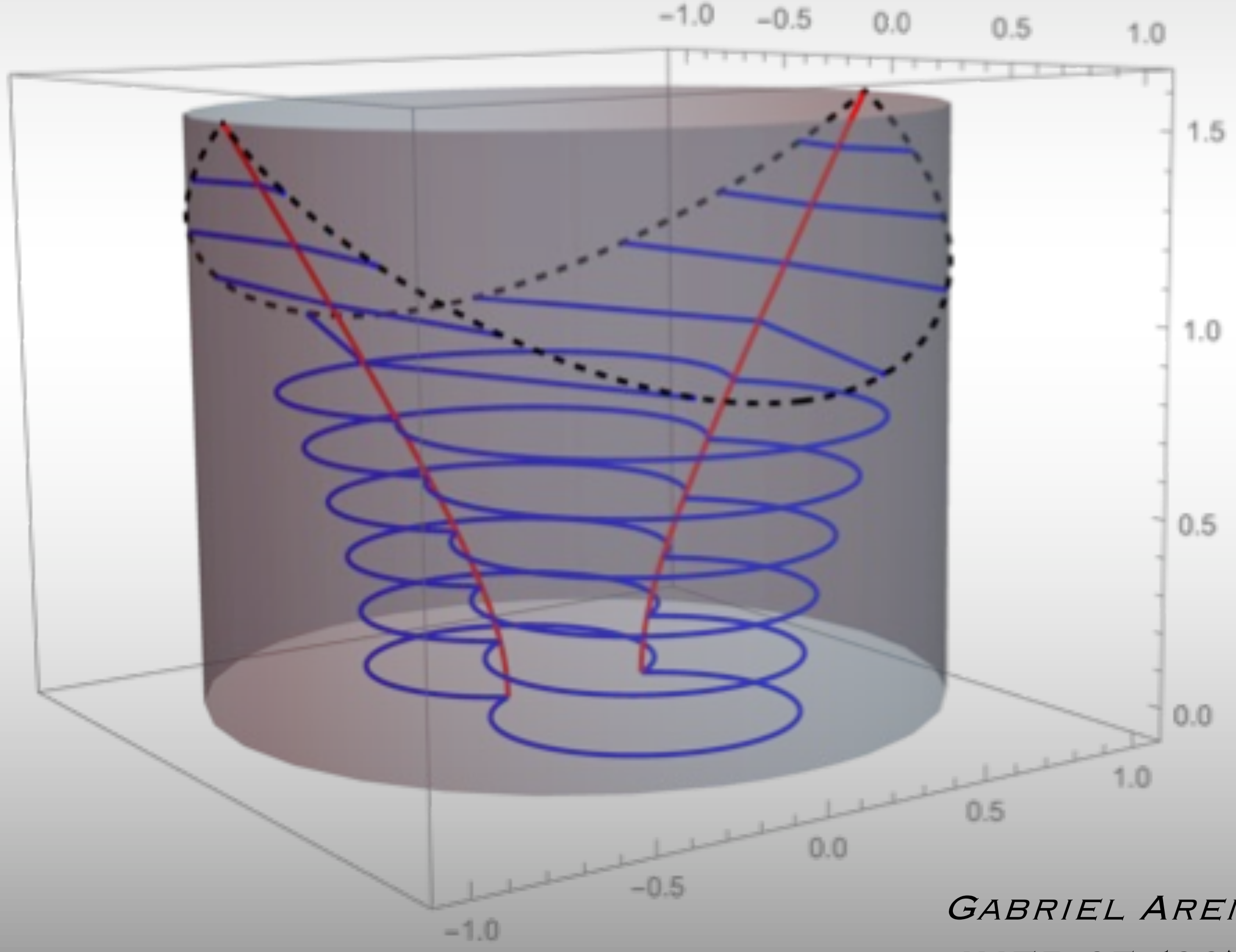
Walls, Bubbles & Israel



Ruth Gregory

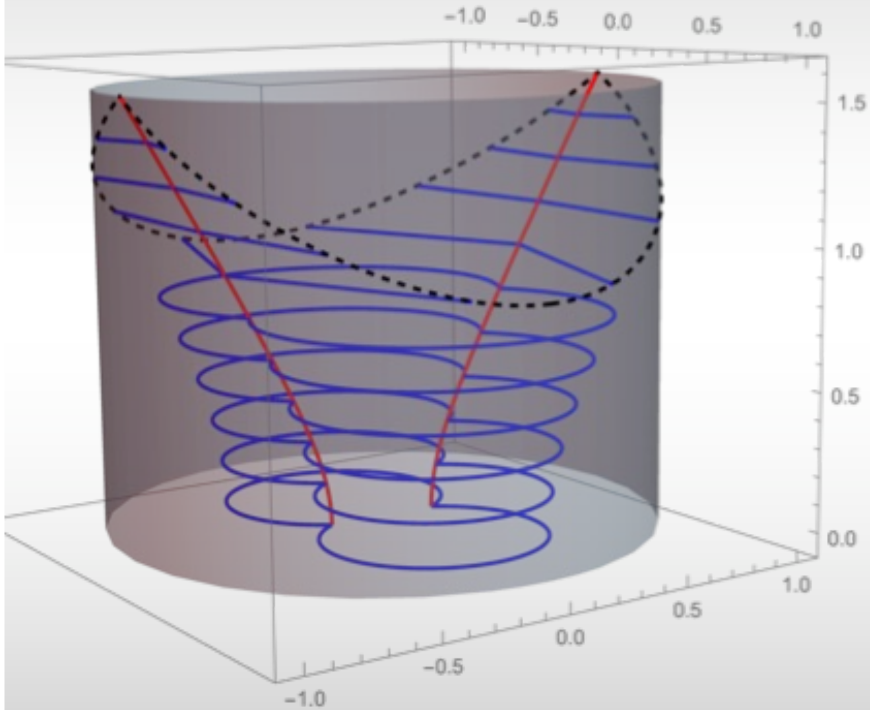
Israel Symposium

18/5/23

Gabriel Arenas-Henriquez, Andy Scoins

JHEP 05 (22) 063 & Thanks to Pavel Krtous

Outline



- The Israel Equations
- On acceleration
- Acceleration in 3D
- Geometry of acceleration

IL NUOVO CIMENTO

RIVISTA INTERNAZIONALE

ORGANO DELLA SOCIETÀ ITALIANA DI FISICA

SOTTO GLI AUSPICI DEL CONSIGLIO NAZIONALE DELLE RICERCHE
E DEL COMITATO NAZIONALE PER L'ENERGIA NUCLEARE

VOL. XLIV B, N. 1 | Serie decima | 11 Luglio 1966

Singular Hypersurfaces and Thin Shells in General Relativity.

W. ISRAEL

Department of Mathematics, University of Alberta - Edmonton (Alberta)

(ricevuto il 18 Ottobre 1965)

Summary. — An approach to shock waves, boundary surfaces and thin shells in general relativity is developed in which their histories are characterized in a purely geometrical way by the extrinsic curvatures of their imbeddings in space-time. There is some gain in simplicity and ease of application over previous treatments in that no mention of « admissible » or, indeed, any space-time co-ordinates is needed. The formalism is applied to a study of the dynamics of thin shells of dust.

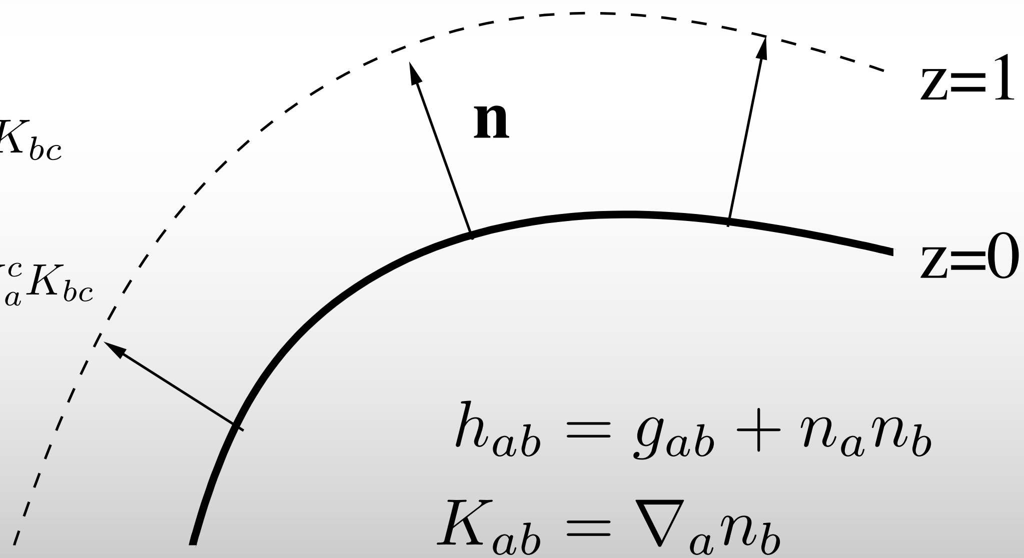
Israel Equations

Israel formulated the Einstein equations for a thin shell as a purely geometric relation – the jump in extrinsic curvature across the shell.

