Skyrmions and Collective Isospin Dynamics

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Skyrmions – A Review

- Skyrme theory is nonlinear, effective field theory (EFT) of pions [T.H.R. Skyrme, 1961]. Its field equations have topological soliton solutions – Skyrmions – with surprising shapes.
- Skyrmions represent nucleons and larger nuclei. No explicit nucleon fields appear. Skyrme theory is "Nuclear Physics without Nucleons" [Iachello].
- Skyrme field

$$U(x) = \sigma(x)\,\mathbf{1}_2 + i\boldsymbol{\pi}(x)\cdot\boldsymbol{\tau}$$

  requires $\sigma^2 + \boldsymbol{\pi}\cdot\boldsymbol{\pi} = 1$, so $U \in SU(2)$. Field is smooth and needs no short-distance cutoff.
- $U \to \mathbf{1}_2$ asymptotically, and $U \simeq -\mathbf{1}_2$ in core of nucleons.

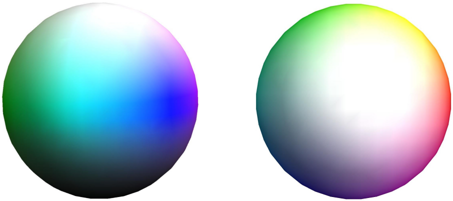
- Topological charge – the topological degree of U over space – is identified with baryon number B (atomic mass number).
- Skyrme field Lagrangian

$$L = \int \frac{1}{2}\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger)\,d^3x + \text{higher order derivative terms} + \text{pion mass term}\,.$$

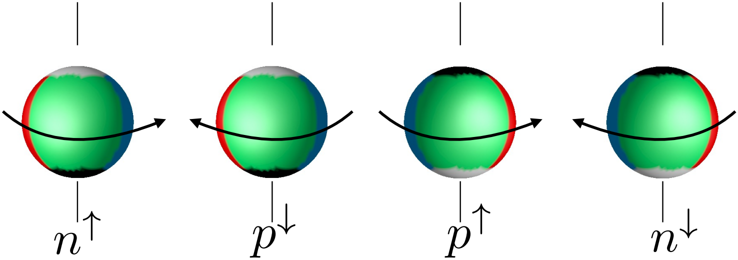
- Solve field equations to determine Skyrmion solutions and their symmetries, energies, spin/isospin moments of inertia, vibrational frequencies.

- Skyrmions assign an intrinsic geometrical shape and pion field structure to nuclei. They spontaneously break the translational, rotational and isorotational symmetries of L.
- An intrinsic (non-spherical) shape is familiar in nuclear physics [Wheeler, Wefelmeier, Bohr–Mottelson]. An intrinsic pion field structure is less familiar.
- Rigid-body quantization restores these symmetries. The ground and excited states of nuclei are classified by (momentum $\mathbf{P}$), spin J and isospin I.
- Evidence for the pion field structure comes from the correlated constraints on spin/isospin.

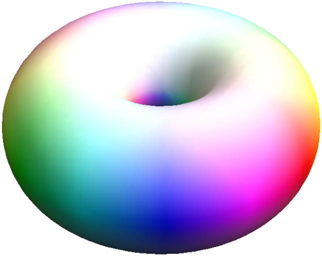
Skyrmions with Small Baryon Numbers



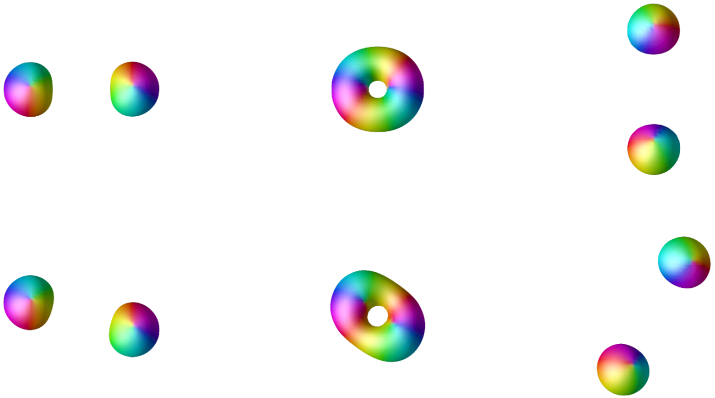
$B = 1$ Skyrmion in two orientations. These attract, clump together and slightly merge to form larger-B Skyrmions. (Colours indicate unit-pion-field values on constant energy density surface.)



