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Multiple Classical Histories as a Solution to the Black Hole Information Loss Paradox

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SM & D-h. Yeom, 1404.1565
P. Chen, MS & D-h.Yeom, 1806.03766, 2005.07011
P. Chen, MS, D-h Yeom, J. Yoon, 2111.01005, 2205.08320

Black Hole Information Loss Paradox

- from entanglement entropy point of view -

$\left(M_{pl}^2 = G^{-1} = 1 \right)$

classical picture

- Black hole ≈ thermal body with $T = T_{BH} \equiv (8\pi M)^{-1}$

  (focus on Schwarzschild BH for simplicity)

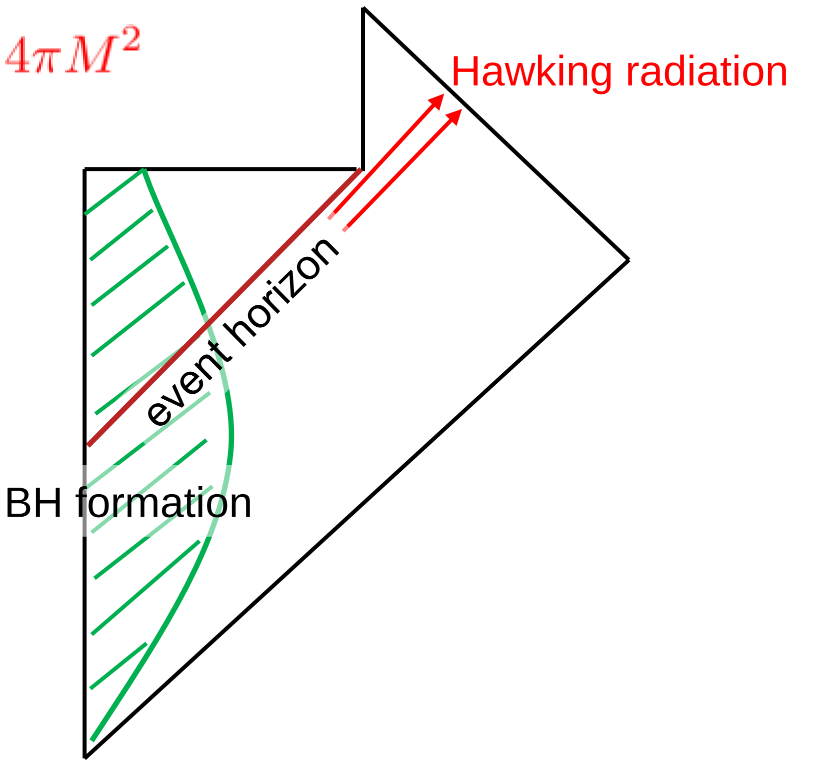
Bekenstein (1972)

Hawking (1974)

- Bekenstein-Hawking entropy: $S_{BH} \equiv \dfrac{A}{4} = 4\pi M^2$

⇓

emit radiation and evaporates away
(Hawking evaporation)



Black Hole Information Loss Paradox

- from entanglement entropy point of view -

quantum picture

- state inside BH (B) is entangled with state outside (A)

$$\rho = |\Psi\rangle\langle\Psi|$$

reduced density matrix: $\rho_A = \mathrm{Tr}_B\,\rho\,,\ \ \rho_B = \mathrm{Tr}_A\,\rho$

- entanglement entropy

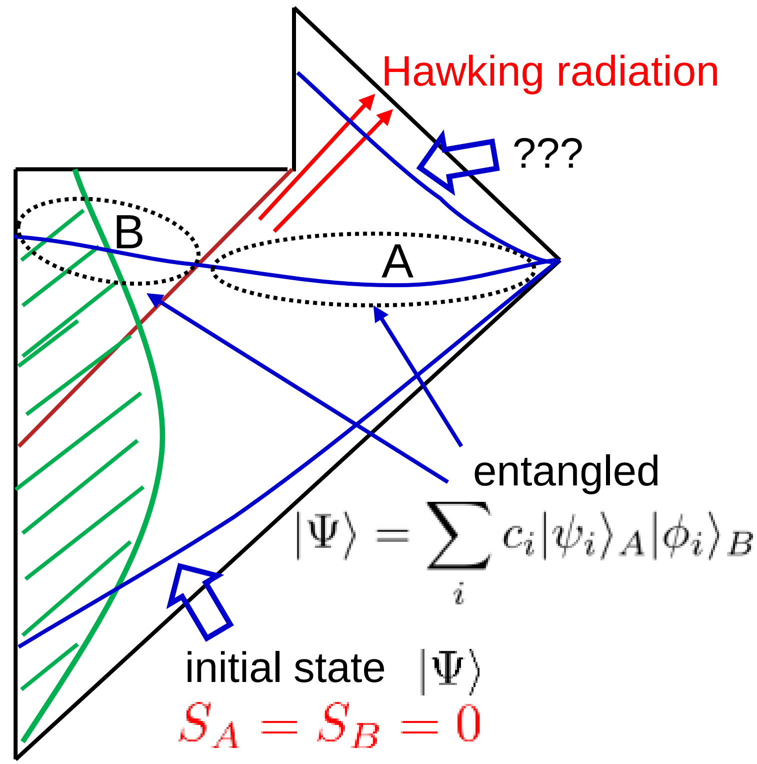
$$S_A = \mathrm{Tr}[\rho_A \ln \rho_A] = S_B$$