$$^{(3)}R^{a}{}_{bcd} = h^{a}_{a'} h^{b'}_{b} h^{c'}_{c} h^{d'}_{d} R^{a'}{}_{b'c'd'} - K^{a}_{c} K_{bd} + K^{a}_{d} K_{bc}$$

$$\frac{\partial K_{ab}}{\partial z} = 8\pi G(T_{ab} - \frac{1}{2} T h_{ab}) - {}^{(3)}R_{ab} - K K_{ab} + 2K^{c}_{a} K_{bc}$$

$$\boxed{[K_{ab}]^{+}_{-} = 8\pi G(S_{ab} - \frac{1}{2} S h_{ab})}$$



$$h_{ab} = g_{ab} + n_a n_b$$

$$K_{ab} = \nabla_a n_b$$

$$S_{ab} = \int_{-}^{+} T_{ab}$$

Example – AdS3

Take AdS_3 in global coordinates

$$ds^2 = \left(1 + \frac{R^2}{\ell^2}\right) dT^2 - \frac{dR^2}{(1 + \frac{R^2}{\ell^2})} - R^2 d\Theta^2$$

And perform the transformation:

$$1 + \frac{R^2}{\ell^2} = \frac{1 + (1 - A^2\ell^2) r^2/\ell^2}{(1 - A^2\ell^2)\Omega^2}\,, \qquad R\sin\Theta = \frac{r\sin\theta}{\Omega}$$

where $\Omega = 1 + Ar\cos\theta$

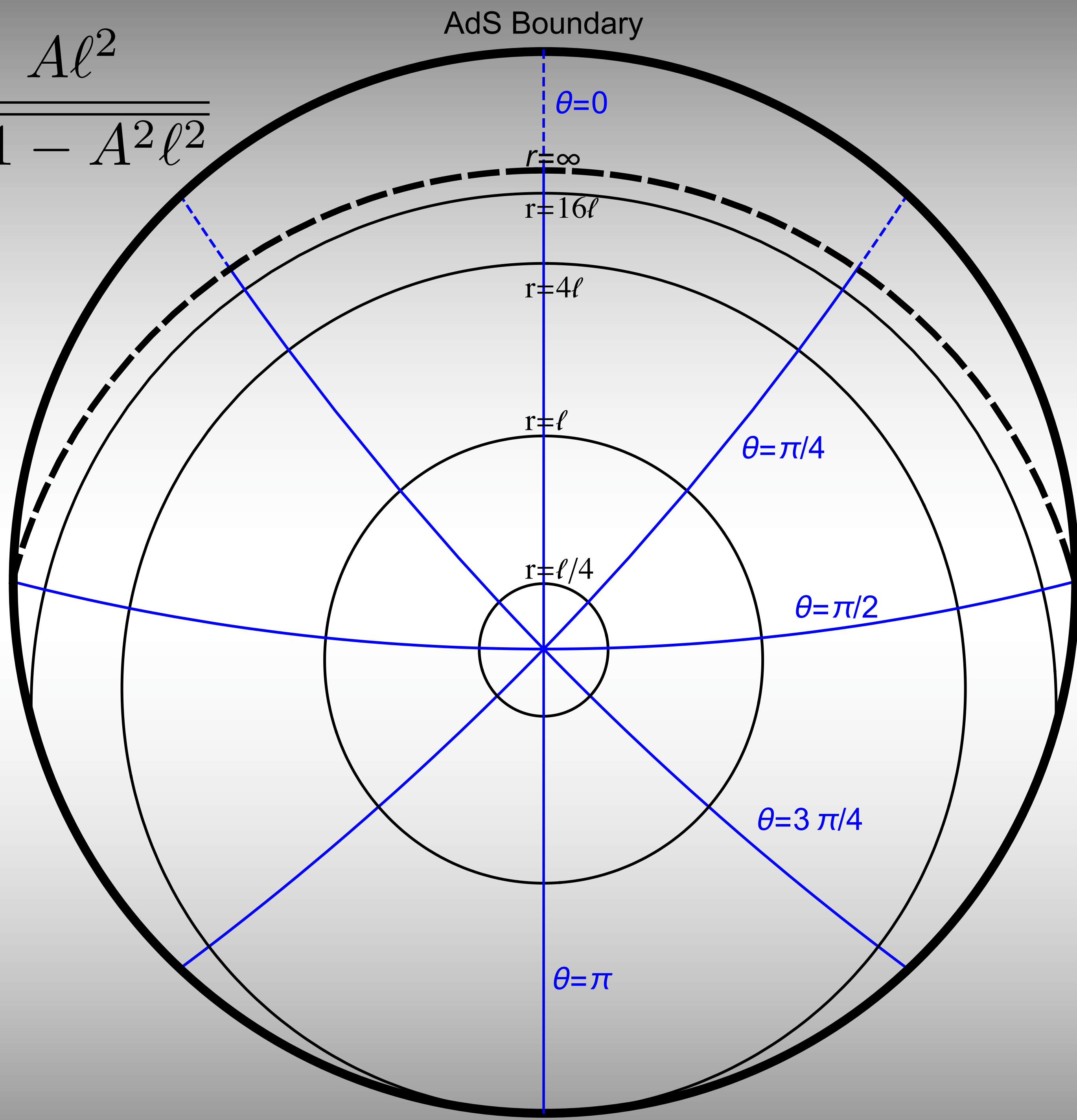
Then

$$ds^2 \to \Omega^{-2}\left[f(r)\frac{dT^2}{1 - A^2\ell^2} - \frac{dr^2}{f(r)} - r^2 d\theta^2\right]$$

where

$$f(r) = 1 - A^2 r^2 + \frac{r^2}{\ell^2}$$

$$r = 0 \leftrightarrow R_0 = \frac{A\ell^2}{\sqrt{1 - A^2\ell^2}}$$



What the transformation has done is shift the "origin" of coordinates to an off-centre position.

Cut Along Theta:

$$ds^2 = \Omega^{-2}\left[f(r)dt^2 - \frac{dr^2}{f(r)} - r^2 d\theta^2\right]$$

$$\theta = \theta_0$$
$$\mathbf{n} = \frac{r}{\Omega}d\theta$$

Cutting along a constant theta line and computing the extrinsic curvature shows that

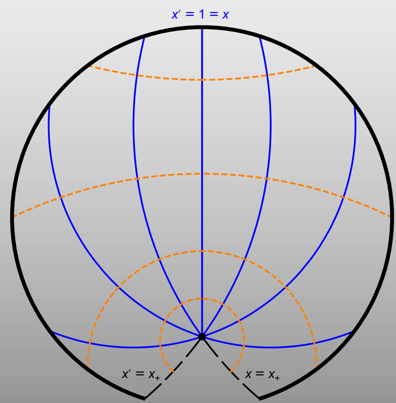
$$K_{ab} = -A\sin\theta_0 h_{ab}$$

So taking two copies of part of this space with $\theta \in [0, \theta_0]$

Shows that we have a domain wall of tension

$$\sigma = A\sin(\theta_0)/4\pi G$$

running from r=0 to the boundary

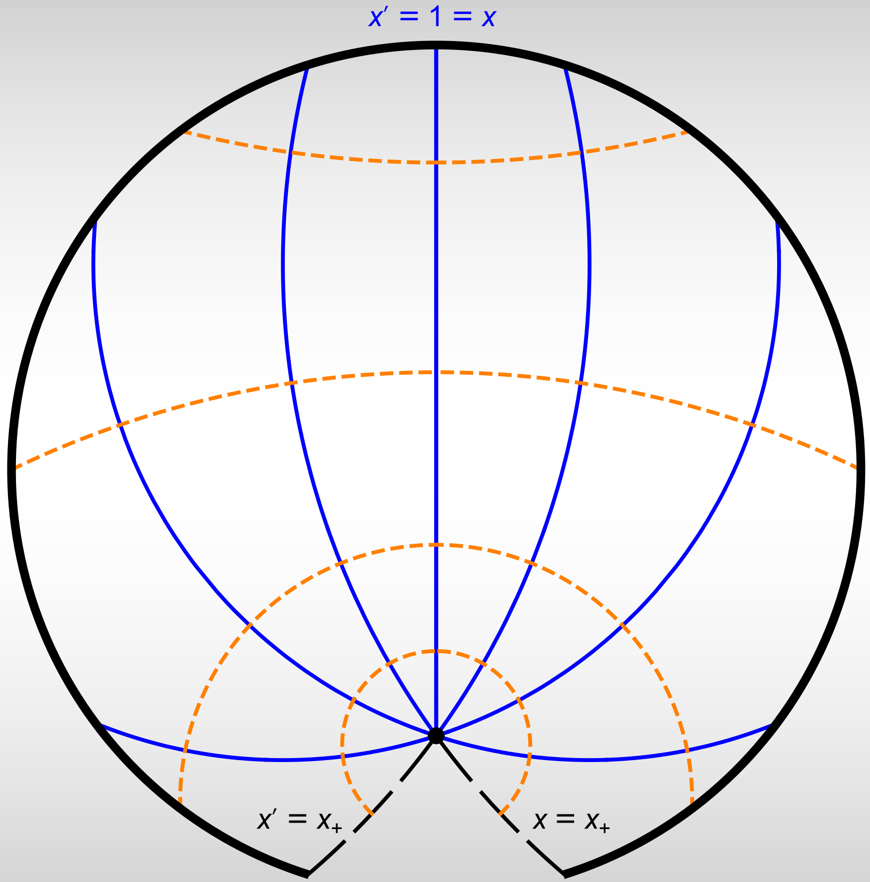


The Spacetime

Since the range of θ is $2\theta_0$, at r=0 there is a conical deficit. This is interpreted as a point particle in 3D.

