Direct and Fusion Reactions including Weakly-bound and Halo Nuclei

2nd Joint Canada-APCTP Meeting on Nuclear Theory

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Collaborations

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Measurements of Interaction Cross Sections and Nuclear Radii in the Light *p*-Shell Region

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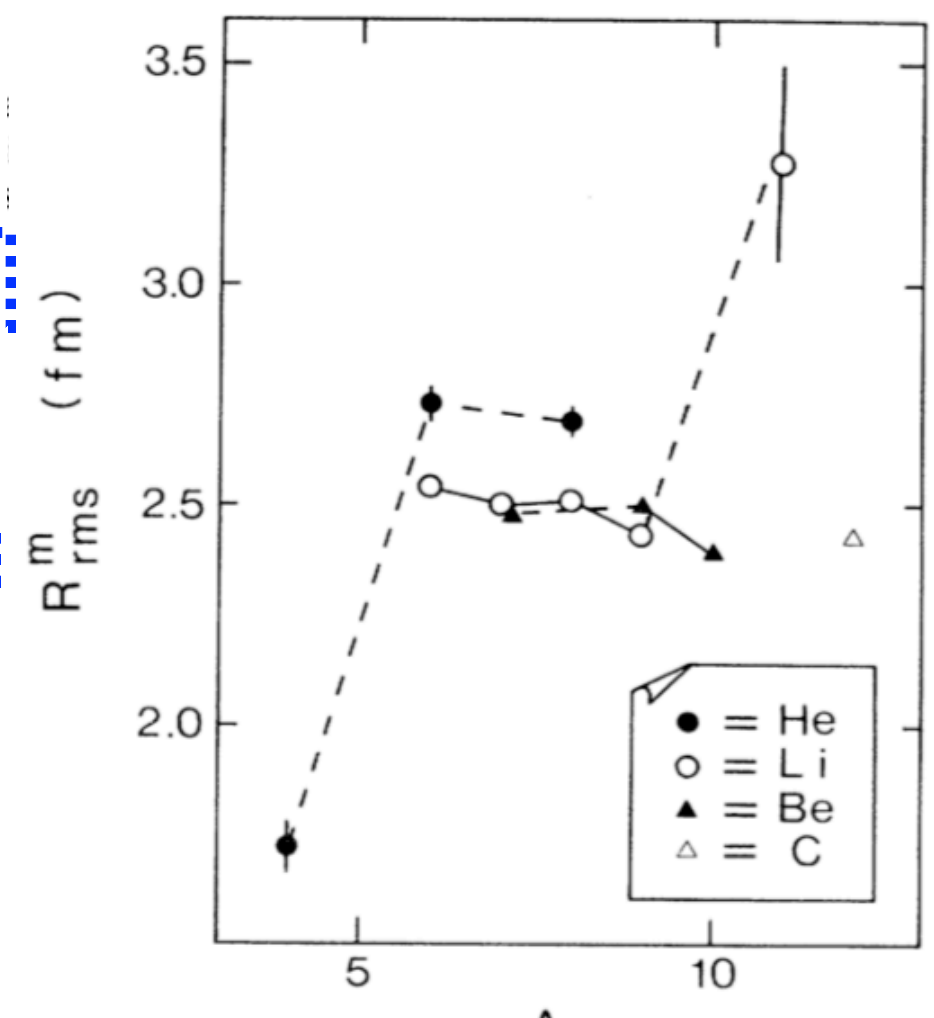
(Received 11 July 1985; revised manuscript received 17 September 1985)

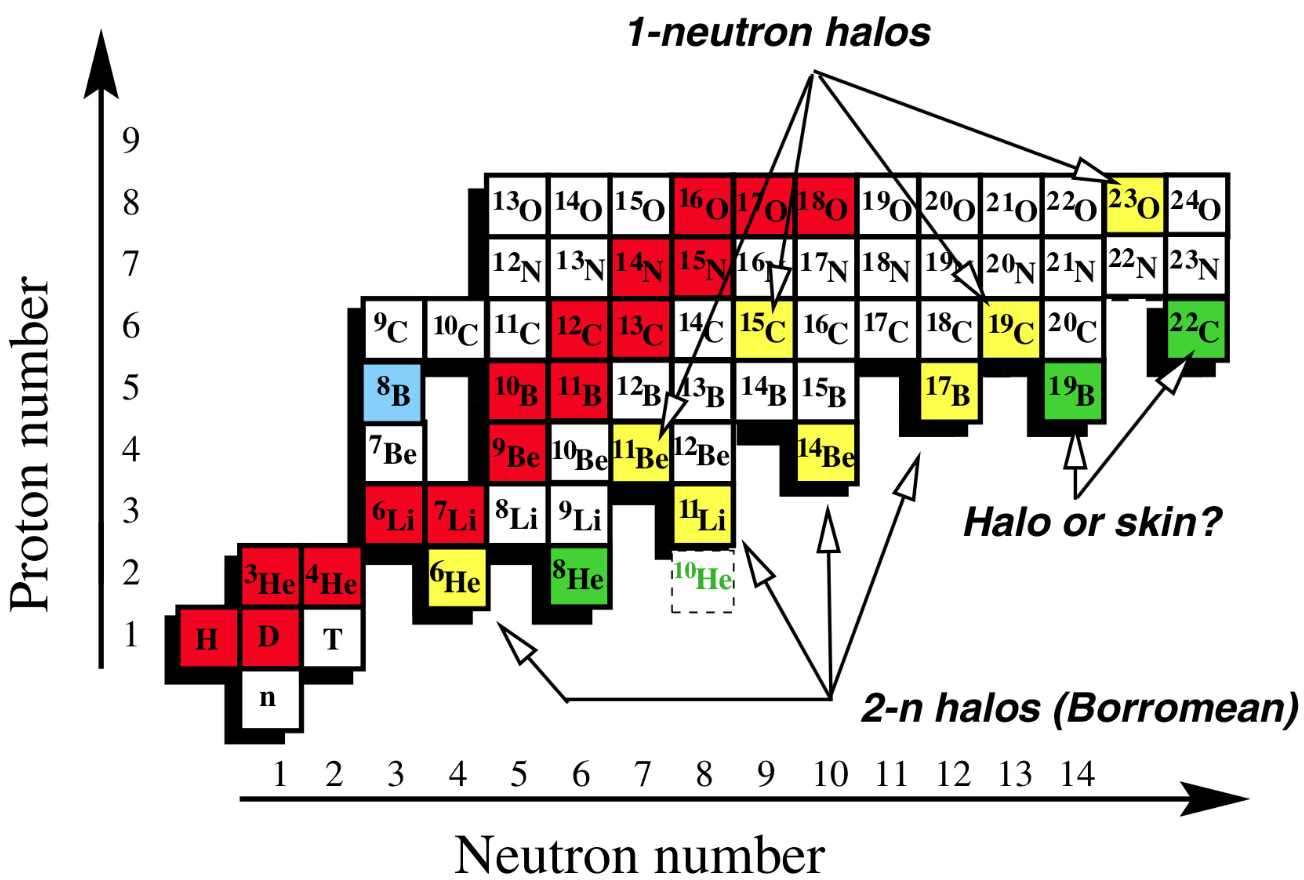
	R_I	e scat. R^e_{rms}	Gaussian R^G_{rms}	Harmonic oscillator R^m_{rms} a	R^c_{rms} a	R^n_{rms} a
^{4}He	1.41 ± 0.03	1.67 ± 0.01	1.72 ± 0.06	1.72 ± 0.06	1.72 ± 0.06	1.72 ± 0.06
^{6}He	2.18 ± 0.02		2.75 ± 0.04	2.73 ± 0.04	2.46 ± 0.04	2.87 ± 0.04
^{8}He	2.48 ± 0.03		2.70 ± 0.03	2.69 ± 0.03	2.33 ± 0.03	2.81 ± 0.03
^{6}Li	2.09 ± 0.02	2.56 ± 0.10	2.54 ± 0.03	2.54 ± 0.03	2.54 ± 0.03	2.54 ± 0.03
^{7}Li	2.23 ± 0.02	2.41 ± 0.10	2.50 ± 0.03	2.50 ± 0.03	2.43 ± 0.03	2.54 ± 0.03
^{8}Li	2.36 ± 0.02		2.51 ± 0.03	2.51 ± 0.03	2.41 ± 0.03	2.57 ± 0.03
^{9}Li	2.41 ± 0.02		2.43 ± 0.02	2.43 ± 0.02	2.30 ± 0.02	2.50 ± 0.02
^{11}Li	3.14 ± 0.16		3.27 ± 0.24	3.27 ± 0.24	3.03 ± 0.24	3.36 ± 0.24
^{7}Be	2.22 ± 0.02		2.48 ± 0.03	2.48 ± 0.03	2.52 ± 0.03	2.41 ± 0.03
^{9}Be	2.45 ± 0.01	2.52 ± 0.01	2.49 ± 0.01	2.50 ± 0.01	2.47 ± 0.01	2.53 ± 0.01
^{10}Be	2.46 ± 0.03		2.38 ± 0.02	2.39 ± 0.02	2.34 ± 0.02	2.43 ± 0.02
^{12}C	2.61 ± 0.02	2.45 ± 0.01	2.40 ± 0.02	2.43 ± 0.02	2.43 ± 0.02	2.43 ± 0.02

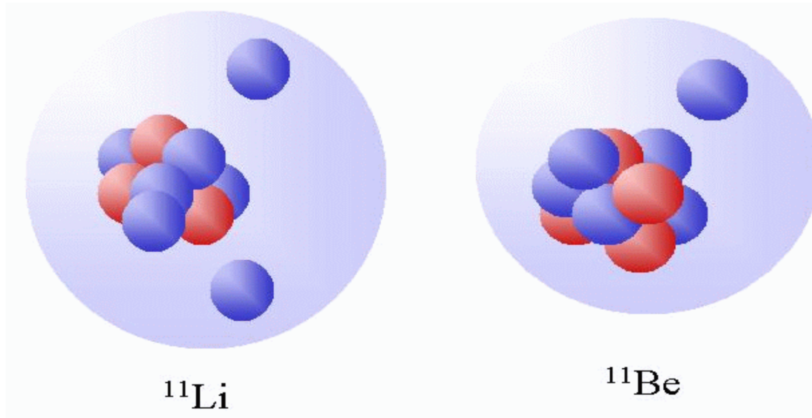
[a]Superscripts *m*, *c*, and *n* indicate the nuclear matter, the charge, and the neutron matter distributions, respectively.

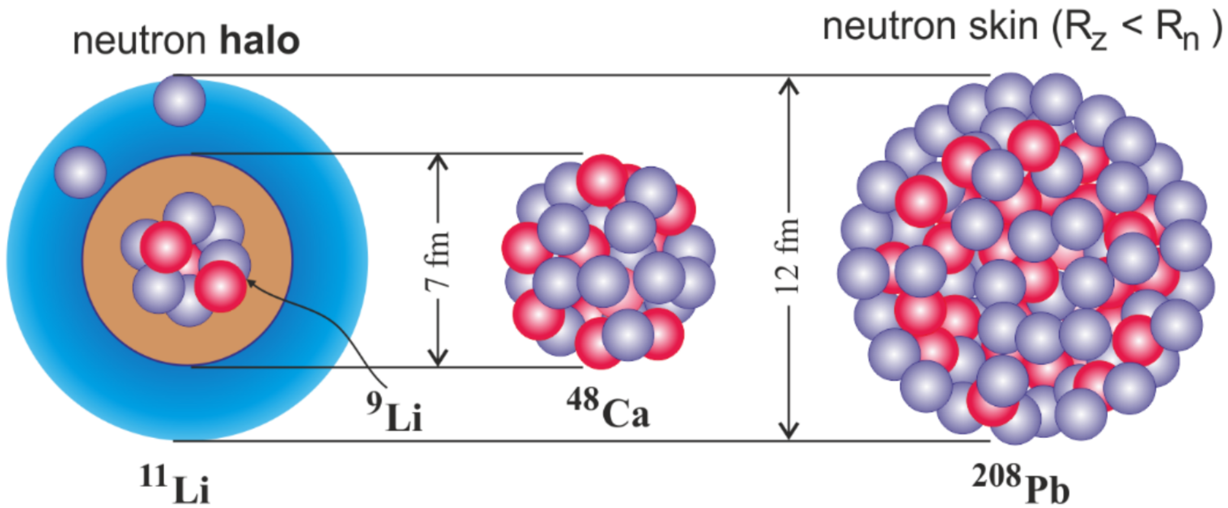


I. Tanihata et al : Phys. Rev. Lett 55, 2676(1985)

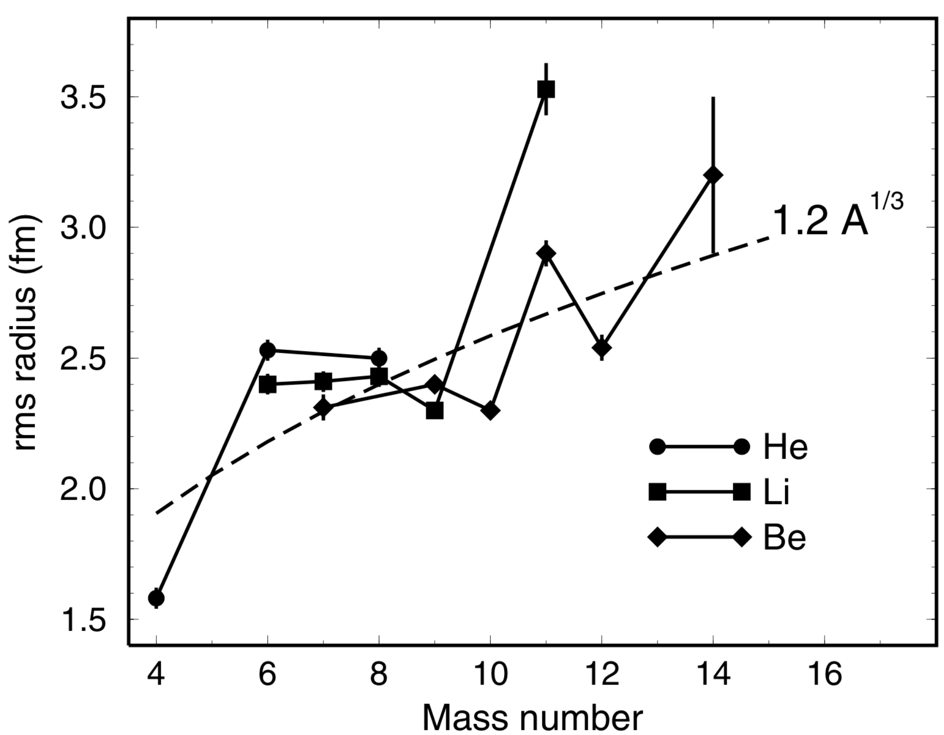
Halo nuclei





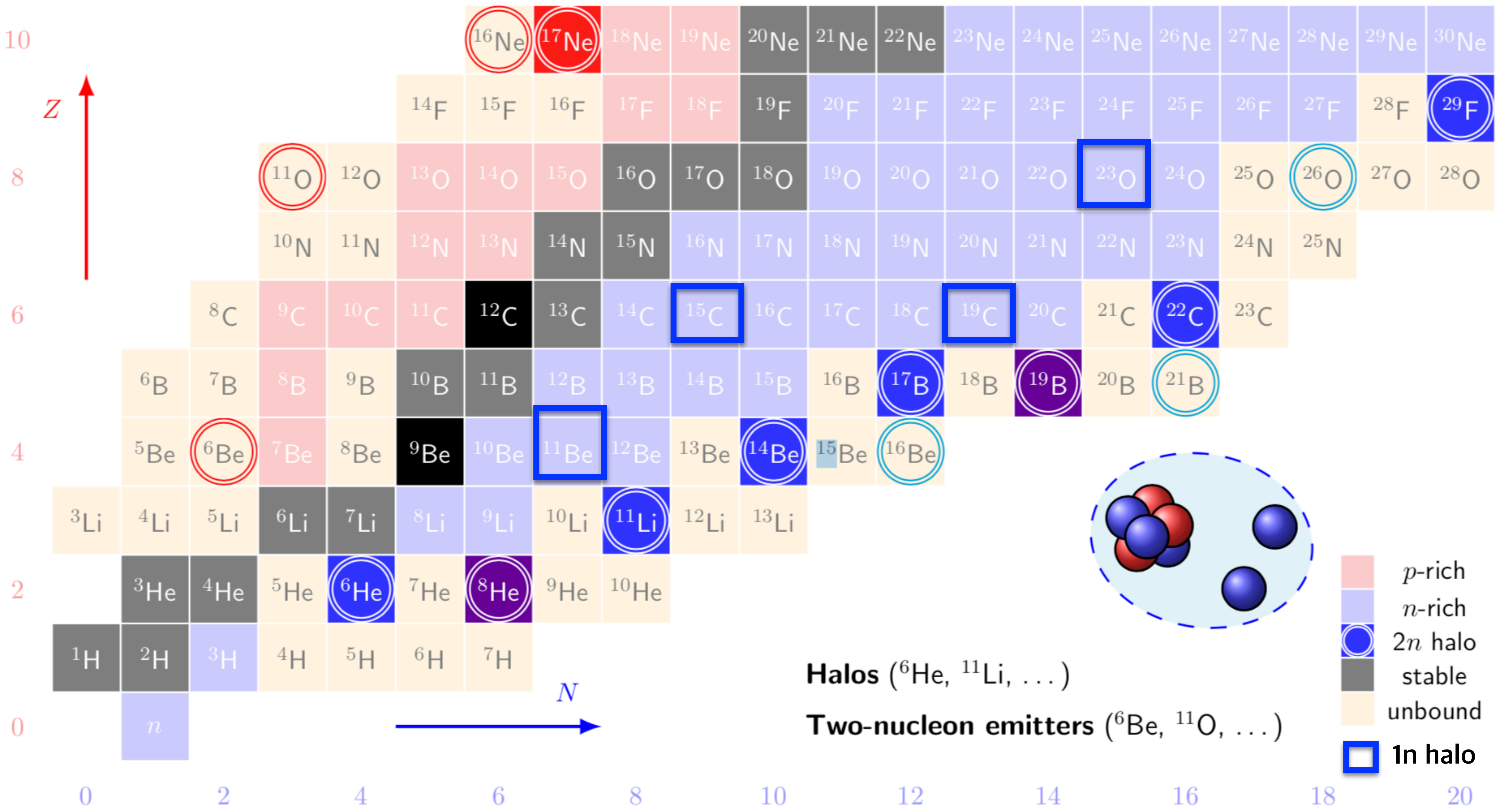


Nucleus	rms matter radii (fm)	
^{4}He	1.57 ± 0.04	1.58 ± 0.04
^{6}He	2.48 ± 0.03	2.71 ± 0.04
^{9}Li	2.32 ± 0.02	2.30 ± 0.02
^{11}Li	3.12 ± 0.16	3.53 ± 0.10
^{12}Be	2.59 ± 0.06	2.54 ± 0.05
^{14}Be	3.16 ± 0.38	3.20 ± 0.30



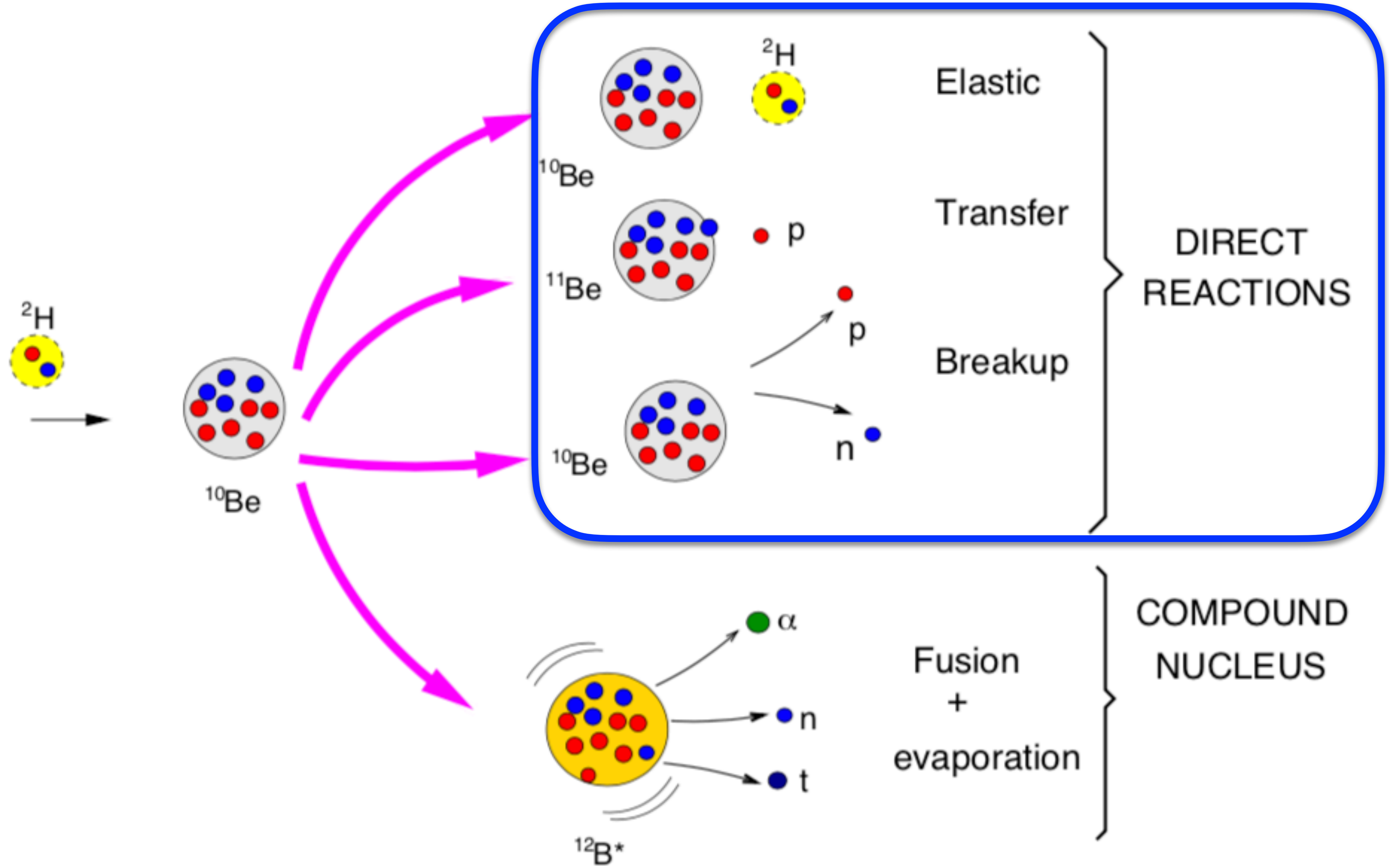
1. I. Tanihata et al : Phys. Lett . B 206, 592 (1998)
2. J.S. AL-Khalili et al : Phys. Rev. C 54 1843 (1996)

