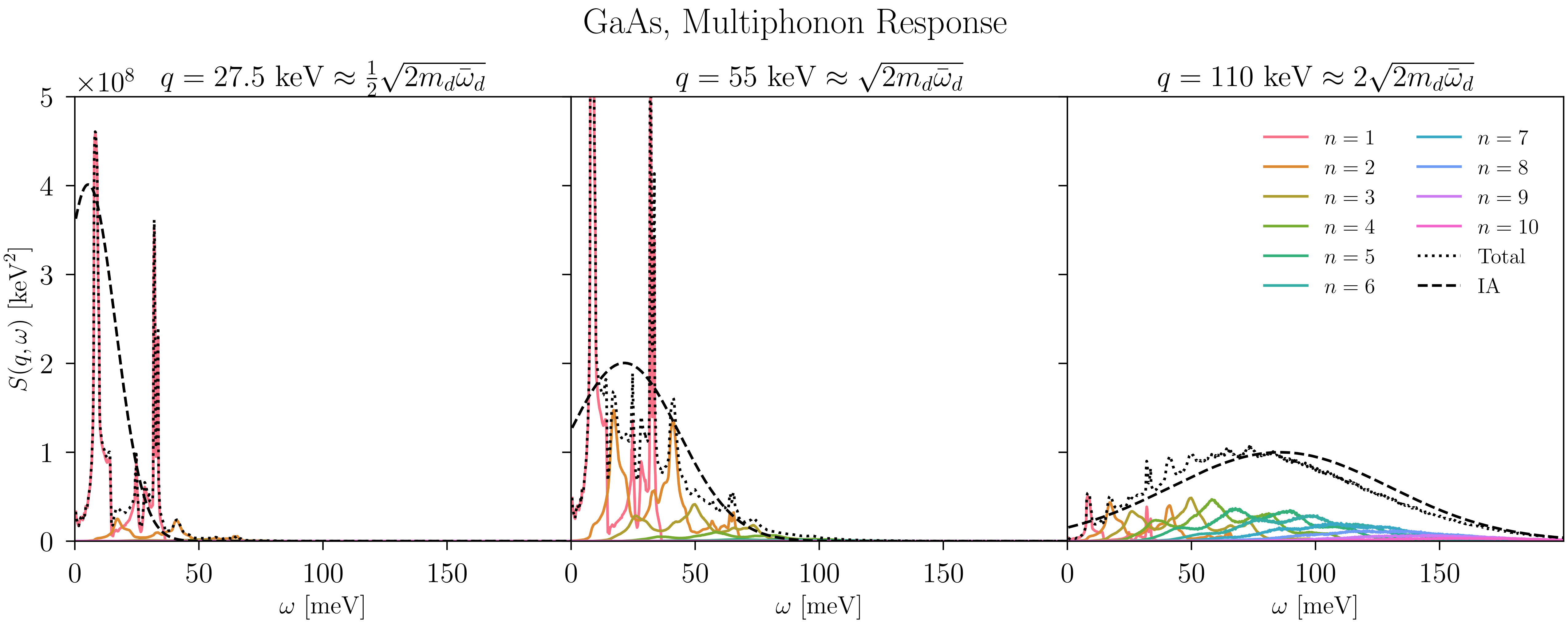
GaAs 2-phonon, ($\omega > 40$ meV)



Incoherent approximation works to within a factor of few for $q < q_{\rm BZ}$, comparing to harmonic crystal result. Anharmonic interactions give another factor of few correction.

This should work better with higher q and n.

Multiphonons become important around $q = \sqrt{2m_N\bar{\omega}_{\text{ph}}}$



$q = \frac{1}{2}\sqrt{2m_N\bar{\omega}_{\text{ph}}}$:

dominated by
n=1 phonon

$q = \sqrt{2m_N\bar{\omega}_{\text{ph}}}$:

contributions from
n=1,2,3,4…

$q = 2\sqrt{2m_N\bar{\omega}_{\text{ph}}}$:

can be approximated by
Gaussian envelope
(Impulse Approximation)

Impulse approximation

When $q \gg \sqrt{2m_N\bar{\omega}_{\rm ph}}$, "re-sum" the n-phonon contributions and directly evaluate by saddle-point approximation:

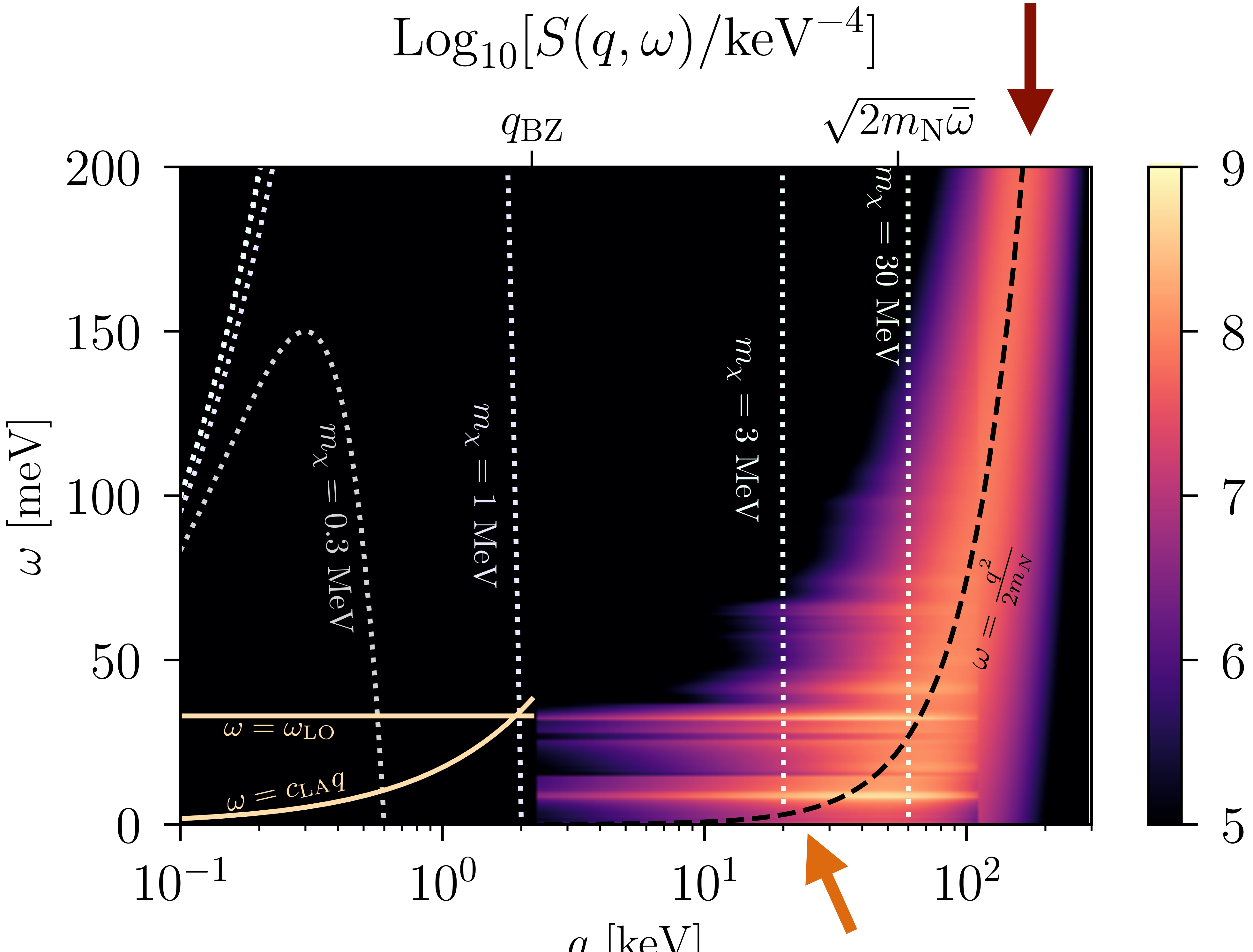
$$S^{\rm IA}(q,\omega) \propto \sum_J f_J^2 \sqrt{\frac{2\pi}{\Delta^2}}\, \exp\left(-\frac{(\omega - \frac{q^2}{2m_N})^2}{2\Delta^2}\right), \quad \Delta^2 = \frac{q^2\bar{\omega}_{\rm ph}}{2m_N}$$

As $\omega \gg \bar{\omega}_{\rm ph}$, $\Delta/\omega \to 0$, take narrow-width limit:

$$S(q,\omega) \propto \sum_J f_J^2\, \delta\left(\omega - \frac{q^2}{2m_N}\right)$$