The spacetime is therefore a point particle “attached” to the boundary by a domain wall.

This is an accelerating particle!



$$m_c = \frac{1}{4}\left(1 - \frac{\theta_0}{\pi}\right)$$

On Acceleration

Acceleration is when an object is not travelling on a geodesic.

$$\nabla_T T \not\propto T$$

For an observer at R=R_0 in AdS:

$$ds^2_{AdS} = -\left(1+\frac{R^2}{\ell^2}\right)dt^2 + \frac{dR^2}{1+\frac{R^2}{\ell^2}} + R^2\left(d\Theta^2 + \sin^2\Theta d\phi^2\right)$$

The tangent vector is purely timelike, but the acceleration is radial:

$$\mathbf{T} = \frac{1}{\sqrt{1+\frac{R_0^2}{\ell^2}}}\frac{\partial}{\partial t} \qquad \mathbf{A} = \nabla_T T = \frac{R_0}{\ell^2}\frac{\partial}{\partial r}$$

Rindler with no Horizon

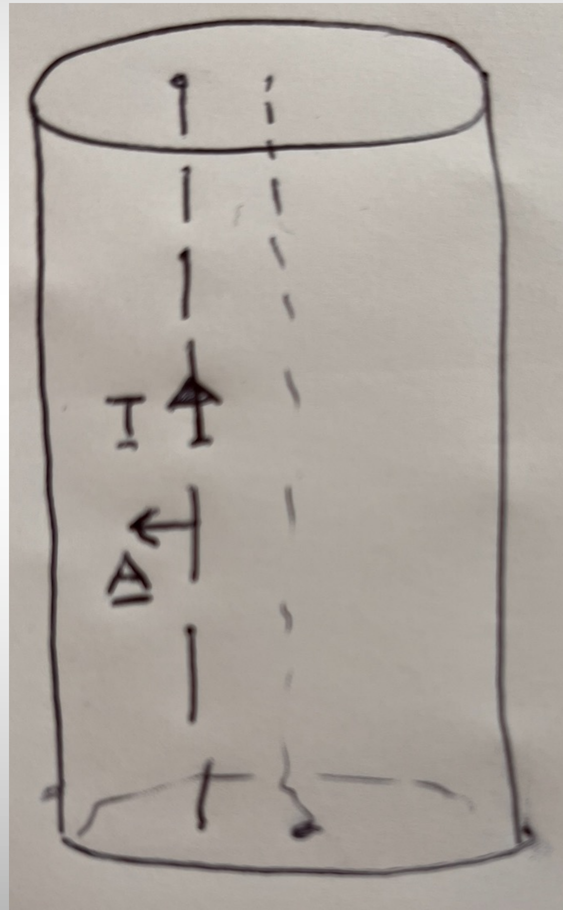
The magnitude of the acceleration is related to R_0

$$\mathbf{T} = \frac{1}{\sqrt{1 + \frac{R_0^2}{\ell^2}}} \frac{\partial}{\partial t}$$

$$|\mathbf{A}|^2 = \frac{R_0^2/\ell^4}{1 + R_0^2/\ell^2}$$

$$\mathbf{A} = \frac{R_0}{\ell^2} \frac{\partial}{\partial r}$$

$$R_0 = \frac{A\ell^2}{\sqrt{1 - A^2\ell^2}}$$



Slowly Accelerating Rindler

Using the coordinate transformation as before

$$1+\frac{R^2}{\ell^2}=\frac{1+(1-A^2\ell^2)r^2/\ell^2}{(1-A^2\ell^2)\Omega^2}\,, \qquad R\sin\Theta=\frac{r\sin\theta}{\Omega}$$

To get back to global AdS

$$ds^2_{AdS}=-\left(1+\frac{R^2}{\ell^2}\right)\alpha^2dt^2+\frac{dR^2}{1+\frac{R^2}{\ell^2}}+R^2\left(d\Theta^2+\sin^2\Theta\frac{d\phi^2}{K^2}\right)$$

And see consistency with (slow) acceleration and pure AdS.

Acceleration in 4D

In 4D, we use the C-metric to describe accelerating black holes

$$ds^2 = \Omega^{-2}\left[f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(\frac{d\theta^2}{g(\theta)} + g(\theta)\sin^2\theta\frac{d\phi^2}{K^2}\right)\right]$$

Where

$$f = \left(1 - \frac{2m}{r}\right)\left(1 - A^2r^2\right) + \frac{r^2}{\ell^2}$$

$$g = 1 + 2mA\cos\theta$$

$$\Omega = 1 + Ar\cos\theta$$

f determines horizon structure – black hole / acceleration / cosmological constant

Hong & Teo, CQG20 3629 (2003)

"C" IN 3?

Look for an exact solution in 3D with the same type of structure:

$$ds^2 = \frac{1}{A^2(x-y)^2}\left[P(y)d\tau^2 - \frac{dy^2}{P(y)} - \frac{dx^2}{Q(x)}\right]$$

The general solution is: $Q(x) = c + bx + ax^2\,,\qquad P(y) = \frac{1}{A^2\ell^2} - Q(y)$

Which, after coordinate rescaling/shifts reduces to:

Class	$Q(x)$	$P(y)$	Maximal range of x
I	$1-x^2$	$\frac{1}{A^2\ell^2} + (y^2-1)$	$\|x\| < 1$
II	x^2-1	$\frac{1}{A^2\ell^2} + (1-y^2)$	$x > 1$ or $x < -1$
III	$1+x^2$	$\frac{1}{A^2\ell^2} - (1+y^2)$	$\mathbb{R}$

Arenas-Henriquez, RG, Scoins: 2202:08823; Anber 0809:2789; Astorino 1101.2616

Class I

Take each in turn. The first class looks very similar to the 4D C-metric ($r=-1/Ay$, $t = \alpha\tau/A$, $x = \cos(\phi/K) = \cos(\theta)$)

$$ds^2 = \frac{1}{\left[1 + Ar\cos\left(\phi/K\right)\right]^2}\left[f(r)\frac{dt^2}{\alpha^2} - \frac{dr^2}{f(r)} - r^2\frac{d\phi^2}{K^2}\right]$$

$$f(r) = 1 + (1 - A^2\ell^2)\frac{r^2}{\ell^2}$$

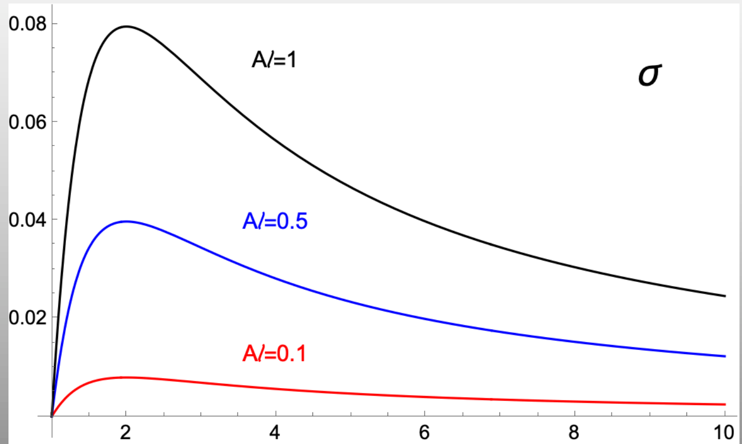
Slow Acceleration	$A\ell < 1$	No horizon
Rapid Acceleration	$A\ell > 1$	Acc. horizon

Slow Acceleration

The presence of K now indicates both a conical deficit (the particle) and a *domain wall* at $\phi=\pm\pi$, i.e. codimension 1 defect. The conical deficit at r=0 has a natural mass:

$$m_c = \frac{1}{4}\left(1 - \frac{1}{K}\right)$$

Because of the nonzero extrinsic curvature along $\phi=\pm\,\pi$, (thanks to A) there is a wall of tension



$$\sigma = \frac{A}{4\pi}\sin\left(\frac{\pi}{K}\right)$$

Particle Mass?