- after evaporation

$S_A = S_B = 0$ because B = void

contradiction!!

$$S_A = S_{rad} = S_{BH}\ !?$$

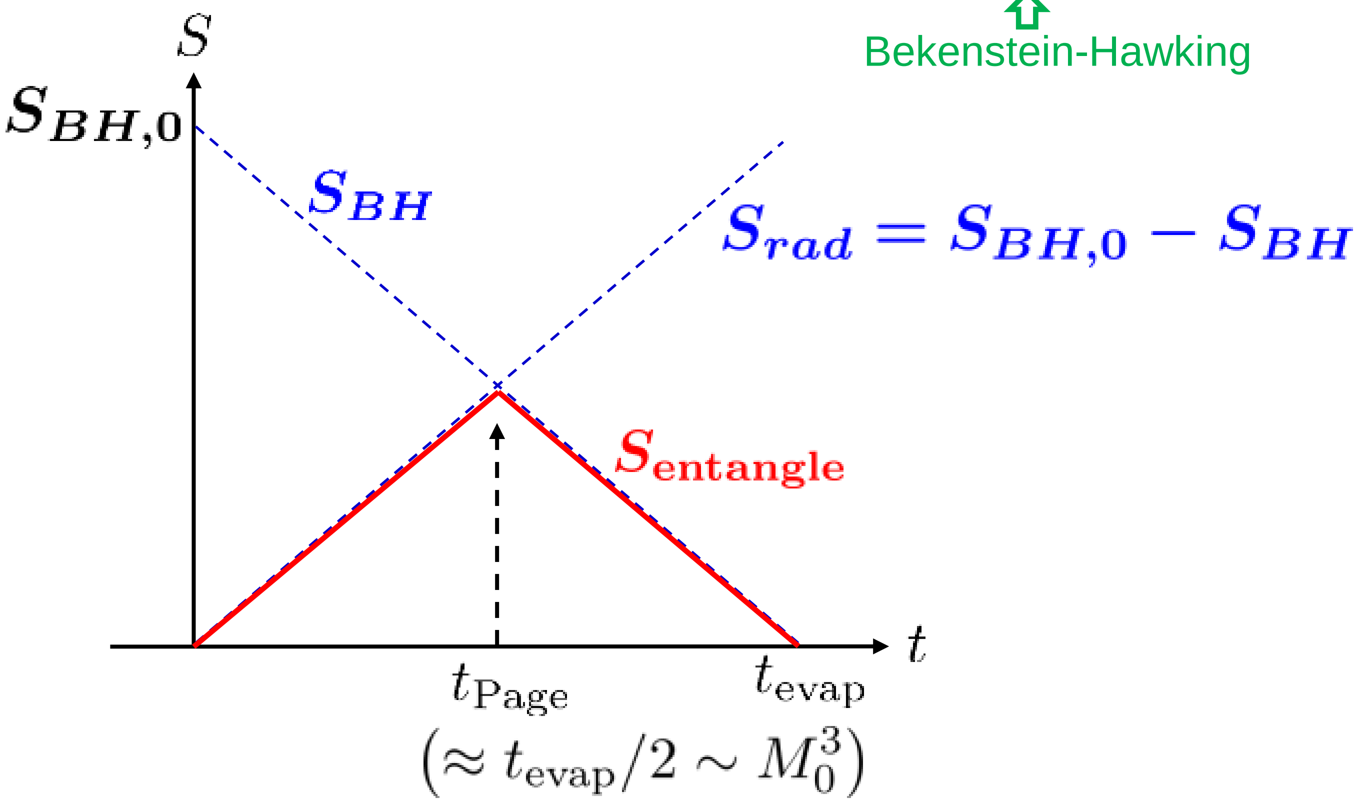


Page curve

D. Page, PRL 71 (1993) 3743

Assertion is that entanglement entropy shouldn't exceed BH entropy

Bekenstein-Hawking



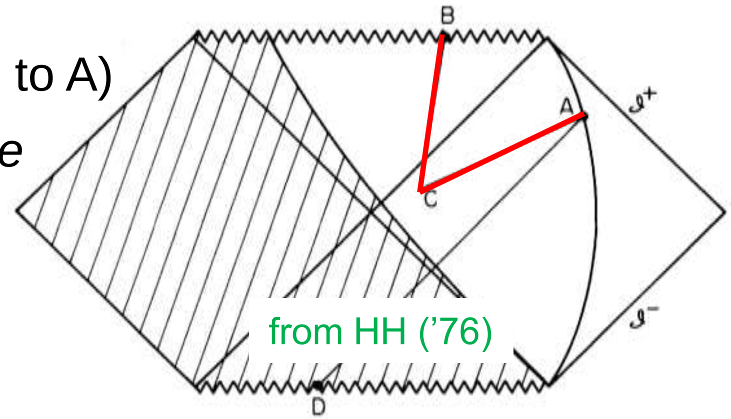
Questions:

- can quantum gravity effect be significant when BH is still macroscopic?
- Is S_{BH} the absolute upper bound of entanglement entropy?

(recent island conjecture computations seem to support these, but …)

Hawking radiation as instantons

- brief (highly biased) historical remarks -



- particle picture (transition amplitude from B to C to A)
  - *Path-integral derivation of black hole radiance*

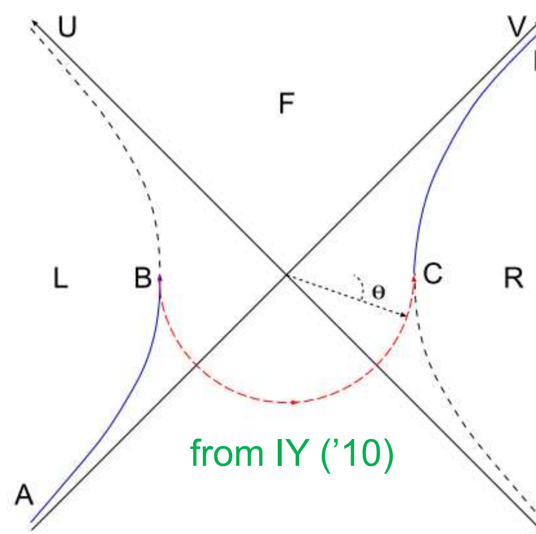
    Hartle & Hawking, PRD13 (1976) 2188

    BH background
  - *Hawking radiation as tunneling*

    Parikh & Wilczek, PRL85 (2000) 5042

    includes backreaction

$$M_{\rm BH} \Rightarrow (M-\omega)_{\rm BH} + \omega$$



- thin shell model (tunneling probability from B to C)