Halo nuclei in nuclear chart



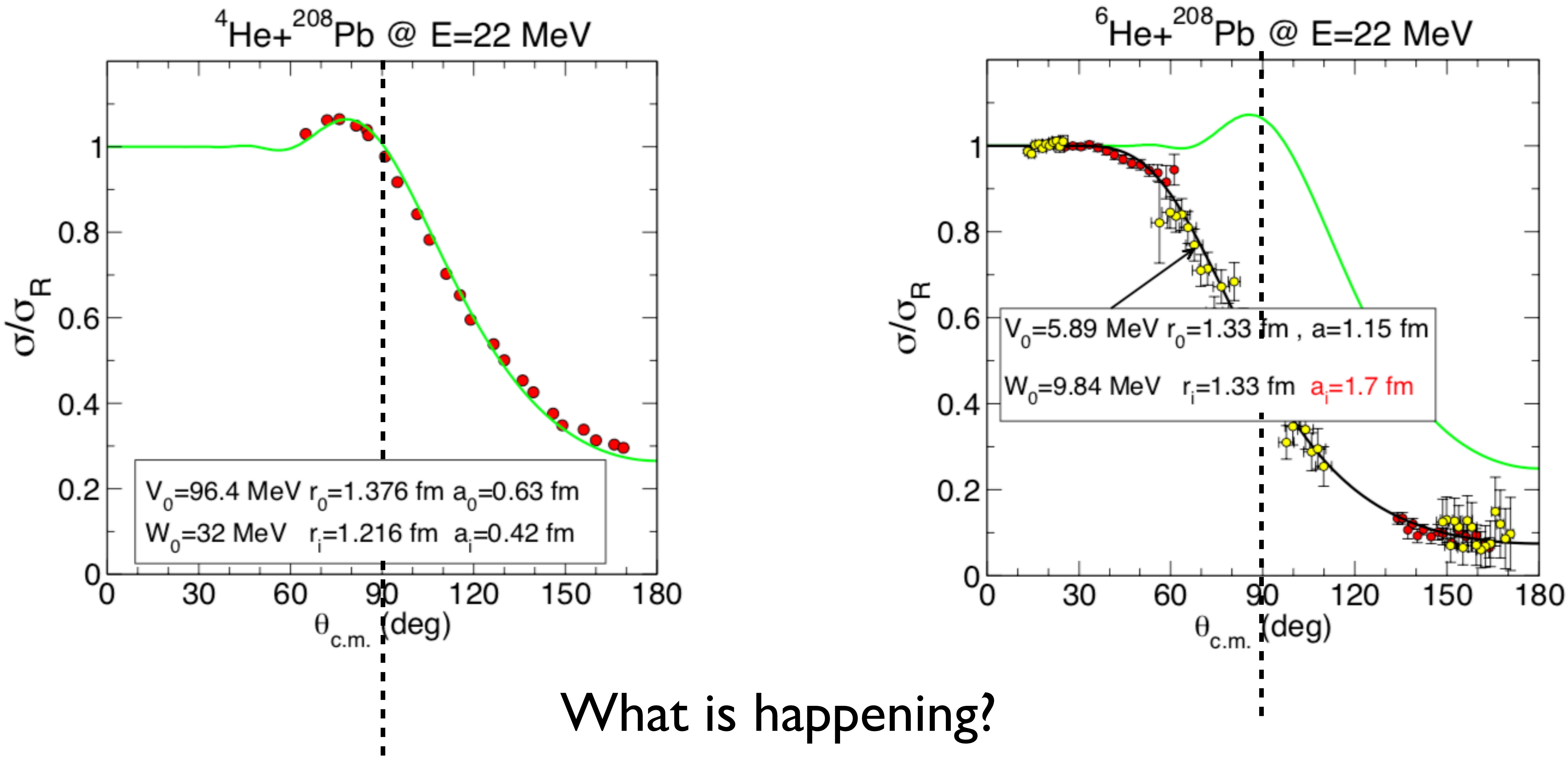
Presentation of J. Casal 2020

Nuclear reaction approach



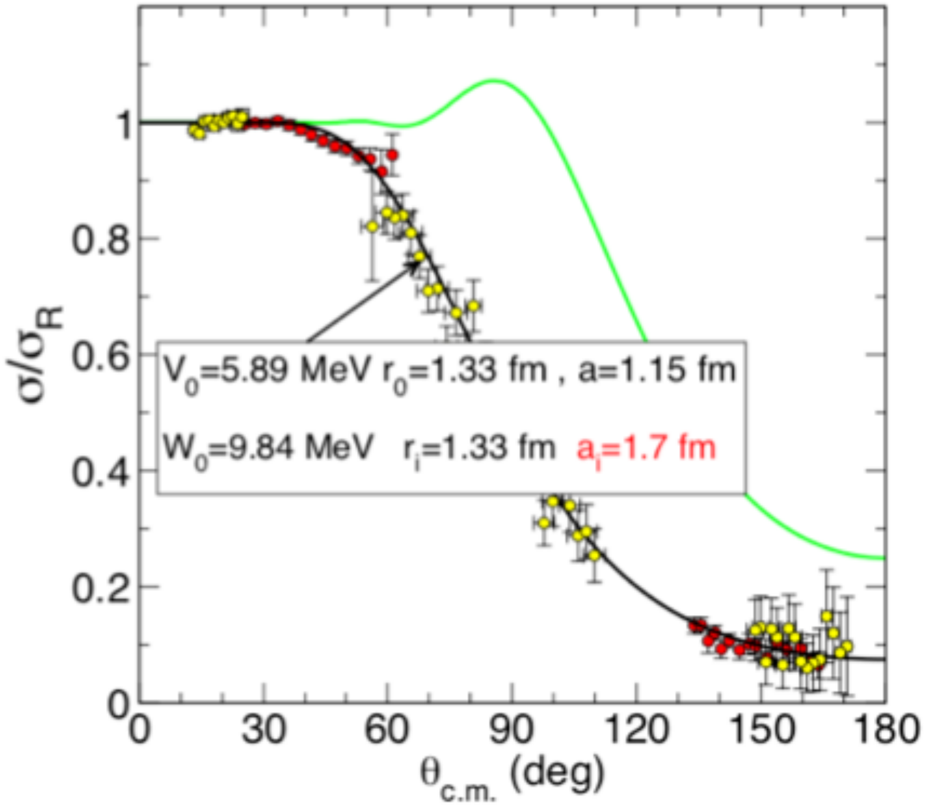
Presentation of A. M. Moro 2014

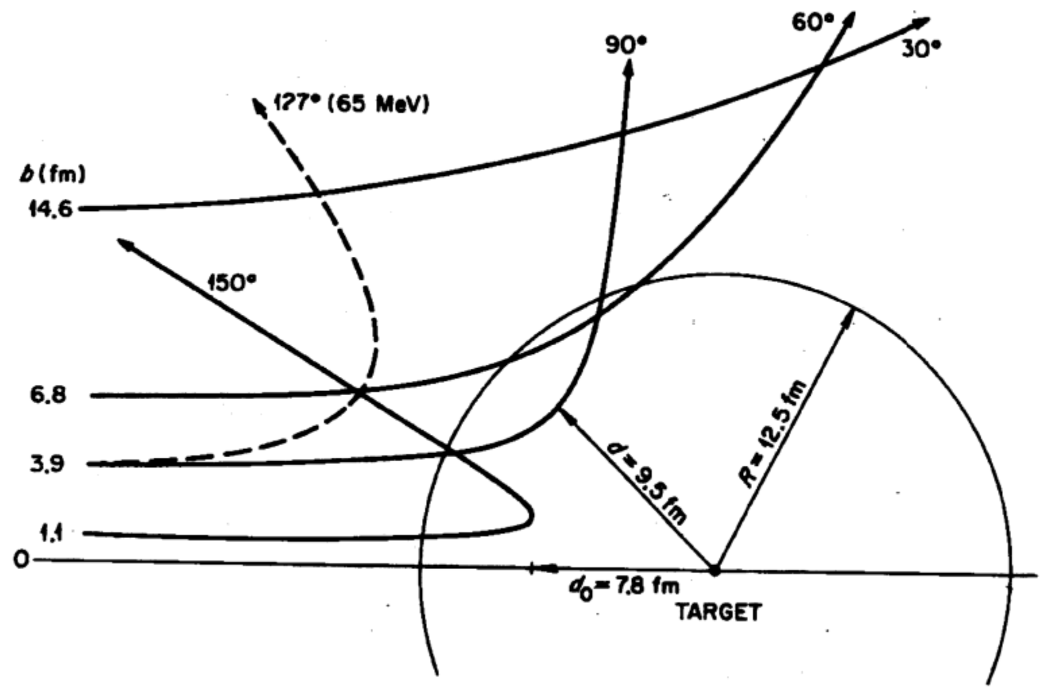
Elastic scattering between Halo and stable nuclei

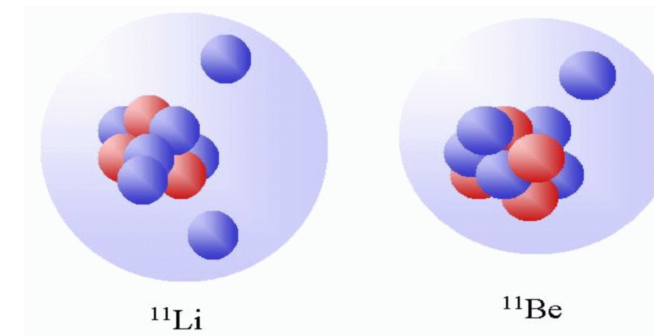


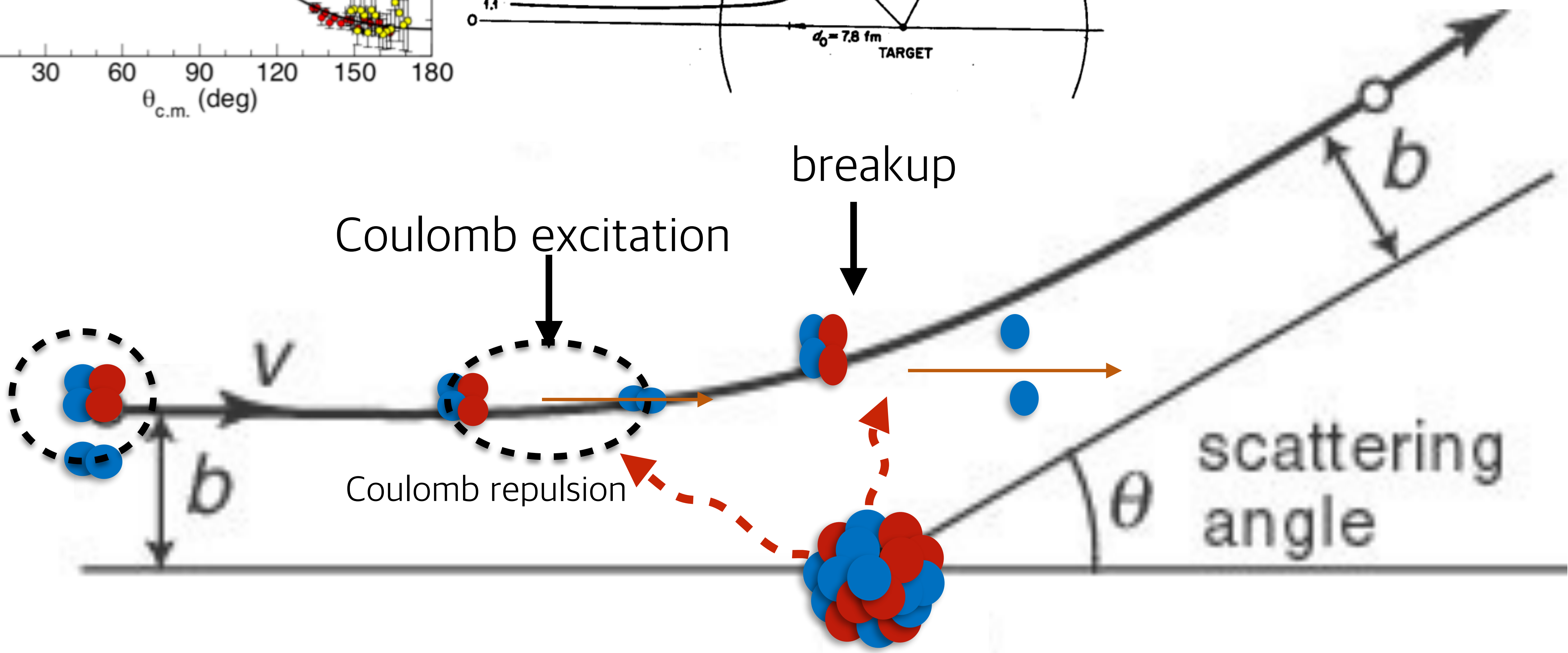
What is happening?

What happens in nuclear reaction with weakly bound nuclei









Generalized Optical potential

P, Q : Feshbach projection operators

P : elastic channel
Q : reaction channel

$$(\mathbf{H} - E)(\mathbf{P} + \mathbf{Q})\Psi = 0$$

$$(\mathbf{PHP} + \mathbf{PHQ})\Psi = E\mathbf{P}\Psi$$

$$(\mathbf{QHP} + \mathbf{QHQ})\Psi = E\mathbf{Q}\Psi$$

$$P\psi = \psi_p \quad PHP = H_{PP}$$

$$Q\psi = \psi_Q \quad PHQ = H_{PQ}$$

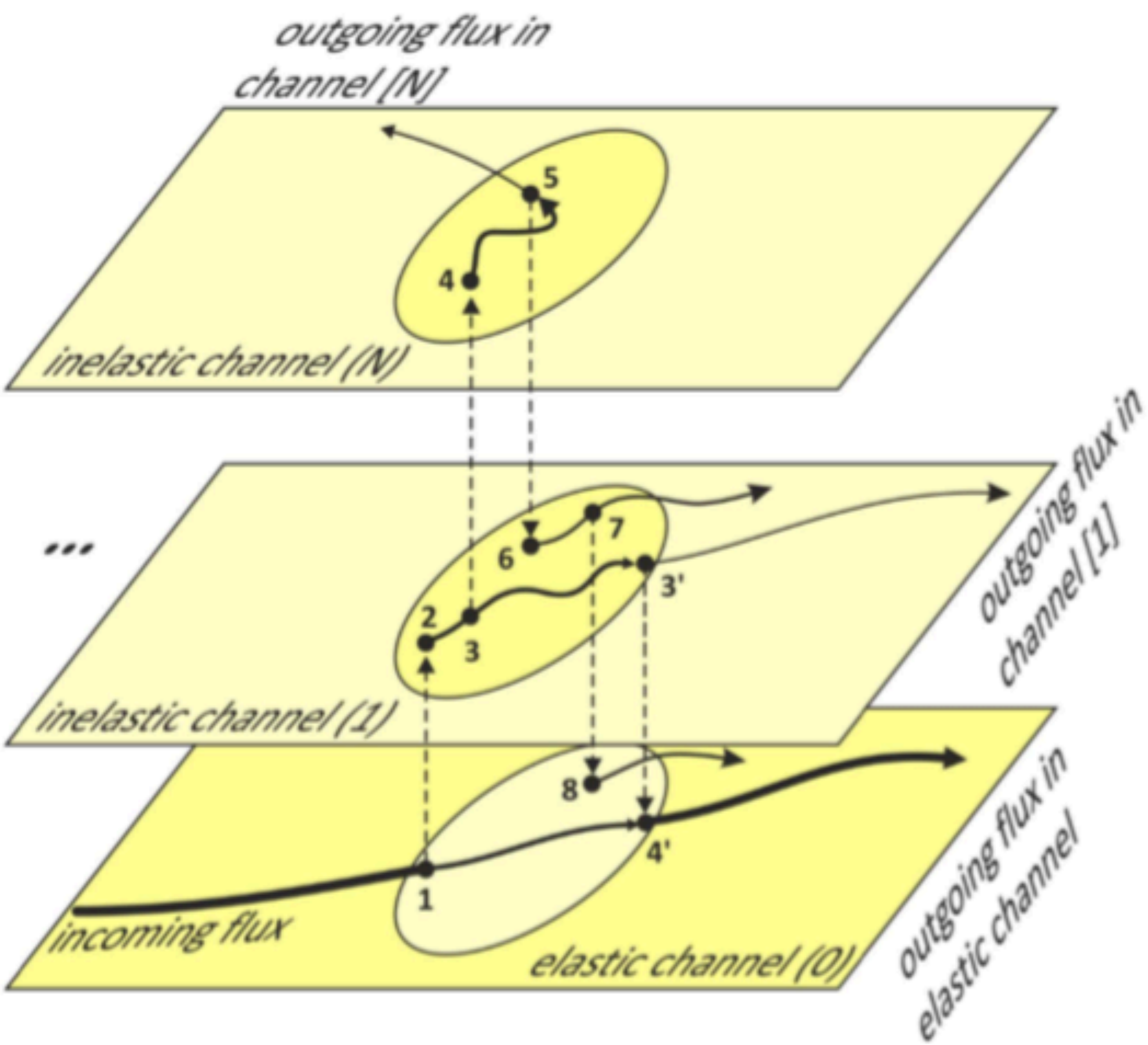
$$\psi_Q = \frac{1}{E - \mathbf{H}_{QQ}} \mathbf{H}_{QP} \psi_P \qquad \mathbf{H}_{PP}\psi_P + \mathbf{H}_{PQ}\psi_Q = E\psi_P$$

$$\left(H_{PP} + H_{PQ}\frac{1}{E - H_{QQ}}H_{QP}\right)\psi_P = E\psi_P$$: Effective Hamiltonian

$$V_{PP} = \langle \varphi_{a,0} | V_{ab} | \varphi_{a,0} \rangle \quad V_{PQ} = \langle \varphi_{a,0} | V_{ab} | \varphi_{a,1} \rangle \quad V_{QP} = \langle \varphi_{a,1} | V_{ab} | \varphi_{a,0} \rangle \quad G_{QQ} = \frac{1}{E - H_{QQ}}$$

$$V_{OM} = \langle \varphi_{a,0} | V_{ab} | \varphi_{a,0} \rangle + \sum_{k=1}^{N} \langle \varphi_{a,0} | V_{ab} | \varphi_{a,k} \rangle \mathbf{G}_{QQ}^{(k)} \langle \varphi_{a,k} | V_{ab} | \varphi_{a,0} \rangle$$

Generalized Optical potential



MULTI CHANNEL CASE

Spectrum of particle **a**'s states

Optical potential is
- ❑ **non-local,**
- ❑ **complex,**
- ❑ **energy-dependent**

$$V_{OM} = \langle \varphi_{a,0} | V_{ab} | \varphi_{a,0} \rangle + \sum_{k=1}^{N} \langle \varphi_{a,0} | V_{ab} | \varphi_{a,k} \rangle \mathbf{G}_{QQ}^{(k)} \langle \varphi_{a,k} | V_{ab} | \varphi_{a,0} \rangle$$

Presentation of A. Denikin : 2017

Phenomenological optical potential

$$V_{OM} = \langle \varphi_{a,0} | V_{ab} | \varphi_{a,0} \rangle + \sum_{k=1}^{N} \langle \varphi_{a,0} | V_{ab} | \varphi_{a,k} \rangle \mathbf{G}_{QQ}^{(k)} \langle \varphi_{a,k} | V_{ab} | \varphi_{a,0} \rangle$$

elastic contribution → real potential

reaction channel contribution
- back to elastic channel → real potential
- go out to other channel → Imaginary potential (treating as optical absorption)

$$V_{OM}^{Ph} = V_R^{el} + (V_R^{re} + iV_I^{re})$$

$$V_{OM}^{Ph} = V + iW \longleftrightarrow \sigma_{total} = \sigma_{el} + \sigma_{non-el}$$

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Dynamic Polarization Potential for Coulomb Excitation Effects on Heavy-Ion Scattering

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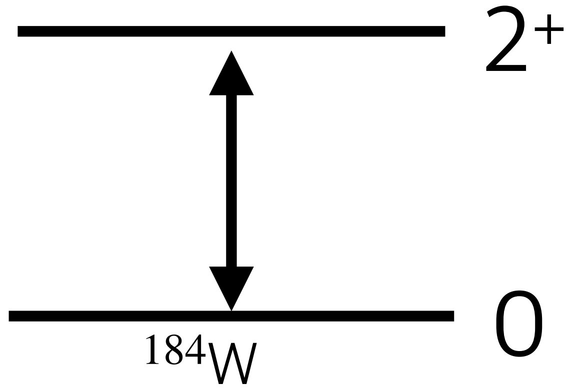
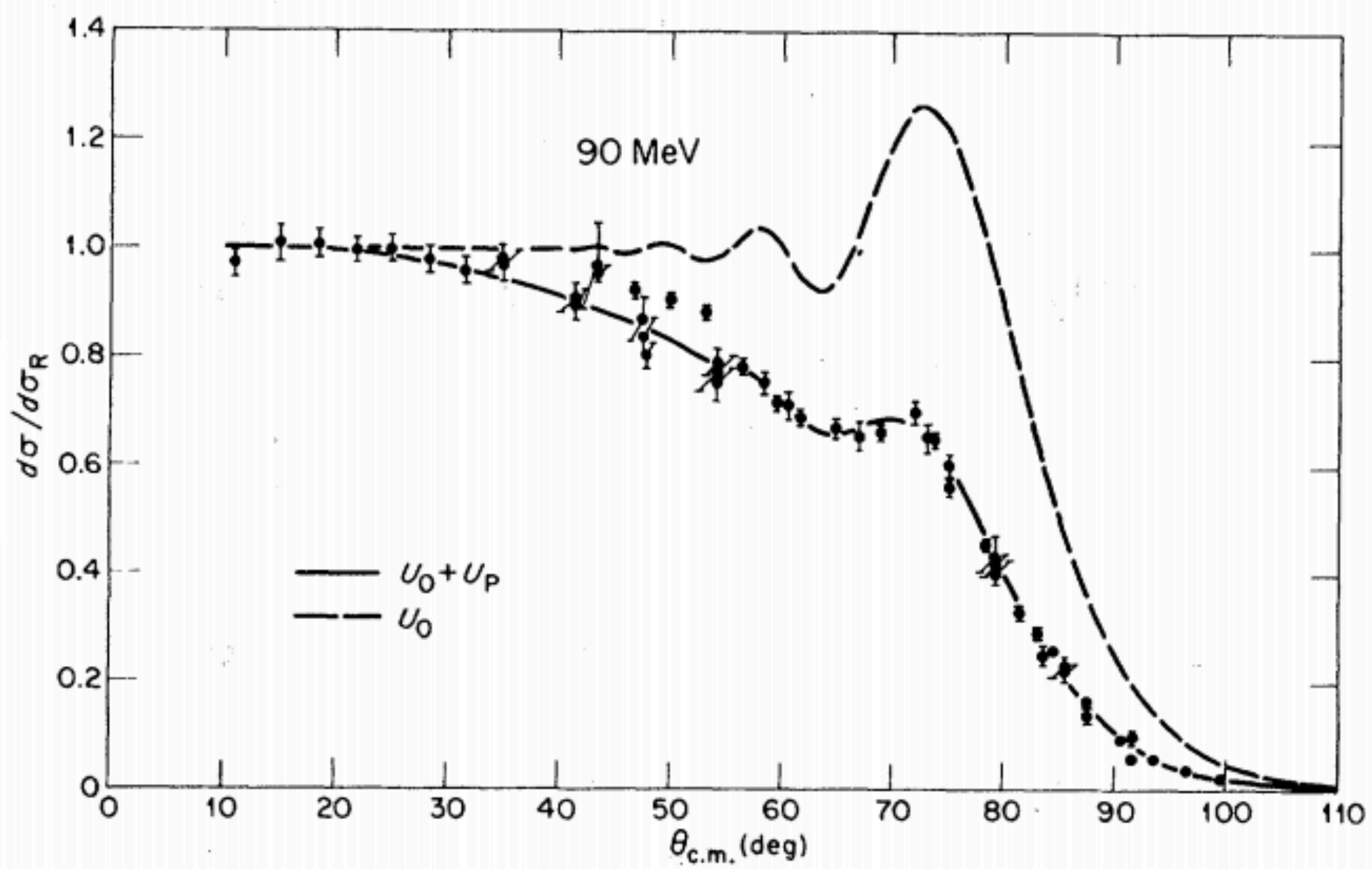
(Received 2 May 1977)

^{18}O + ^{184}W including ^{18}O+^{184}W* effect



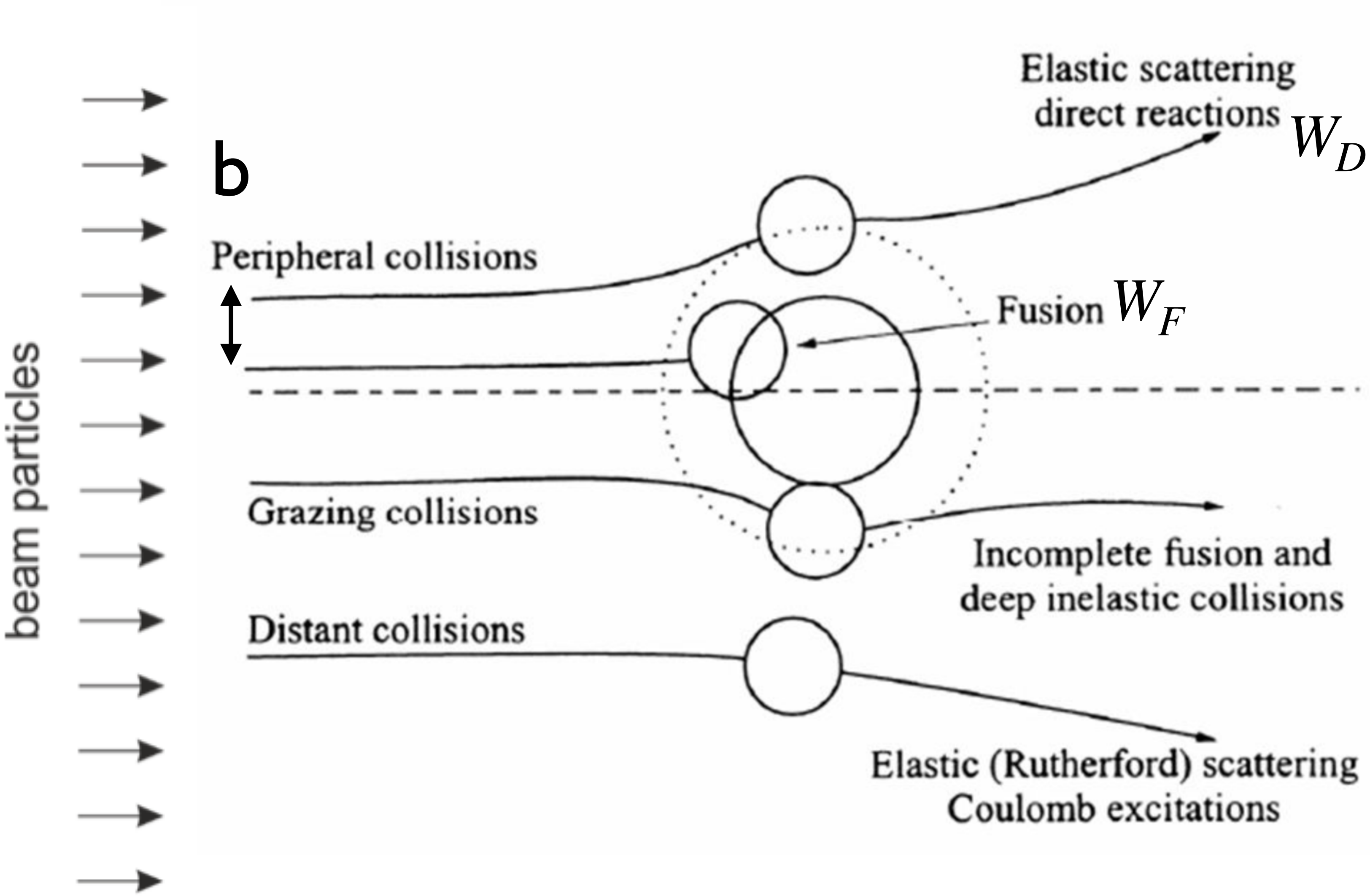
$$\mathrm{Im}\,U_p(R) = \begin{cases} -\left[1 - \frac{2}{7}\left(\frac{R_c}{R}\right)^2 - \frac{1}{21}\left(\frac{R_c}{R}\right)^4\right] K(R)\frac{W_p}{R^5} & \text{for } R \geq R_c \\ -\frac{2}{3}K(R)W_p R^4/R_c{}^9 & \text{for } R < R_c, \end{cases}$$

$$W_p = C(\mu Z_p{}^2/k)B(E2, 0 \rightarrow 2)g_2(\xi) \text{ MeV fm}^5$$



Phenomenological extended optical potential

$$V_{OM}^{Ph} = V + iW = V_N + V_{inel} + V_{bu} \sim \left(V_N + iW_F\right) + \left(V_{inel} + iW_{inel}\right) + \left(V_{bu} + iW_{bu}\right)$$