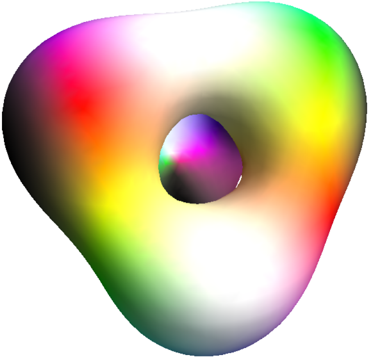
Classically spinning $B = 1$ Skyrmions, modelling quantized spin/isospin $\frac{1}{2}$ nucleons [Foster and NSM]. Spin/isospin $\frac{3}{2}$ delta resonances spin faster. Wavefunctions change sign under 2π rotation [Skyrme].



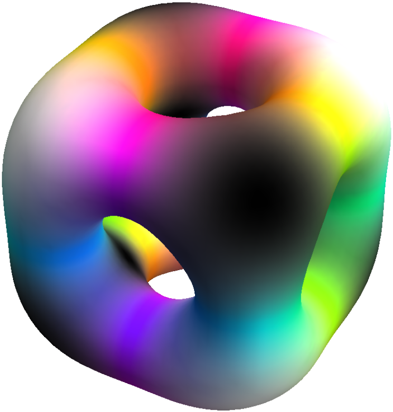
$B = 2$ Skyrmion



Scattering $B = 1$ Skyrmions [Foster and Krusch]



$B = 3$ Skyrmion [Braaten et al.]



$B = 4$ Skyrmion

Runge Colouring Scheme

- Figures show a surface of constant energy density (away from the Skyrmion centres where $\sigma = -1$).
- The *unit pion field* $\hat{\pi}$ is indicated using the *Runge colour sphere*.
- White: $\hat{\pi} = (0, 0, 1)$,
- Black: $\hat{\pi} = (0, 0, -1)$,
- Red, Green, Blue: $\hat{\pi} = \left(1, 0, 0\right)\,,\ \left(\cos(\frac{2\pi}{3}), \sin(\frac{2\pi}{3}), 0\right)\,,\ \left(\cos(\frac{4\pi}{3}), \sin(\frac{4\pi}{3}), 0\right).$

Rigid-body Quantization – Spin J and Isospin I

- Skyrmions quantized as rigid bodies represent nuclei. Skyrmion symmetries and topology constrain ground state spin/isospin [Finkelstein and Rubinstein; Adkins, Nappi and Witten; Braaten and Carson; Walhout; Krusch].
- $B = 1$: Proton and neutron, with spin $J = \frac{1}{2}$ and isospin $I = \frac{1}{2}$. Excited states (Delta-resonances) have $J = I = \frac{3}{2}$.
- $B = 2$: ^{2}H (Deuteron), with $J = 1$ and $I = 0$.
- $B = 3$: ^{3}H and ^{3}He, with $J = \frac{1}{2}$ and $I = \frac{1}{2}$.
- $B = 4$: ^{4}He (Alpha particle), with $J = I = 0$.

Quantized $B = 4$ Cube

- $B = 4$ Skyrmion has cubic symmetry and Finkelstein–Rubinstein constraints (for two generators)

$$\begin{aligned} e^{i\frac{2\pi}{3}\frac{1}{\sqrt{3}}(L_1+L_2+L_3)} e^{i\frac{2\pi}{3}K_3}|\Psi\rangle &= |\Psi\rangle \\ e^{i\frac{\pi}{2}L_3} e^{i\pi K_1}|\Psi\rangle &= |\Psi\rangle \,. \end{aligned}$$

  L_i, K_i are spin and isospin operators w.r.t. body-fixed axes.
- Use basis states $|J, L_3\rangle \otimes |I, K_3\rangle$. Only certain linear combinations are allowed.
- Space-fixed projections J_3, I_3 not constrained – get full spin/isospin multiplets.
- Parity operator (effect of inversion in space and isospace) is $\mathcal{P} = e^{i\pi K_3}$.