  - *Band-aid for information loss from black holes*

    Israel & Yun, PRD82 (2010) 124036
  - *Stationary bubbles and their tunneling channels toward trivial geometry*

    Chen, Domenech, MS & Yeom, JCAP 04 (2016) 013

Hawking radiation as instantons

- our strategy -

- take a conservative approach, and consider semi-classical wavefunction
- semi-classical histories are described by WKB wavefunctions

$$\Psi[g,\phi] \sim e^{-iS[g,\phi]} \quad \text{or} \quad e^{-S_E[g,\phi]}$$

$S[g,\phi]\,,\, S_E[g,\phi]$ (>>1) : on-shell action

- Hawking radiation may be regarded as quantum tunneling via instantons

$$e^{-B} = e^{-4\pi[M^2-(M-\omega)^2]} \approx e^{-\omega/T_{BH}}$$

P. Chen, MS & D-h. Yeom, 1806.03766

B : bounce action for emission of quantum ω

- This implies there should also be a tunneling channel/instanton from BH geometry to no BH (ie, trivial) geometry with the probability

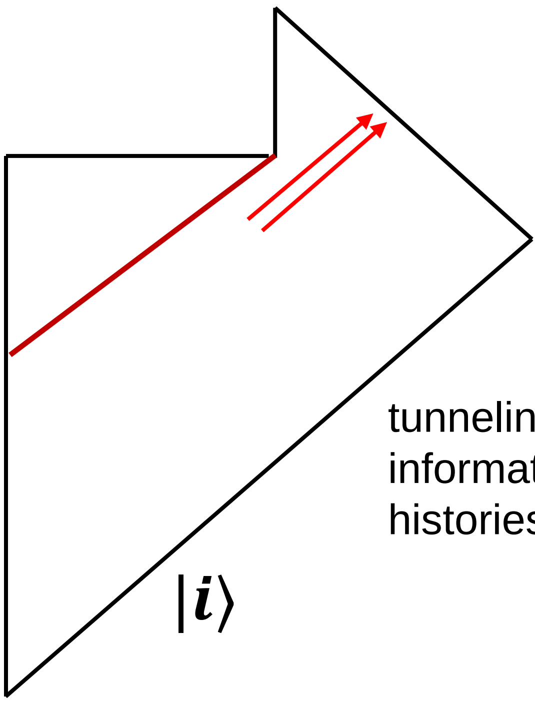
$$\sim\ e^{-B} = e^{-4\pi M^2}$$

tunneling between semi-classical histories!

