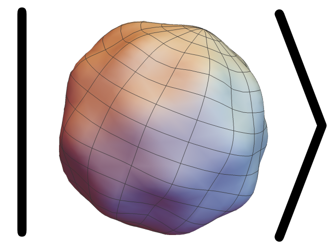
Counting States of Quantum Space

Ted Jacobson

(on work with various combinations of Batoul Banihashemi, Andy Svesko, and Manus Visser)

Werner Israel Memorial Symposium
University of Victoria, May 18, 2023



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Motivations

- Why the minus sign in the Gibbons-Hawking (GH) first law of de Sitter space?
- What does the GH entropy of de Sitter space count?
- Can we generalize the dS count to quasi-local quantum gravity?
- Is the maximal vacuum entanglement hypothesis valid?

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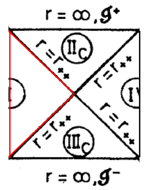
The first law in de Sitter spacetime

Gibbons & Hawking, 1977



$$\int \delta T_{ab} K^a d\Sigma^b = -\kappa_c \delta A_C (8\pi)^{-1}$$

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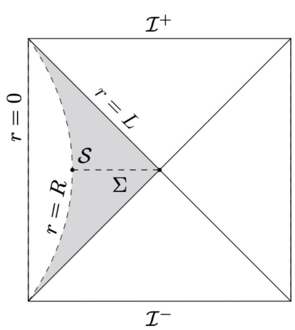
First law of dS with a York boundary

Banihashemi, Svesko, TJ, Visser (2208.11706)



$$T\delta S_{\mathrm{BH}} = \delta E_{\mathrm{BY}} - \delta E_{\mathrm{matter}}$$

$$E_{\mathrm{BY}} = -\frac{1}{8\pi G}\oint_{\mathcal{S}} d^2x\sqrt{\sigma}\,k \quad \text{and} \quad E_{\mathrm{matter}} = \int_{\Sigma} T_{\mu\nu}\xi^{\mu} d\Sigma^{\nu}$$

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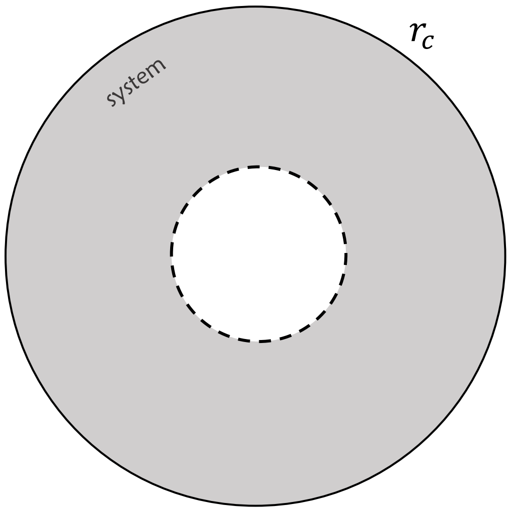

Shrink the boundary to zero size

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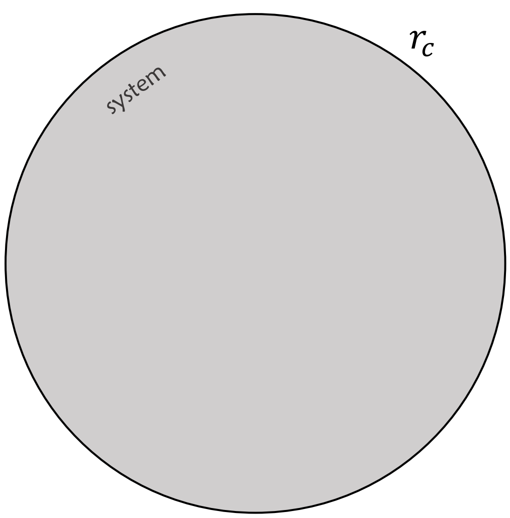

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First law of dS with a York boundary

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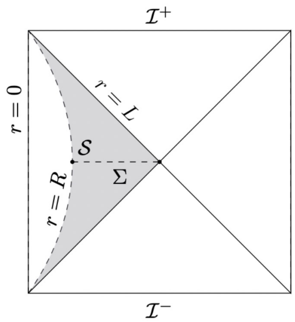

$$T\delta S_{\rm BH} = \delta E_{\rm BY} - \delta E_{\rm matter}$$

$E_{\rm BY} = -\frac{1}{8\pi G}\oint_{\mathcal{S}} d^2x\sqrt{\sigma}\,k$ and $E_{\rm matter} = \int_\Sigma T_{\mu\nu}\xi^\mu d\Sigma^\nu$

As $R\to 0$, $E_{\rm BY}\to 0$ and $T\to T_{\rm GH} = \hbar/2\pi L$

$$\Longrightarrow \quad T_{\rm GH}\delta S_{\rm BH} = -\delta E_{\rm matter}$$

Demystifies the minus sign.