reproducing free nuclear recoils

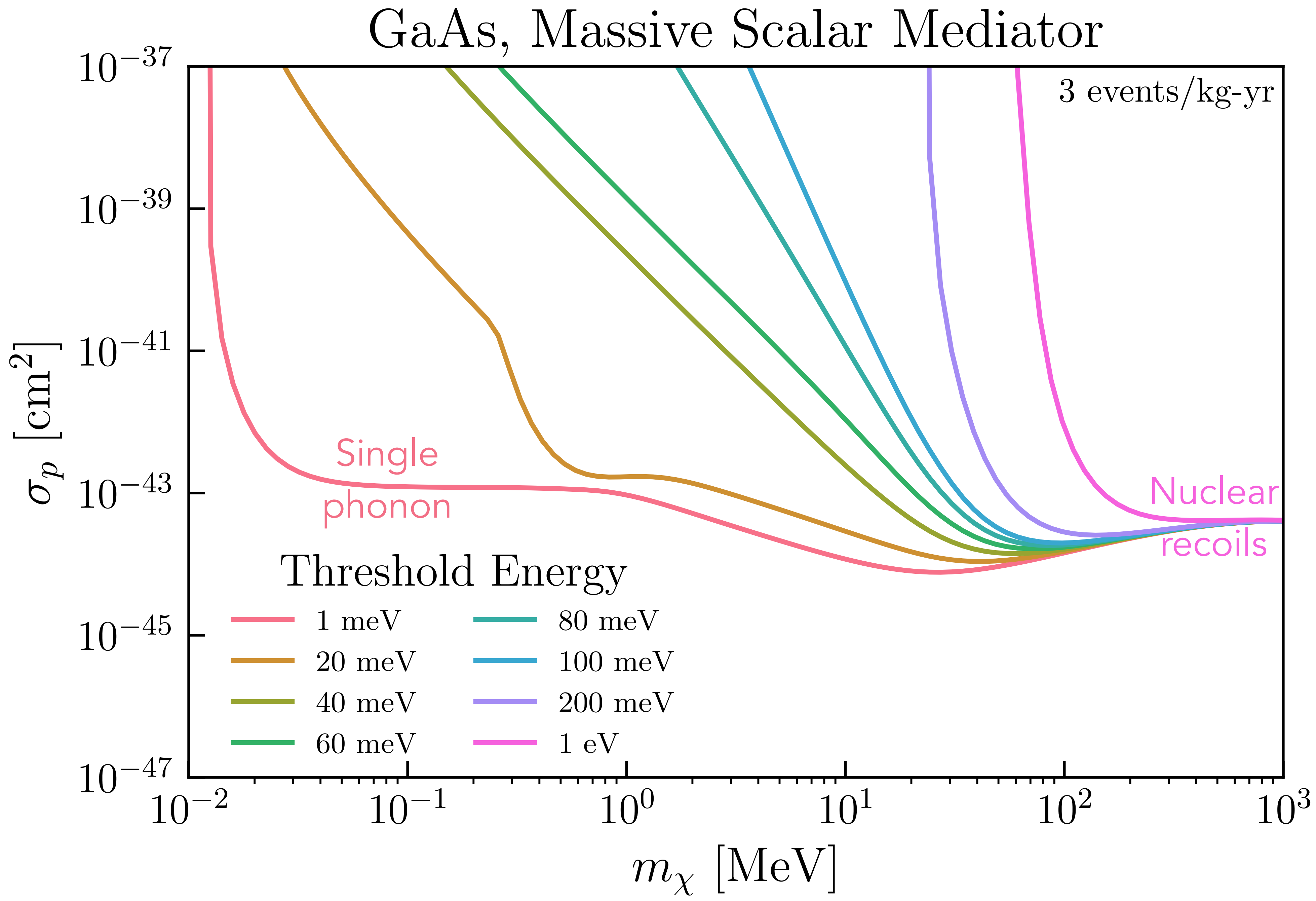
Free nuclear recoil limit



Multiphonon response

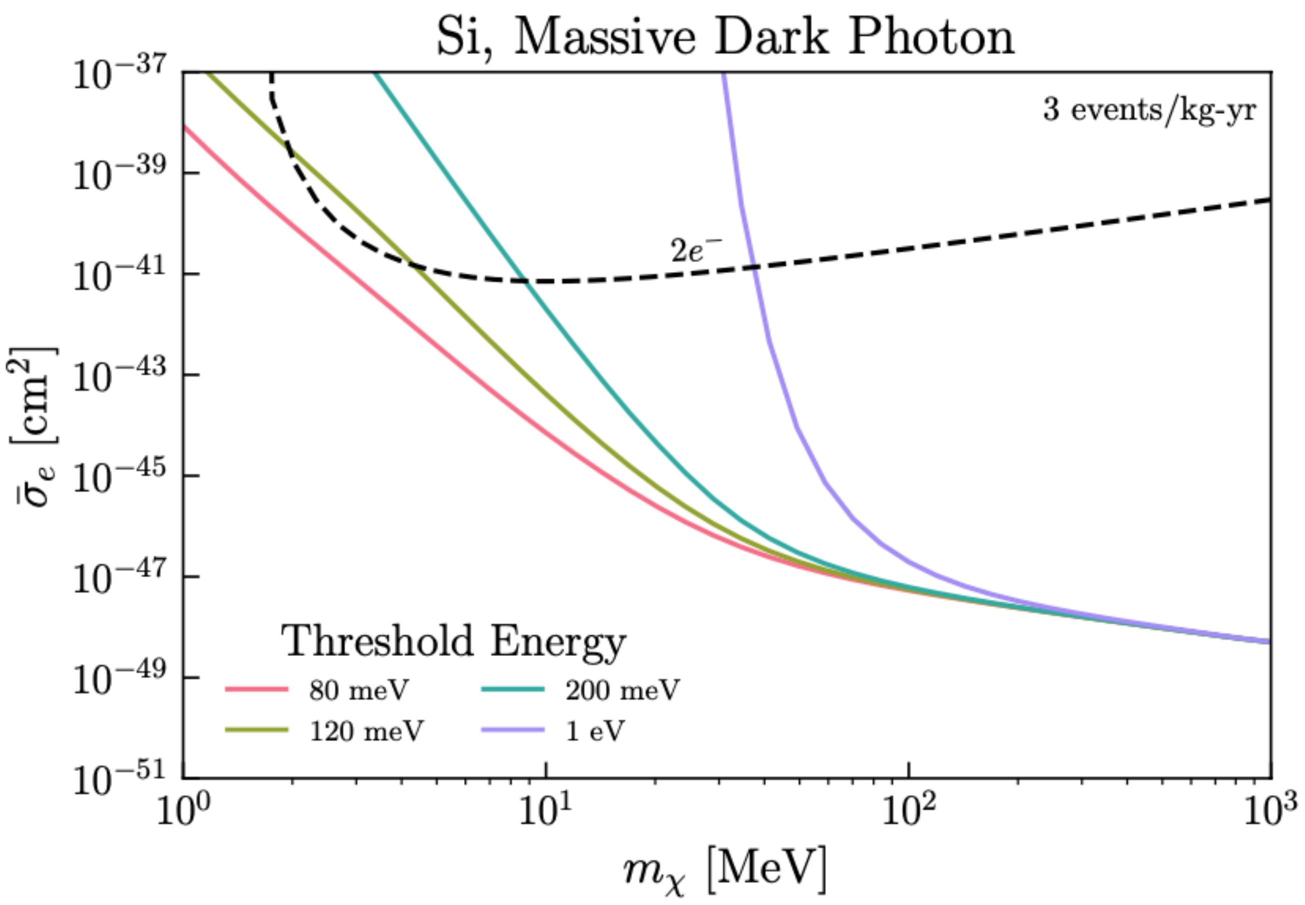
Campbell-Deem, Knapen, TL, Villarama 2205.02250