Can follow a Fefferman-Graham prescription, giving the expected boundary metric:

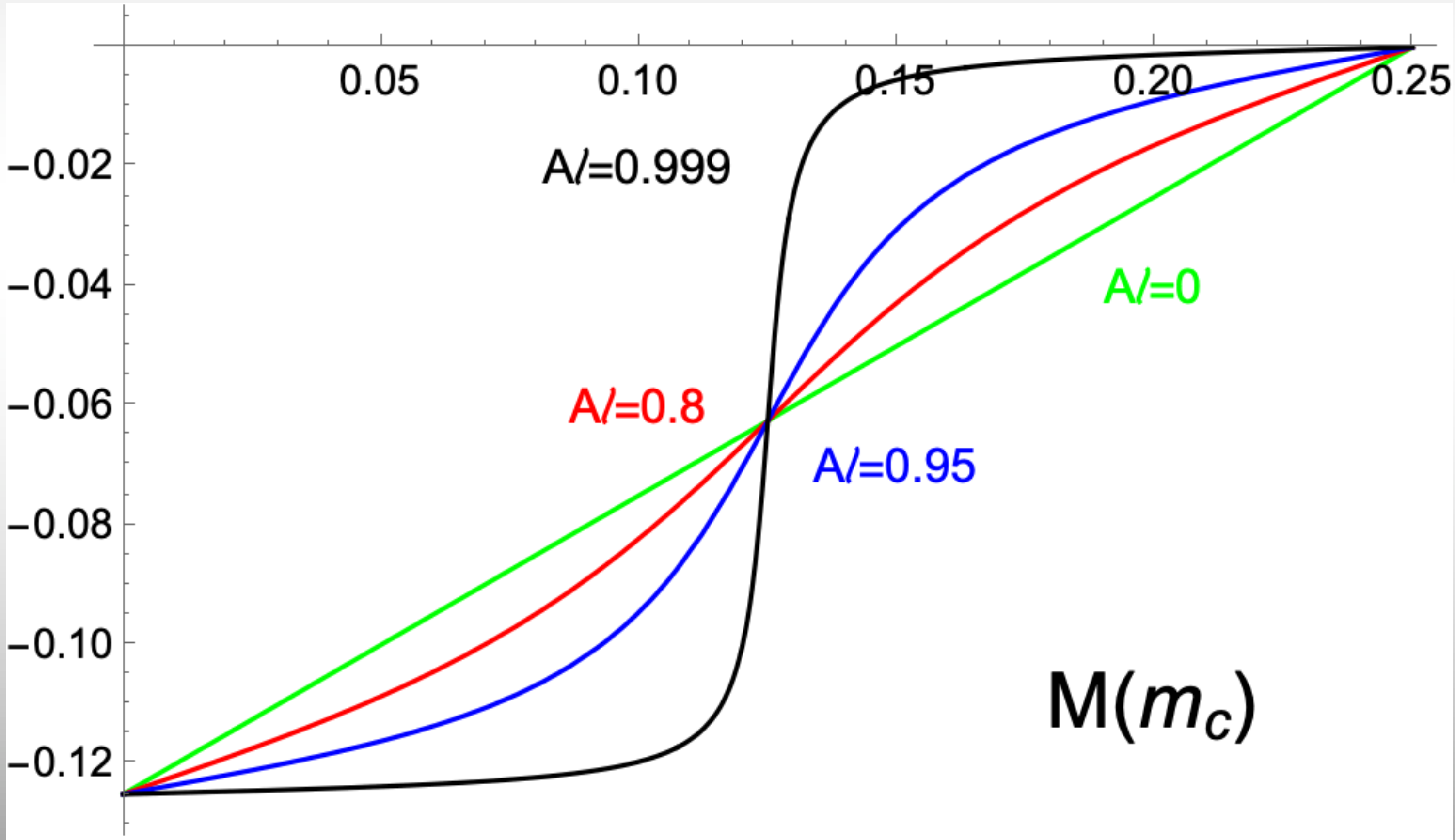
$$\gamma_0 = \frac{\omega(\xi)^2}{A^2}\left[d\tau^2 - A^2\ell^2\frac{d\xi^2}{1-\xi^2}\right]$$

And find the mass from the stress tensor:

$$M = -\frac{1}{8\pi}\left(\frac{\pi}{2} - \arctan\left[\frac{\cot\left(\frac{\pi}{K}\right)}{\sqrt{1-A^2\ell^2}}\right]\right)$$

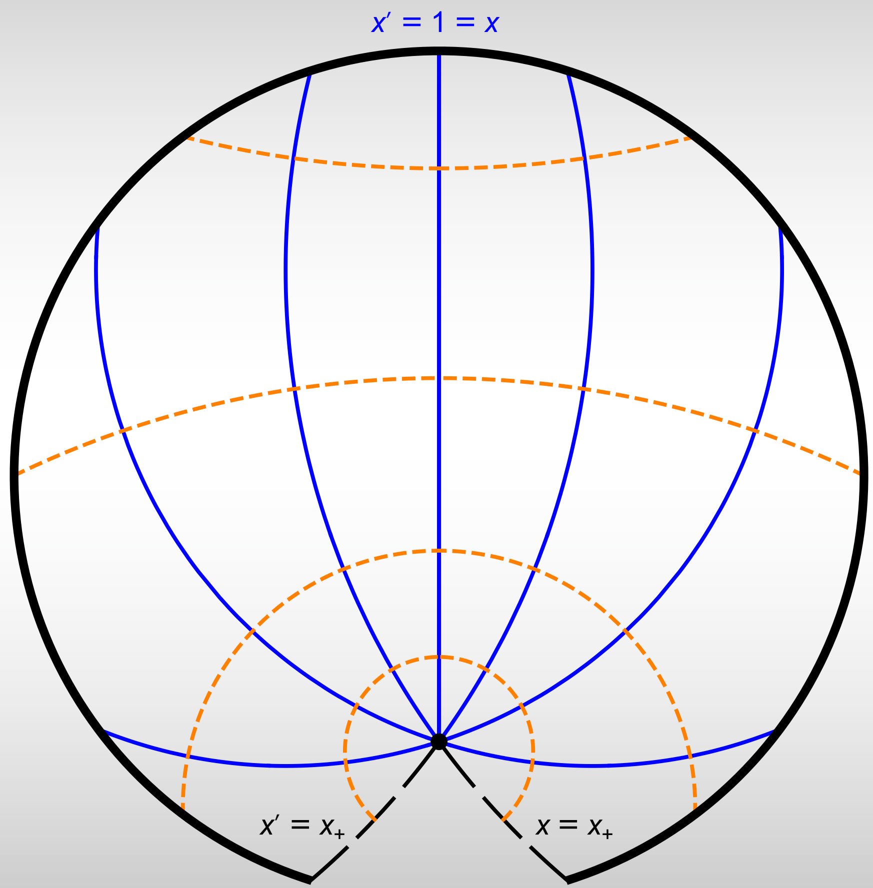
Holographic Mass

Compare to "particle" mass from conical deficit:

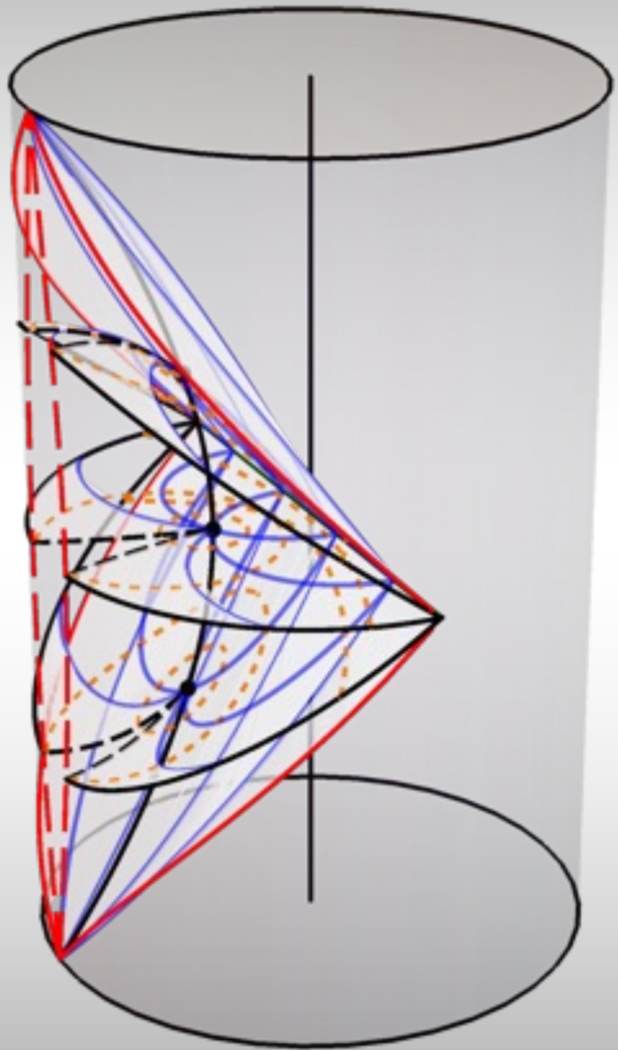


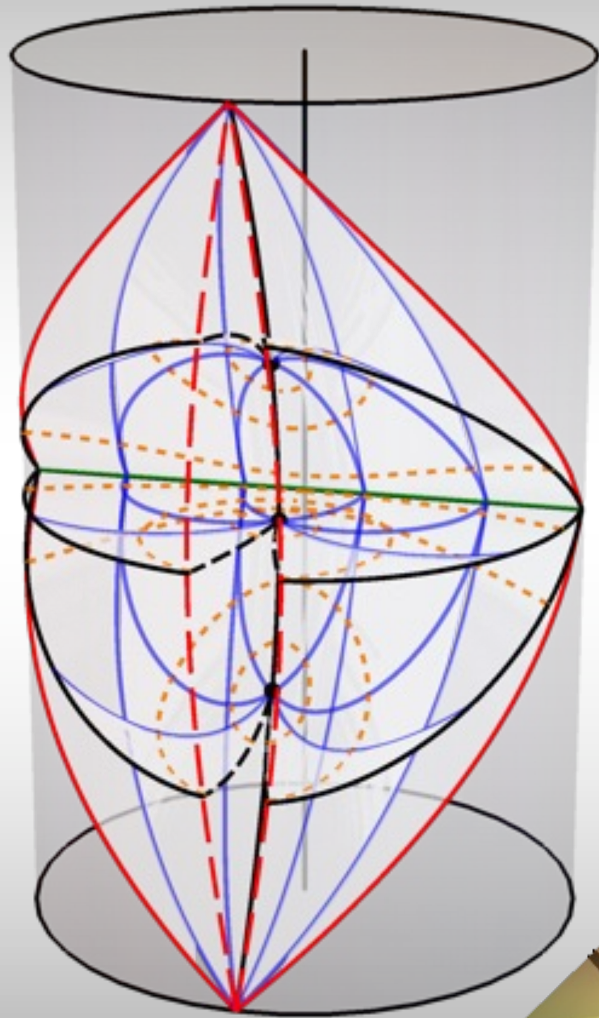
New Solutions?

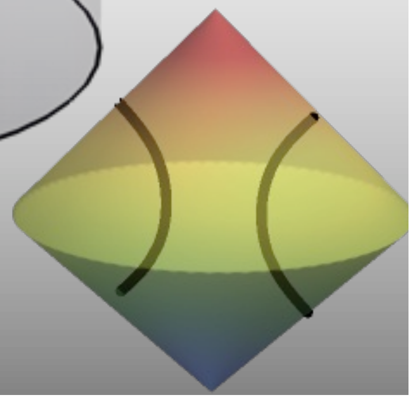
Although these have been derived as “new” solutions, we know that in 3D, gravity does not propagate, so any ”vacuum” solution has to be locally equivalent to AdS. The transformation formulae for the various solutions are quite lengthy, but give an interesting alternative viewpoint, and help with understanding the “BTZ” family of solutions.