$W_D \sim W_{bu}$

W_F

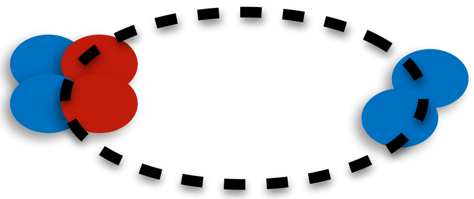
$\sim W_{inel}$

$$T_{BU;inel;l} = \frac{8}{hv}\int_0^\infty |\chi_l^+(r)|^2 W_{inel;Bu} dr$$

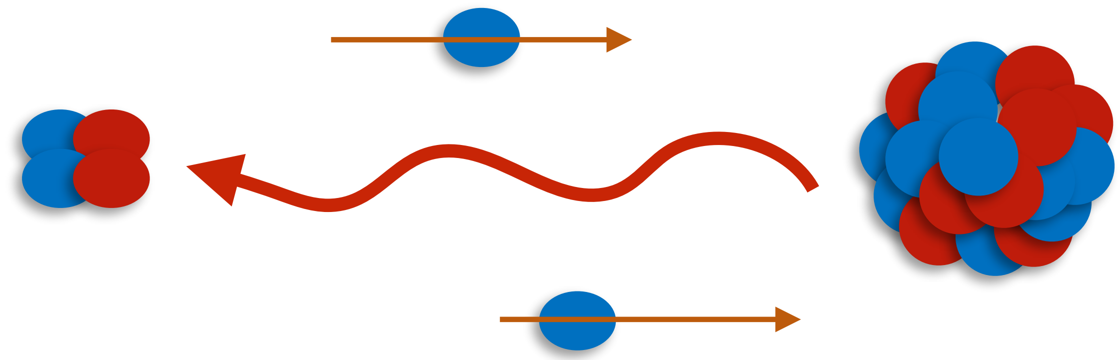
$$\frac{d\sigma_{\mathrm{BU,inel}}}{d\Omega} = \frac{kD_0}{16\pi}\frac{1}{\cos(\frac{\theta_{\mathrm{c.m.}}}{2})\sin^3(\frac{\theta_{\mathrm{c.m.}}}{2})}\sum_l \frac{\pi}{k}(2l+1)\; T_{\mathrm{BU,inel};l}$$

$$\sigma_{total} = \sigma_{el} + \sigma_{non-el} = \sigma_{el} + \left[\sigma_{fusion} + \sigma_{inel} + \sigma_{bu}\right]$$

Construction potential with weakly bound nuclei



and



elastic scattering + inelastic scattering + Coulomb breakup

contribution of dipole Coulomb excitation

$$V_{inel} + V_{bu} \sim U^{inel}_{CDE} + \left(U^{br}_{CDE} + V_{residue} + iW_{residue}\right)$$ fitting (Long rage effect and so on.)

$$U_{\mathrm{CDE}} = U^{\mathrm{inel}}_{\mathrm{CDE}} + U^{\mathrm{br}}_{\mathrm{CDE}} = (V^{\mathrm{inel}}_{\mathrm{CDE}} + iW^{\mathrm{inel}}_{\mathrm{CDE}}) + (V^{\mathrm{br}}_{\mathrm{CDE}} + iW^{\mathrm{br}}_{\mathrm{CDE}})$$

$$U^{\mathrm{inel}}_{\mathrm{CDE}}(r) = \frac{4\pi}{9}\frac{Z_t^2 e^2}{\hbar v}\frac{B(E1;\varepsilon_x^{1st})}{(r-a_0)^2 r} \times [g(\frac{r}{a_0}-1,\xi) + if(\frac{r}{a_0}-1,\xi)$$

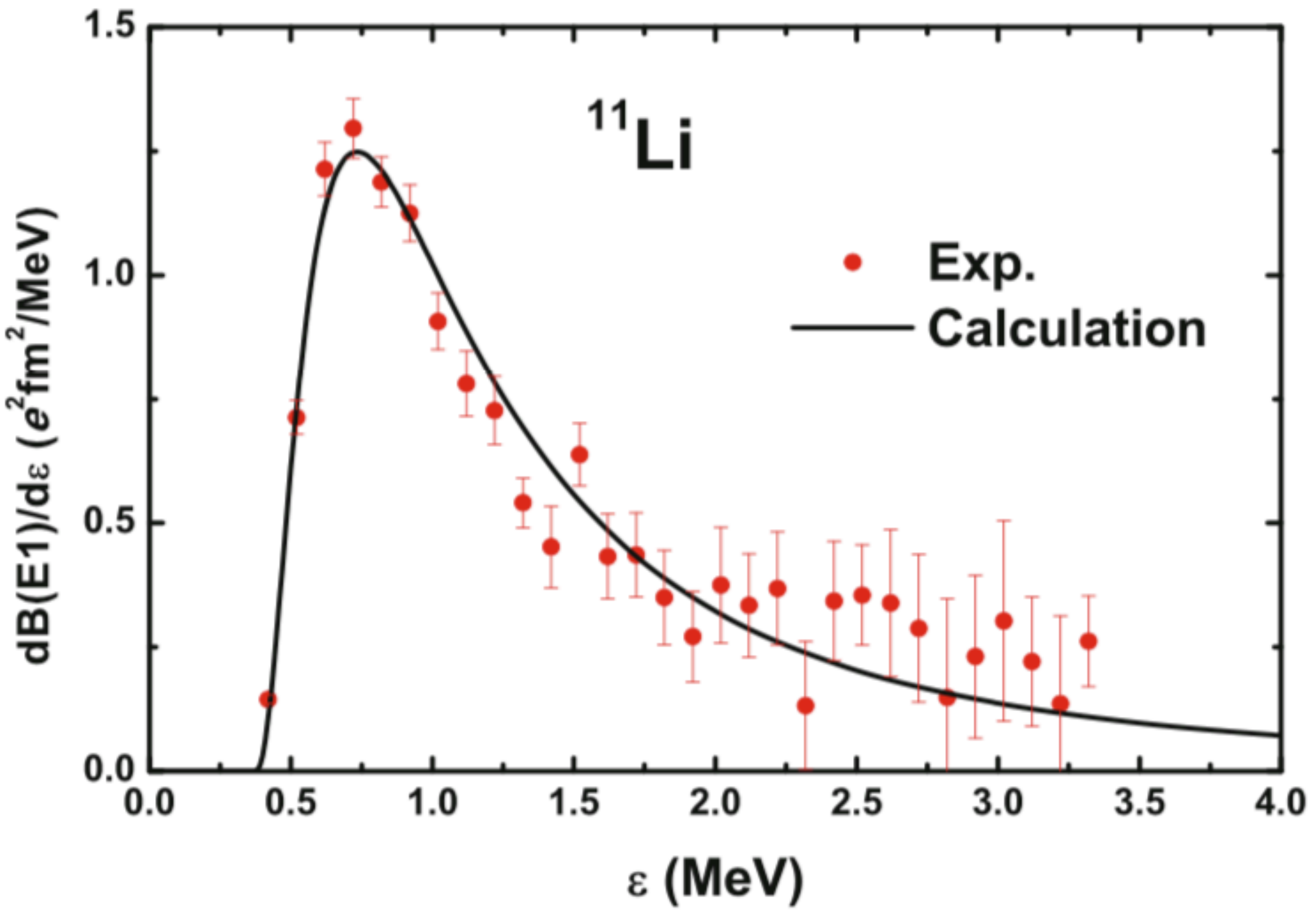
$$U^{\mathrm{br}}_{\mathrm{CDE}}(r) = \frac{4\pi}{9}\frac{Z_t^2 e^2}{\hbar v}\frac{1}{(r-a_0)^2 r}\int_{\varepsilon_b}^{\infty} d\varepsilon \frac{dB(E1)}{d\varepsilon} \times g(\frac{r}{a_0}-1,\xi) + if(\frac{r}{a_0}-1,\xi)]$$

real part imaginary part

M. Andres et al, Nucl. Phys. A 612 82 (1997)

Example : Coulomb dipole excitation(CDE) of ^{11}Li.

$$U^{\rm br}_{\rm CDE}(r) = \frac{4\pi}{9}\frac{Z_t^2 e^2}{\hbar v}\frac{1}{(r-a_0)^2 r}\int_{\varepsilon_b}^{\infty} d\varepsilon \frac{dB(E1)}{d\varepsilon} \times \left[g(\frac{r}{a_0}-1,\xi) + i f(\frac{r}{a_0}-1,\xi)\right]$$



$$\frac{dB(E1)}{d\varepsilon} = N\frac{(\varepsilon - S_{2n})^3}{(\varepsilon + S_{2n}^{\rm eff} - S_{2n})^{11/2}} = N\frac{(\varepsilon - S_{2n})^3}{[\varepsilon + (a-1)S_{2n}]^{11/2}},$$

$$f\left(\frac{r}{a_0}-1,\xi\right) = 4\xi^2\left(\frac{r}{a_0}-1\right)^2 e^{-\pi\xi} K''_{2i\xi}\left[2\xi\left(\frac{r}{a_0}-1\right)\right]$$

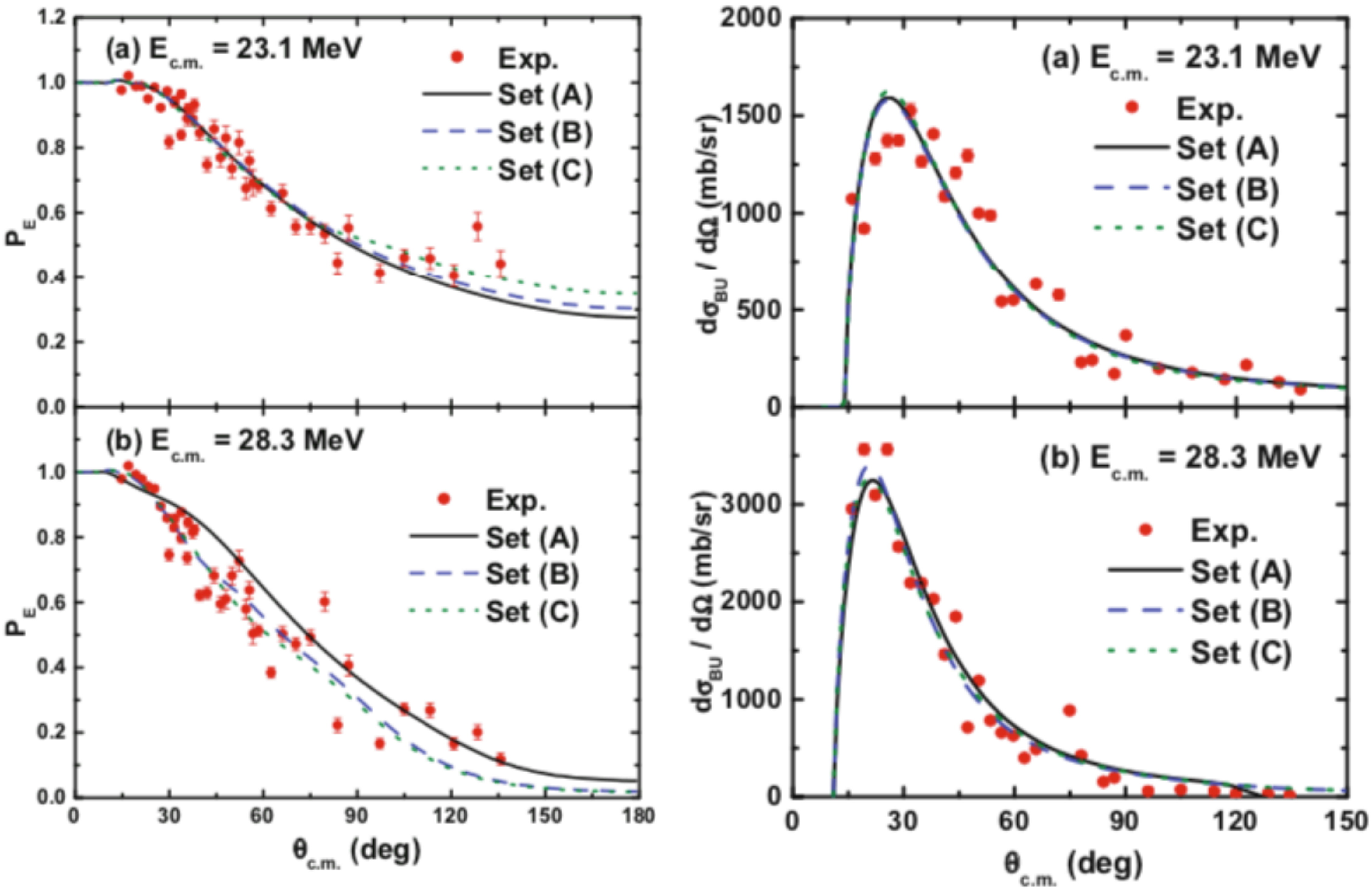
$$\xi = a_0\varepsilon/\hbar v \qquad a_0 \equiv Z_1 Z_2 e^2/2E_{\rm c.m.}$$

$$g\left(\frac{r}{a_0}-1,\xi\right) = \frac{P}{\pi}\int_{-\infty}^{\infty}\frac{f\left(\frac{r}{a_0}-1,\xi\right)}{\xi-\xi'}d\xi'$$

Real and imaginary parts are connected by dispersion relation

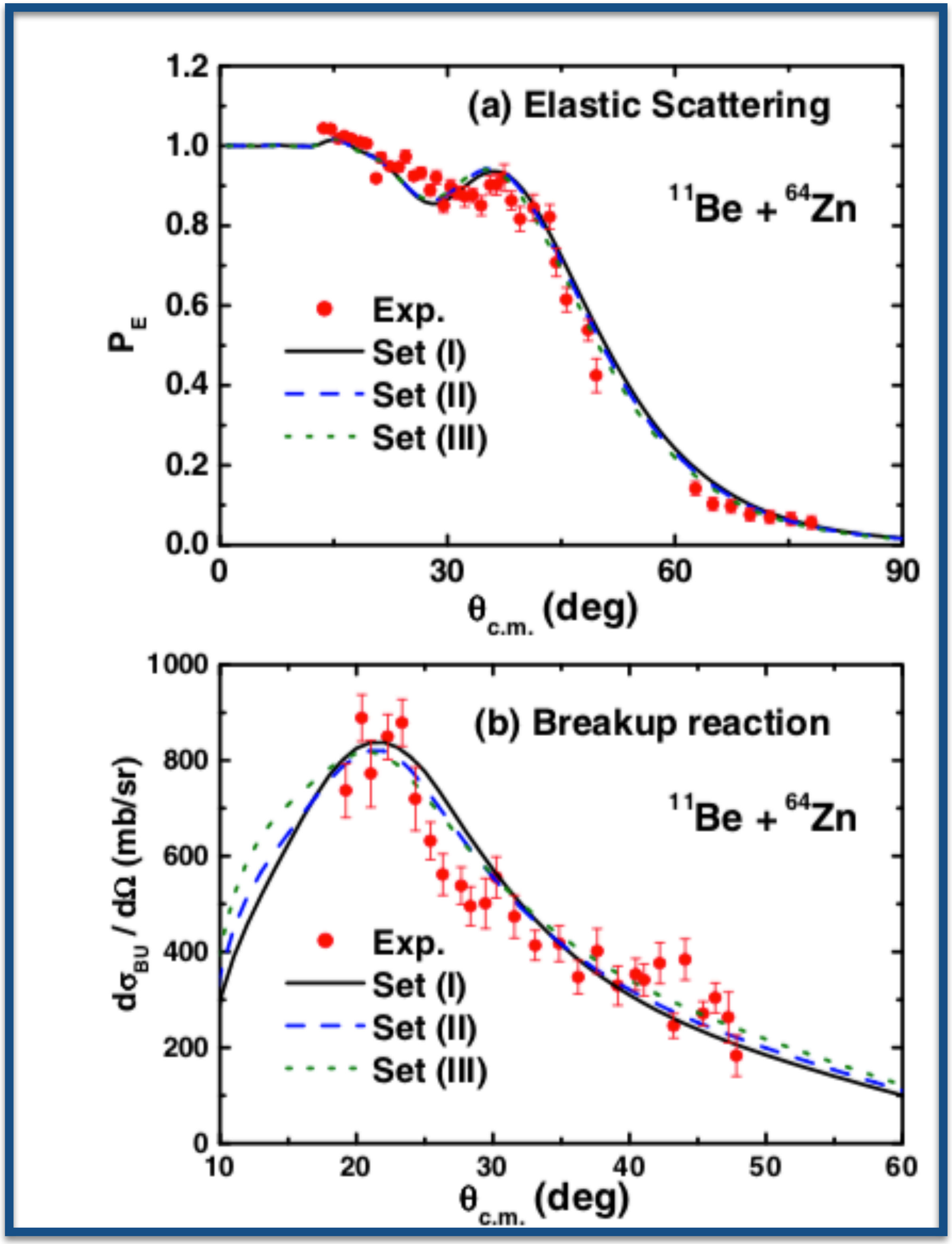
T. NAkamura et al : Phys. Rev. Lett 96 252502 (2012)
T. H. Kim, W.Y. So, K. S. Kim K. S. Choi, Kyoungsu Her and Myung-Ki Cheoun: JKPS 73 533 (2018)

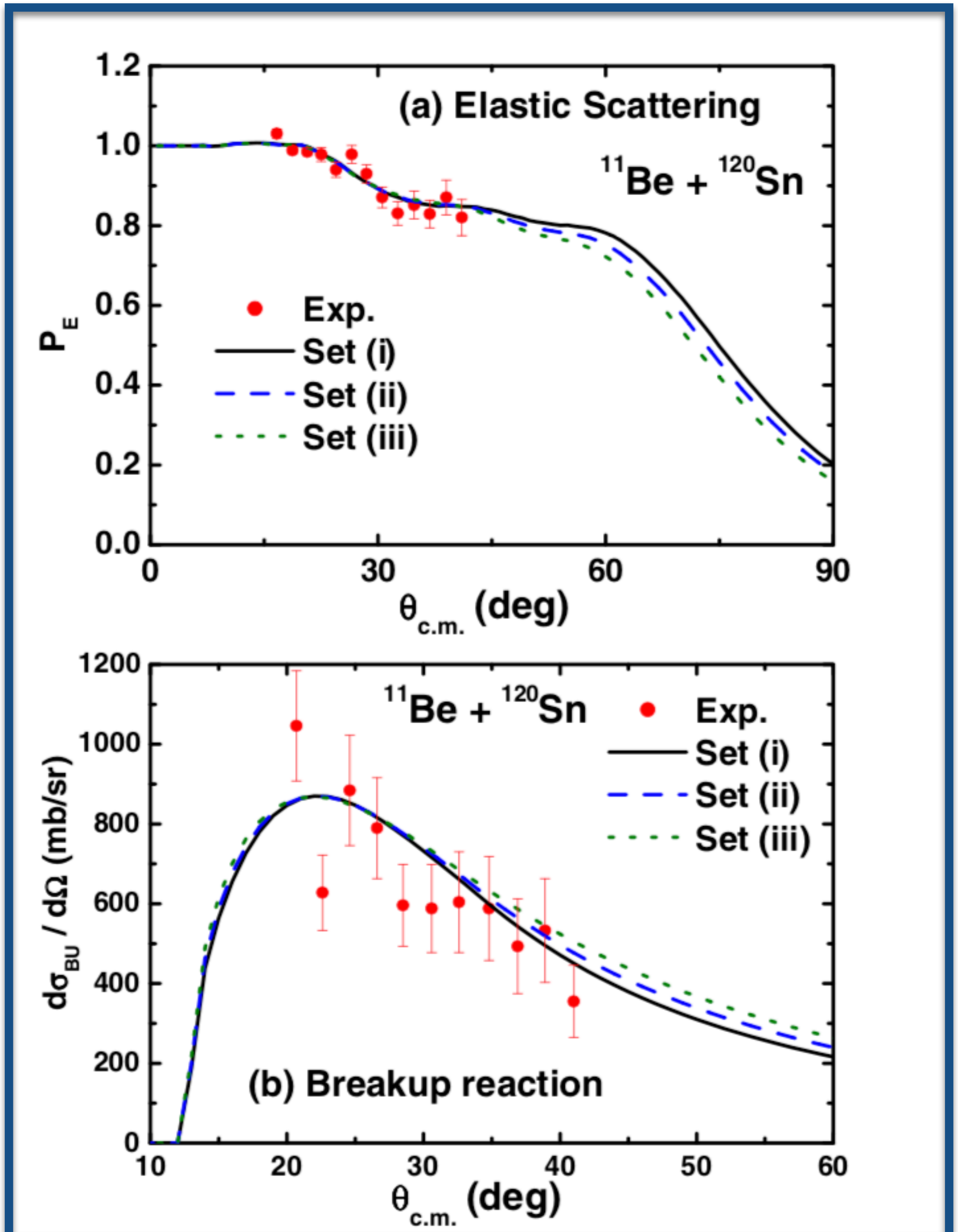
$^{11}Li + ^{208}Pb$ case (elastic and break up)



T. H. Kim, W. Y. So, K. S. Kim K. S. Choi, Kyoungsu Her and Myung-Ki Cheoun: JKPS 73 533 (2018)

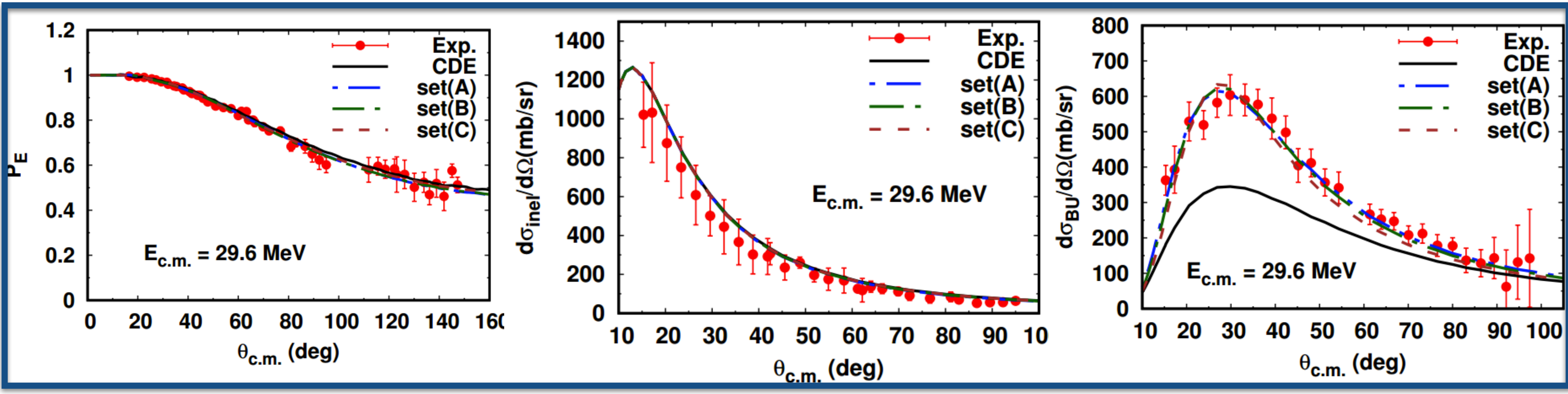
$^{11}Be + {}^{64}Zn$ and $^{11}Be + {}^{120}Sn$ case (elastic and break up)

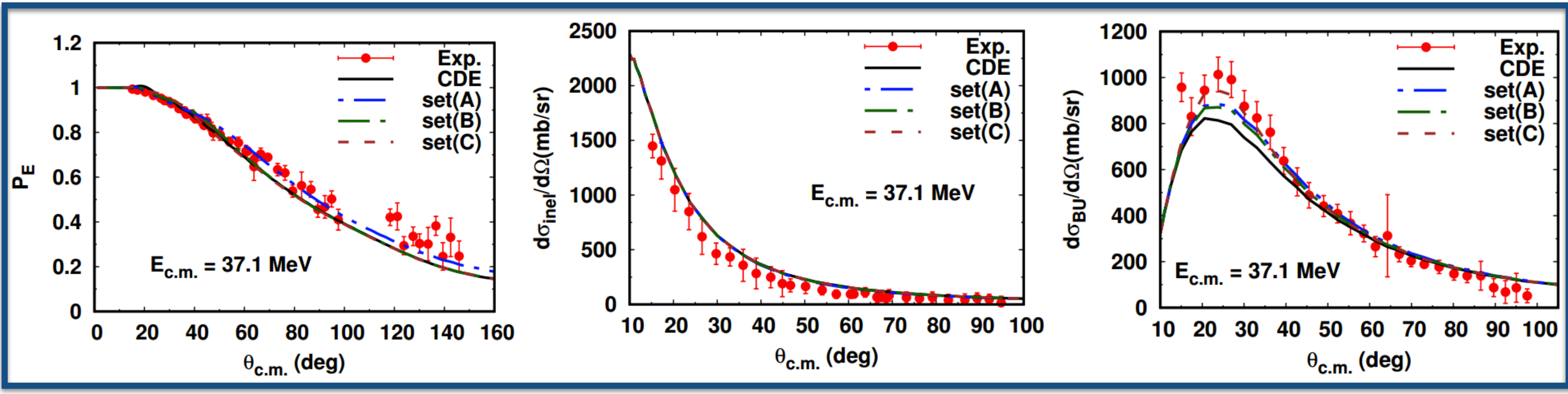




W.Y. So, K. S. Kim K. S. Choi and Myung-Ki Cheoun: Phys. Rev. C 93 054624 (2016)