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Path integral representation of Z

In QM, $Z = \mathrm{Tr} e^{-\beta H} = \int_{\text{x period }\beta} \mathcal{D}x\; e^{-I_E/\hbar}$ $\quad I_E = \int d\tau\left[\frac{1}{2}m(dx/d\tau)^2 + V(x)\right]$

"Euclidean" action

In QFT, $x(\tau) \longrightarrow \phi(\vec{x},\tau)$, path in the space of field configurations.

In GR, $\longrightarrow g_{\mu\nu}(x)$, (Euclidean?) (complex?) spacetime metric; must include GHY boundary term, respect constraints & fix gauge.

linearized case: Schleich (1987), Hartle & Schleich (1987)

Let's first see how this works for a nonrotating thermal gravitational ensemble, then a black hole, and then a cosmological horizon…

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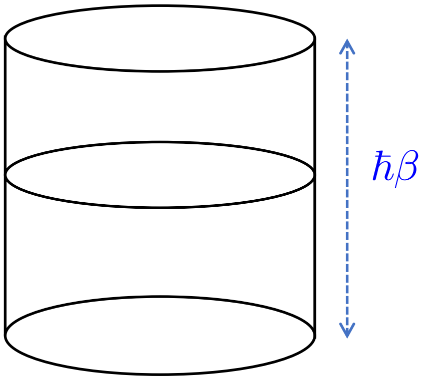

The boundary might be at asymptotic infinity, or a finite boundary (York), with suitable boundary conditions.

Could describe a thermal gas of gravitons; or, with matter added, a star in thermal equilibrium with a heat bath. How about an ensemble that dominated by a black hole?

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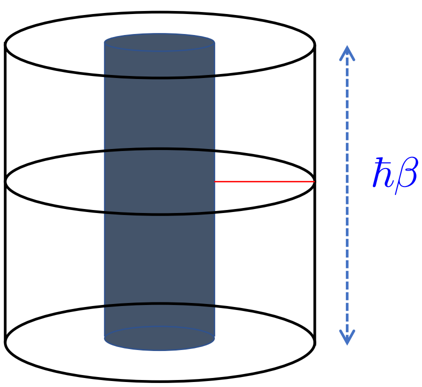


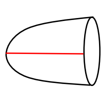
If the action is very large compared to Planck's constant, the path integral can perhaps be estimated as:

$$Z \sim \exp(-I_E^{\text{minimum, or saddle}}/\hbar)$$

$$S = \left(\beta \frac{d}{d\beta} - 1\right) I_E^{\text{saddle}}/\hbar = \frac{\text{horizon area}}{4\hbar G}$$

Gibbons & Hawking obtained this from the *outer* boundary term (integral of extrinsic curvature) at infinity, so the appearance of the horizon area is surprising, but it can also be obtained from a horizon contribution via other computational methods (snip the tip, or treat mass and period variations separately).

Euclidean:



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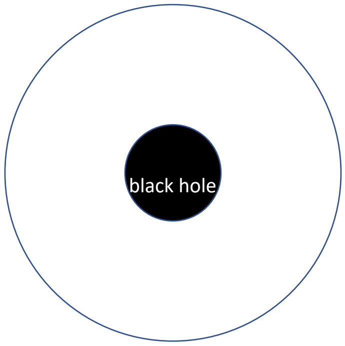