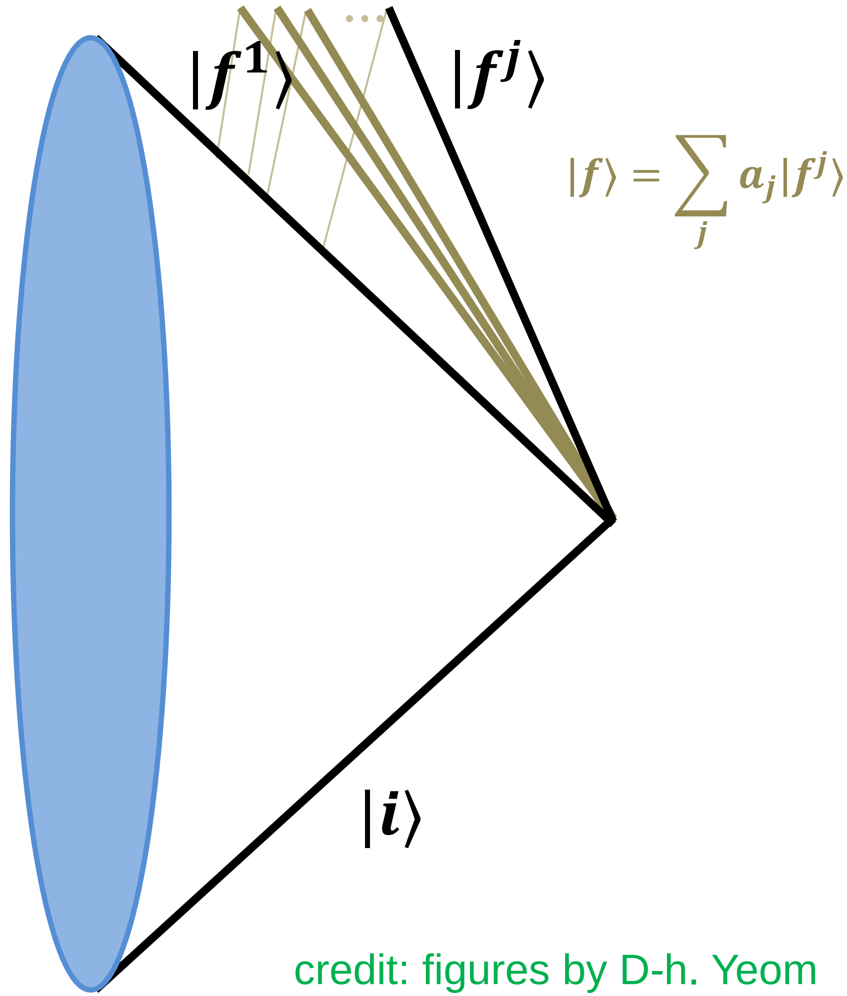
J. Hartle & T. Hertog, 1502.06770

Black hole evaporation as multiple classical histories

$$\langle f^j | i \rangle = \int_{i \to f^j} Dg D\phi \, e^{-S_E}$$



tunneling to
information-preserving
histories



credit: figures by D-h. Yeom

Black hole evaporation as multiple classical histories

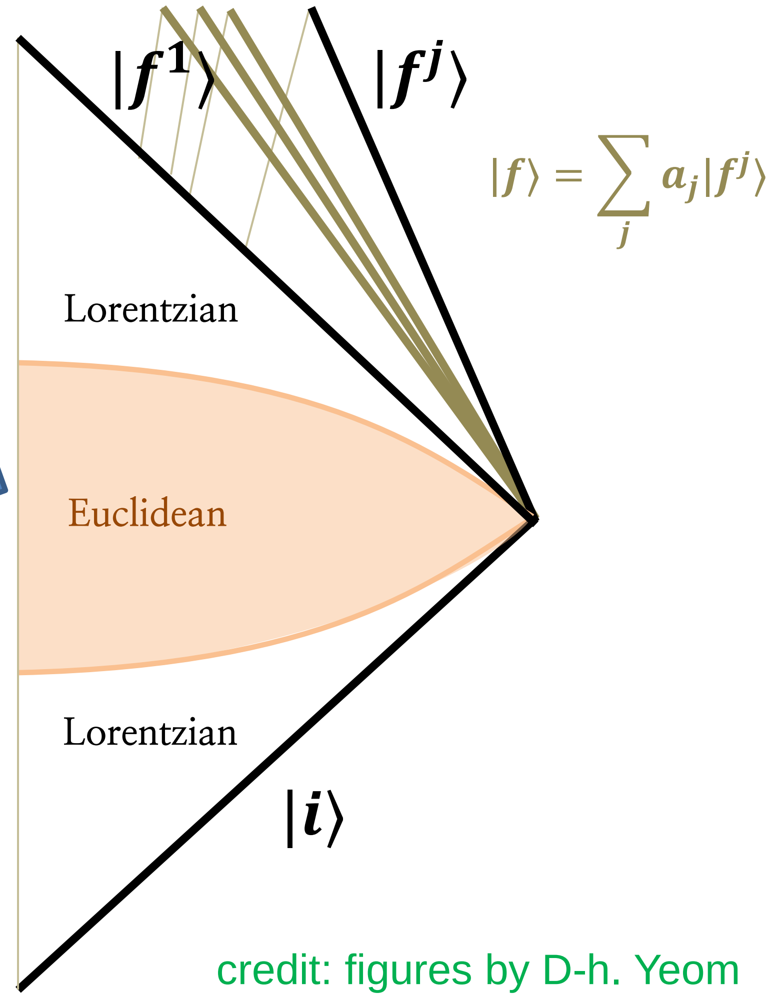
$$\langle f^j | i \rangle = \int_{i \to f^j} Dg D\phi \, e^{-S_E^{\text{on-shell}}}$$

- tunneling mediated by instantons
- tunneling probability ~ e^{-B}
  B : bounce action

$$B = S(M) = 4\pi M^2$$

for BH of mass M

P. Chen, MS & D-h. Yeom, 1806.03766



credit: figures by D-h. Yeom