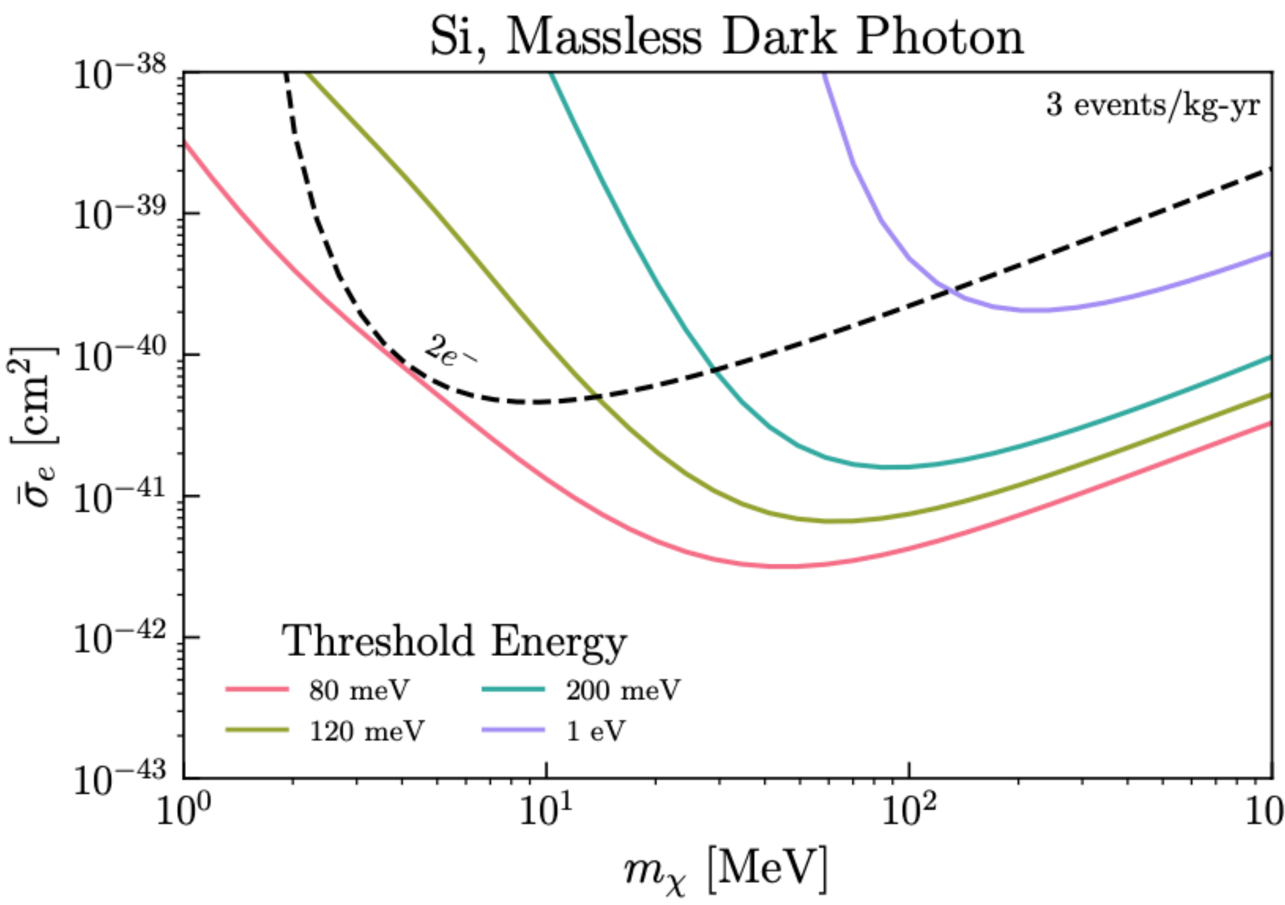
DM scattering rate



Campbell-Deem, Knapen, TL, Villarama 2205.02250
Harmonic oscillator model: Kahn, Krnjaic, Mandava 2011.09477

Dark photon mediator