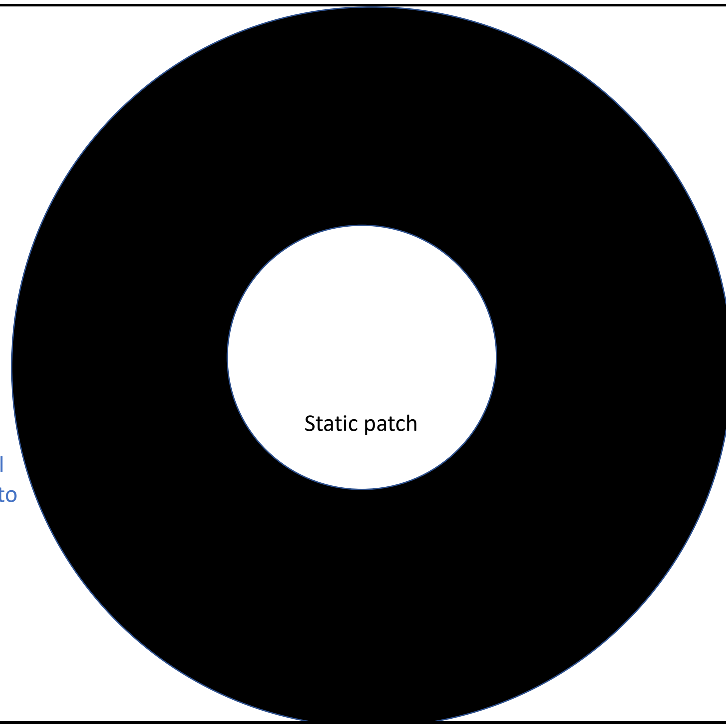

No boundary in the patch surrounded by a cosmological horizon, so there is nowhere to specify the ensemble…
So what is the meaning of Z?

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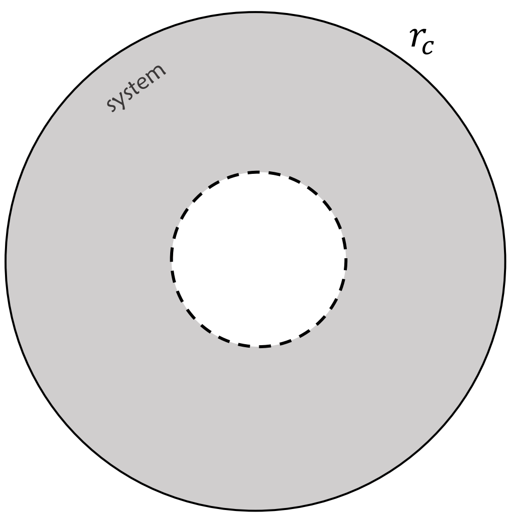
Gibbons-Hawking sphere partition function for de Sitter



Shrink the boundary to zero size

Topology $D^2 \times S^2 \rightarrow S^4$

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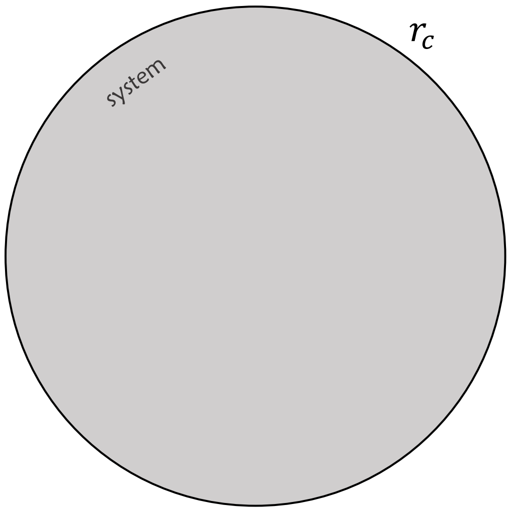

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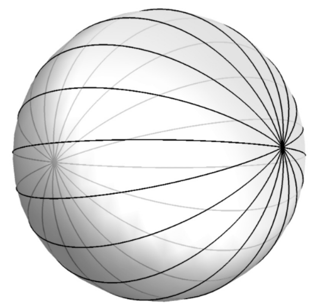

2-dimensional version

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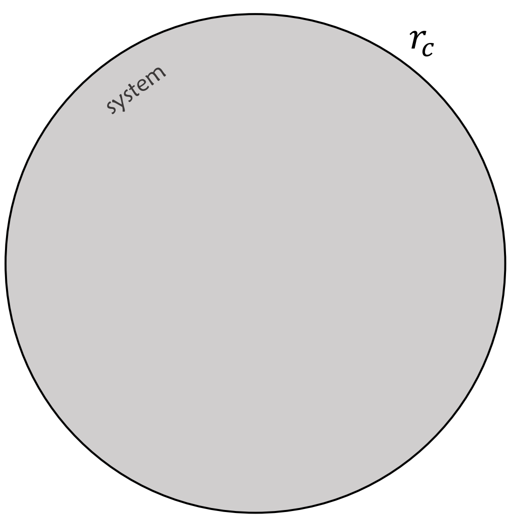
Gibbons-Hawking sphere partition function for de Sitter



Z becomes simply the path integral over "all metrics" on S^4

$$Z = Tr\, e^{-\beta H_{BY}} \rightarrow Tr\, I$$

= dimension of Hilbert space of states of a ball of space

Banihashemi & TJ (2204.05324)