Rapidly accelerating light Particle

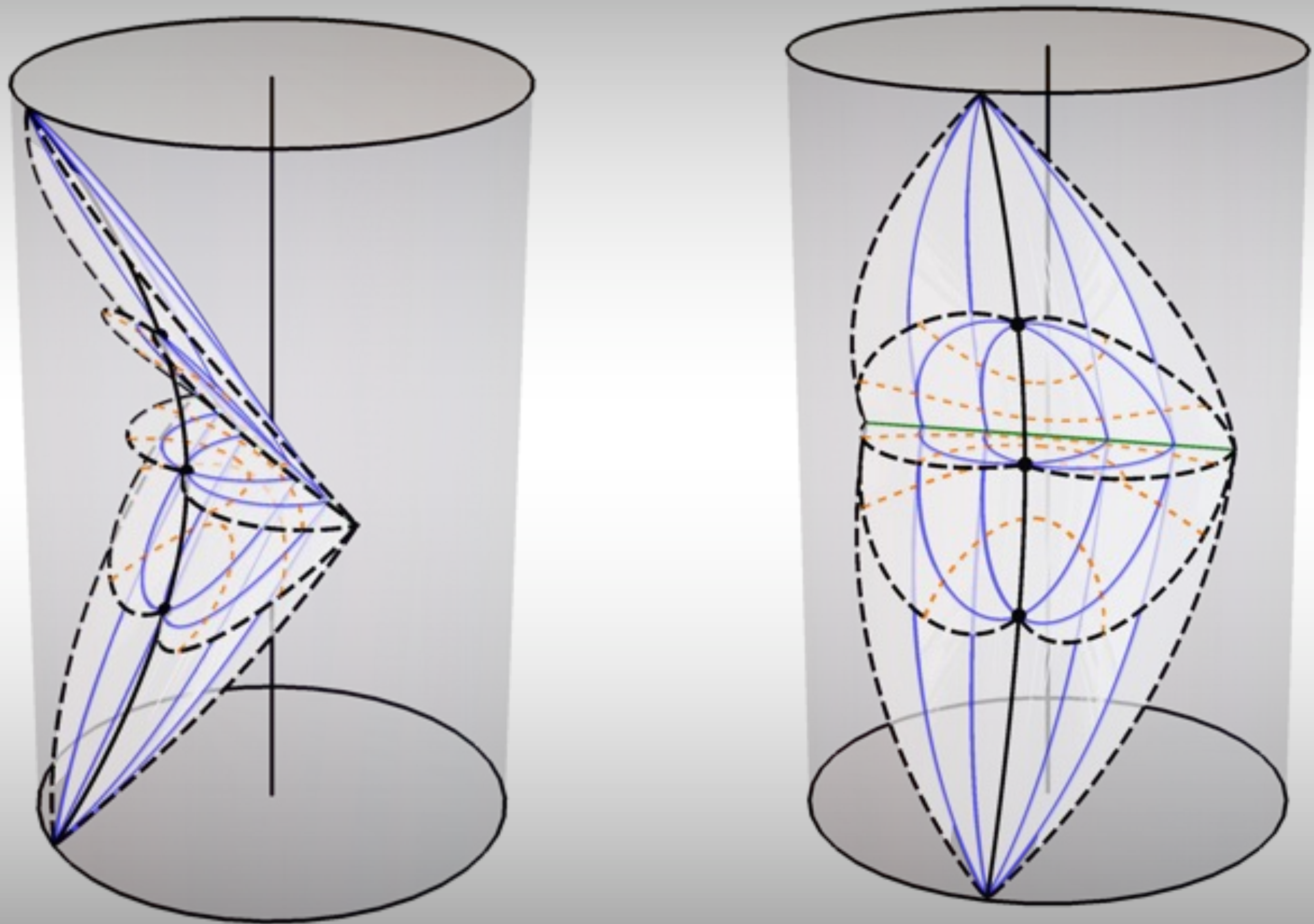




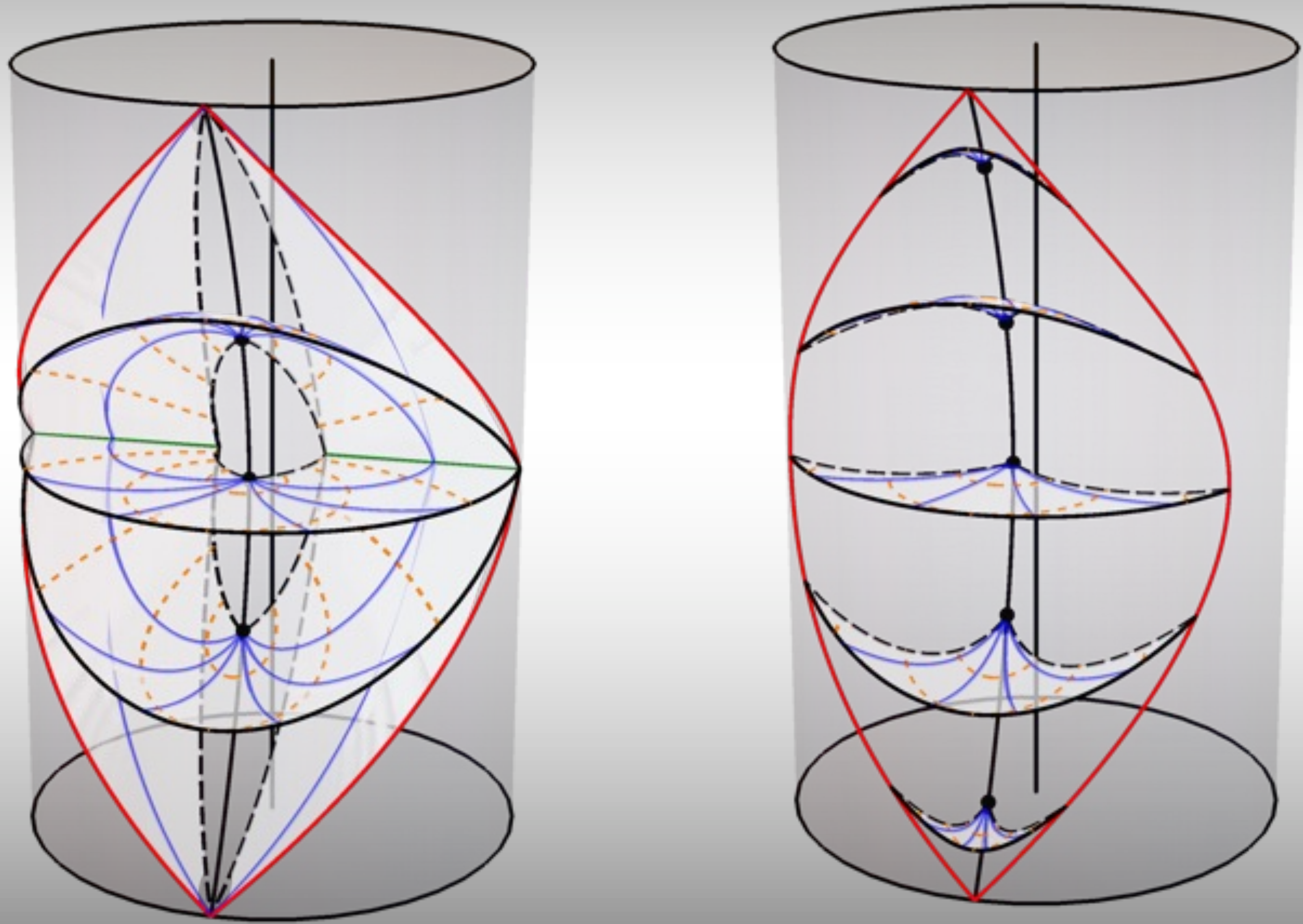


Main difference to 4D: no accelerating partner!

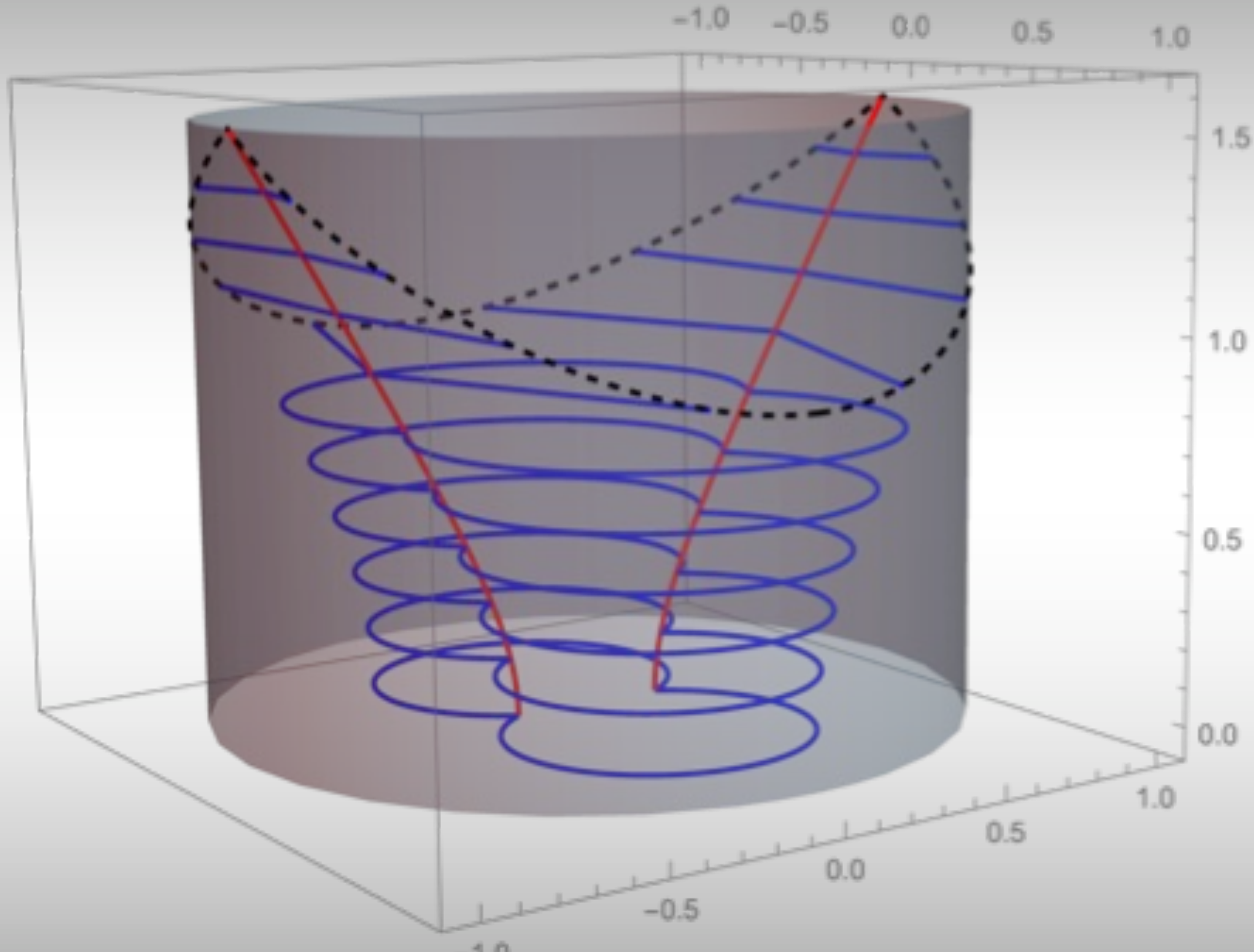
Rapidly accelerating heavy Particle



Acceleration with Struts



Rapidly accelerating heavy particle – full bulk.

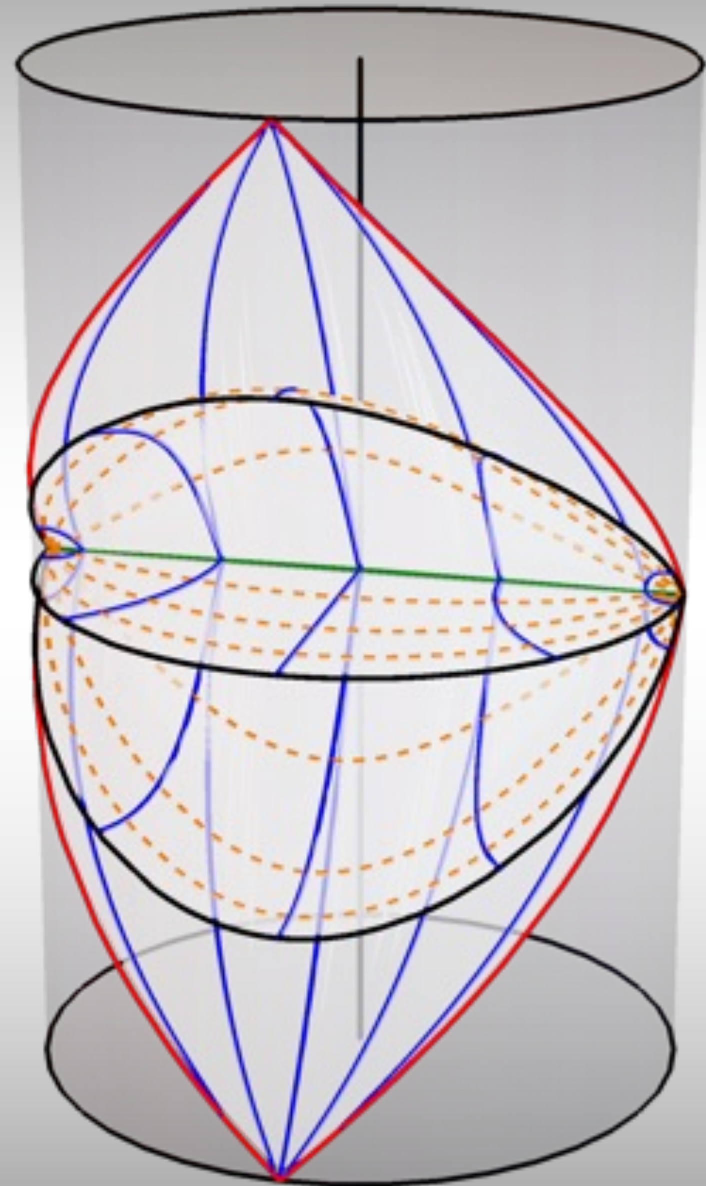


Would like to understand bulk and holographic nature of 3D solutions, as well as thermal back-reaction.

