Overview of Known nEDM Systematics

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nEDM Measurement Sensitivity

Statistics: uncertainty σ_{stat} in the measured frequency due to uncertainty in the neutron counts

$$\sigma_{\rm stat} = \frac{\hbar}{2\alpha E t \sqrt{N}}$$

Systematics: uncertainty or bias σ_{sys} in the measured frequency due to imperfections in the experiment compared to the ideal case

Total: combine statistics and systematics in quadrature

$$\sigma = \sqrt{\sigma_{\text{stat}}^2 + \sigma_{\text{sys}}^2} = \sqrt{2} \times 10^{-27} = 1.41 \times 10^{-27} e\text{cm}$$

90% confidence interval: assuming a normal distribution

$$d_{\rm n,meas} - 1.645\sigma < d_{\rm n} < d_{\rm n,meas} + 1.645\sigma$$

With $d_{n,meas} = 0$:

$$|d_n| < 2.32 \times 10^{-27} e \text{cm}$$

RAL/Sussex/ILL Measurement

Expected statistics	1.34x10 ⁻²⁶ <i>e</i> cm
Actual statistics	1.53x10 ⁻²⁶ <i>e</i> cm
Estimated systematics	0.99x10 ⁻²⁶ <i>e</i> cm
Total uncertainty	1.82x10 ⁻²⁶ <i>e</i> cm
Measured d _n	-0.21x10 ⁻²⁶ <i>e</i> cm
90% centred Cl	d _n < 3.0x10 ⁻²⁶ ecm

- Actual statistics were enhanced by additional noise in the frequency measurement, suggested to be attributed to:
 - Imprecision in measured points on curve
 - Undetectable changes in gradient
 - Rapid depolarization of mercury
- Statistics were 14% worse than expected, final result was 10% larger

Sources of Systematic Uncertainty

Effects that produce a signal that looks like an EDM (false EDM)

- 1. Magnetic field gradient geometric phase
- 2. Permanent magnetic dipole effects
- 3. GP correction effects
- 4. Comagnetometer effects
- 5. Motional field effects
- 6. HV effects
- 7. Fitting adjustment

Effect	Shift (10 ⁻²⁷)	σ (10 ⁻²⁷)
Door dipole field	-7.10	0.70
Hg door dipole	0.00	6.00
Quadrupole field (& Earth rotation)	3.30	1.40
Light shift (geometric) Light shift (direct)	(3.50) 0.00	0.80 0.20
Nonuniform Hg depolarization	0.00	0.01
Hg EDM	-0.02	0.06
vxE translational	0.00	0.60
vxE rotational	0.00	0.50
Second order vxE	0.00	0.00
Electric forces	0.00	0.40
Leakage currents	0.00	0.10
AC fields	0.00	0.01
Uncompensated B drift	0.00	0.34
χ^2 = 1.2 adjustment	0.00	6.80
Total	-3.80	9.90

A Revised Experimental Upper Limit on the Electric Dipole Moment of the Neutron, 2015

Essential Equations

• Precession equation:

$$\hbar\omega_{\pm} = |2\mu B \pm 2dE|$$

• Neutron EDM:

$$d_n = \frac{\hbar(\omega_+ - \omega_-)}{4E}$$

• False EDM signals appear from inexact cancellation between the two frequencies

• Hg-n EDM coupling from B fluctuation
correction (subscript f denotes false effects):
$$\omega_{n}^{*} = \omega_{n,meas} - \frac{\gamma_{n}}{\gamma_{Hg}} \omega_{Hg}$$
$$d_{n,meas} = \left(d_{n} + d_{n,f} + \left|\frac{\gamma_{n}}{\gamma_{Hg}}\right| \left(d_{Hg} + d_{Hg,f}\right)\right)$$