- Lowest-energy allowed states:

  Isospin 0 ($^4\mathrm{He}$) with $J^P = 0^+, 4^+$

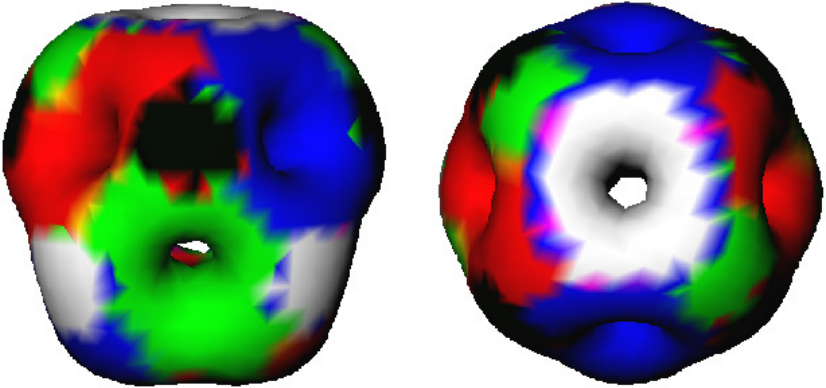
  Isospin 1 ($^4\mathrm{H}\,,^4\mathrm{He}\,,^4\mathrm{Li}$) with $J^P = 2^-$

  Isospin 2 (4-neutron) with $J^P = 0^+$.
- Physical interpretation:

  Isospin 0: Lowest state is $^4\mathrm{He}$ ground state; highly-excited 4^+ state not observed.

  Isospin 1: Multiplet of well-known resonances at 24 MeV.

  Isospin 2: State $|0,0\rangle \otimes |2,0\rangle$ matches 4-neutron resonance recently observed at $\sim$ 30 MeV.
- Further $^4\mathrm{He}$ resonances are modelled by vibrating $B = 4$ cube [Rawlinson].



$B = 6$ Skyrmion

$B = 6$ States

- $B = 6$ Skyrmion has D_{4d} symmetry and Finkelstein–Rubinstein constraints [Wood]

$$
\begin{aligned}
e^{i\frac{\pi}{2}L_3} e^{i\pi K_3} |\Psi\rangle &= |\Psi\rangle \\
e^{i\pi L_1} e^{i\pi K_1} |\Psi\rangle &= -|\Psi\rangle \,.
\end{aligned}
$$

- Parity $\mathcal{P} = e^{i\frac{\pi}{4}L_3} e^{-i\frac{\pi}{2}K_3}$.
- Allowed isospin 0 states (^{6}Li):

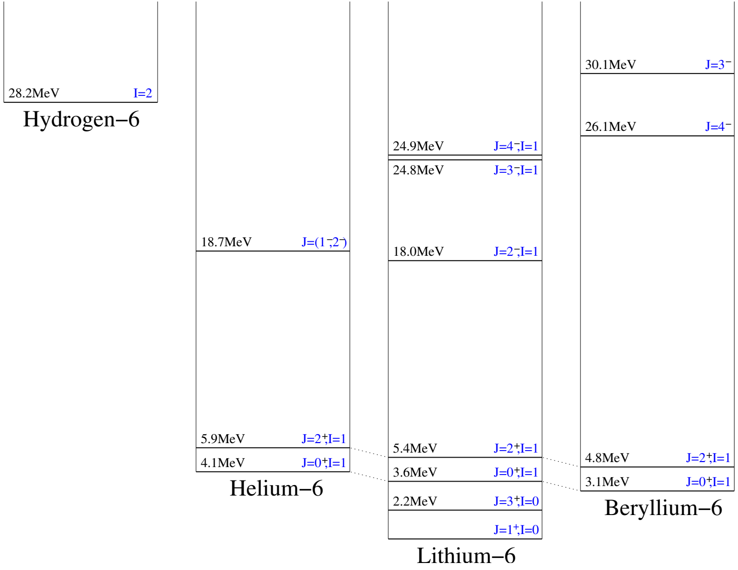
$$
J^P = 1^+, 3^+, 4^-, 5^+, 5^-, \cdots \,,
$$

isospin 1 states (^{6}He,^{6} Li,^{6} Be):

$$
J^P = 0^+, 2^+, 2^-, \cdots \,,
$$

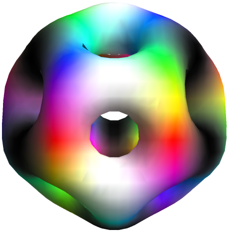
and isospin 2 state $^6\mathrm{H}$ with predicted $J^P = 0^-$.