Page curve under multiple classical histories

- state having multiple classical histories

$$|\Psi\rangle = p|\Phi\rangle + \sum_j q^j |\Psi^j\rangle \, ; \quad |\Phi\rangle = \sum_a c_a |\phi_a\rangle \, , \; |\Psi^j > = \sum_a d_a^j |\psi_a^j\rangle$$

$$\sum_a |c_a|^2 = 1, \; \sum_a |d_a^j|^2 = 1$$

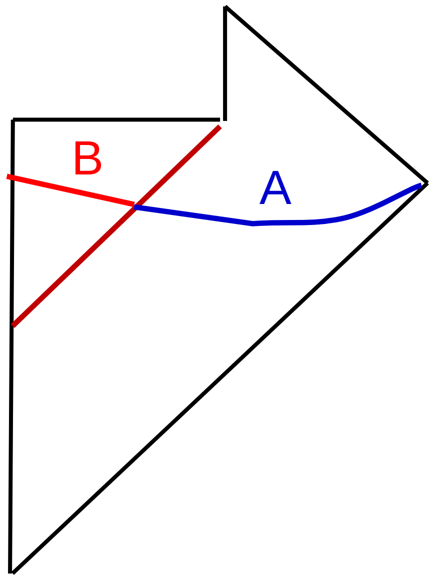
$$p^2 + \sum_j (q^j)^2 = 1;$$

(decoherence between classical states)

j: classical states
a: quantum states in state j

- if we divide each classical state to two subsystems, A and B, the entanglement entropy is given by

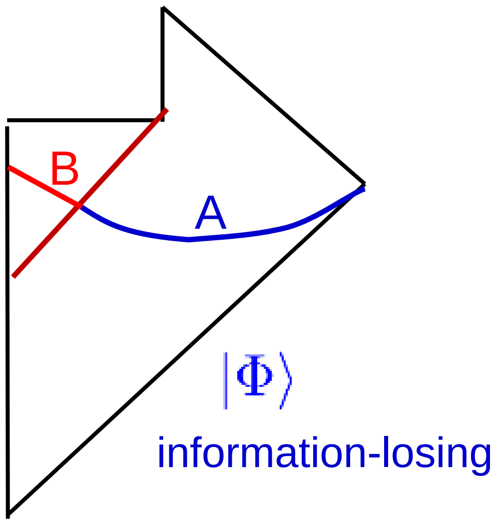
$$S_{\rm ent} \approx p^2 S_{\rm ent}^{\Phi} + \sum_j (q^j)^2 S_{\rm ent}^{\Psi_j}$$