Coupling given by *q*-dependent effective charge *Z(q)*

Single phonon reach estimated by dielectric response or directly computed in DFT

Future steps

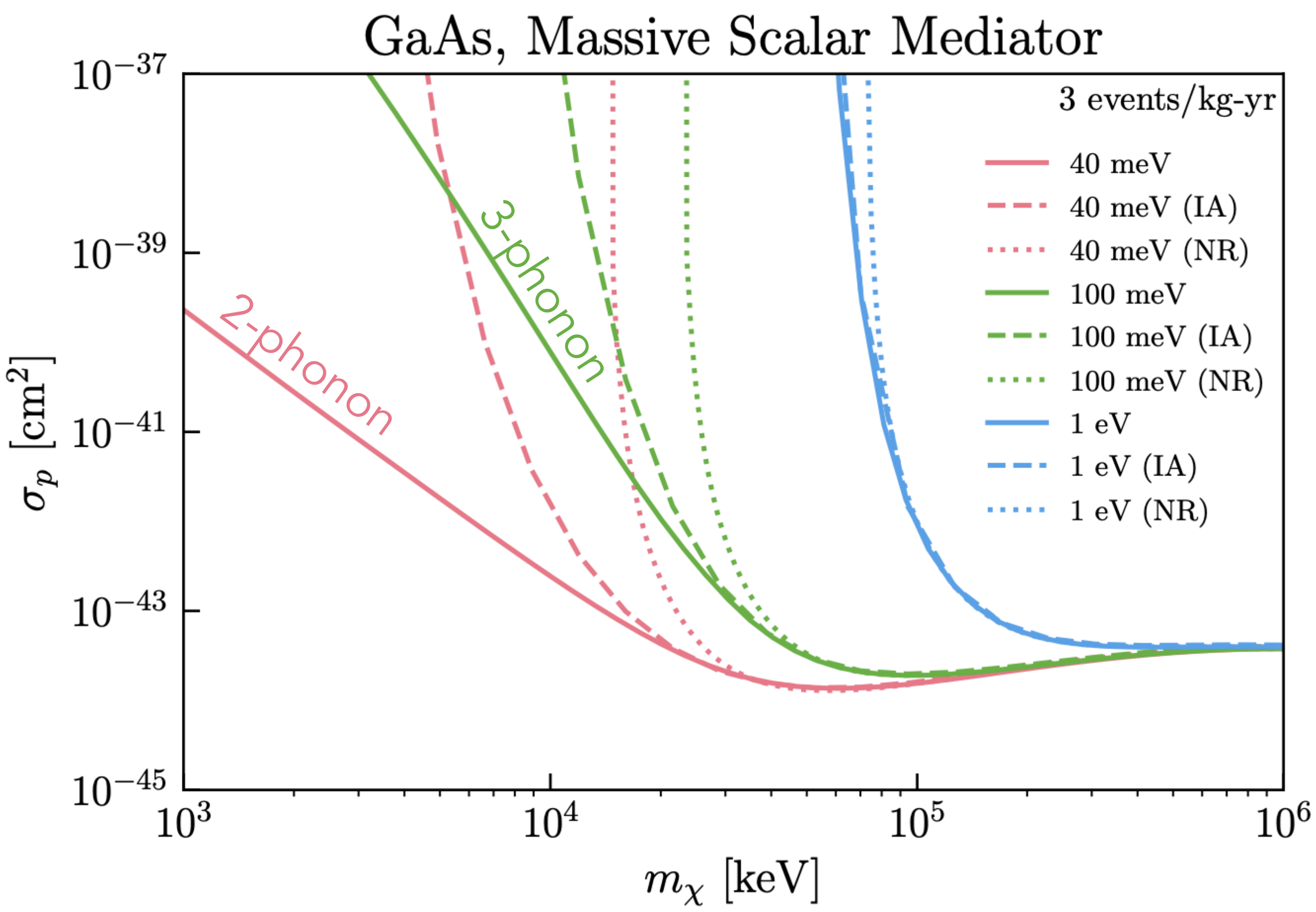
Pinning down $S(q, \omega)$:

Quantify theoretical uncertainties and validity of approximations

Detailed look at two (or three) phonon rates

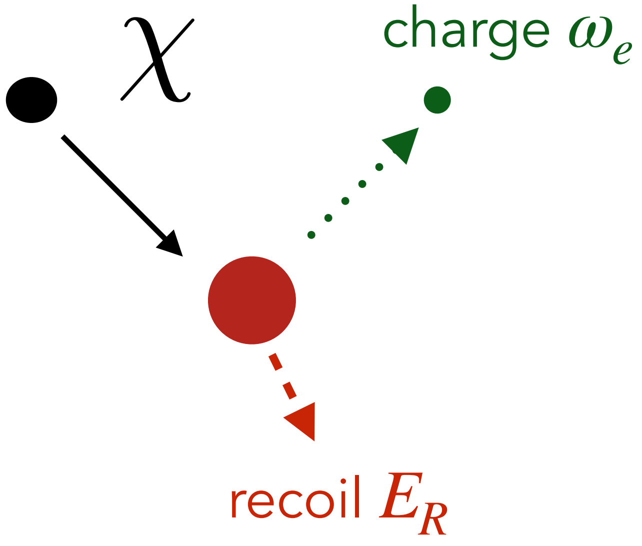
Experimental calibration?

Above eV scale, rates pretty quickly converge to the impulse approximation, nuclear recoils