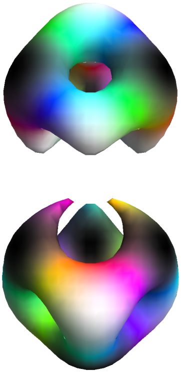
- Quite good fit to $B = 6$ nuclei.



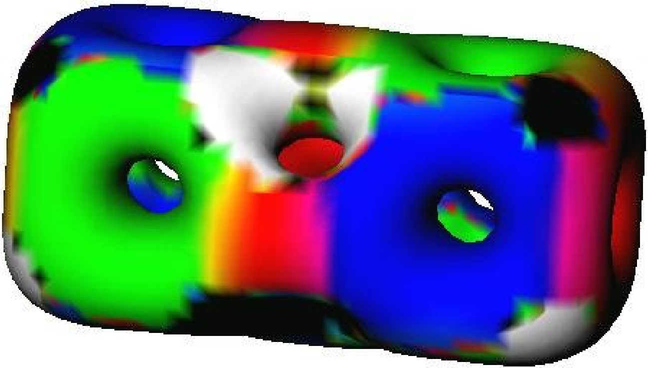
Energy levels of $B = 6$ nuclei

Some Higher *B* Skyrmions

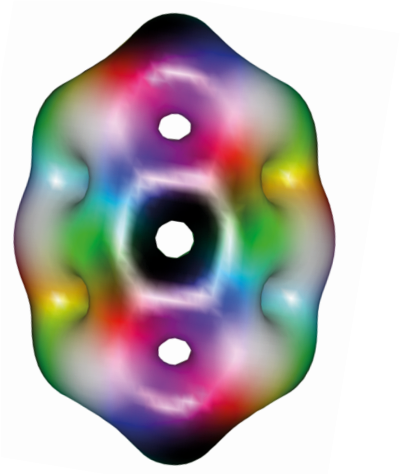




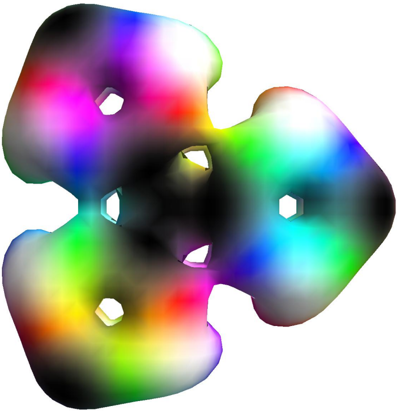
$B = 7$ Skyrmion and its deformation into clusters. Deformed Skyrmion models $\frac{3}{2}^-$ ground states of $^7\mathrm{Be}/^7\mathrm{Li}$. The quantized icosahedral Skyrmion models excited $\frac{7}{2}^-$ states, and 'ground' $\frac{3}{2}^-$ states of isospin quartet including $^7\mathrm{He}$.



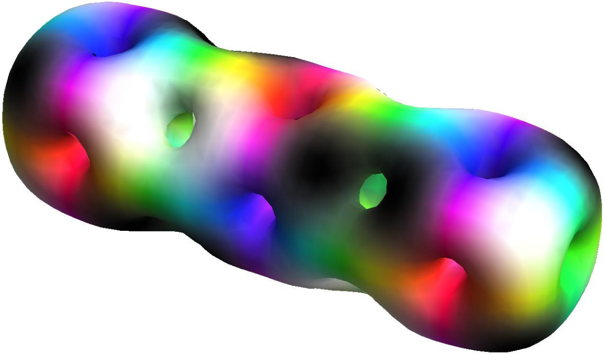
$B = 8$ Skyrmion ($m_\pi = 1$)