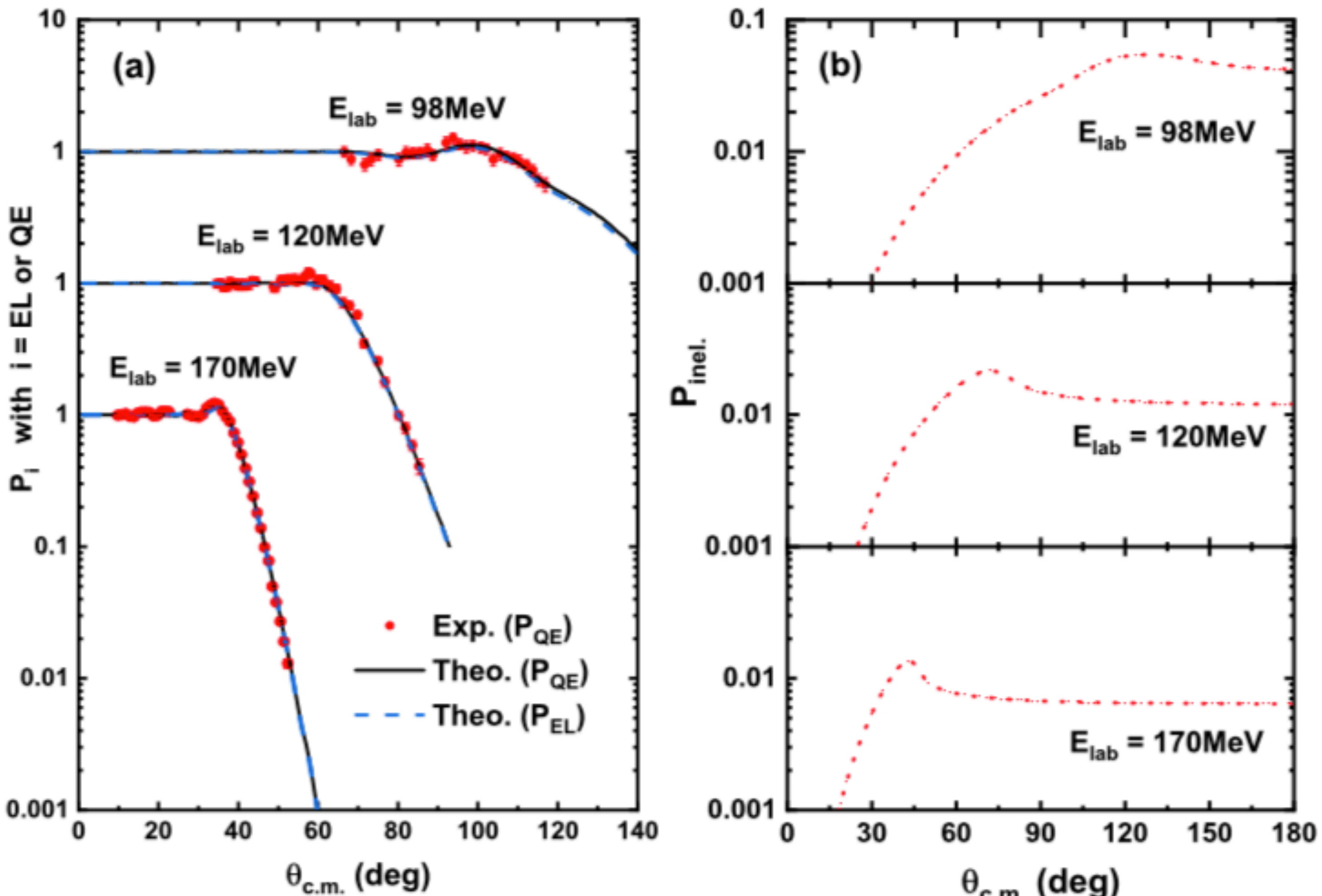
^{11}Be + ^{197}Au case (elastic, inelastic and break up)

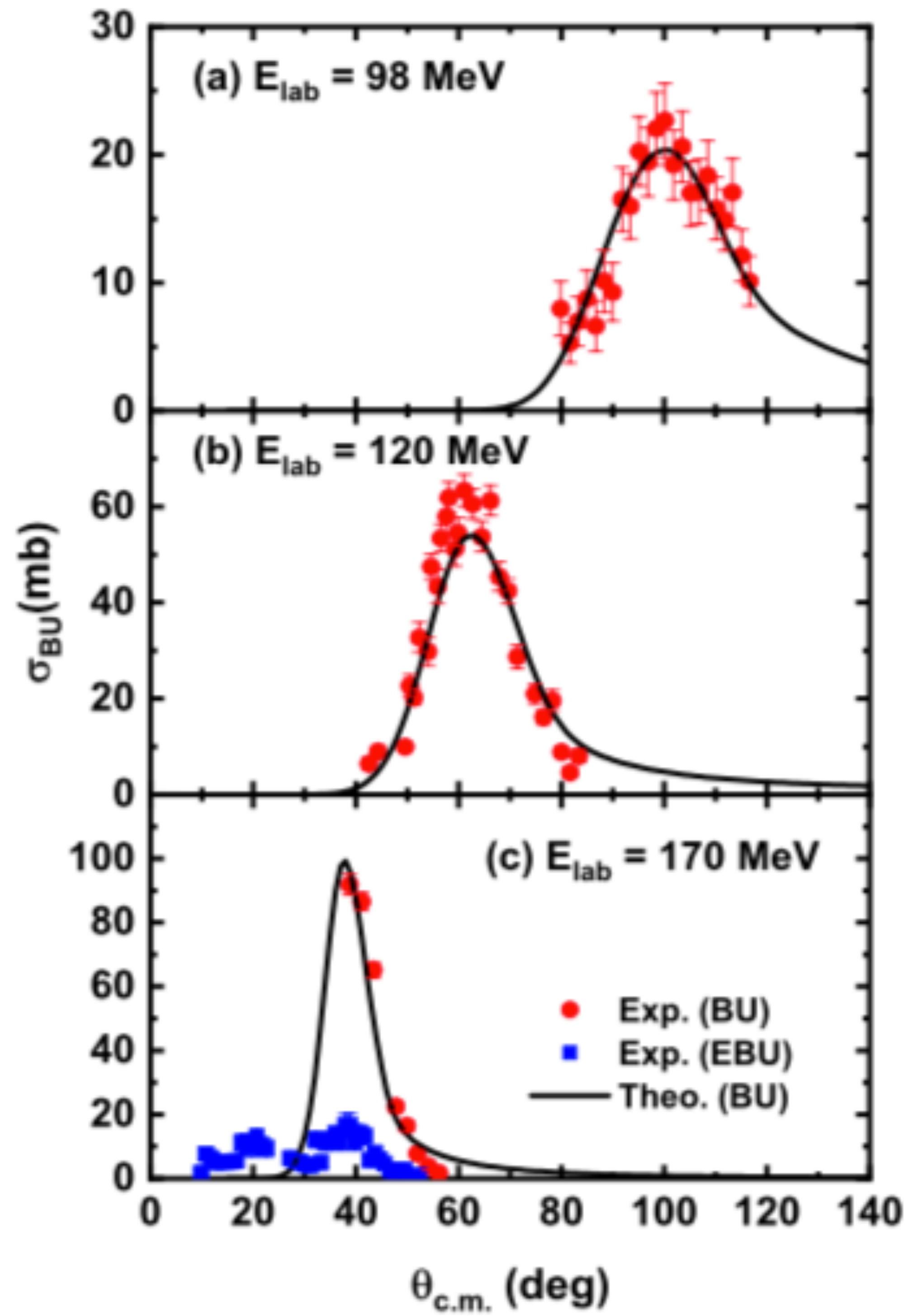


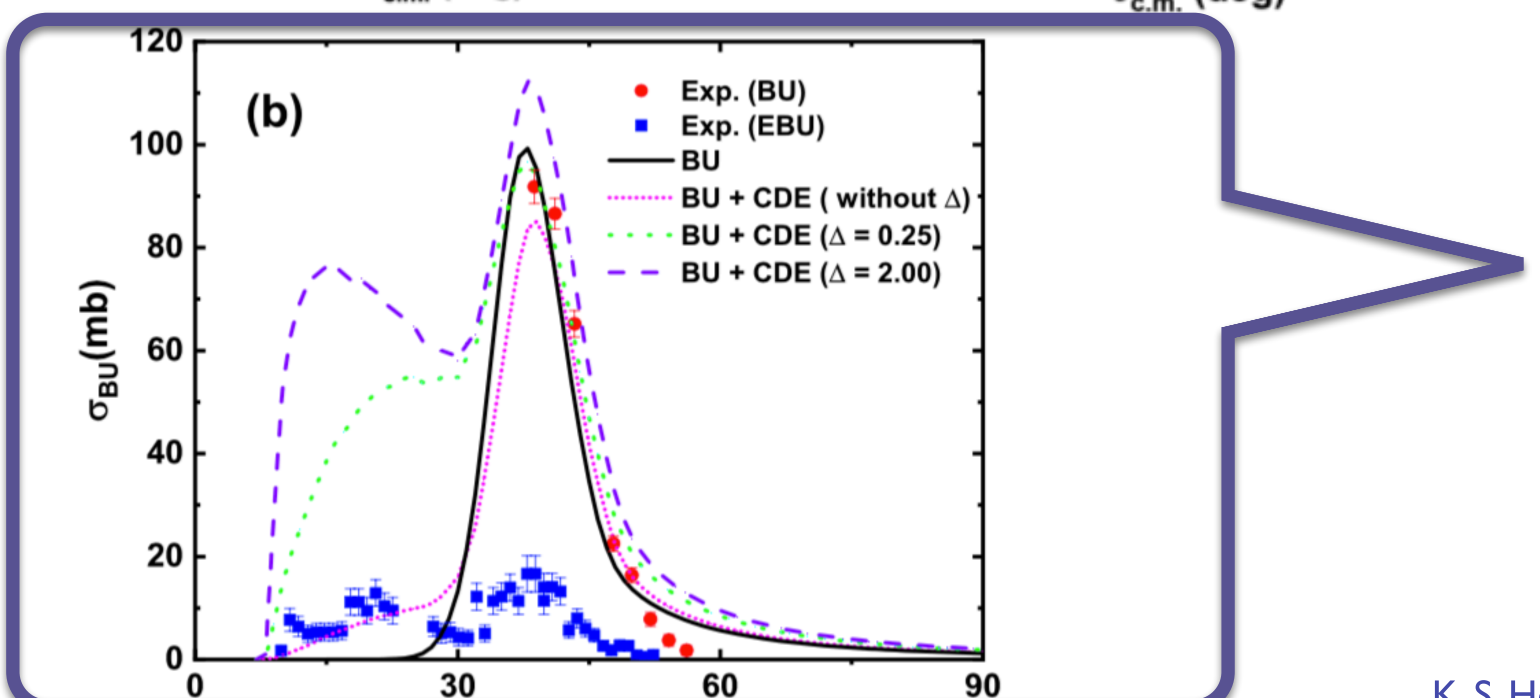


K. S. Heo, M.K.Cheoun, K.S. Choi, K.S. Kim, W.Y. So EPJ 56,42(2020)
V. Pesudo et al. Phys. Rev. Lett 118 152502 (2017)

^{17}F + ^{208}Pb case (elastic, inelastic and break up)

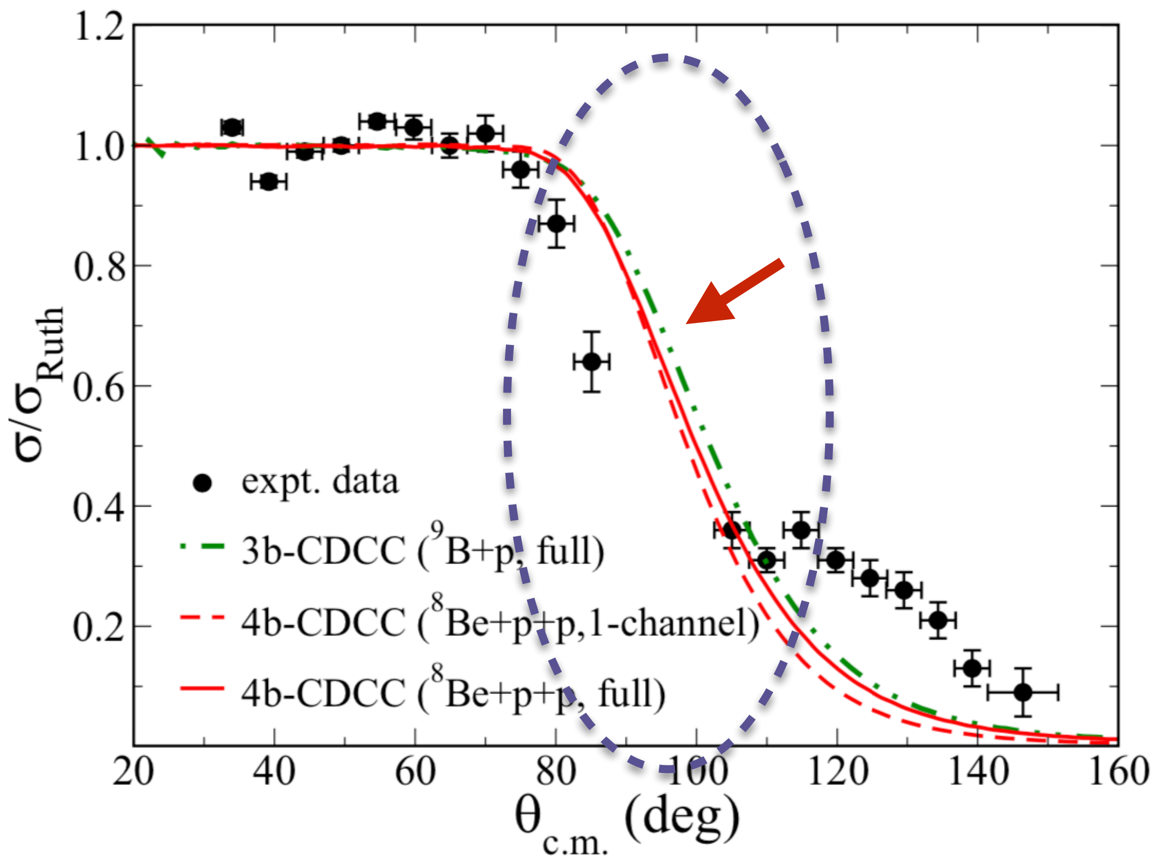




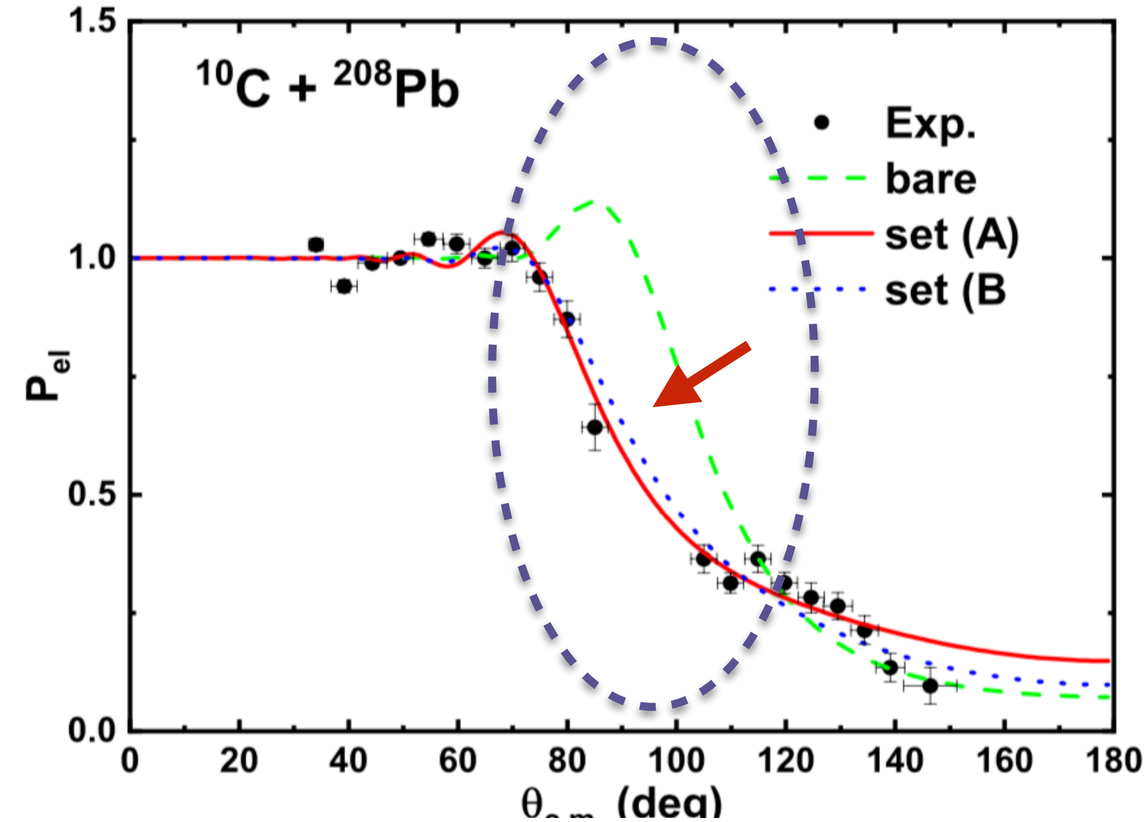


K. S. Heo, M.K.Cheoun, K.S. Choi, K.S. Kim, W.Y. So PRC 105 014601 (2022)

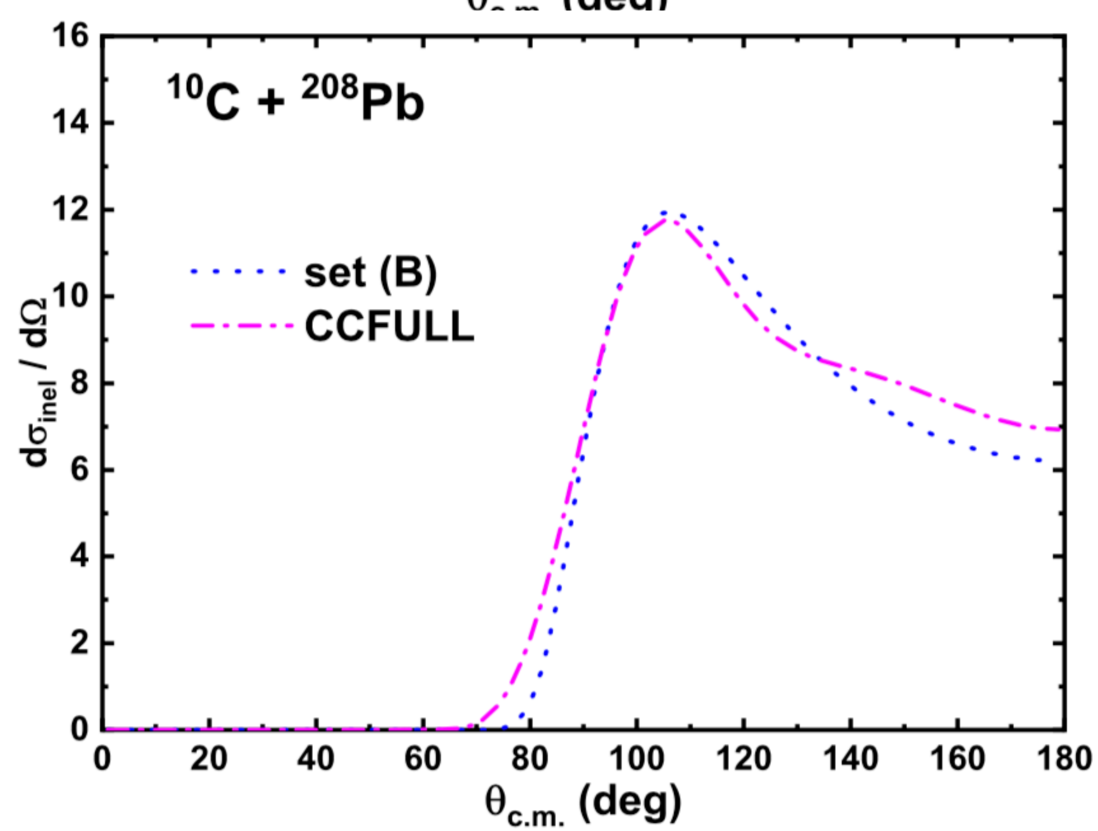
$^{10}C + {}^{208}Pb$ case (elastic and inelastic)



R. Linares et al. Phys. Rev. C 103, 044613 (2021)

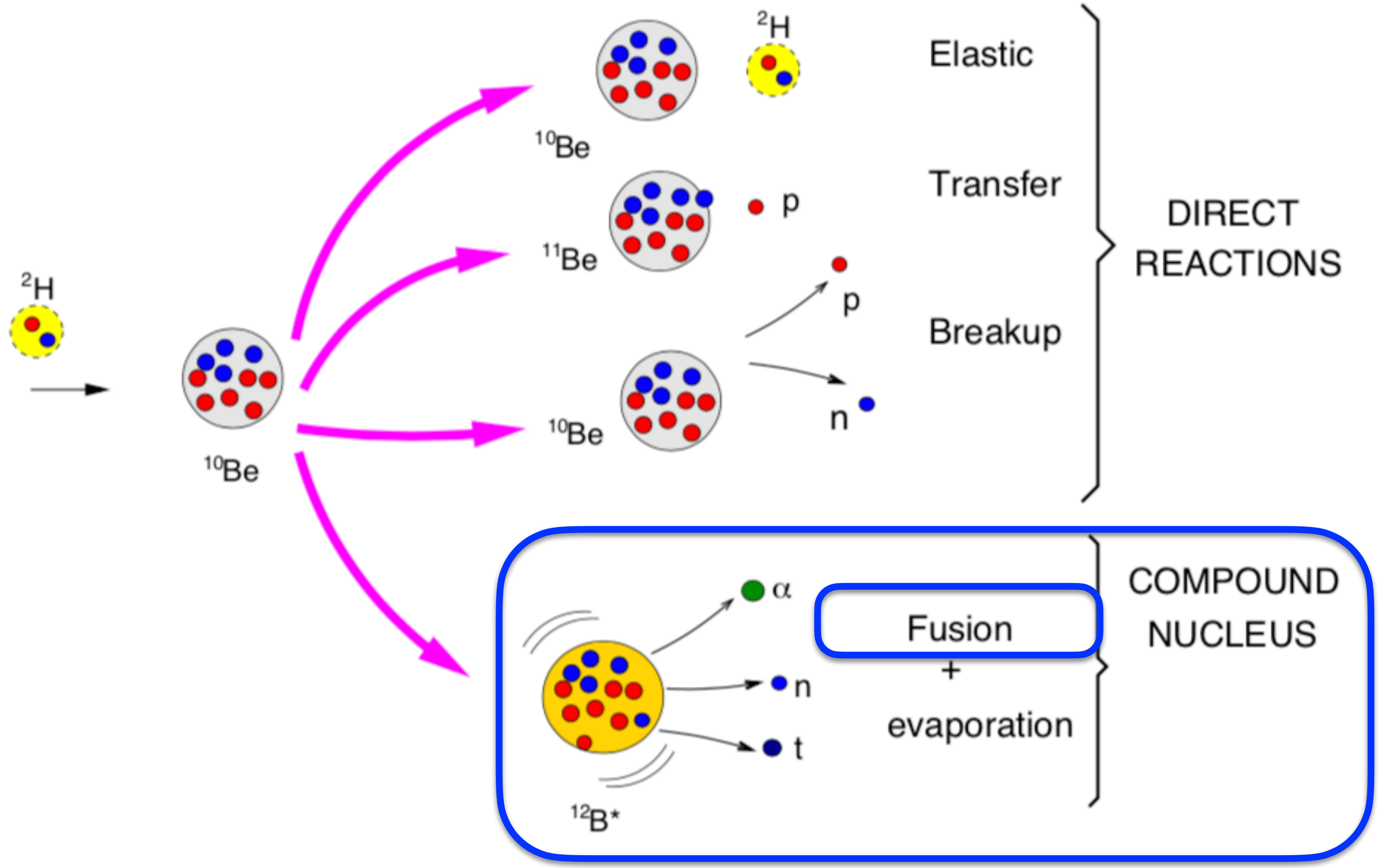


	set	σ_{DR}				σ_F	σ_R
		σ^N_{inel}	σ^C_{inel}	σ^N_{br}	σ^C_{br}		
		(mb)	(mb)	(mb)	(mb)	(mb)	(mb)
Present work	(A)	391.8				362.8	755.0
	(B)	65.2	24.4	305.2	0.4	353.4	748.6
Ref. [16]							753.0
CCFULL [43]		65.0					



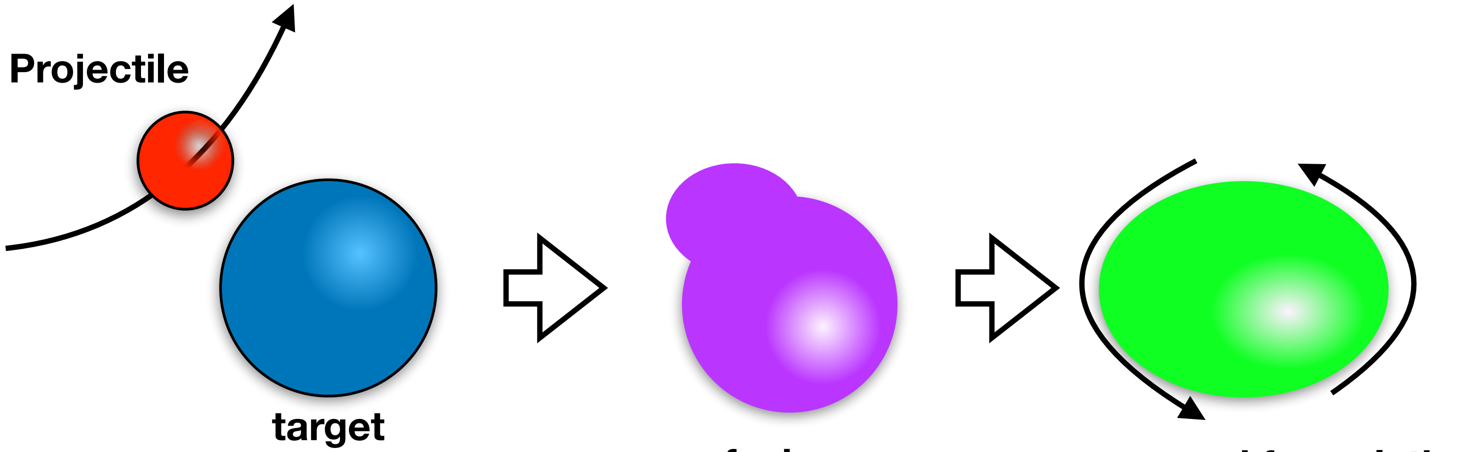
K. S. Heo, M.K.Cheoun, K.S. Choi, K.S. Kim, W.Y. So submitted.