Migdal effect

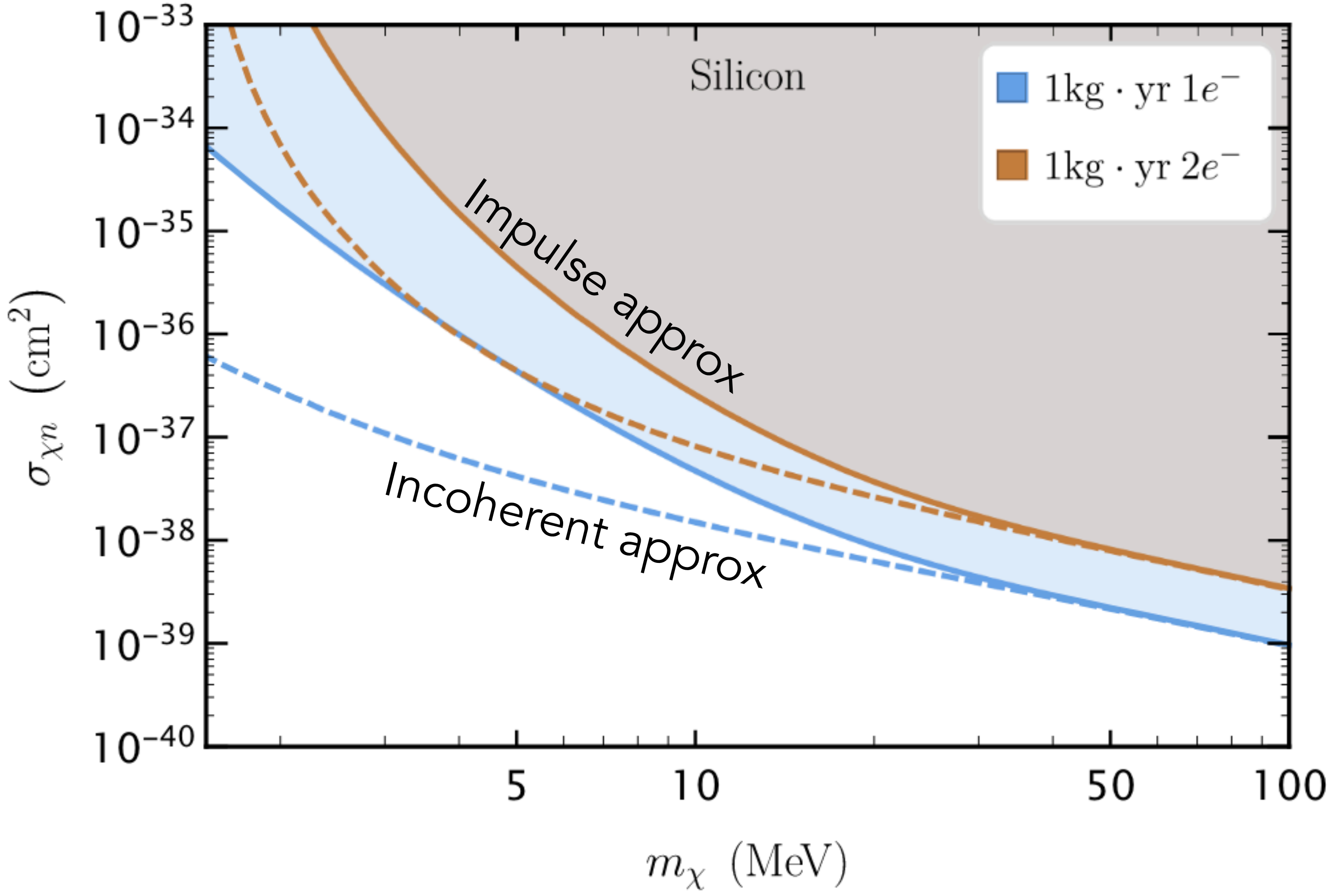
DM-nucleus scattering with
charge emission



$$\frac{d\sigma}{dE_R d\omega_e} \approx \frac{d\sigma_N}{dE_R}\frac{dP}{d\omega_e}$$