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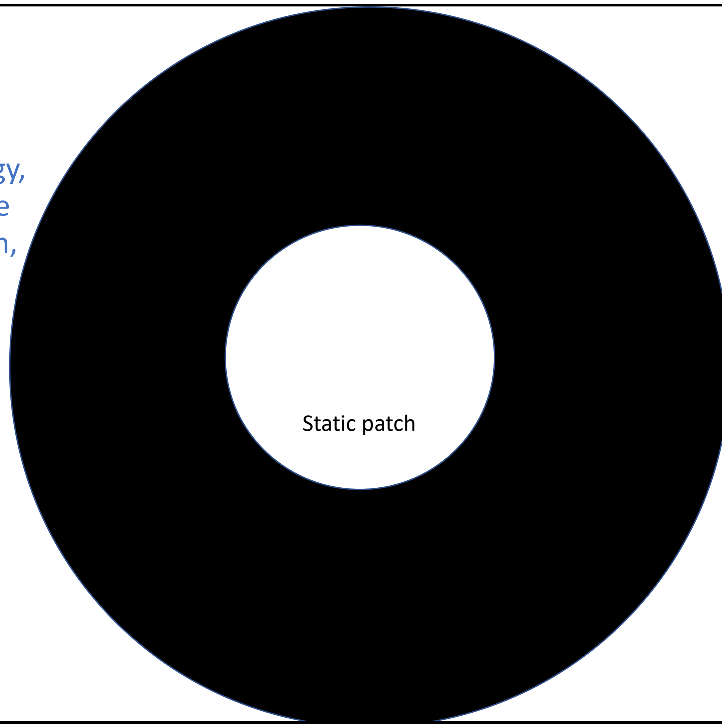
Gibbons & Hawking figured that since there is no boundary, the mean energy, angular momentum, and charge of the grand canonical ensemble must vanish, so the free energy is just -TS, hence the entropy is:

$$S = -I_E^{\text{4-sphere}}/\hbar$$



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Gibbons-Hawking sphere partition function for de Sitter

Euclidean de Sitter (round sphere) is the stationary point with the lowest action.

Size and temperature are properties of the saddle, determined by the cosmological constant.

$Z \approx \exp(A/4\hbar G), \quad A = 4\pi L^2$ in 4d

So the semiclassical de Sitter entropy already captures --- <u>at least at leading order in $\hbar$</u> --- the log of the dimension of the Hilbert space of a ball of space!

Supports the Banks & Fischler hypothesis (~ 2000).

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How does semiclassical GR know about the count of horizon microstates $A/4\hbar G$??

It knows the net count because that's encoded in G.

Analogy: the entropy of 1 kg water increases by about 10^{24} when the temperature is raised by one degree Kelvin at room temperature. We know this without knowing anything about structure of water molecules and the quantum mechanics of hydrogen bonds, because we measure a the heat capacity

What's special about gravity is the *universality* of this entropy area density, and the fact that the *same* parameter determines the entropy and the strength of gravity: *gravity is thermodynamics of the vacuum*.

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What are the microstates??

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How does semiclassical GR know about the count of horizon microstates $A/4\hbar G$??

It knows the net count because that's encoded in G (which includes matter contributions).

Analogy: the entropy of 1 kg water increases by about 10^{24} when the temperature is raised by one degree Kelvin at room temperature. We know this without knowing anything about structure of water molecules and the quantum mechanics of hydrogen bonds, because we measure a the heat capacity

What's special about gravity is the *universality* of this entropy area density, and the fact that the *same* parameter determines the entropy and the strength of gravity: *gravity is thermodynamics of the vacuum*.

What are the microstates??

Are they the ``edge states'' of an underlying UV complete theory?

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Partition function for a volume of space

TJ, Manus Visser (2212.10607)

Are not all horizons the same? Should "area/4 = log dimension" apply to *any* causal diamond?

To specify a *generic* diamond/ball of space, must somehow fix its size. How?

- Fixed edge area: no saddle exists; because fluctuations too large ?
  (The dS static patch saddle is exceptional.)
- Fixed Euclidean spacetime volume: not a condition on the state at one time.
- Fixed spatial volume V: admits a (mildly singular) saddle, gives log dimension = A/4!

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Cotler & Jensen, 2021
Gravitational constrained instantons

Constrained sphere partition function

$$Z[V,\Lambda] = \text{``}\int \mathcal{D}\lambda\,\mathcal{D}g\,\exp\left[\frac{1}{16\pi\hbar G}\int d^Dx\sqrt{g}(R-2\Lambda) + \frac{1}{\hbar}\int d\phi\,\lambda(\phi)\left(\int d^{D-1}x\sqrt{\gamma} - V\right)\right]\text{''}$$

λ contour parallel to imaginary axis.