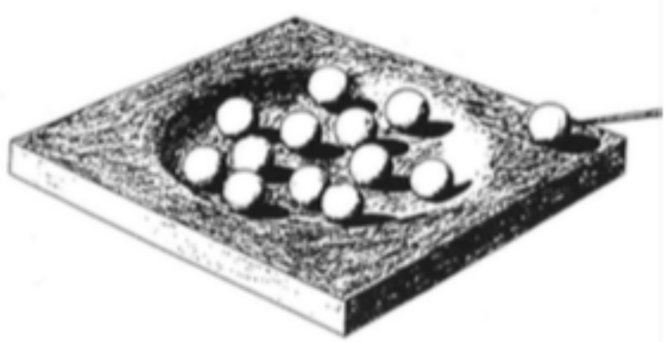
Nuclear reaction approach



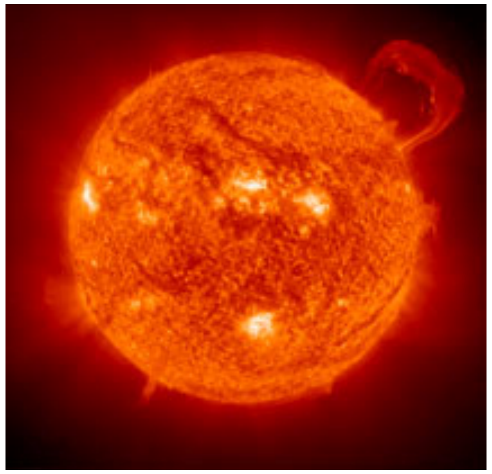
Presentation of A. M. Moro 2014

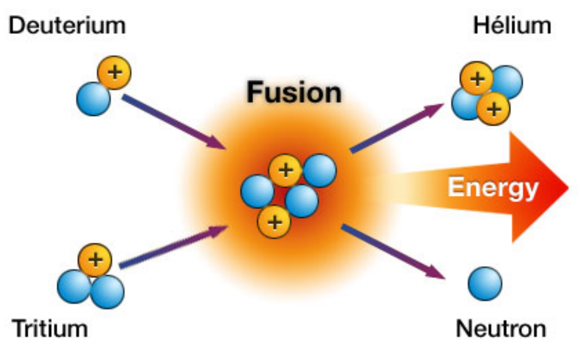
Fusion reaction: compound nucleus formation



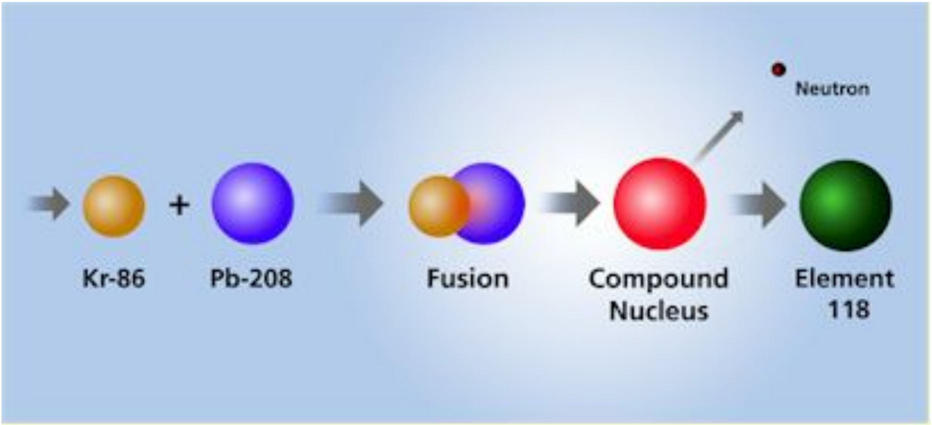


cf. Bohr '36

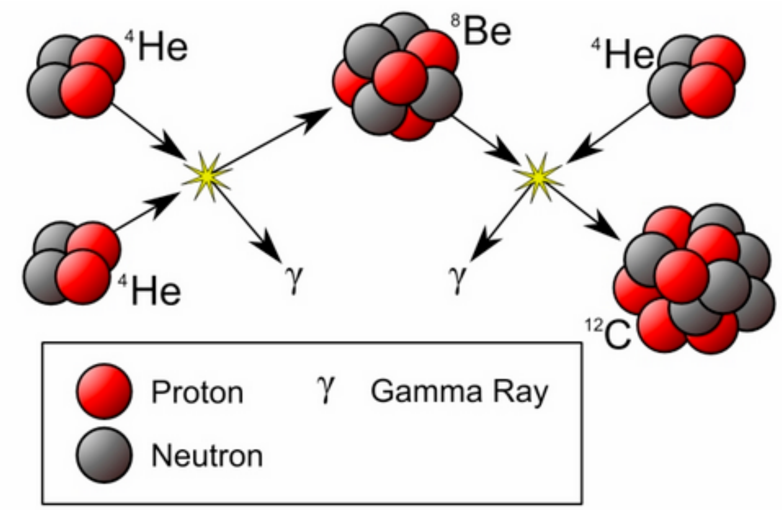




energy production in stars



super heavy elements



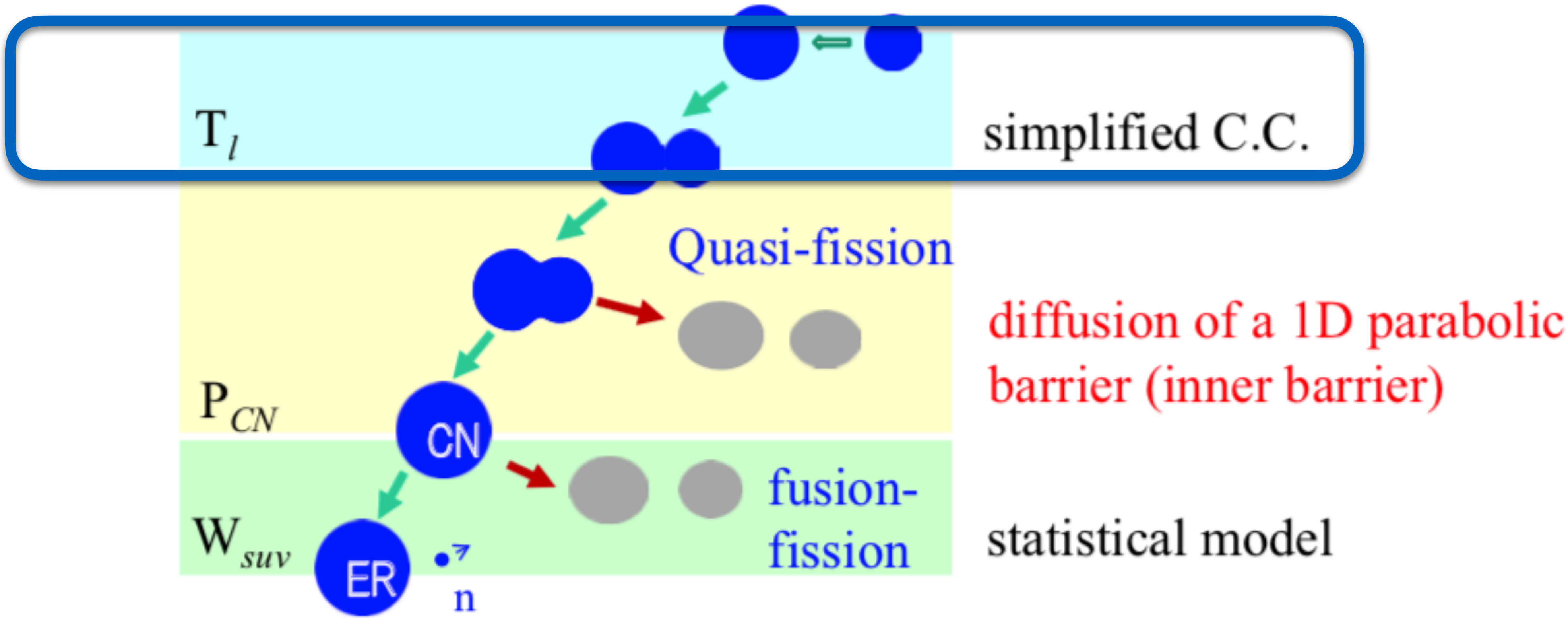
nucleosynthesis

https://fusionforenergy.europa.eu/
https://www.thphys.uni-heidelberg.de

Extension of the fusion-by-diffusion model

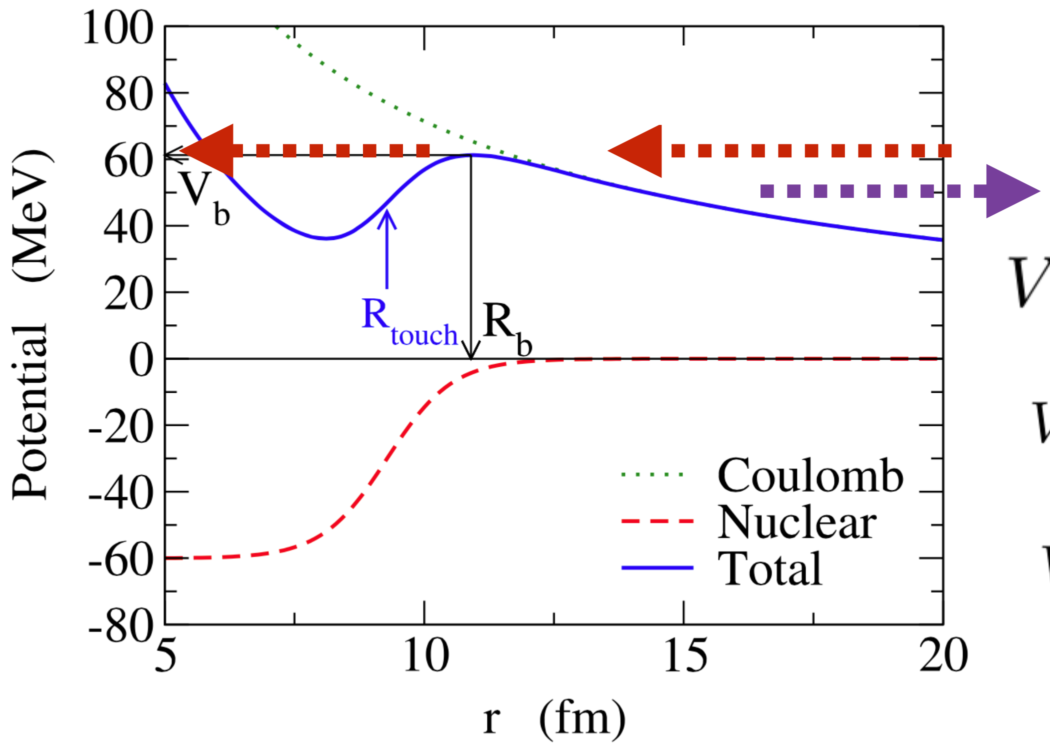
K.H., PRC98 ('18) 014607

Fusion-by-diffusion model
W.J. Swiatecki et al., Acta Phys. Pol. B34 ('03) 2049
PRC71 ('05) 014602



$$\sigma_{\rm ER}(E) = \frac{\pi}{k^2}\sum_l (2l+1)T_l(E)P_{\rm fus}(E,l)P_{\rm sur}(E^*,l).$$

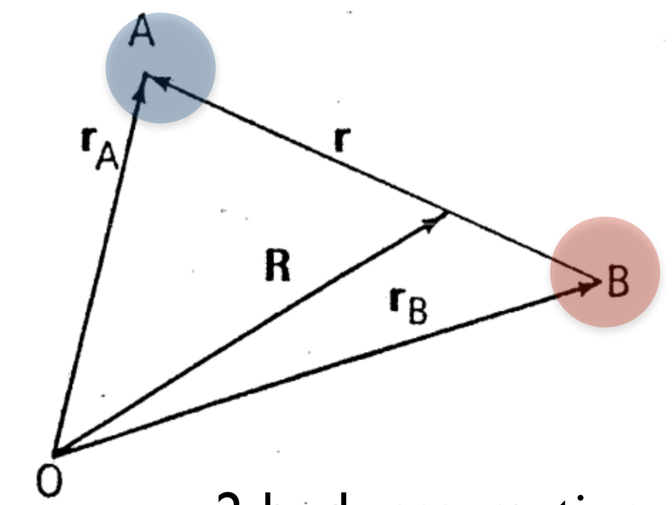
One-dimensional potential model (Barrier penetration model(BPM))



$$V(r) = V_N(r) + V_C(r)$$

$$V_C(r) = \frac{Z_P Z_T e^2}{r}$$

$$V_N(r) = -\frac{V_0}{1+\exp[(r-R_0)/a]}$$



2-body assumption

$$u_l(r) = \sqrt{\frac{k}{k_l(r)}}\,\mathcal{T}_l \exp\left(-i\int_{r_{\rm abs}}^{r} k_l(r')dr'\right),$$

: incoming wave boundary condition (IWBC)

$r \leq r_{\rm abs}$

$$k_l(r) = \sqrt{\frac{2\mu}{\hbar^2}\left(E - V(r) - \frac{l(l+1)\hbar^2}{2\mu r^2}\right)}.$$

$$k = \sqrt{2\mu E/\hbar^2}$$

$$H_l^{(-)}(kr) - S_l\, H_l^{(+)}(kr), \qquad r \to \infty$$

$H_l^{(+)}(kr)$ and $H_l^{(-)}(kr)$ are the outgoing and the incoming Coulomb waves

$$P_l(E) = 1 - |S_l|^2 = |\mathcal{T}_l|^2$$

$$\sigma_{\rm fus}(E) = \frac{\pi}{k^2}\sum_l (2l+1)P_l(E).$$

Fusion reaction with Coupled-Channel method in BPM

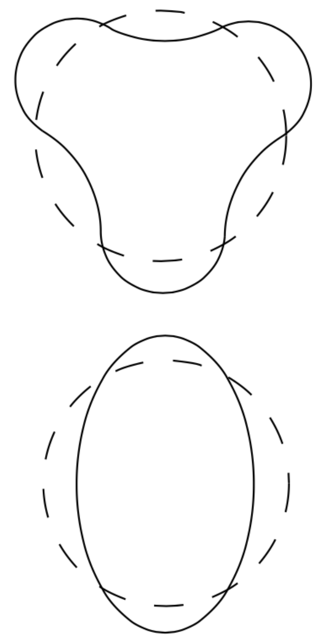
(Barrier penetration model)

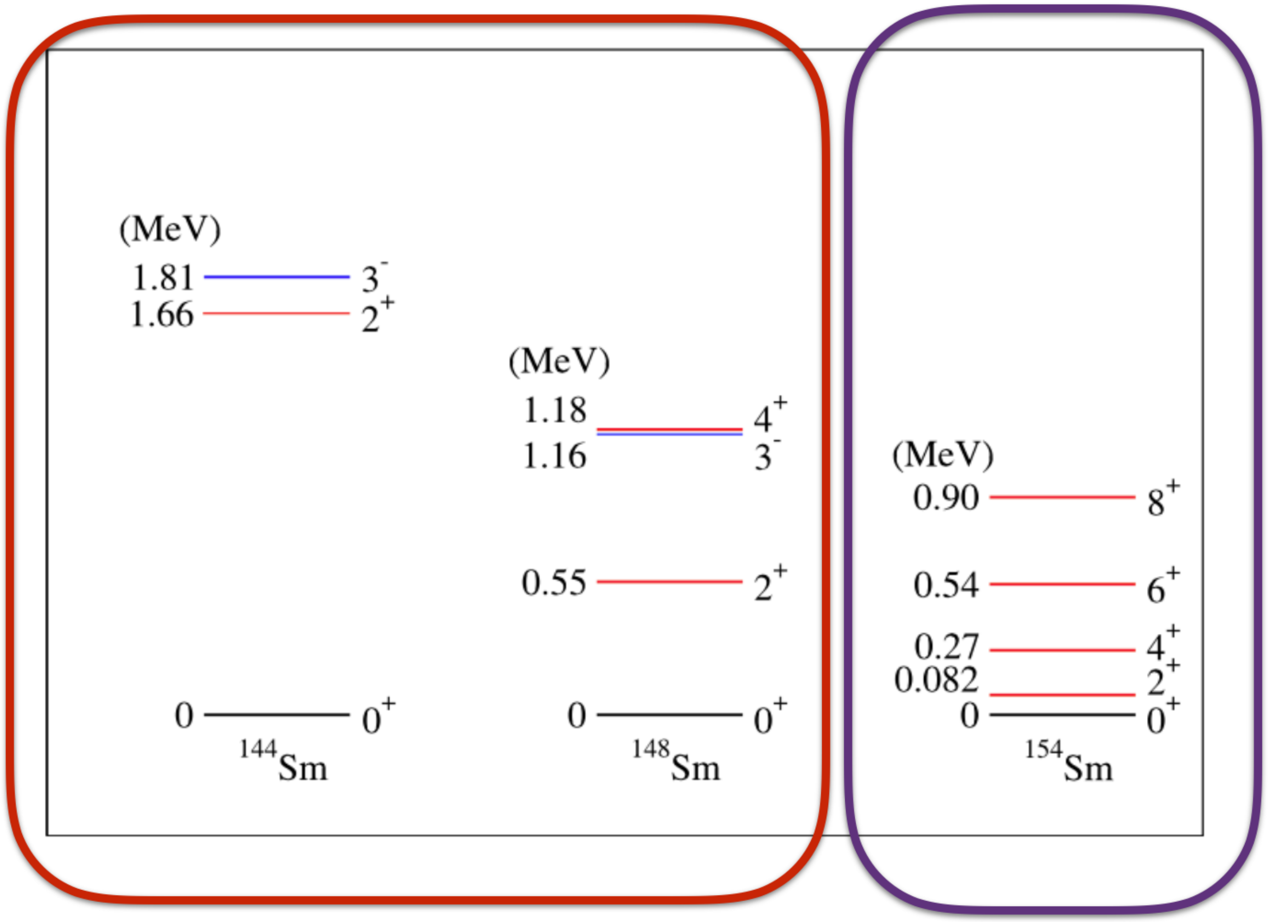
Coupled-channel equations

$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{J(J+1)\hbar^2}{2\mu r^2}+V_N^{(0)}(r)+\frac{Z_P Z_T e^2}{r}+\epsilon_n-E\right]\psi_n(r)+\sum_m V_{nm}(r)\psi_m(r)=0,$$

Coupled with excited states

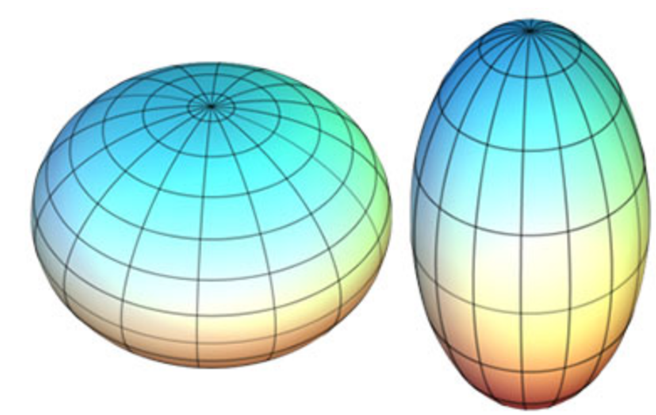
$$(2^+\otimes 3^-)^{J^-}$$





vibrational motion

rotational motion



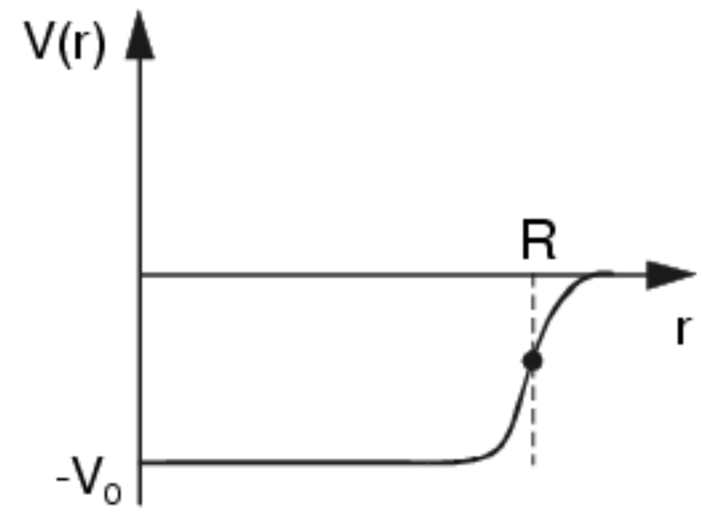
Oblate and Prolate nuclei

K.Hagino et al : CPC 123, 143 (1999)

AKyuz-Winther(AW) Potential(Global potential)

$$V_N(r) = -\frac{V_0}{1+\exp[(r-R_0)/a]},$$

(typical Wood-Saxon potential form)