↑ DM-nucleus scattering

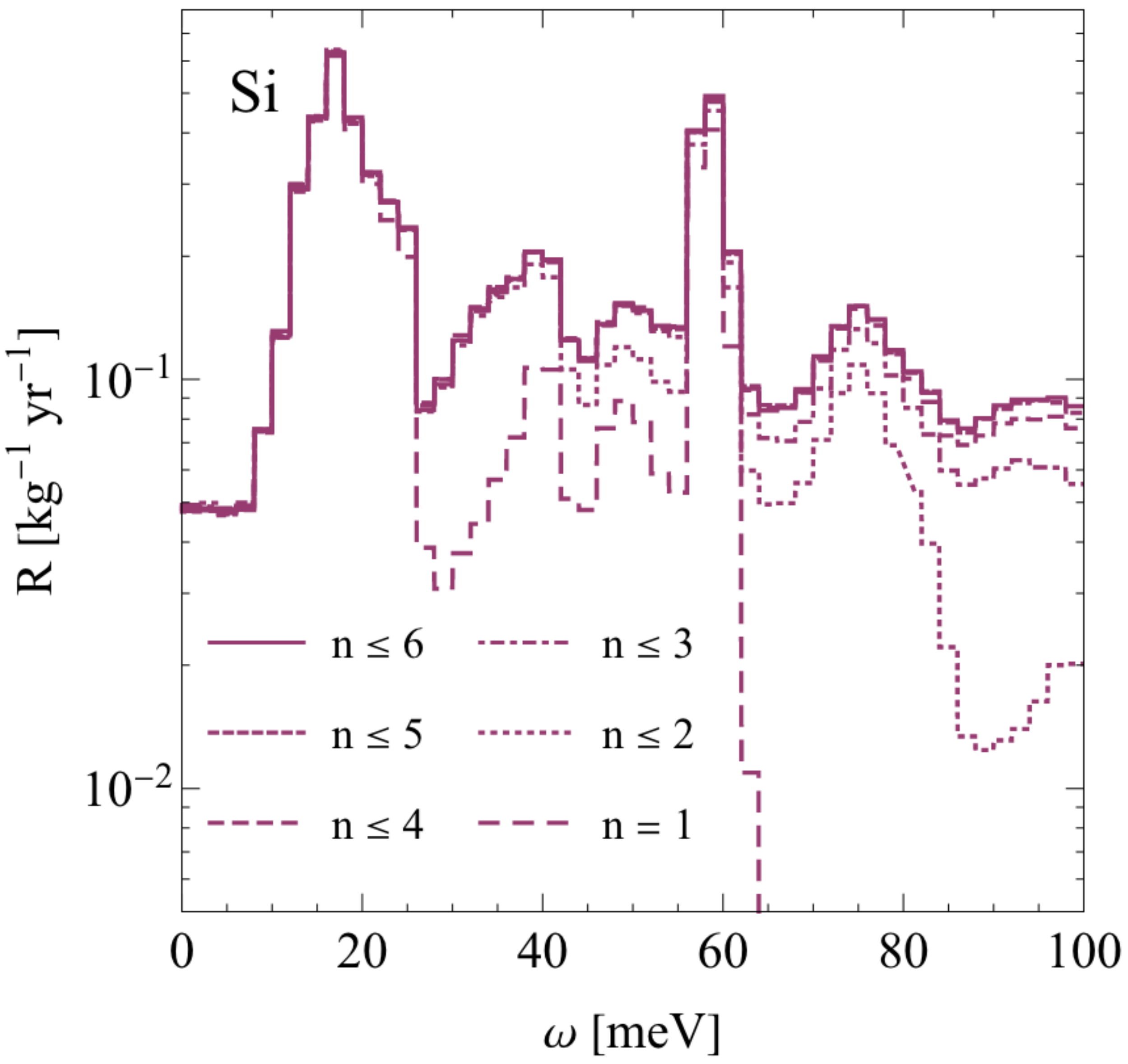
↑ Probability for charge excitation



From Liang, Mo, Zheng, Zhang 2205.03395
Knapen, Kozaczuk, Lin 2011.09496

Backgrounds

Coherent scattering of high energy (~MeV) photons off ions

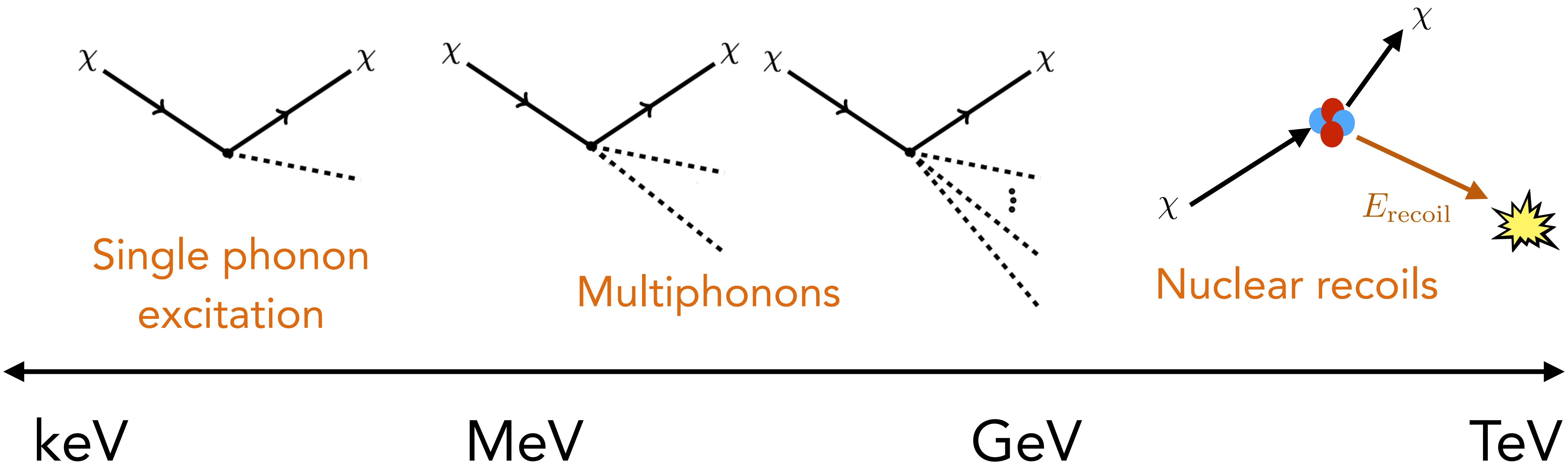


A. Robinson 1610.07656

Figure from Berghaus, Essig, Hochberg, Shoji, Sholapurkar 2112.09702

DM scattering in crystals

First steps towards describing DM-nucleus scattering into multiphonons.



Single phonon excitation

Multiphonons

Nuclear recoils

keV

MeV

GeV

TeV

Dark matter mass