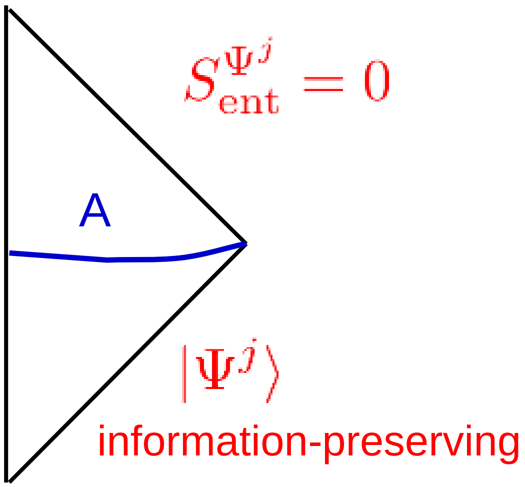


- if we identify $|\Phi\rangle$ with the evaporating BH (ie information-losing) history and $|\Psi^j\rangle$ with no BH (ie information-preserving) histories

$$S_{\rm ent} \approx p^2 S_{\rm ent}^{\Phi} + \sum_j (q^j)^2 S_{\rm ent}^{\Psi_j} = p^2 S_{\rm ent}^{\Phi}$$

because B = void for information-preserving histories





- p^2 is given by the persistent probability of the (evaporating) BH (ie, information-losing) history

- Page curve is obtained if p^2 initially increases and $p^2 \to 0$ at late times

BH persisting probability

- for a single instanton transition from BH with mass M to trivial geometry

Chen, Domenech, MS & Yeom, 1704.04020

$$\Gamma_1 = e^{-B}\,;\ B = S_E(\text{instanton}) - S_E(\text{background}) = \frac{A_{\text{ini}} - A_{\text{fin}}}{4}$$

$$= \frac{A_{\text{ini}}}{4} \equiv S\,; \qquad S = 4\pi M^2$$

initial and final BH surface areas

- for n-instanton transition

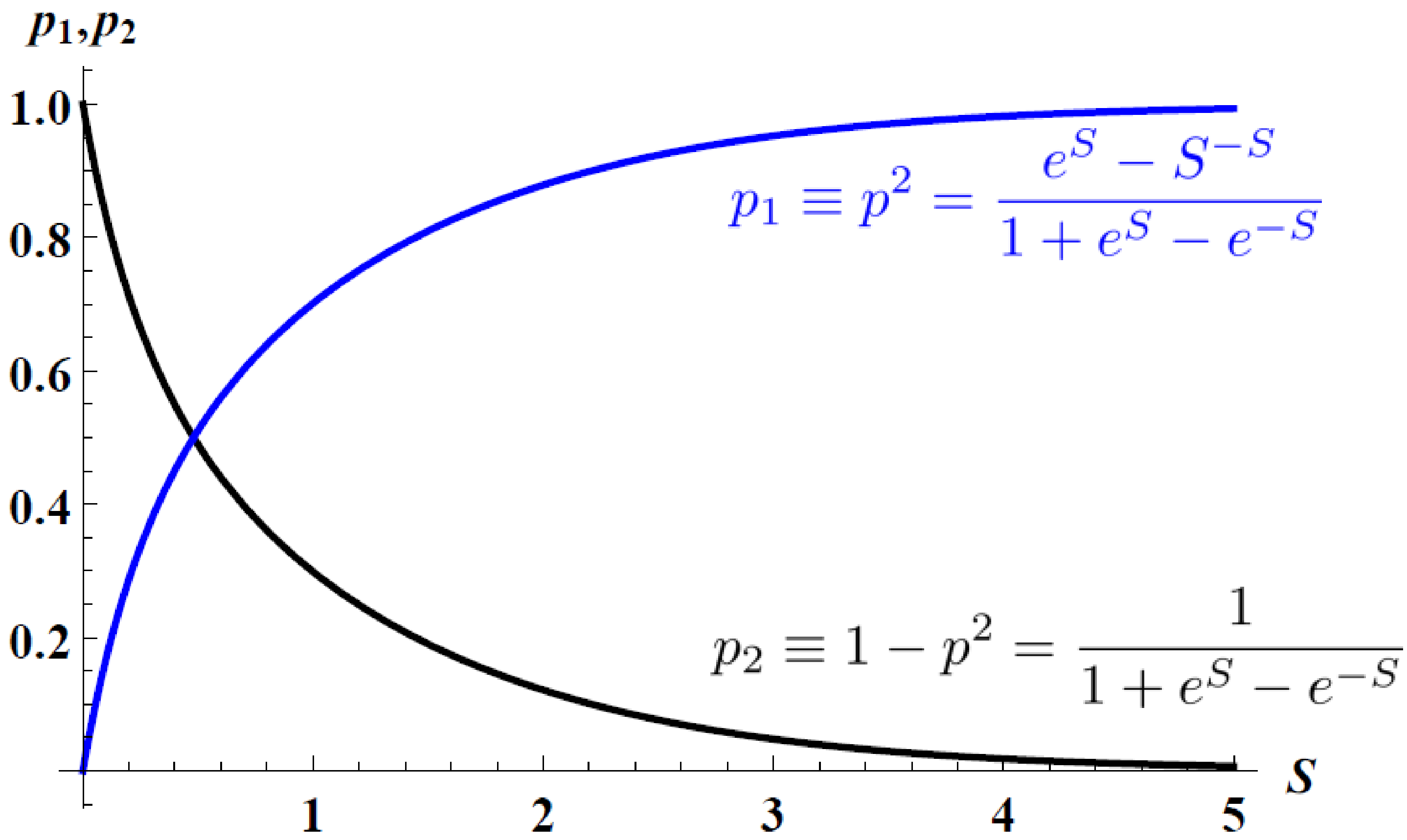
$$\Gamma_n = e^{-B_n}\,;\ B_n = n\,S_E(\text{instanton}) - S_E(\text{background}) = (2n-1)S$$