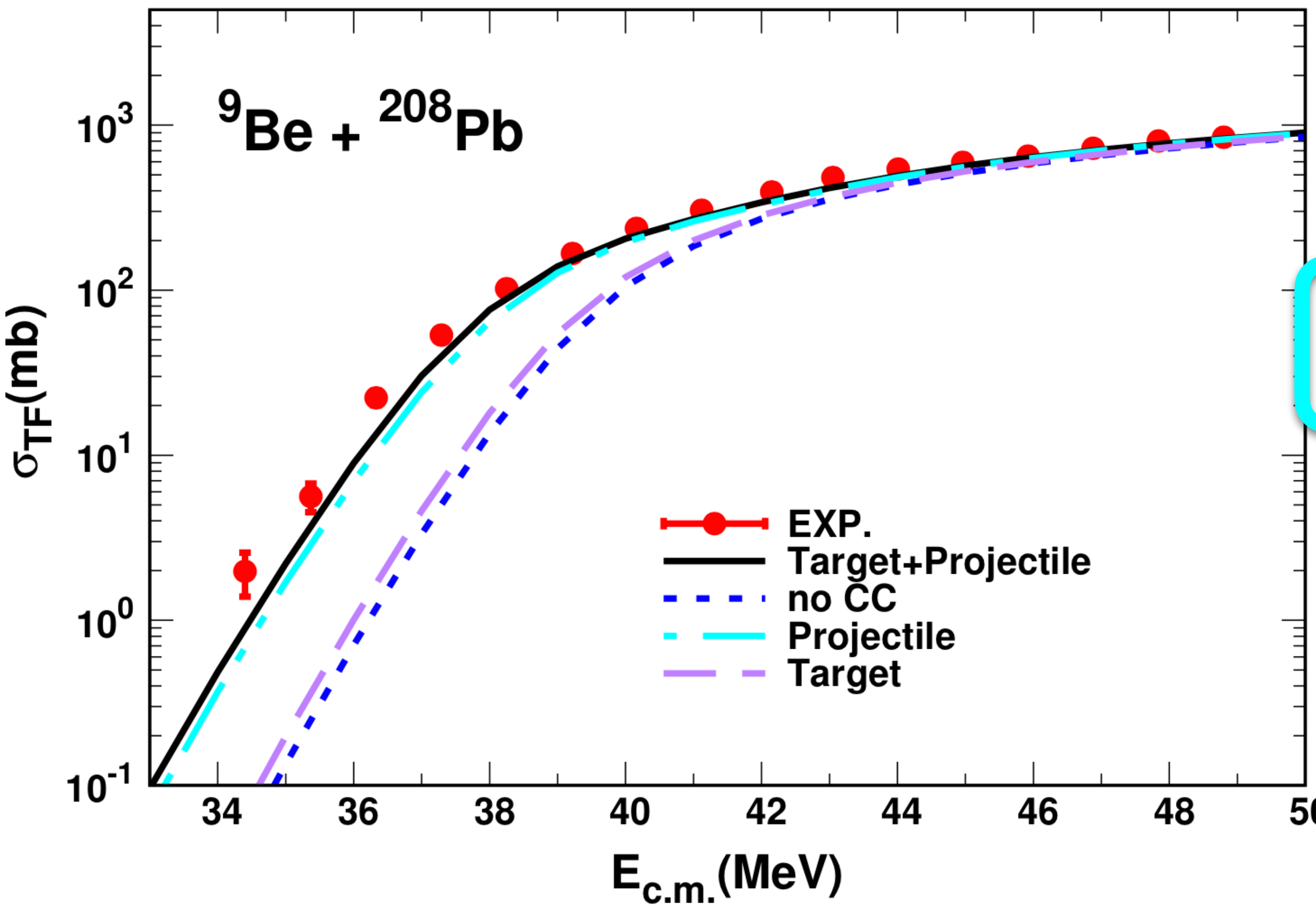


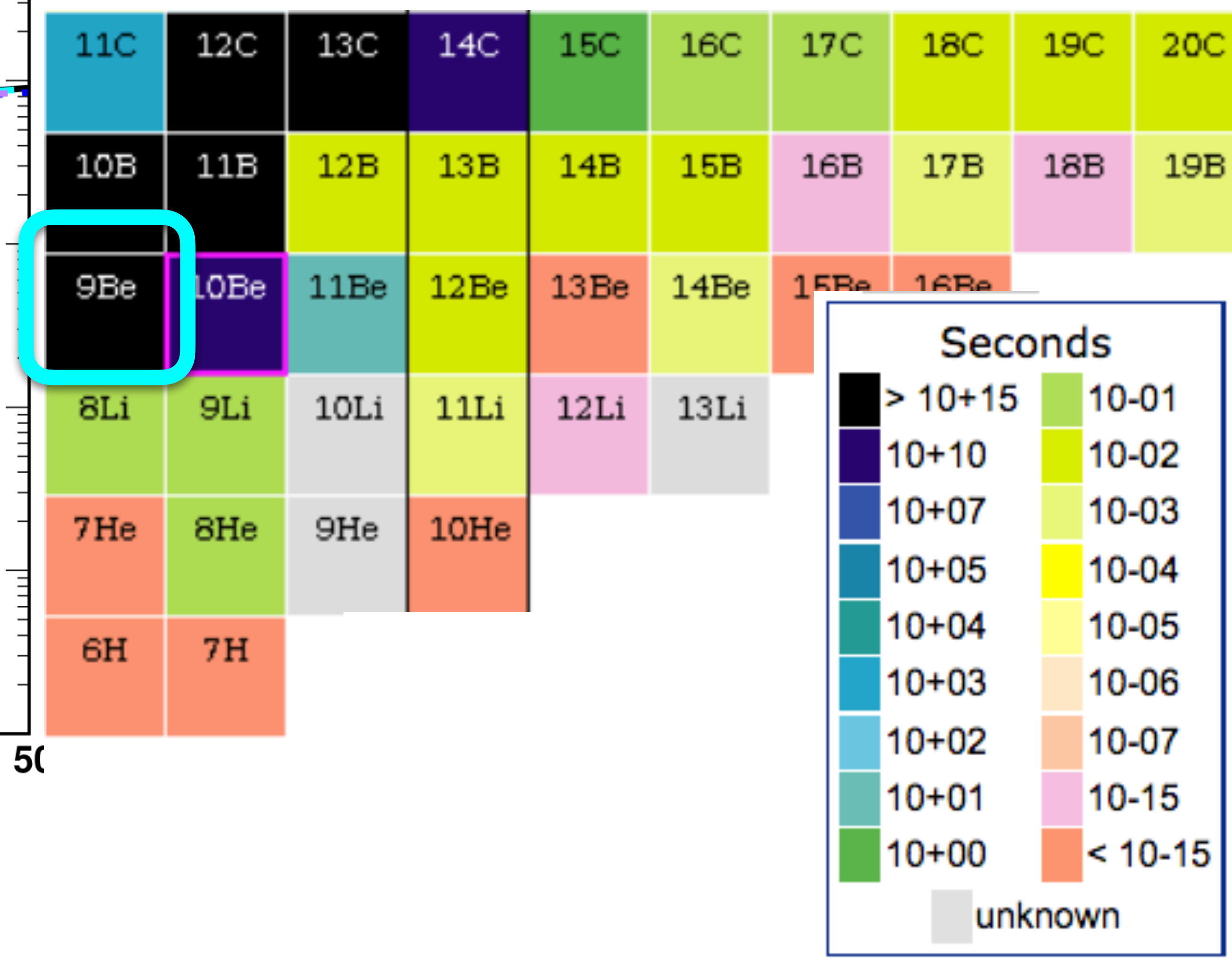
attractive nuclear force

$$\begin{aligned}
V_0 &= 16\pi\gamma\bar{R}a, \\
R_0 &= R_P + R_T, \\
R_i &= 1.20 A_i^{1/3} - 0.09 \text{ fm}, \qquad (i = P, T) \\
\bar{R} &= R_P R_T/(R_P + R_T), \\
\gamma &= 0.95\left[1 - 1.8\left(\frac{N_P - Z_P}{A_P}\right)\left(\frac{N_T - Z_T}{A_T}\right)\right] \text{ MeV fm}^{-2}, \\
1/a &= 1.17\left[1 + 0.53\left(A_P^{-1/3} + A_T^{-1/3}\right)\right] \text{ fm}^{-1},
\end{aligned}$$

R. A. Broglia and A. Winther, Heavy-Ion Reactions (Addison-Wesley, New York, 1991).

Total fusion cross section of ^{9}Be + ^{208}Pb



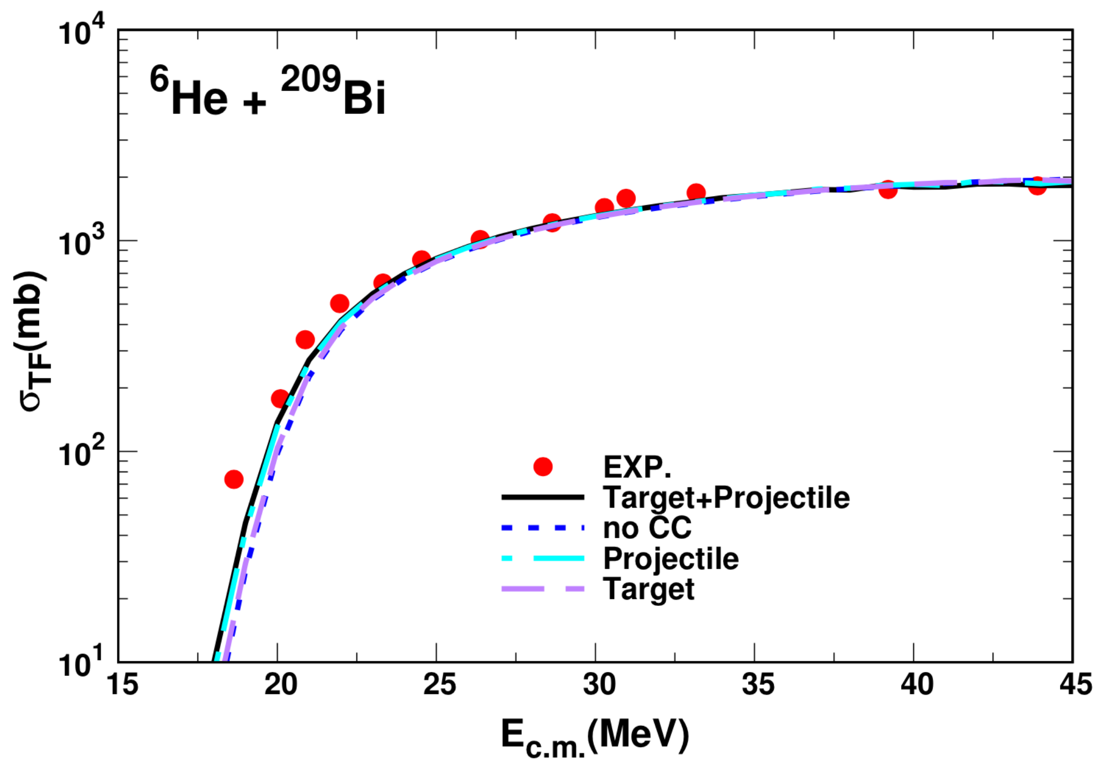


M. Dasgupta et al : Phys. Rev. C 70, 024606 (2004)

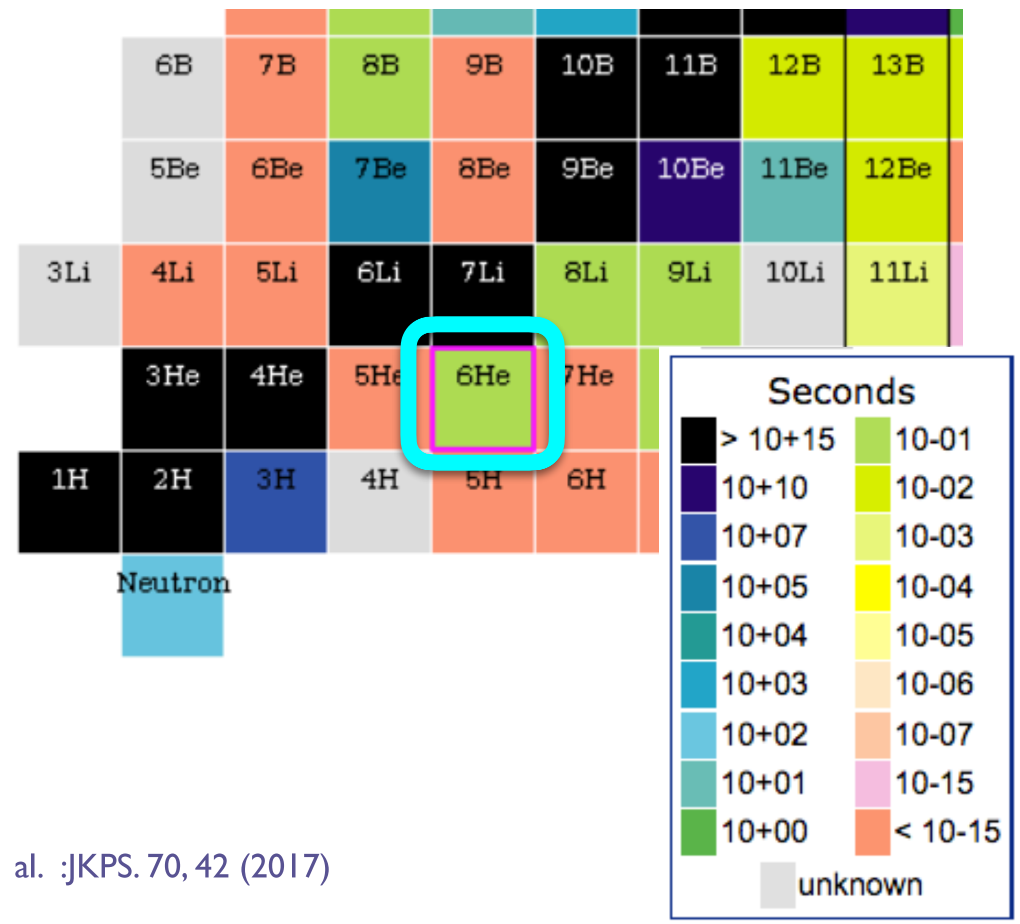
System	$V_N^{(0)}$ V_0 (MeV)	r_0 (fm)	a_0 (fm)	Target excitation ε_x(Eλ; Band) (MeV)	β_λ	Projectile excitation ε_x(Eλ; Band) (MeV)	β_2
^{9}Be+^{208}Pb	52.104	1.178	0.636	2.616(E3; Vi.)	0.111	2.43(E2; Ro.)	0.875

Ki-Seok Choi et. al. :JKPS. 70, 42 (2017)
R. H. Spear :At. Data Nucl. Data Tables 42, 55 (1989)
V.V. Parkar et al : Phys. Rev. C 82, 054601 (2010)

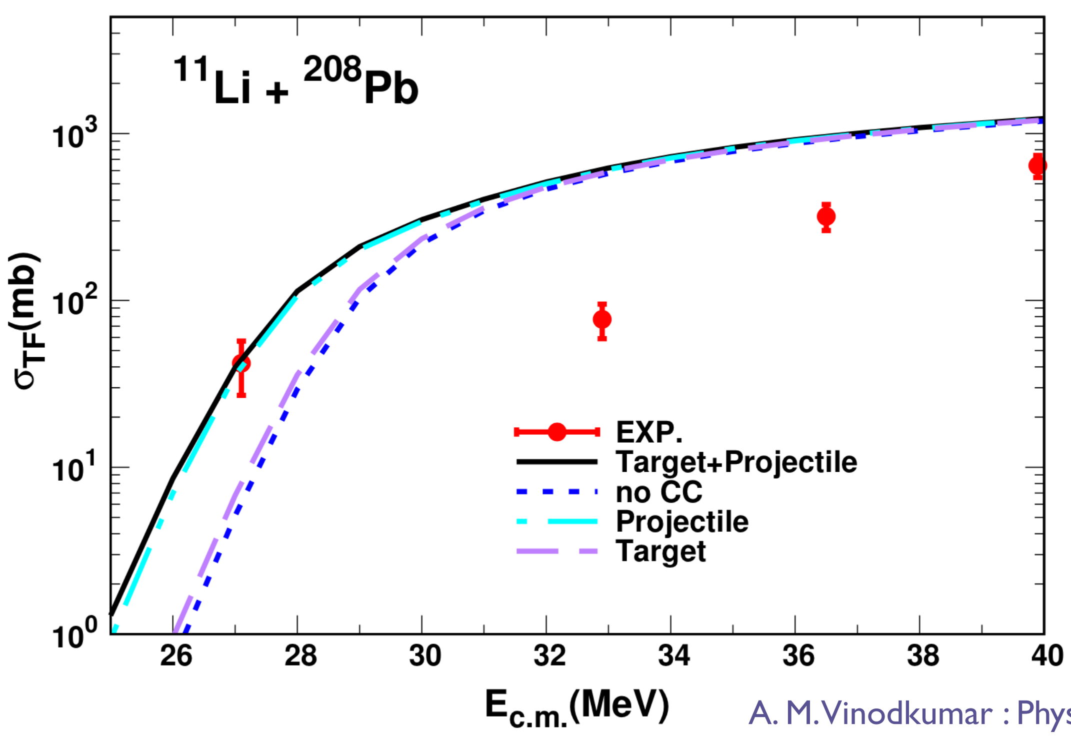
Total fusion reaction : $^{6}He + {}^{209}Bi$ and $^{11}Li + {}^{208}Pb$

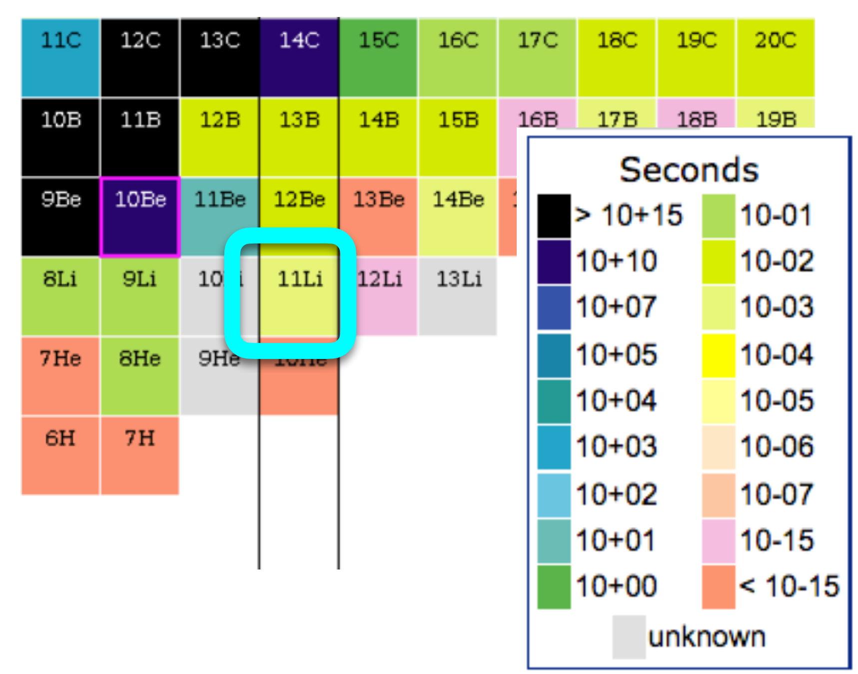


A. A. Hassan et al : Bull, Rus. Acad. Sci. Phys. 70, 1558 (2006)



Ki-Seok Choi et. al. :JKPS. 70, 42 (2017)





A. M. Vinodkumar : Phys. Rev. C 87, 044603 (2006)

NATURE 2984—22/9/2004—VBICKNELL—121305

No enhancement of fusion probability by the neutron halo of ^{6}He

R. Raabe[2,1], J. L. Sida[1*], J. L. Charvet[1], N. Alamanos[1], C. Angulo[3], J. M. Casandjian[4], S. Courtin[5], A. Drouart[1], D. J. C. Durand[1], P. Figuera[6], A. Gillibert[1], S. Heinrich[1], C. Jouanne[1], V. Lapoux[1], A. Lepine-Szily[7], A. Musumarra[6], L. Nalpas[1], D. Pierroutsakou[8], M. Romoli[8], K. Rusek[9] & M. Trotta[8]

[1]*DSM/DAPNIA, CEA Saclay, F-91191 Gif sur Yvette Cedex, France*
[2]*Instituut voor Kern- en Stralingsfysica, University of Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium*
[3]*Centre de Recherches du Cyclotron, UCL, Chemin du cyclotron 2, B-1348 Louvain-la-Neuve, Belgium*
[4]*GANIL, BP 55027, F-14076 Caen Cedex 5, France*
[5]*Institut de Recherches Subatomiques, IN2P3-CNRS, F-67037 Strasbourg, Cedex 2, France*
[6]*INFN, Laboratori Nazionali del Sud, Via Santa Sofia 44, 95123 Catania, Italy*
[7]*Instituto de Fisica, Universidade de São Paulo, CP 66318, 05389-970, São Paulo, Brazil*
[8]*University of Napoli and INFN Sezione di Napoli, I-80125, Napoli, Italy*
[9]*Department of Nuclear Reactions, The Andrzej Soltan Institute for Nuclear Studies, Hoza 69, PL-00-681 Warsaw, Poland*

* Present address: DAM/DPTA/ Service de Physique Nucléaire, CEA DIF, F-91680 Bruyères-le-Châtel, France

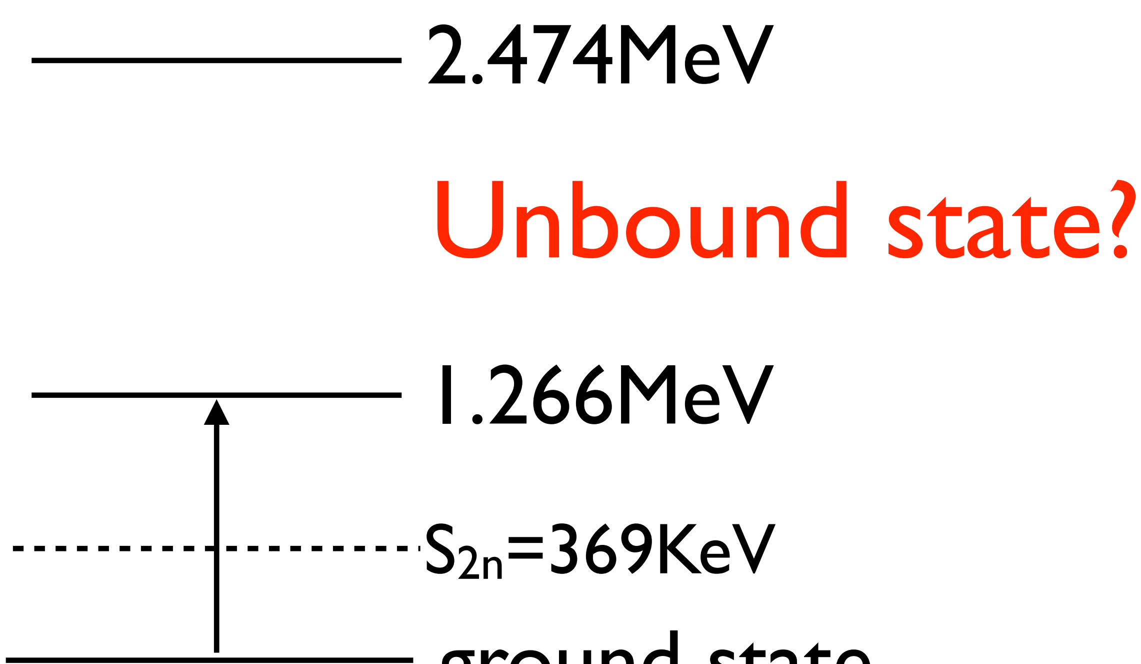
the projectile in the field of the target); however, its effect on fusion is strongly disputed. If considered as any other channel, it should enhance the sub-barrier cross-section[5,13,14]; on the other hand, the breakup process could just prevent the capture of the whole projectile, thereby inhibiting fusion[6,15–17]. Here we use 'fusion' in the sense of complete fusion[14,17–19] (the whole projectile and target fuse together; evaporation of fragments may follow) as distinct from incomplete fusion (only a fragment of the projectile is transferred to the target; in light nuclei this is not distinguishable from a direct transfer).

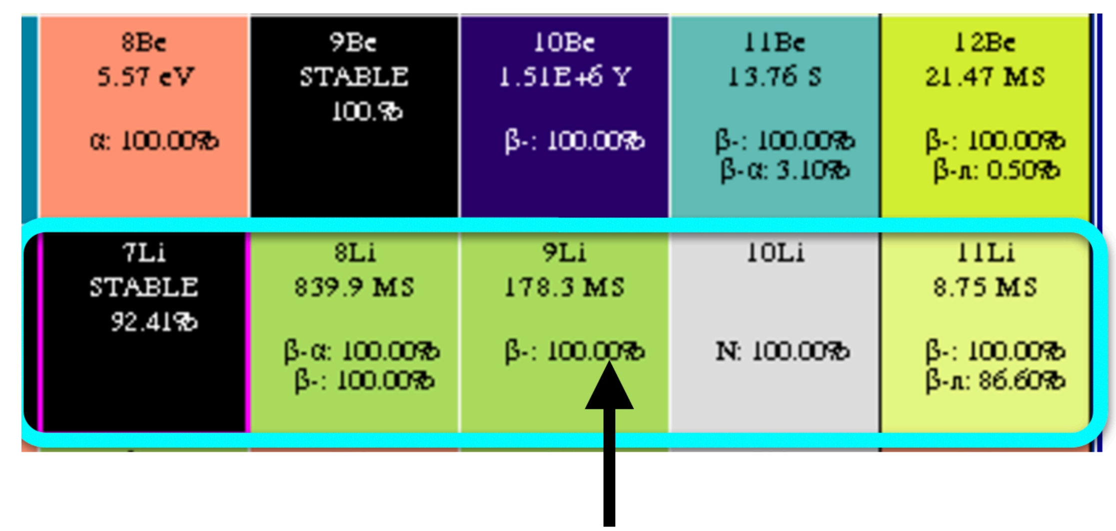
The halo-nucleus ^{6}He presents the peculiar properties mentioned above, and has a relatively small breakup threshold (the two-neutron separation energy is $S_{2n} = 0.973$ MeV). It is a favourable candidate for experimental studies, because it is available as a beam of accelerated ions at various facilities. However, the weak intensity of these radioactive beams sets a limit on the accuracy attainable in cross-section measurements, and the results obtained so far have not been conclusive. An enhancement was reported in the ^{6}He + ^{209}Bi fusion cross-section at energies below the potential barrier[20], and then related to the presence of strong breakup and/or transfer channels[21]. Our previous measurement with ^{6}He on ^{238}U target nuclei[22] also showed a large probability for the sum of all processes leading to the fission of the residual nucleus. The measurement reported here, however, allows the identification of the contribution of incomplete fusion (transfer) channels, which are found to be the most important in the total reaction cross-section.

Raabe et. al. :Nature. 431, 823 (2004)

^{11}Li level information

E_{level} (keV)	XREF	Jπ	$T_{1/2}$
0	ABCDE GH	3/2-	8.75 ms *14* % β^- = 100
1266 *42*	A EF HI		0.53 MeV *15*
2474 *54*	A C EF HI		1.26 MeV *30*
3.70E3 *13*	F		< 200 keV
4.86E3 *60*	A C H		< 100 keV
6.23E3 *60*	A C H		< 100 keV
11.3E3	A		





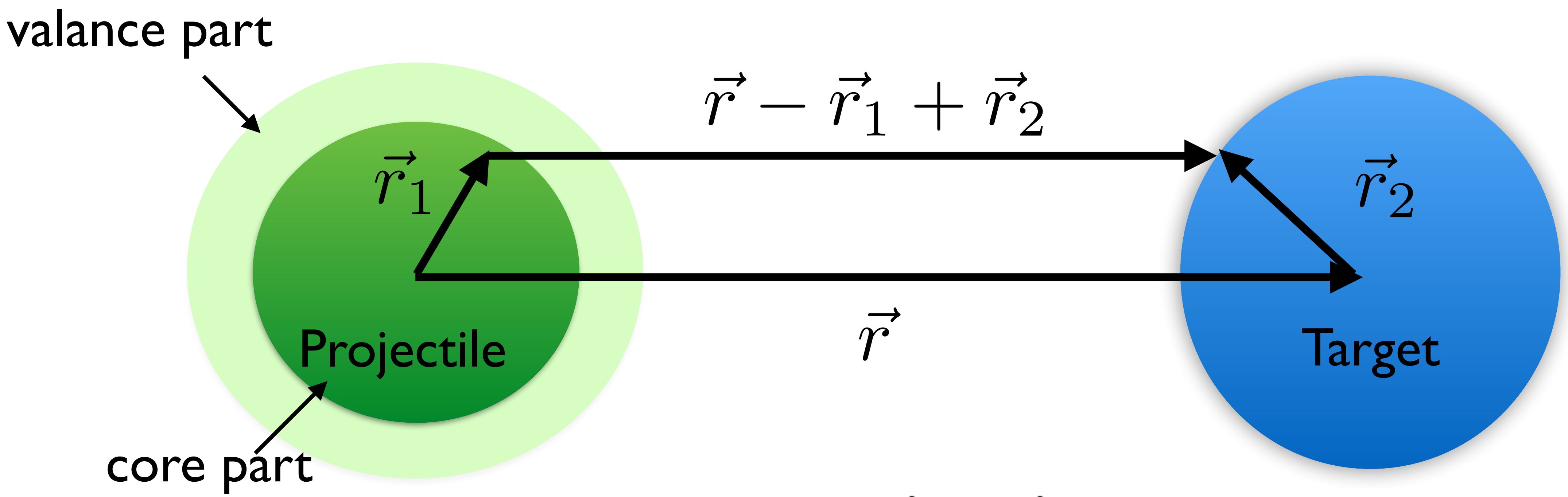
transfer channel will be opened

^{9}Li is also unstable!