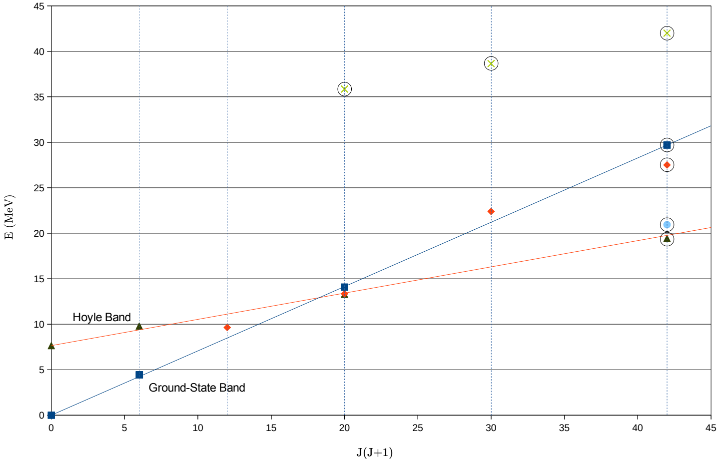
$B = 10$ Skyrmion



$B = 12$ Skyrmion with D_{3h} symmetry



$B = 12$ Skyrmion with D_{4h} symmetry



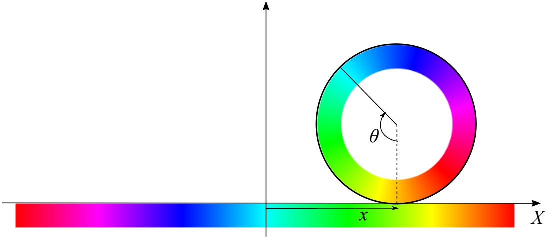
Carbon-12 states in the ground state band and Hoyle band

Beta-Decay of Nuclei

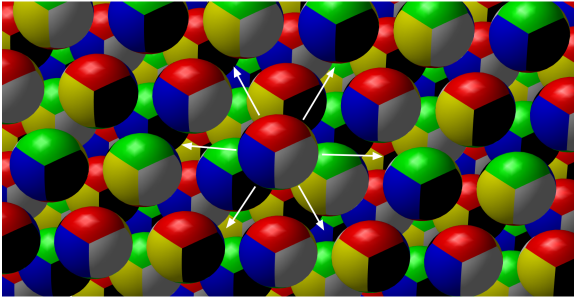
- Not yet calculated using Skyrmions, except for $B = 1$ [Adkins, Nappi and Witten] and $B = 3$ [Carson].
- Matrix elements involve isospin lowering/raising operator acting on rigid-body state, and an integral depending on the classical Skyrmion solution. This is for the dominant allowed transition within a single isospin multiplet, if energetically available.
- Calculations of beta-decay of $^{6}\mathrm{He}$, $^{12}\mathrm{N}$ and $^{14}\mathrm{C}$ are feasible, using known Skyrmions.

Spin-Orbit Coupling

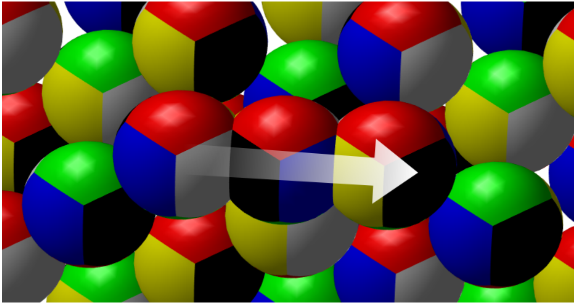
- Nucleon-nucleon potentials including spin-orbit coupling partly understood using Skyrmions [Harland and Halcrow]. A $B = 1$ Skyrmion interacting with a planar Skyrmion surface also studied [Harland and NSM]. The pion field structure is essential.
- The $B = 1$ Skyrmion prefers to roll (classically) over the surface. This corresponds to spin and orbital angular momentum being parallel for a $B = 1$ Skyrmion orbiting a magic nucleus.
- Quantum calculations are more difficult. 2nd-order perturbation theory, or a tight-binding approximation, are needed.