$$\Rightarrow \quad \Gamma = \sum_{n=1}^{\infty} e^{-(2n-1)S} = \frac{1}{e^S - e^{-S}}$$

- persistent probability: p^2

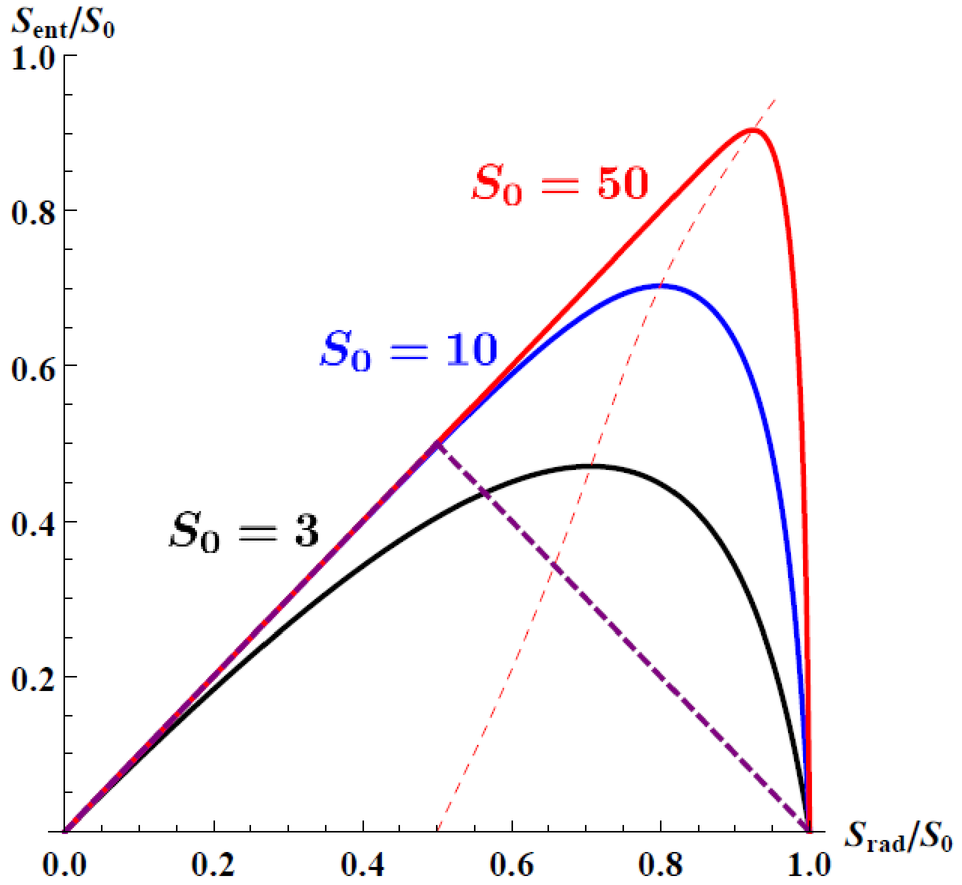
$$p^2\Gamma = 1 - p^2$$

$$\Rightarrow \begin{cases} p^2 = \dfrac{1}{1+\Gamma} = \dfrac{e^S - e^{-S}}{1 + e^S - s^{-S}} & \text{BH geometry} \\[2ex] 1 - p^2 = \dfrac{1}{1+\Gamma} = \dfrac{1}{1 + e^S - s^{-S}} & \text{trivial geometry} \end{cases}$$



The probability to tunnel to a trivial geometry dominates at late times when $M << M_0$