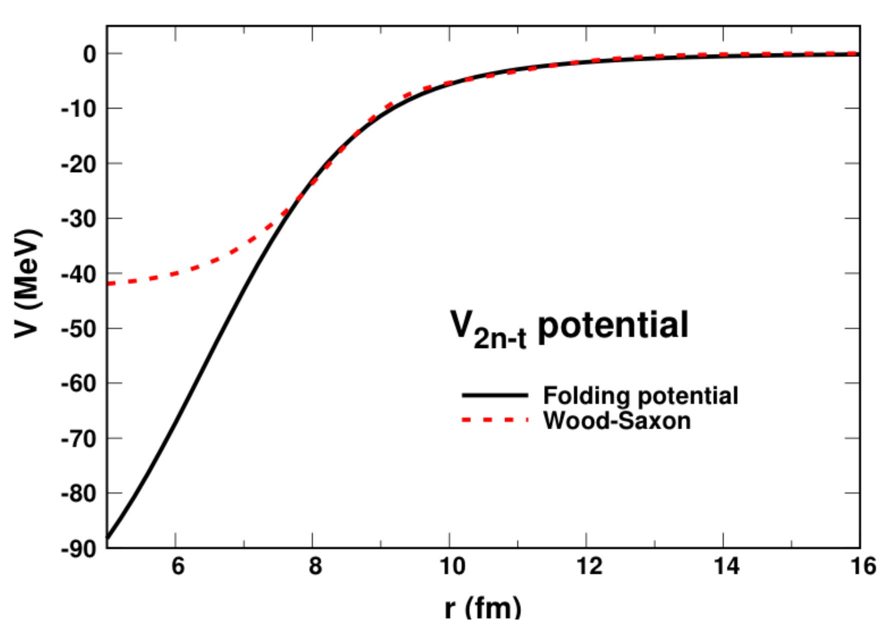
dominant channel : ^{9}Li + ^{210}Pb

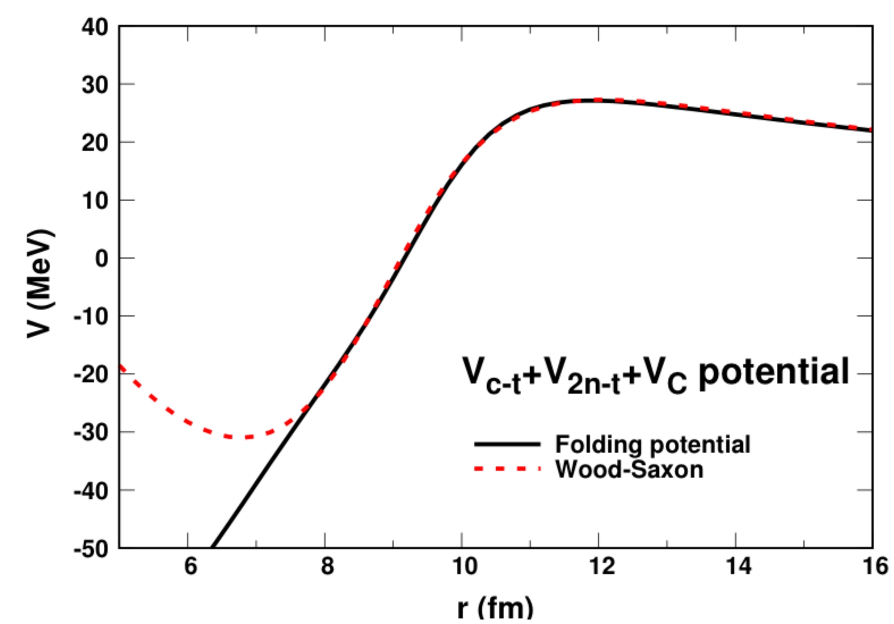
J. Kelley et al : Nucl. Phys. A 880, 88 (2012)

Folding Potential for ^{11}Li + ^{208}Pb



$$
\begin{aligned}
V(r) &= \int d\mathbf{r}_1 \int d\mathbf{r}_2 \left(\rho_c(r) + \rho_{2n}(r)\right) \rho_t(r_2) V_{NN}(r_{12}) \\
&= V_{ct}(r) + V_{2nt}(r), \sim V_{WS}^{ct}(\vec{R}) + V_{WS}^{2nt}(\vec{R})
\end{aligned}
$$





Modification of coupled channel equation

$$\begin{pmatrix} K+V_1-E & \underline{F_{1\to 2}(r)} & 0 \\ \underline{F_{1\to 2}(r)} & K+V_2-(E-Q_2) & \underline{F_{2\to 3}(r)} \\ 0 & \underline{F_{2\to 3}(r)} & K+V_2-(E-Q_2-Q_{23}) \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = 0$$

Ch1 : $^{11}\mathrm{Li} + ^{208}\mathrm{Pb}$

2-neutron transfer

Ch2 : $^{9}\mathrm{Li} + ^{210}\mathrm{Pb}$

2-neutron transfer

Ch3 : $^{7}\mathrm{Li} + ^{212}\mathrm{Pb}$

4-neutron transfer ignorance

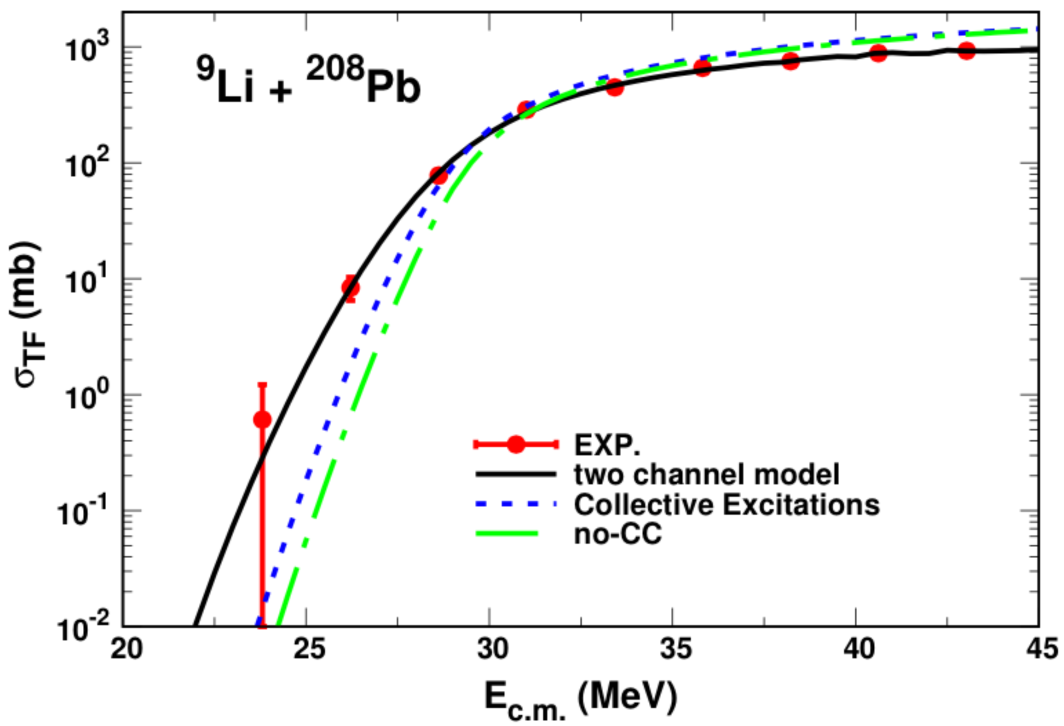
$F_{1\to 2}(r)$ 4 fitting parameters

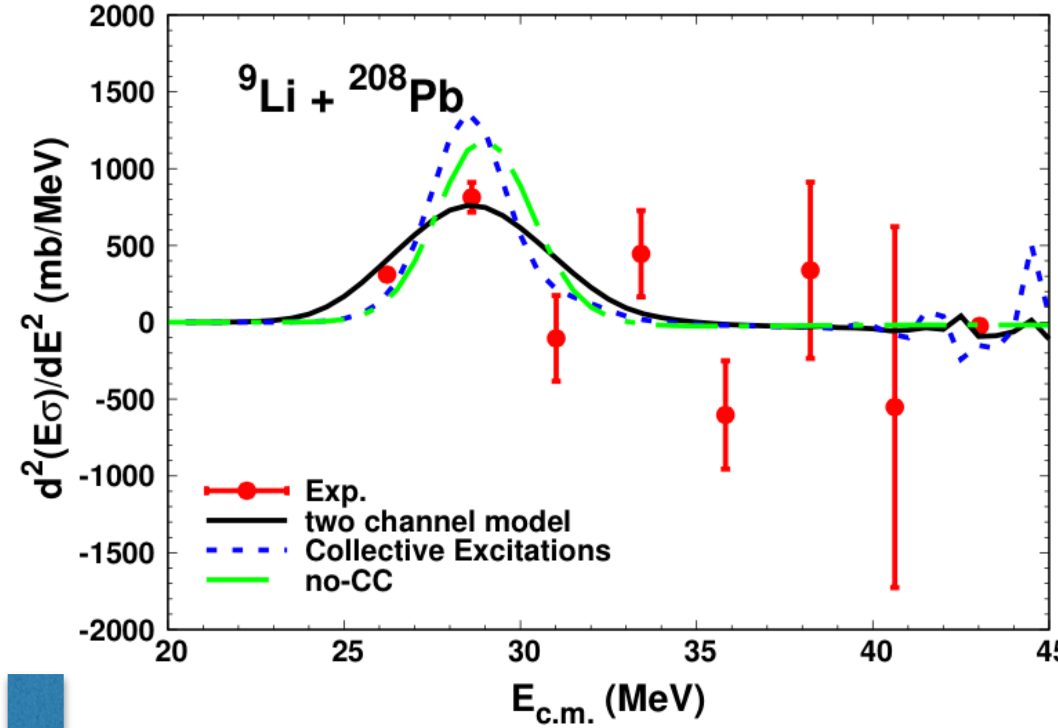
Channel	Q (MeV)	F_t (MeV fm)	r_{coup} (fm)	a_{coup} (fm)
$1 \to 2$	+8.346	40.227	1.666	0.857
$2 \to 3$	−3.204	51.367	1.357	0.264

$^{11}\mathrm{Li} + ^{208}\mathrm{Pb}$			$^{9}\mathrm{Li} + ^{210}\mathrm{Pb}$			$^{7}\mathrm{Li} + ^{212}\mathrm{Pb}$		
Ch 1			Ch 2			Ch 3		
V_0	r_0	a_0	V_{02}	r_{02}	a_{02}	V_{03}	r_{03}	a_{03}
90.309	1.090	0.852	47.304	1.178	0.636	47.298	1.177	0.626

folding potential — AW potential — AW potential

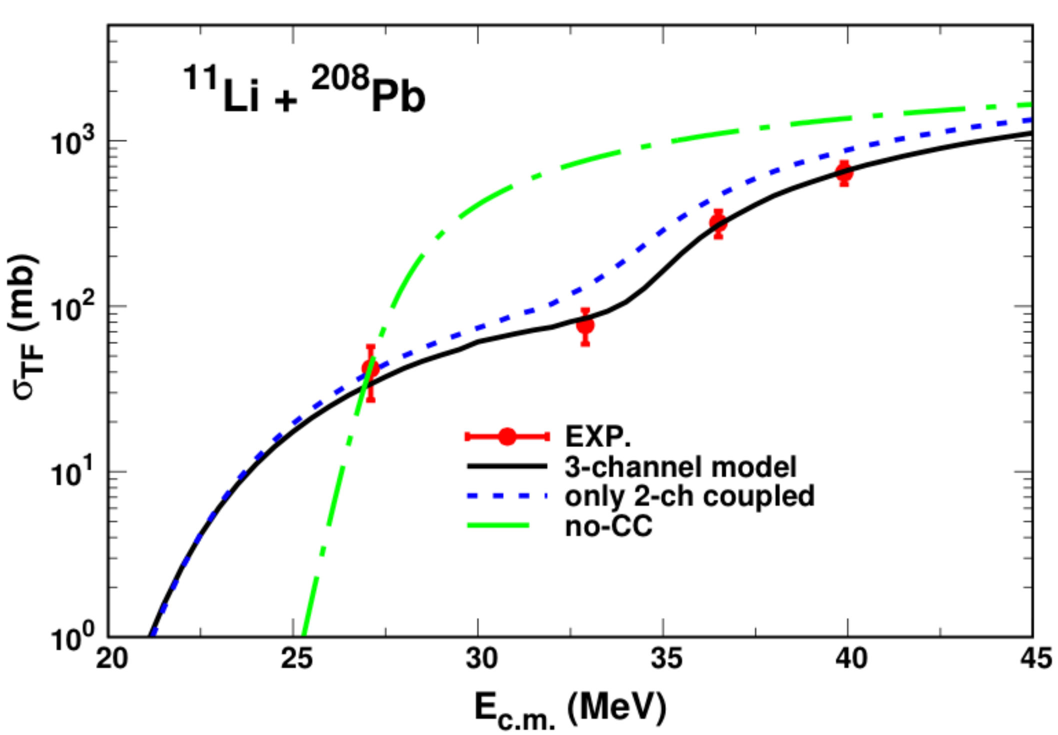
$^{11}Li + ^{208}Pb$ fusion reaction with transfer contribution

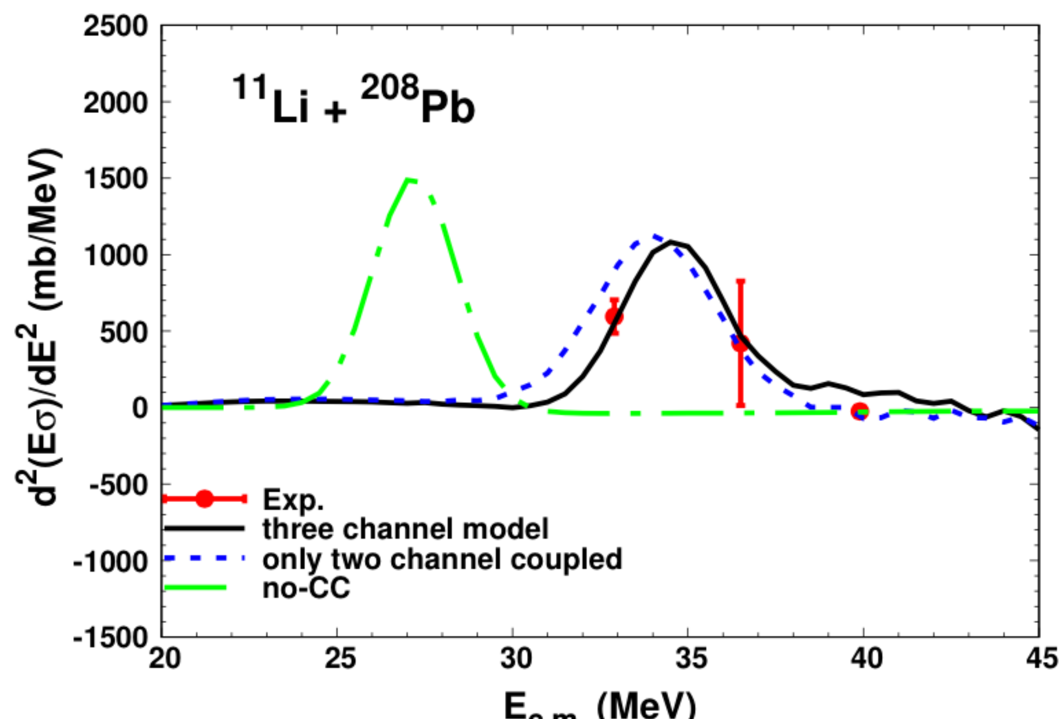




A. M.Vinodkumar : Phys. Rev. C 80, 054609 (2009)

sequential fitting for form factors





Ki-Seok Choi Myung-Ki Cheoun W.Y. So K. Hagino , K.S. Kim :PLB. 780, 455 (2018)

A. M.Vinodkumar : Phys. Rev. C 87, 044603 (2006)

^{15}C + ^{232}Th fusion reaction case

PRL 106, 172701 (2011) PHYSICAL REVIEW LETTERS week end 29 APRIL

Fusion Reactions with the One-Neutron Halo Nucleus ^{15}C

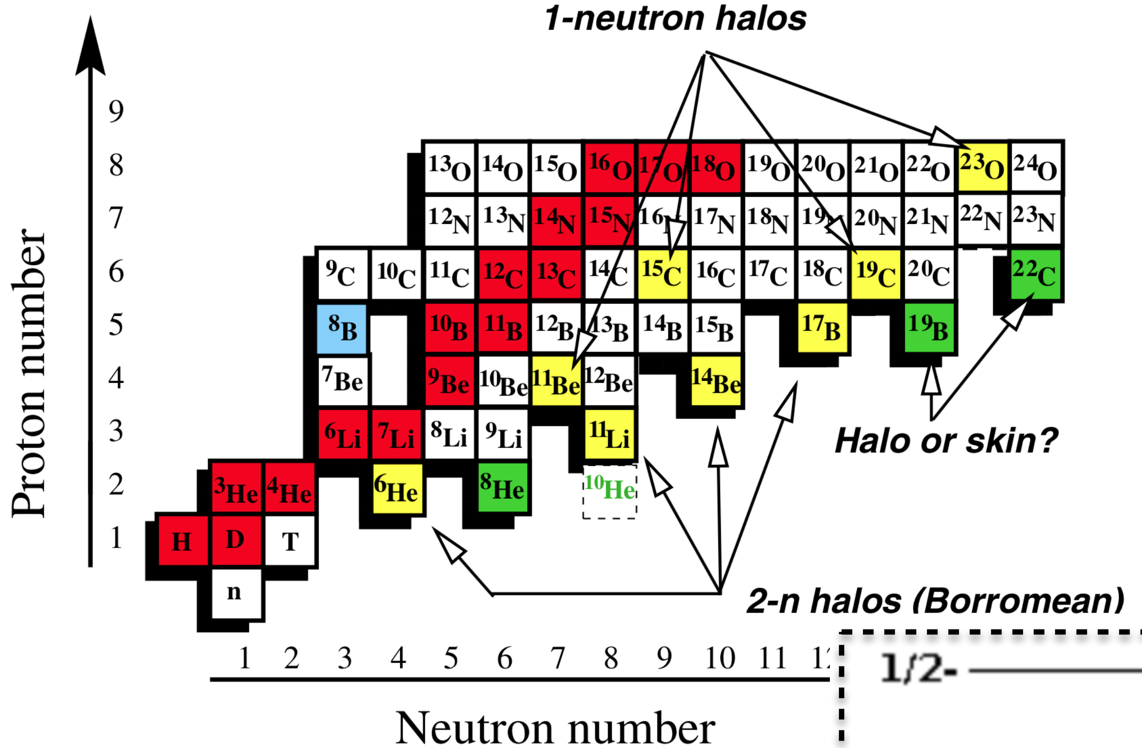
M. Alcorta,[1] K. E. Rehm,[1] B. B. Back,[1] S. Bedoor,[2] P. F. Bertone,[1] C. M. Deibel,[1,3] B. DiGiovine,[1] H. Esbensen,[1] J. P. Greene,[1] C. R. Hoffman,[1] C. L. Jiang,[1] J. C. Lighthall,[1,2] S. T. Marley,[1,2] R. C. Pardo,[1] M. Paul,[4] A. M. Rogers,[1] C. Ugalde,[1,5,6] and A. H. Wuosmaa[2]

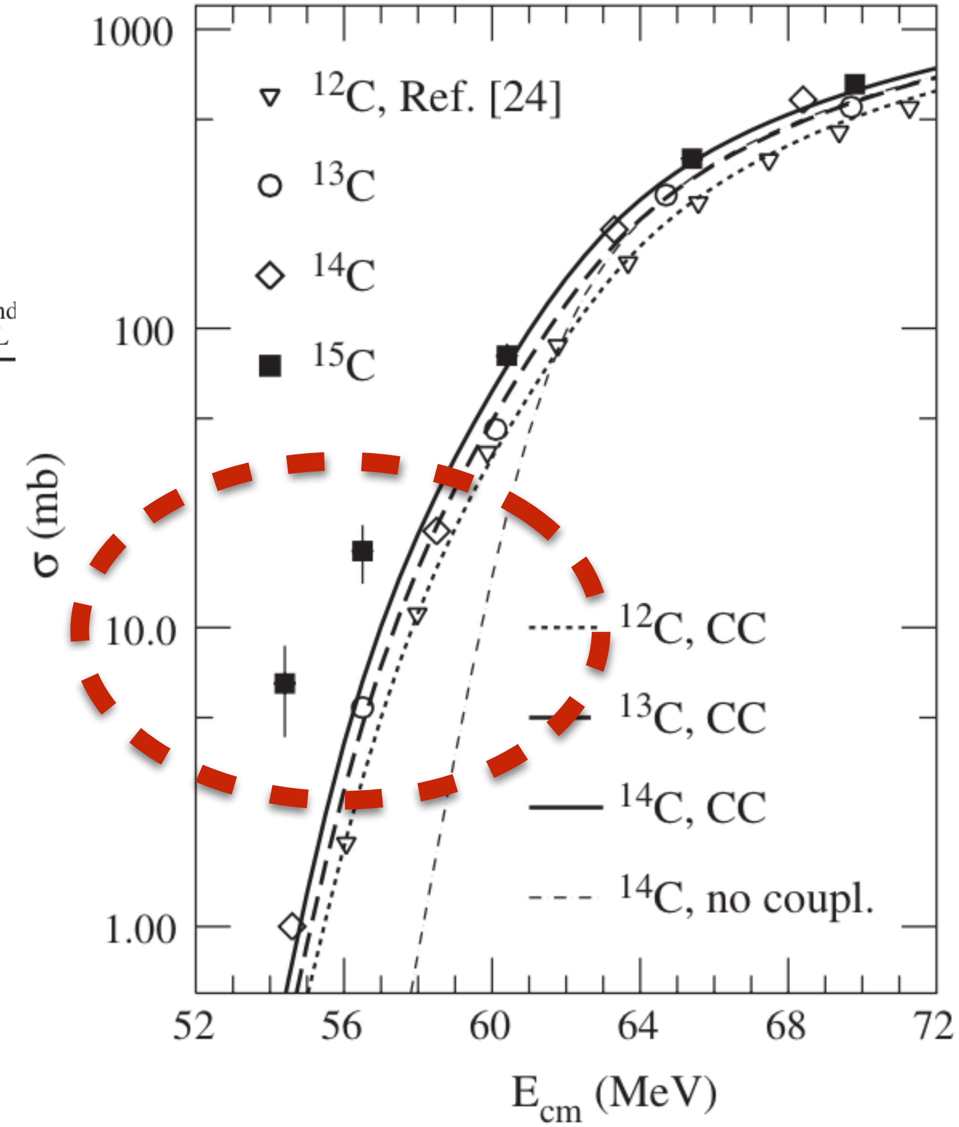
[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[2]*Western Michigan University, Kalamazoo, Michigan 49008, USA*
...ising, Michigan 48824, USA
...1904, Israel
...ago, Illinois 60637 USA
...60637 USA

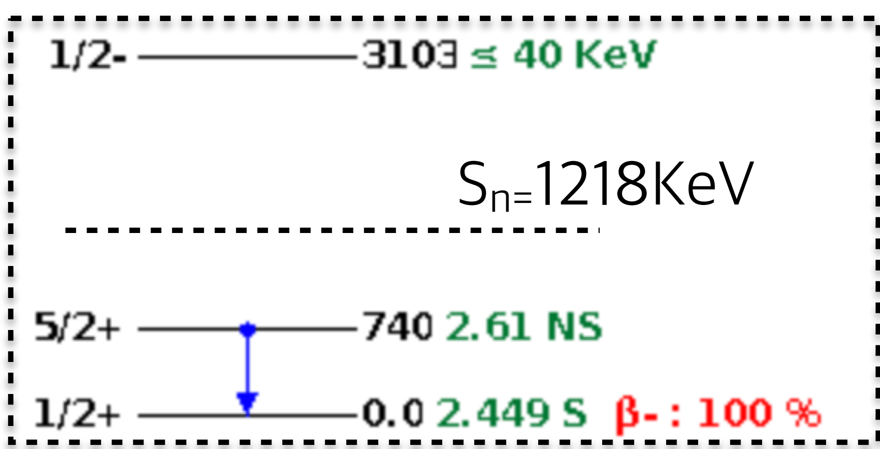
...12 July 2013)

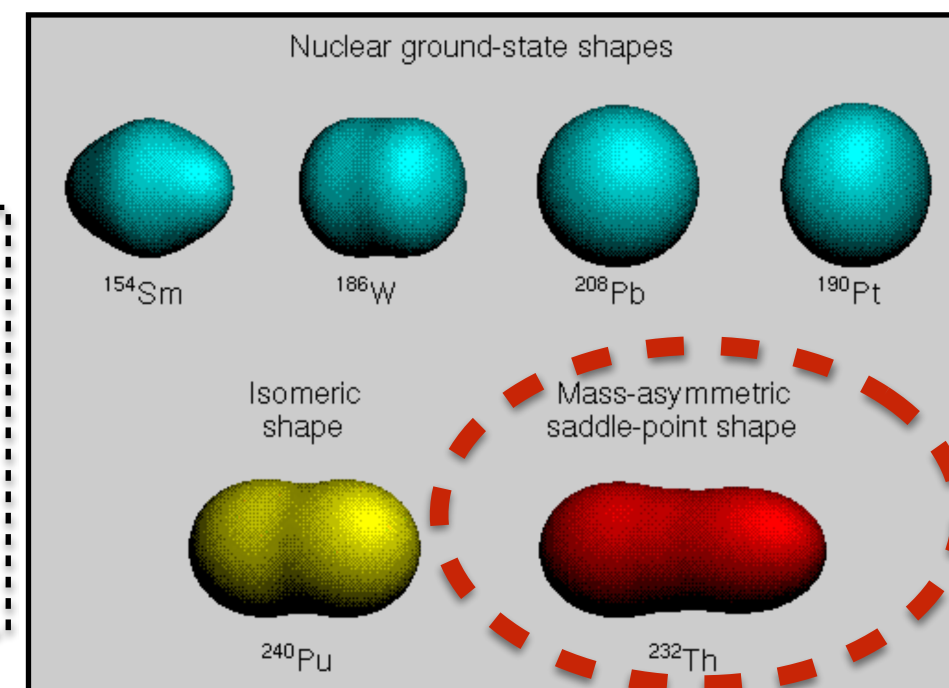
...shell nucleus ^{14}C, makes it ...studied the cross section for ...mb barrier and compared it ...experiment. At sub-barrier ...ed for ^{15}C, while the cross