Coloured disc (cog) rolling on a coloured rail (Halcrow)



A Skyrmion above a half-filled FCC crystal of Skyrmions
(Harland and NSM)

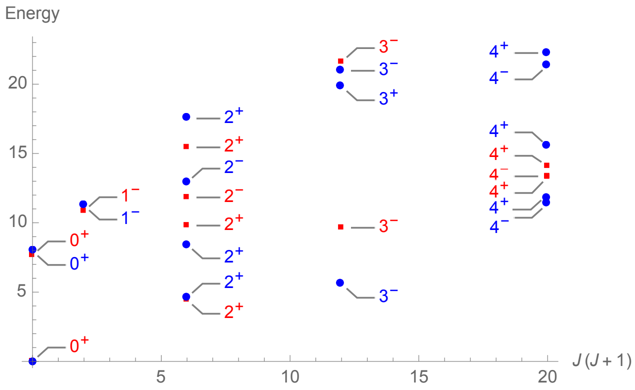


The path of a rolling Skyrmion

Conclusions

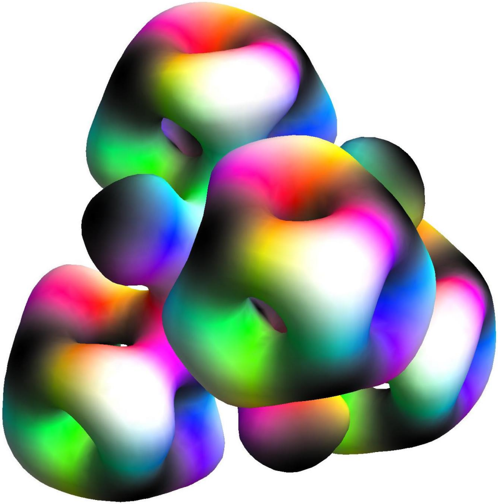
- Skyrmions give correlated intrinsic shapes and pion field structures to nuclei.
- Rigid-body quantization restores rotational/isorotational symmetry. Nuclear states acquire constrained spin/isospin combinations. Results up to Carbon-12 and its isobars mostly satisfactory.
- Energy spectrum calculated using spin/isospin moments of inertia. Isospin energy matches Bethe-Weizsäcker asymmetry energy. **Collective isospin dynamics is key signature of Skyrmion models of nuclei.**
- Vibrational excitations of Skyrmions needed to model e.g. Helium-4 resonances, and Calcium-40 spectrum.
- Spin-orbit coupling and beta-decay matrix elements depend on pion field structure – further test of Skyrmion picture.

EXTRA SLIDES

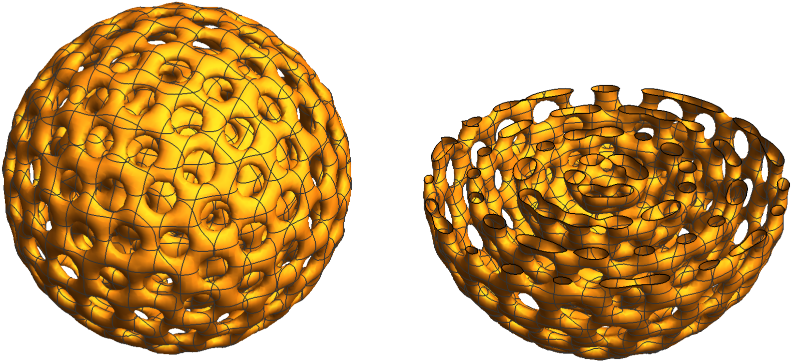


Carbon-12 energy levels [Rawlinson], allowing for interpolation between triangular and chain Skyrmions: Experiment, Skyrme model

- Bending mode between triangle and chain Skyrmions is excited in Hoyle state, and needed to model 1^- and 2^- states of Carbon-12.
- $B = 10$ Skyrmion is "molecule" with 2-alphas/2-nucleons. Good for Boron-10, but rigid-body quantization misses negative-parity states. Bending mode probably needed to model 1^-, 2^-, 3^- states?
- Two joined-up $B = 10$ Skyrmions may model some states of Neon-20.
- Gudnason and Halcrow have website "Database of Skyrmion Vibrations", showing vibrational modes up to $B = 8$.



$B = 20$ Skyrmion [Lau, Halcrow]



Icosahedral $B = 208$ Skyrmion [Halcrow]