BTZ

Recall the BTZ black hole is an identification of the Rindler wedge:

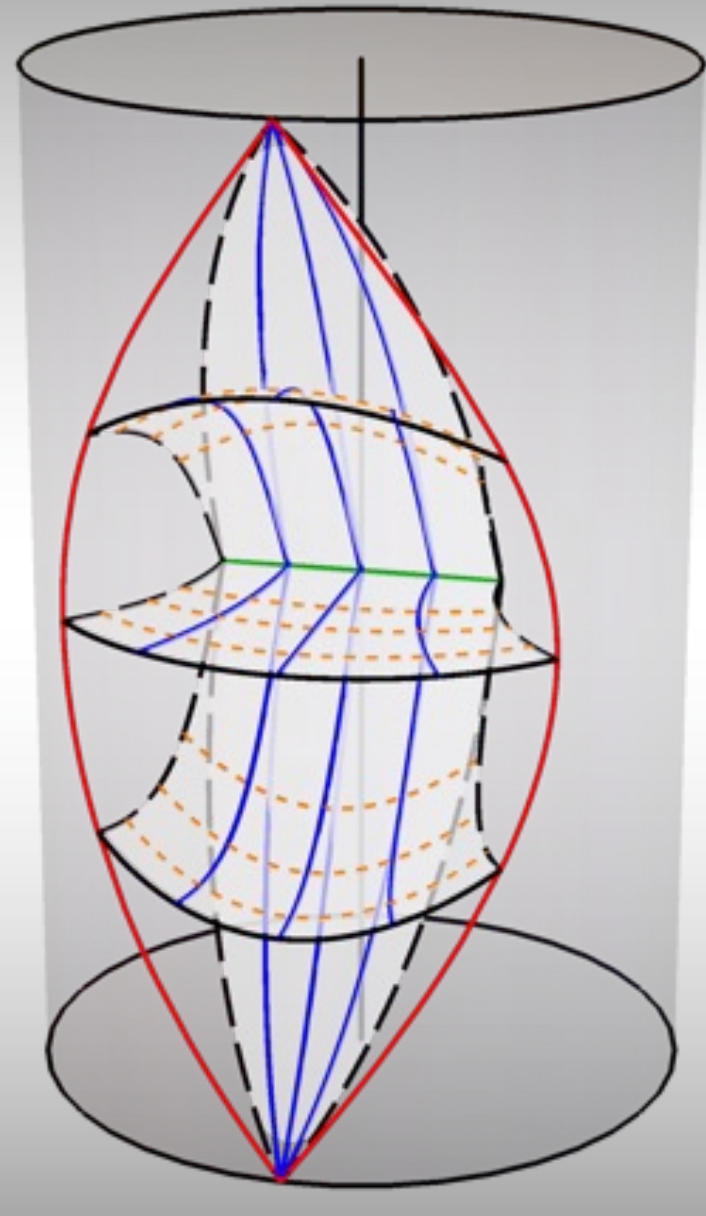
Blue lines are constant ϕ, and have zero extrinsic curvature,



BTZ

Recall the BTZ black hole is an identification of the Rindler wedge:

Blue lines are constant ϕ, and have zero extrinsic curvature, so can cut and paste along ϕ–lines to form the BTZ black hole.



BTZ's as class II

Looking at BTZ from the exact solution perspective:

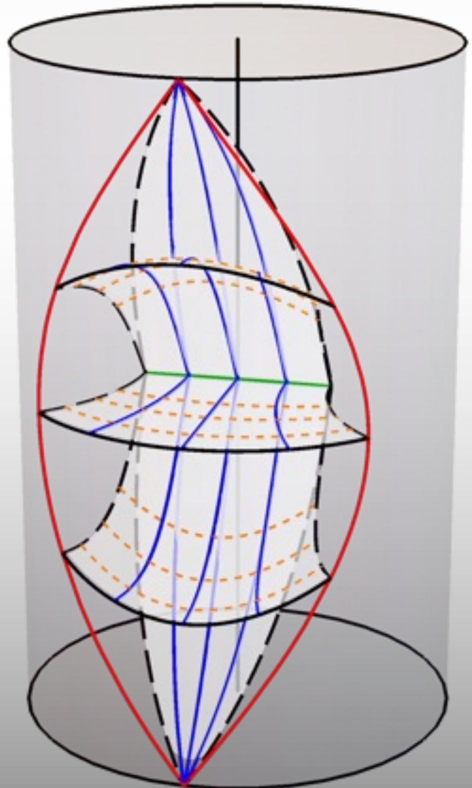
$$ds^2 = \frac{1}{\Omega(r,\psi)^2}\left[F(r)\frac{d\tilde{t}^2}{\alpha^2} - \frac{dr^2}{F(r)} - r^2 d\psi^2\right] ,$$

$$F(r) = -m^2(1-\mathcal{A}^2 r^2) + \frac{r^2}{\ell^2} ,$$

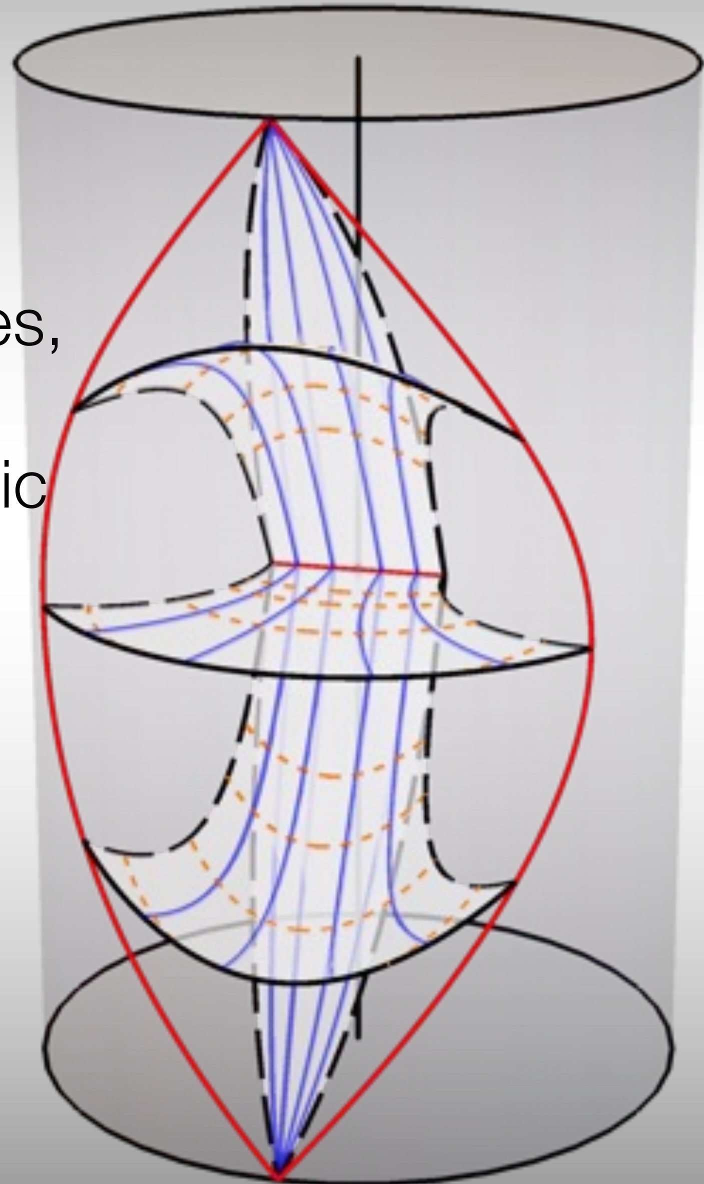
$$\Omega(r,\psi) = 1 + \mathcal{A} r \cosh(m\psi)$$

$$\begin{pmatrix} K = 1/m \\ A = m\mathcal{A} \end{pmatrix}$$

BTZ's

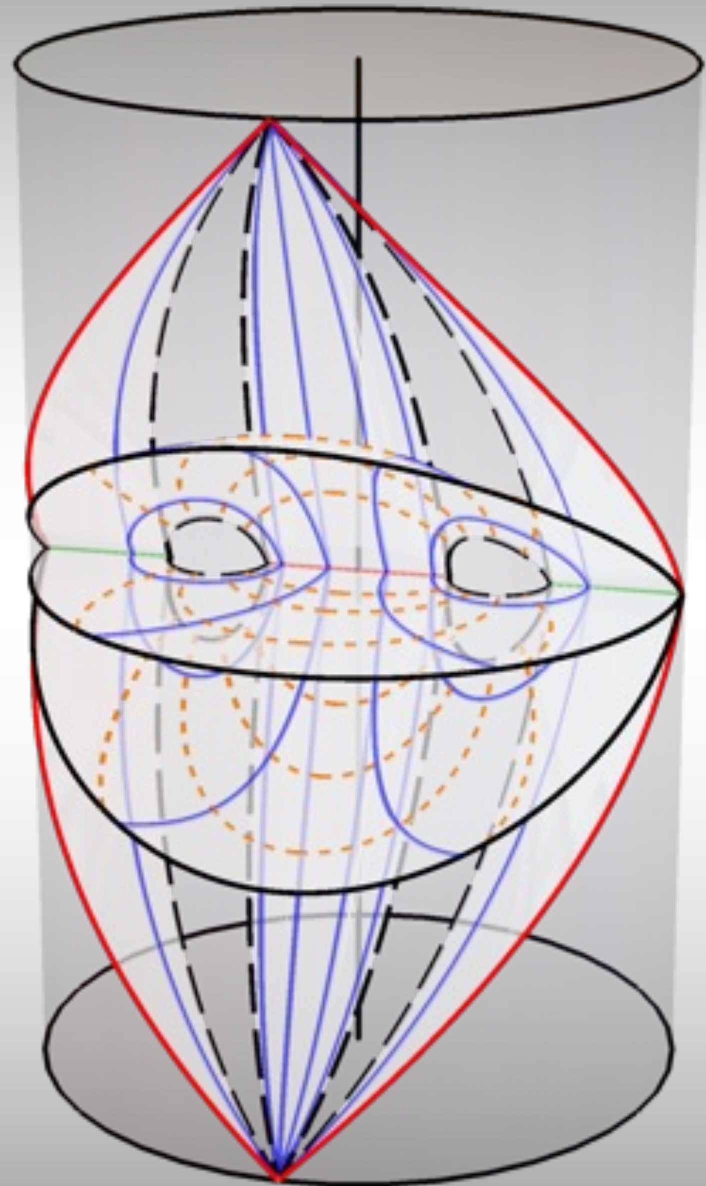


Adding A in the class 2's skews the constant ϕ lines, changing the way AdS is sliced and adding extrinsic curvature to constant ϕ–lines – here is a slightly distorted BTZ (slow acceleration)