1. B_{0,z} gradient phase shift

- **Contribution**: 0.00x10⁻²⁷*e*cm
- Ramsey-Bloch-Siegert shift from transverse magnetic field:

$$\Delta \omega \propto \gamma^2 \mathbf{B}_{xy}^2 \propto \left(\frac{\partial B_{0z}}{\partial z}\frac{\mathbf{r}}{2}\right)^2 + \left(\frac{\mathbf{E} \times \mathbf{v}}{c^2}\right)^2 + 2\frac{\partial B_{0z}}{\partial z}\frac{\mathbf{r}}{2} \cdot \frac{\mathbf{E} \times \mathbf{v}}{c^2}$$

• False EDM from RBS shift

$$d_{\mathrm{n,f}} = -\frac{J\hbar}{2} \frac{\partial B_{0z}}{\partial z} \frac{1}{B_{0z}^2} \frac{v_{xy}^2}{c^2} \left[1 - \frac{\omega_r^2}{\omega_0^2} \right]^{-1}$$
$$d_{\mathrm{Hg,f}} = \frac{J\hbar}{2} \frac{\partial B_{0z}}{\partial z} \frac{\gamma^2 R^2}{c^2} \left[1 - \frac{\omega_0^2}{\omega_r^2} \right]^{-1}$$

- Hg effect estimated to be ~50 times larger than n effect
- **Requirement**: correct phase shift by different analysis systems
 - Sussex crossing point analysis (exploit n-Hg height difference to correct to zero gradient)
 - Dual comagnetometer (solve simultaneously for both z field and gradient)
 - Magnetometer array (fit entire B field)

1. B_{0,z} gradient phase shift

• Crossing point analysis: neutrons sample the lower part of the cell, so measure a different B field compared to Hg in the presence of a gradient

$$R' = \left| \frac{\omega_{\rm n}}{\omega_{\rm Hg}} \frac{\gamma_{\rm Hg}}{\gamma_{\rm n}} \right| - 1 = \pm \Delta h \frac{\partial B_{0z}}{\partial z}$$
$$d_{\rm n,meas} = d'_{\rm n} \pm k(R' - R_0')$$

- R₀' corresponds to zero gradient. This gives two lines (B up and B down).
- Trim coils are used to adjust the ratio for each run to produce the two lines of data.
- Finding the crossing point from the resulting data gives a gradient corrected nEDM.



2. Permanent magnetic dipole effects

- **Contribution**: 6.04x10⁻²⁷*e*cm
- Dipoles enhance the phase shift (second equation for axially centred dipole)

$$d_{n,f} = -\frac{\hbar}{4} \left\langle \frac{\partial B_{dip,z}}{\partial z} \right\rangle_{V} \frac{1}{B_{0z}^{2}} \frac{v_{xy}^{2}}{c^{2}} \left[1 - \frac{\omega_{r}^{*2}}{\omega_{0}^{2}} \right]^{-1}$$
$$d_{Hg,f} = \frac{\hbar}{4} \left\langle \frac{\partial B_{dip,z}}{\partial z} \left(1 + \frac{R^{2}}{(z_{0} + z)^{2}} \right) \right\rangle_{V} \frac{\gamma^{2}R^{2}}{c^{2}} \left[1 - \frac{\omega_{0}^{2}}{\omega_{r}^{*2}} \right]^{-1}$$

- Also introduces direct effect due to differential sampling of the nonuniform field
- Requirement: survey and reduce PMDs introduced by equipment (eg. valves)

3. GP Correction Effects:

- **Contribution**: 1.61x10⁻²⁷*e*cm
- Effects which introduce uncertainty through the process of correcting for the gradient phase shift
- Crossing point analysis: effects that cause a differential shift (in the ratio of measured magnetic fields) with B field reversal
 - Quadrupole or other transverse magnetic fields
 - Earth rotation
 - Localized losses of Hg and n
 - Hg indirect light shift
- Currently limited understanding of what effects may be for new systems
- **Requirement:** develop deeper understanding of systematic effects in the gradient correction process and design correction systems to have small uncertainty, be able to survey the field within the MSR to high precision