Foliate S^D by constant ϕ, $(D-1)$-balls with induced metric $\gamma_{ab} = g_{ab} - N^2\phi_{,a}\phi_{,b}$

$$N \equiv (g^{ab}\phi_{,a}\phi_{,b})^{-1/2}$$

Saddle point field equation:

$$G_{ab} + \Lambda g_{ab} = 8\pi G T_{ab} \qquad \text{with} \qquad T_{ab} = \frac{\lambda}{N}\gamma_{ab} \equiv P\gamma_{ab}$$

Perfect fluid with vanishing "energy density"

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(D-2)-spherically symmetric, static saddle

$$ds^2 = N^2(r)d\phi^2 + h(r)dr^2 + r^2 d\Omega^2_{D-2}\,, \qquad h(r) = (1 - r^2/L^2)^{-1}$$

$N(r)$ is determined by the Euclidean Tolman-Oppenheimer-Volkoff (TOV) equation, with boundary conditions:

1) $N = 0$ at some $r = R_V$, the horizon (so P and therefore Ricci curvature diverges!)

2) $N' = -\sqrt{h}$ at $r = R_V$, for no conical singularity

Λ = 0 solution: $P(r) = -\frac{D-2}{4\pi G}\frac{1}{R_V^2 - r^2}$, $\lambda = -\frac{1}{8\pi G}\frac{D-2}{R_V}$, $R_V = [(D-1)V/\Omega_{D-2}]^{1/(D-1)}$

$$\boxed{ds^2 = \frac{1}{4R_V^2}(R_V^2 - r^2)^2 d\phi^2 + dr^2 + r^2 d\Omega^2_{D-2}}$$

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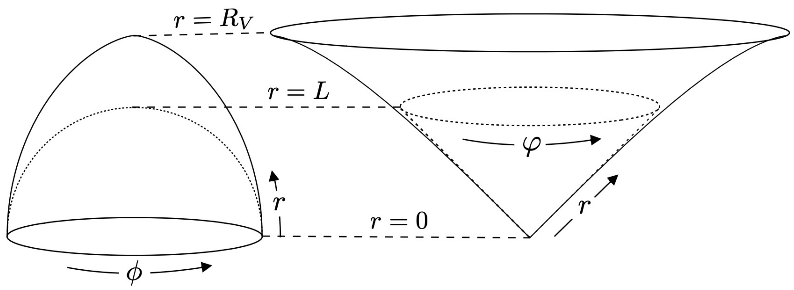
$$ds^2 = \frac{1}{4R_V^2}(R_V^2 - r^2)^2 d\phi^2 + dr^2 + r^2 d\Omega_{D-2}^2$$



There's a 1/(r – R_V) curvature singularity at the horizon, but *the action is finite and equal to* $-A_V/4\hbar G$

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Regulation of the saddle singularity in EFT?

Suppose the singularity is regulated by higher derivative terms in the action, governed by a UV length ℓ:

$$I = -\frac{1}{16\pi G}\int d^D x\sqrt{g}(R + \ell^2 R^2 + \dots)$$

The R^2 term contributes to the field equation $\sim \ell^2 \partial_r^2 R \sim \frac{\ell^2}{\rho^2} R\,,$ where $\rho \equiv r - R_V$

which is of the same order as the Einstein term when $\rho \sim \ell$, at which point $R \sim \frac{1}{R_V \ell}$.

If the curvature saturates at this value, then EFT remains effective, and the higher curvature corrections

to the entropy are of order $\ell^2 R \sim \frac{\ell}{R_V} \ll 1$ relative to the Bekenstein-Hawking term.

We conjecture that this is what happens . . .

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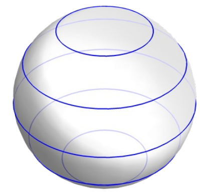
$\Lambda > 0$



The solution with positive cosmological constant is similar, BUT:

1) If $V = V(dS_{\text{static patch}})$, then the saddle is dS, which is smooth
2) If V is larger than the dS spatial hemisphere, *the entropy decreases as volume increases*
3) There is *no saddle* if V is larger than the full dS spatial sphere
4) The integral over all V is dominated by the dS saddle

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Questions

- Do higher curvature corrections indeed regulate the horizon singularity?
- What about the subleading (one-loop and higher curvature) corrections?
  - Do particular excited states indeed always have less entropy than the log of the dimension of the Hilbert space as calculated by the path integral?
  - Can the ``maximal vacuum entanglement hypothesis'' --- that entanglement entropy in small balls at fixed volume is maximized in the semiclassical vacuum state --- be verified?
- Why fixed volume?
- Can this result for the dimension of the Hilbert space be recovered in discrete approaches to quantum gravity? (Regge calculus, spin foams, CDT, …)

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