Rapid BTZ's

Because the distorted ϕ–lines now wrap back to the Rindler wedge horizon, for some values of ϕ we get an “additional” horizon (different portions of the bulk Rindler horizon).



Novel BTZ

Hiding within class I is a new BTZ-like solution. If Al>1, have a horizon at

$$y_h^2 = 1 - \frac{1}{A^2\ell^2}$$

For the accelerating particle, we usually take y<-y_h with y ~ -1/Ar, but can also have $y \in (y_h, x) \qquad x \in (x_+, 1)$

To make this look more familiar, take

$$\tau = \frac{At}{\alpha}, \qquad y = \frac{1}{A\rho}, \qquad x = \cos(\phi/K)$$

where

$$K = \pi/\arccos(x_+) > \pi/\arccos(y_h) > 2$$

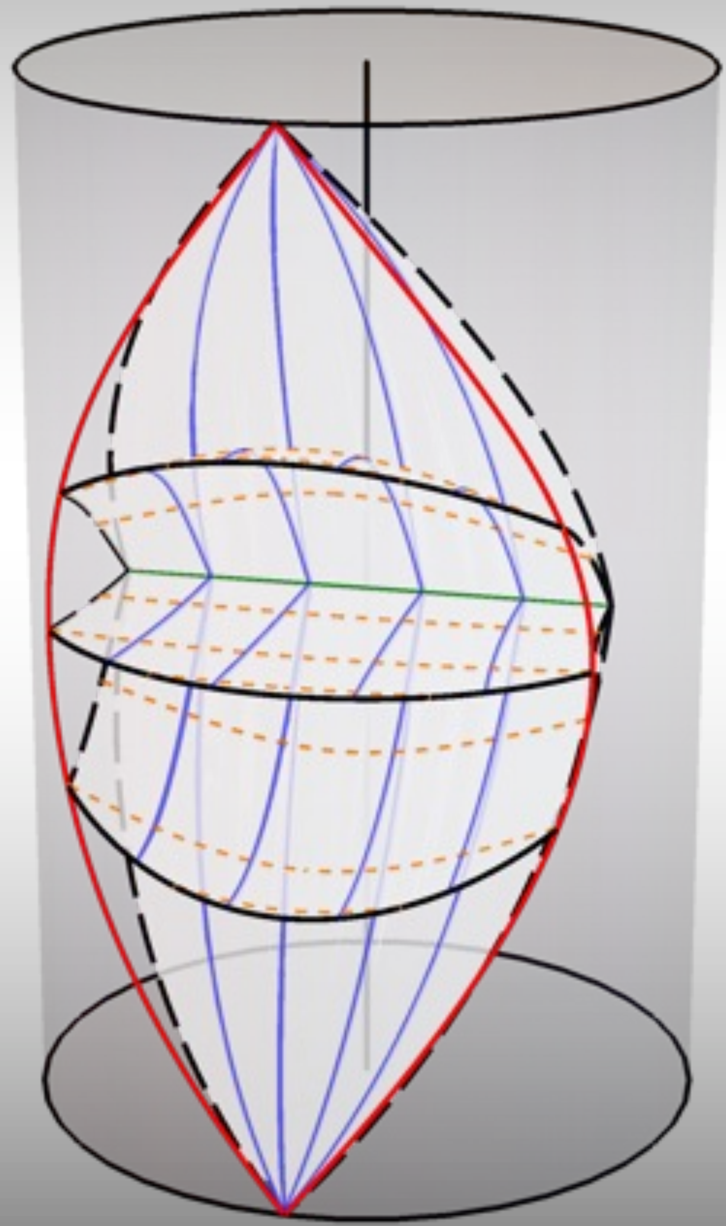
So that this solution is clearly disconnected from the non-accelerating solutions.

$$P(y) = \frac{1}{A^2\ell^2} - 1 + y^2$$

$$ds^2 = \frac{1}{\left[A\rho\cos\left(\frac{\phi}{K}\right) - 1\right]^2}\left(f(\rho)\frac{dt^2}{\alpha^2} - \frac{d\rho^2}{f(\rho)} - \rho^2\frac{d\phi^2}{K^2}\right)$$

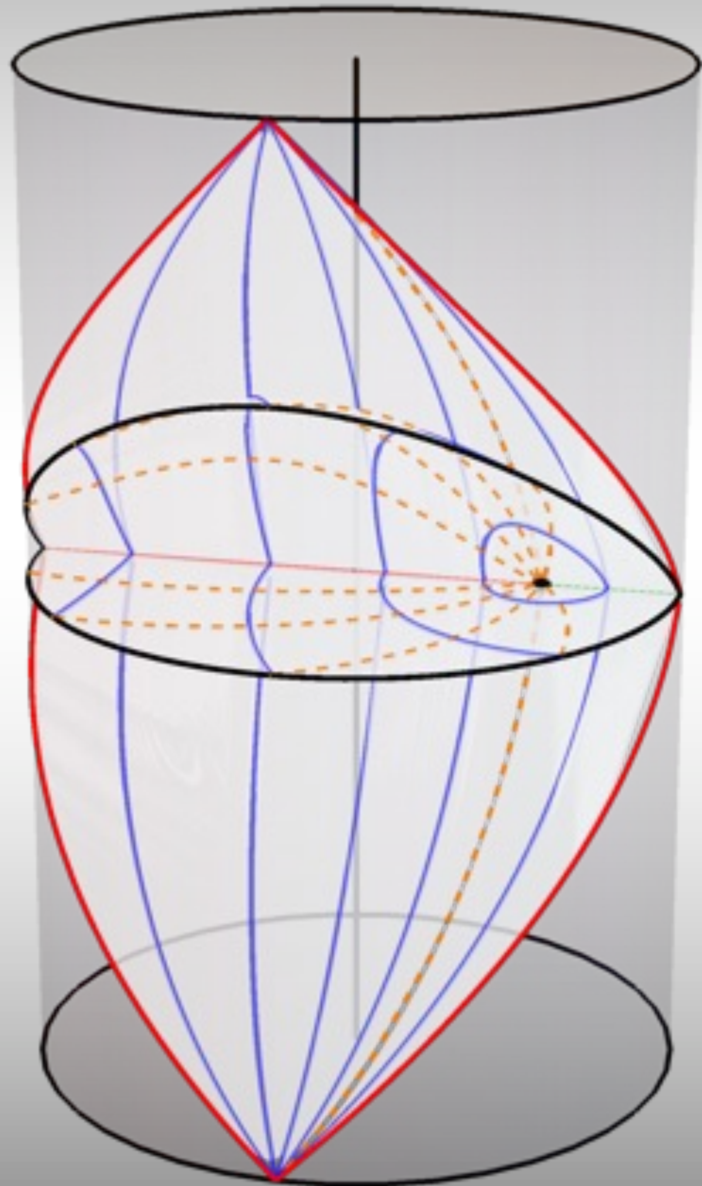
$$f(\rho) = 1 - (A^2\ell^2 - 1)\rho^2/\ell^2$$

Plotting this solution in global coordinates shows a clear parallel with BTZ. This time however, there is no continuous link to the BTZ metric.



Class III - braneworld

Finally, the class 3 solutions don't allow for an identification of a single bulk with a wall, instead we take 2 bulk copies a la Randall-Sundrum to form a braneworld.



Recap

- Have classified all possible 3D AdS solutions with “acceleration”.
- Class I are (mostly) accelerating “particles”
- Class II are generalized accelerating BTZ.
- Class III are braneworld-type solutions
- Have found a new solution (in Class I) that is BTZ in nature but disconnected parametrically from BTZ.

Thank you Werner!

- Domain walls
- Braneworlds
- Instantons
- PBH seeded vacuum decay