4. Comagnetometer Effects:

- **Contribution**: 0.21x10⁻²⁷*e*cm
- Effects which cause a shift in the Hg frequency proportional to E
 - Hg EDM
 - Nonuniform depolarization rates
 - Direct light shift effect
- For a single comagnetometer more recent results indicate that this class of effects should be negligible at the 10⁻²⁷ level:
 - $|d_n| < 7.4 \times 10^{-30} e$ cm (95% CL) should make the Hg EDM negligible
 - An effect from nonuniform depolarization was already ruled out at the 10⁻²⁹ level
 - Studies of the light shift from a modern comagnetometer estimated the size of the direct light shift as 5.6x10⁻³¹ ecm
- **Requirement**: achieve the expected negligible contribution from Hg frequency shifts for a single magnetometer, develop understanding of the impact of such shifts for a dual comagnetometer

5. Motional Field Effects

- **Contribution**: 0.78x10⁻²⁷*e*cm
- Effects appearing directly from the motional magnetic field seen by the neutrons (not a phase shift)

$$\boldsymbol{B}_{\boldsymbol{v}} = \frac{\boldsymbol{E} \times \boldsymbol{v}}{c^2}$$

- Mostly requires a net motion of the neutrons in the cell
- Causes E-proportional shifts in the B₀ field seen by the neutrons due to:
 - Inhomogeneity of the E field
 - Differences in the E field for different orientations
 - Misalignment between fields (including gravitational)
- **Requirement**: understand and minimize the various imperfections in the design and assembly of the spectrometer

6. HV Effects

- **Contribution**: 0.53x10⁻²⁷*e*cm
- Effects that are caused by the reversal of the high voltage polarity
- Things which reverse direction/orientation with HV reversal:
 - Leakage currents
 - Residual magnetization of the MSR at the HV feedthrough
- Things which cause a false signal if the HV magnitude is different for different polarities:
 - Electrostatic forces between the electrodes
 - Ripple of the HV supply
- Requirement: attempt to minimize asymmetry between the two HV polarity configurations, reduce leakage currents, understand interaction of HV with MSR

7. Global Fit Adjustment

- **Contribution**: 6.80x10⁻²⁷*e*cm
- An increase in the (statistical) uncertainty of the gradient phase corrected nEDM measurement to account for the quality of the fit in the crossing point analysis ($\chi^2 = 1.2$), attributed to random fluctuations in the magnetic field
- Depends on how precise the measurement/correction are to begin with, making it difficult to target for reduction.
- If we assume a similar χ^2 with an order of magnitude better statistics we might expect an order of magnitude reduction.
- **Requirement**: ???

Systematics Summary and Goals

Effects	Contribution (10 ⁻²⁷ ecm)	Target (10 ⁻²⁷ ecm)
Gradient phase shift	0.00	0.00
Permanent magnetic dipoles	6.04	0.60
GP correction	1.61	0.20
Comagnetometer	0.21	0.10
Motional field	0.78	0.40
High voltage	0.53	0.25
Fitting adjustment	6.80	0.60
Total	9.90	1.00

- Targeting development efforts to reduce the major effects by an order of magnitude, minor effects by factor of 2 should give systematic uncertainty ~1.00x10⁻²⁷ ecm
- Combined with ~1.00x10⁻²⁷ ecm statistics this will result in a new bound on the nEDM of $|d_n| < 2.32 \times 10^{-27} ecm$

Additional Physics Requirements

- Several nEDM subsystems have statistics requirements
 - Losses
 - Polarization
 - Lifetime
 - Detector effectiveness
 - Electric field magnitude
- Other sources of uncertainty (degraded statistical precision) in the Ramsey measurement need to be addressed:
 - Comagnetometer accuracy
 - Clock precision
- Need subsystems to be reliable with regards to behaviour that would increase time and personnel demands
 - "Ruined" data runs due to sparks, etc.
- With a higher sensitivity, other systematics may appear that have not been previously studied
 - "Johnson-Nyquist Noise Studies on the n2EDM Experiment", Ping-Jung Chiu

References

- A revised experimental upper limit on the electric dipole moment of the neutron, Pendlebury et al., 2015
- Geometric-phase-induced false electric dipole moment signals for particles in traps, Pendlebury et al., 2004
- Apparatus for measurement of the electric dipole moment of the neutron using a cohabiting atomic-mercury magnetometer, Baker et al., 2013