BH mass when it's formed



$$S \simeq p_1 S_1 + p_2 S_2$$

$S_2 = 0$

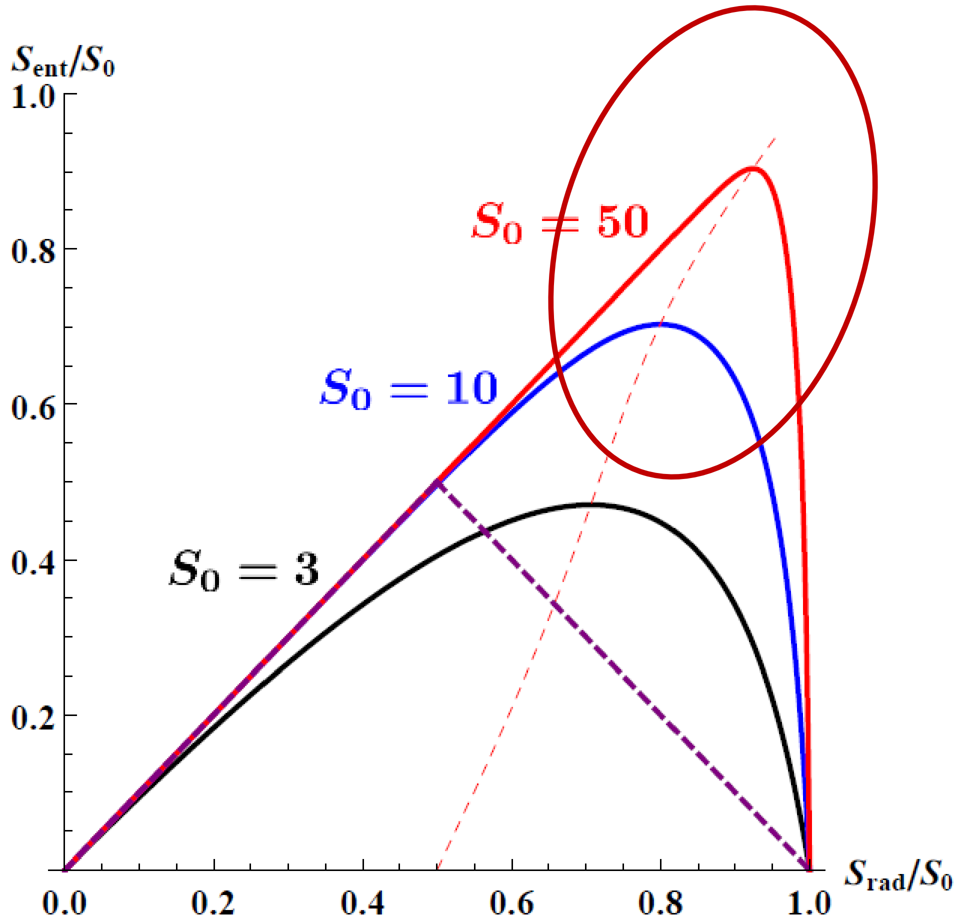
entropy due to Hawking radiation

$$S_1 = 4\pi(M_0^2 - M^2)$$

semi-classical approx. is valid for $S_0 >> 1$

e.g. for M_0=1 g, $S_0 \sim 10^{10}$, $S_{max} \sim 23$

- Page curve is obtained, albeit being modified, with the maximum at very late time when $S = S_{max} \sim \ln S_0$ for $S_0 >> 1$
- S_{max} is still large enough to be semi-classical for macroscopic BHs



$$S \simeq p_1 S_1 + p_2 S_2 \quad (S_2 = 0)$$

entropy due to Hawking radiation

$$S_1 = 4\pi(M_0^2 - M^2)$$

(semi-classical approx. is valid for S_0>>1

e.g. for M_0=1 g, S_0 ~10^{10} , S_{max}~23)

- S_{ent} violates Bekenstein-Hawking bound after the (original) Page time
- existence of a BH remnant / monster state ?

Chen, Ong & Yeom, 1412.8366

quotes from Israel & Yu ('10)

at the end of sec. 4

charge are evaporated preferentially. All of this raises questions worth exploring further, in particular the speculative possibility of stable, information-storing black hole remnants.

2^{nd} paragraph of sec. 5

Thus, the degrees of freedom in the outgoing radiation equal (or even exceed, on the thin-shell model) the maximum information capacity of the hole, as measured by the Bekenstein-Hawking entropy. This provides evidence, purely on the basis of counting, that unitarity could be preserved and that the radiation has enough room to accommodate all of the information. (But this has not been a matter of universal agreement[3].)

These words make me feel much closer to Werner, though he must have been thinking much deeper than I.

Summary

- We propose resolution to BH information loss paradox with Euclidean path integral approach under semi-classical approximation
- Hawking radiation as quantum tunneling implies the existence of a tunneling channel / instanton to trivial geometries
- This further implies the existence of multiple classical histories
- Trivial geometries dominate the wavefunction at late times
- Page curve for the entanglement entropy is obtained, albeit being modified, which violates the Bekenstein-Hawking bound

Relation (consistency/controversy?) to the island conjecture?