...rs: 25.60.Pj, 24.10.Eq, 25.70.Jj



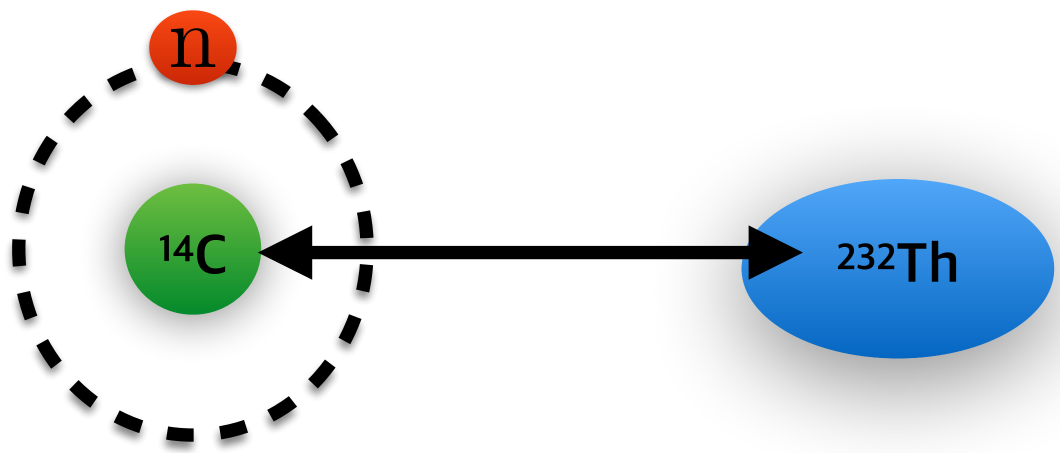


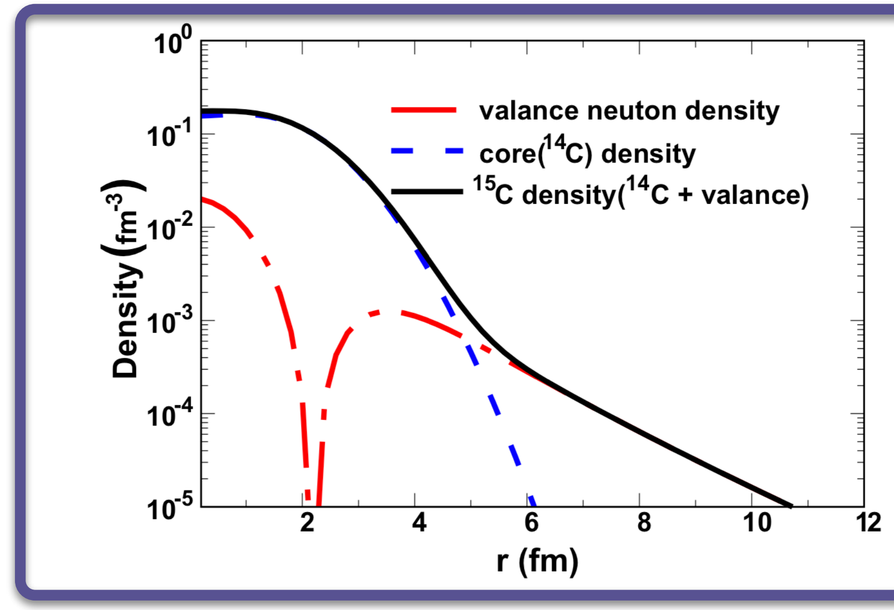




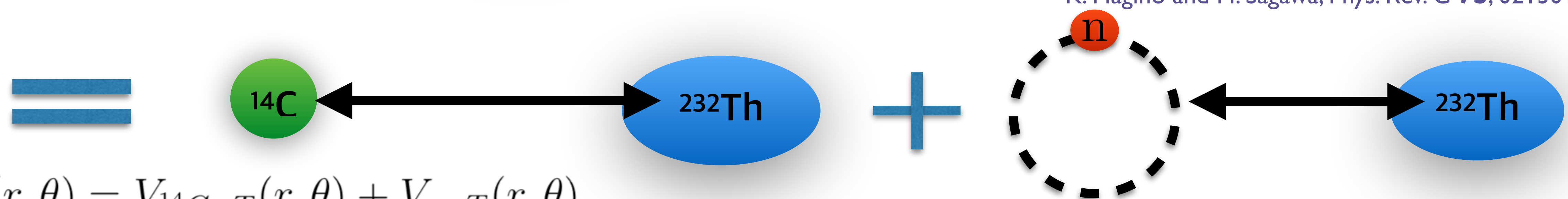
M. Alcorta et. al : Phys. Rev. Lett 106, 172701 (2011)

Construction of Potential





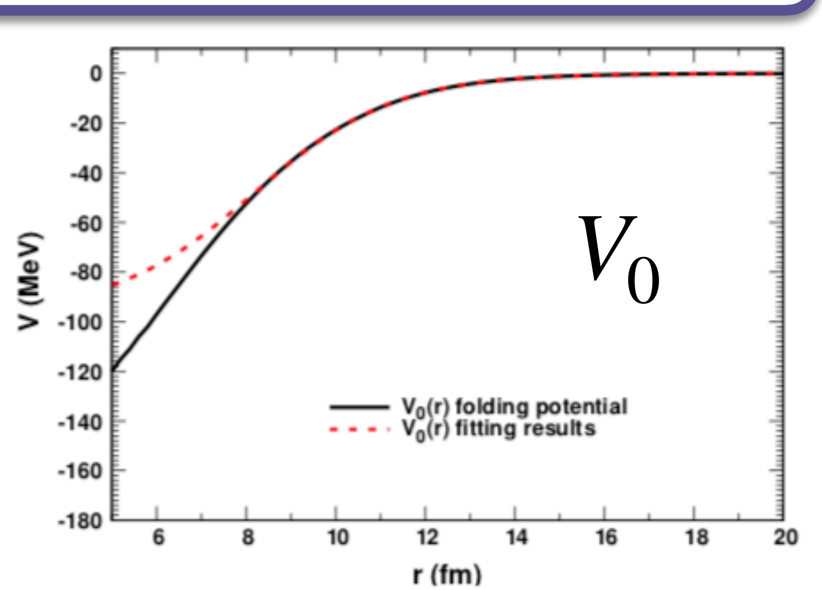
K. Hagino and H. Sagawa, Phys. Rev. C **75**, 021301 (2007).

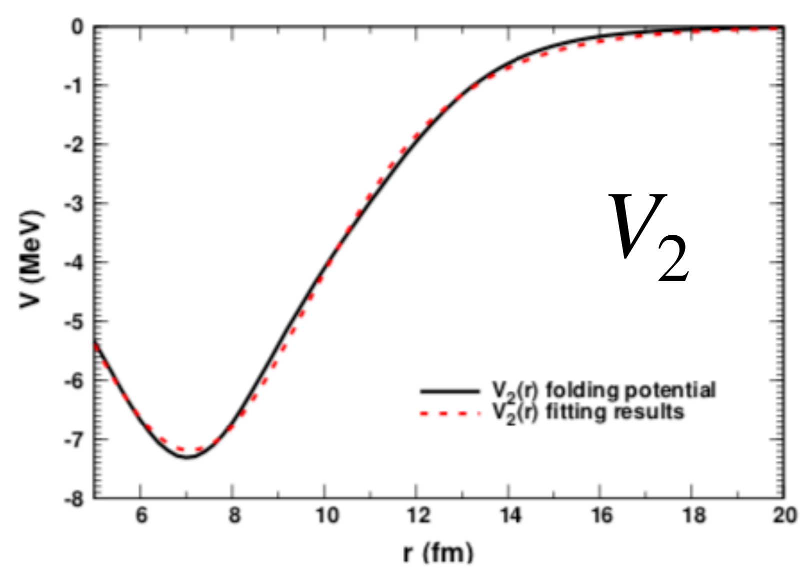


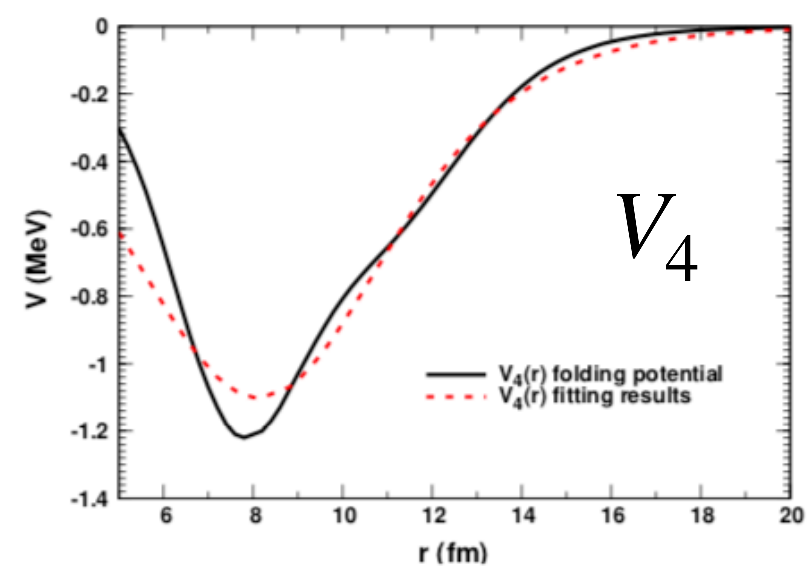
$$V_{^{15}C-T}(r,\theta) = V_{^{14}C-T}(r,\theta) + V_{n-T}(r,\theta)$$

$$\frac{V_0}{1 + e^{(r - R_0 - R_0\beta_2 Y_{20}(\theta))/a}}$$

$$\sum_\lambda V_\lambda(r) Y_{\lambda 0}(\theta) \rightarrow V_0(r)Y_{00}(\theta) + V_2(r)Y_{20}(\theta) + V_4(r)Y_{40}(\theta)$$





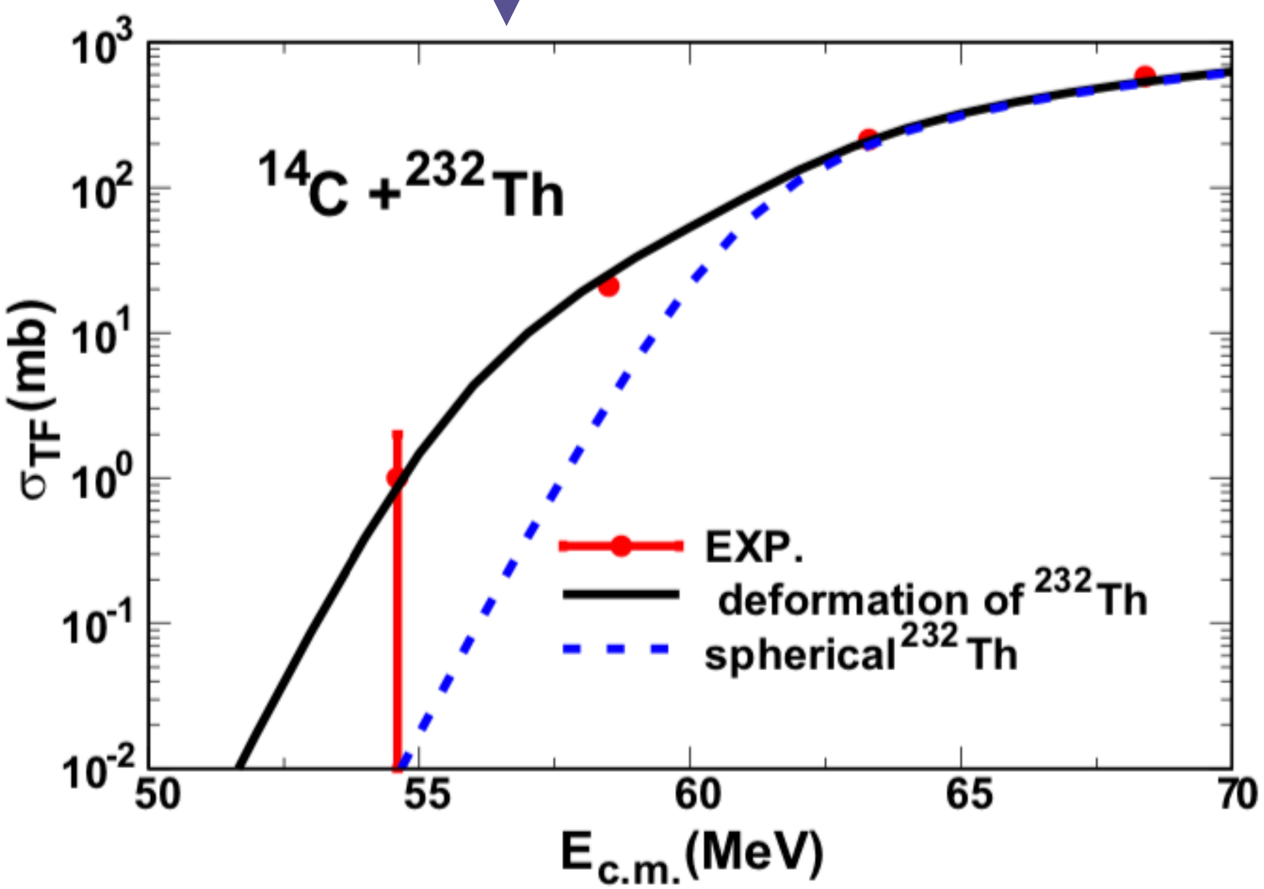


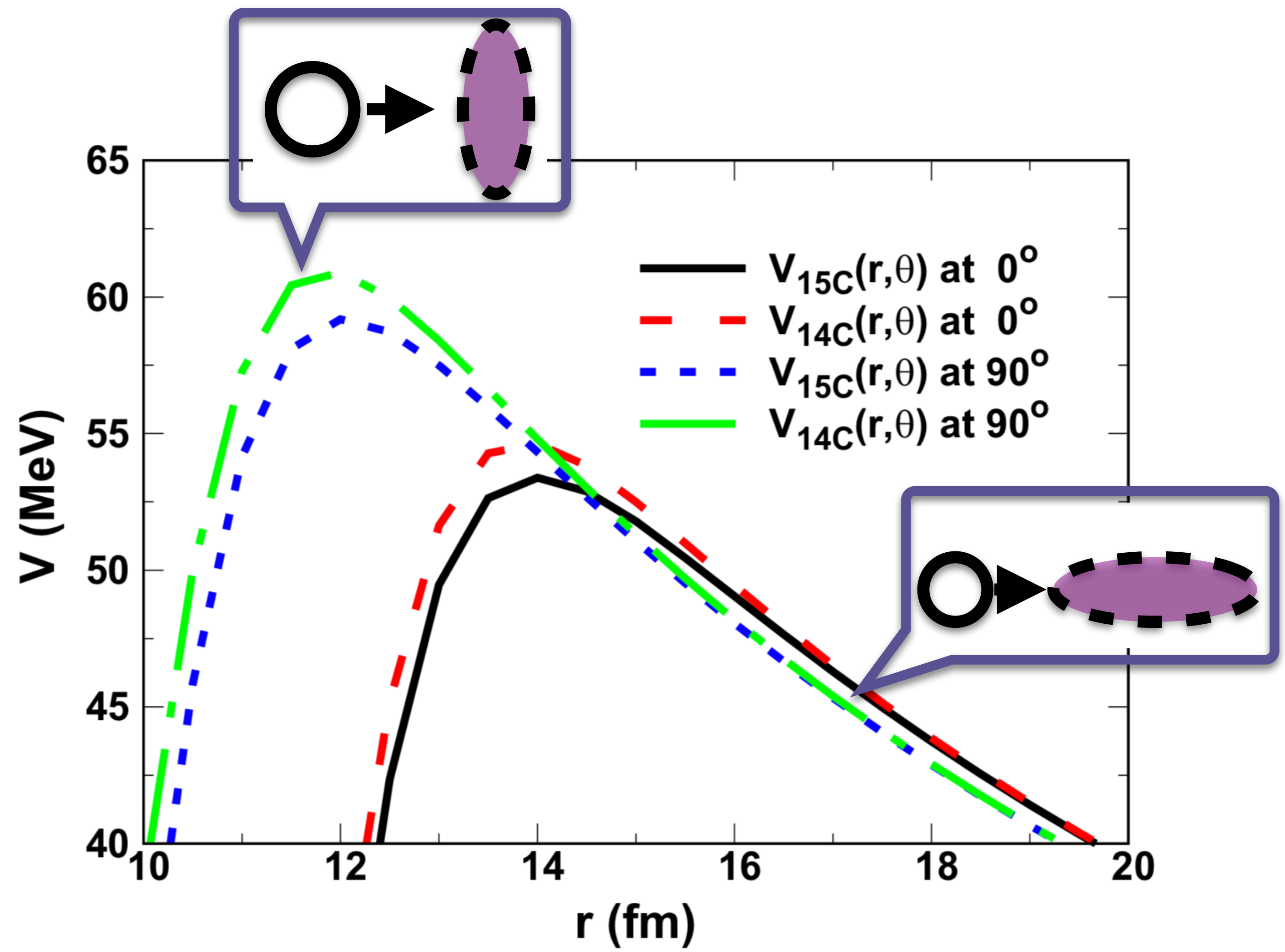
Construction of Potential

$$V_{^{15}C-T}(r,\theta) = V_{^{14}C-T}(r,\theta) + V_{n-T}(r,\theta)$$

$$\frac{V_0}{1+e^{(r-R_0-R_0\beta_2 Y_{20}(\theta))/a}}$$

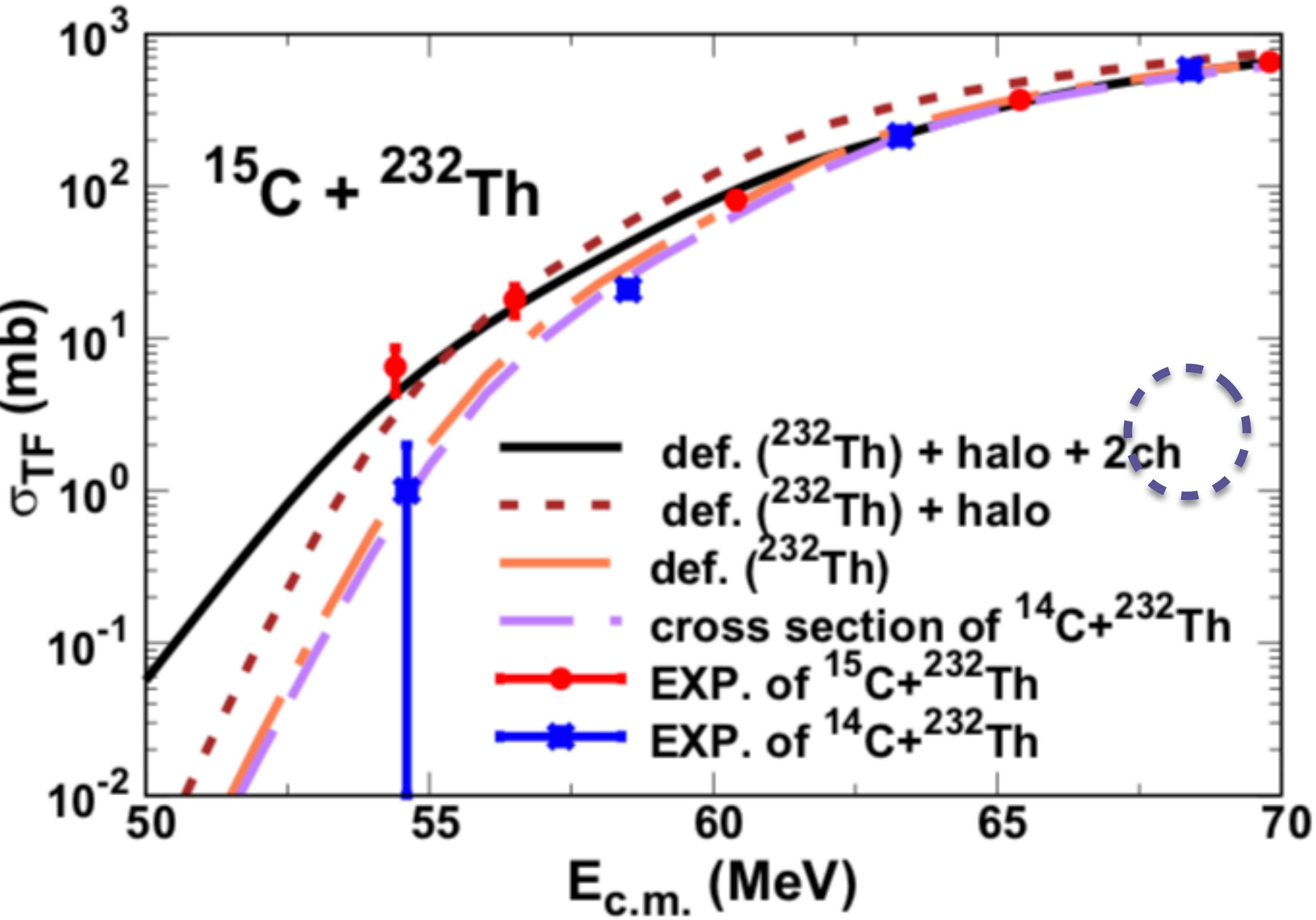
$$\sum_{\lambda} V_\lambda(r) Y_{\lambda 0}(\theta) \rightarrow V_0(r)Y_{00}(\theta) + V_2(r)Y_{20}(\theta) + V_4(r)Y_{40}(\theta)$$



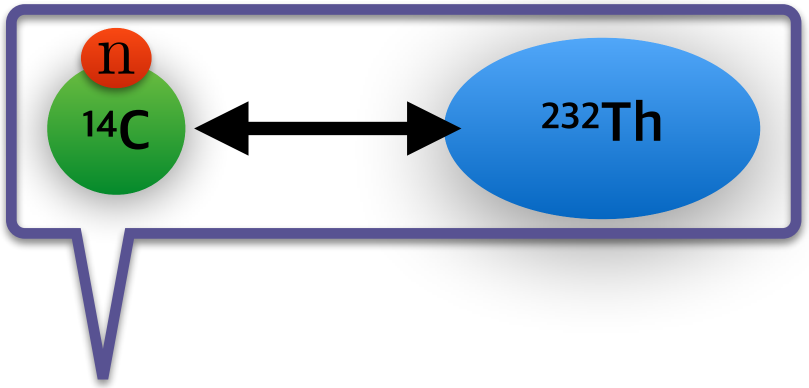


M. Alcorta et. al. : Phys. Rev. Lett. 106, 172701 (2011)
Ki-Seok Choi, K.S. Kim, Myung-Ki Cheoun, W.Y. So, K. Hagino: Phys. Rev. C 103, 034611 (2021)

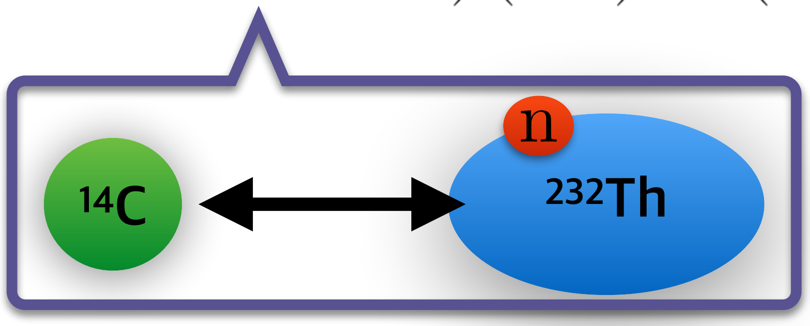
Fusion cross section for the $^{15}C + ^{232}Th$



$$^{15}C + ^{232}Th$$



$$\begin{pmatrix} K + V_1(r,\theta) & F_{1\to 2}(r) \\ F_{1\to 2}(r) & K + V_2(r,\theta) - Q \end{pmatrix} \begin{pmatrix} \psi_1(r) \\ \psi_2(r) \end{pmatrix} = E \begin{pmatrix} \psi_1(r) \\ \psi_2(r) \end{pmatrix}$$



$$^{14}C + ^{233}Th \sim ^{14}C + ^{232}Th$$

$$F_{1\to 2}(r) = F_t \frac{d}{dr}\left(\frac{1}{1 + \exp[(r - R_{coup})/a_{coup}]}\right)$$

Q (MeV)	F_t (MeV fm)	R_{coup} (fm)	a_{coup} (fm)
0	27.5	14.638	0.69

M. Alcorta et. al. : Phys. Rev. Lett. 106, 172701 (2011)
Ki-Seok Choi, K.S. Kim, Myung-Ki Cheoun, W.Y. So, K. Hagino: Phys. Rev. C 103, 034611 (2021)

Summary

- We introduced general formalism for direct and fusion reactions including halo nuclei in our group.
- Experimental data including weakly bound nuclei as halo nuclei are successfully reproduced using extended optical model formalism.
- Also, we looked around approaches for description of fusion reaction, and verified contributions of halo properties in fusion process.
- More experimental data including halo and exotic nuclei are expected by improved facilities.