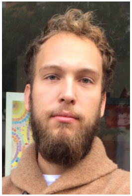
Hydro, Heavy Ions, and Soft Pions

Derek Teaney
Stony Brook University

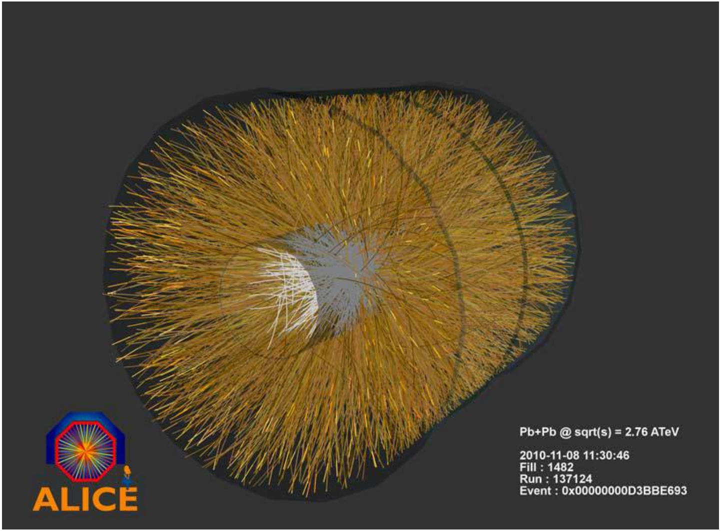
Stony Brook University

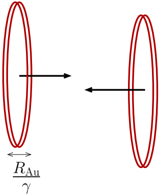
- Eduardo Grossi, Alex Soloviev, DT, Fanglida Yan: PRD, arXiv:2005.02885
- Eduardo Grossi, Alex Soloviev, DT, Fanglida Yan: PRD, arXiv:2101.10847
- Adrien Florio, Eduardo Grossi, Alex Soloviev, DT, PRD, arXiv:2111.03640
- Adrien Florio, Eduardo Grossi, DT, coming soon



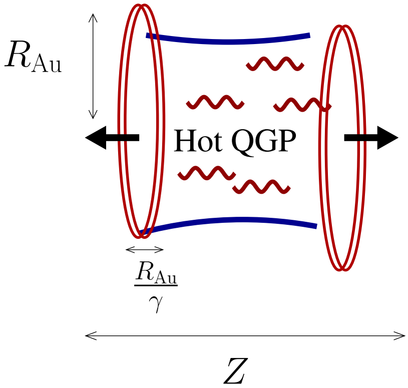


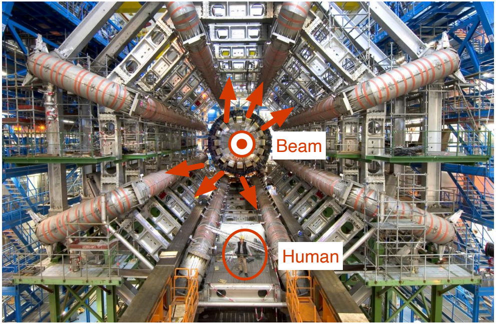
Colliding Nuclei and Creating Plasma of Quarks and Gluons (QGP)





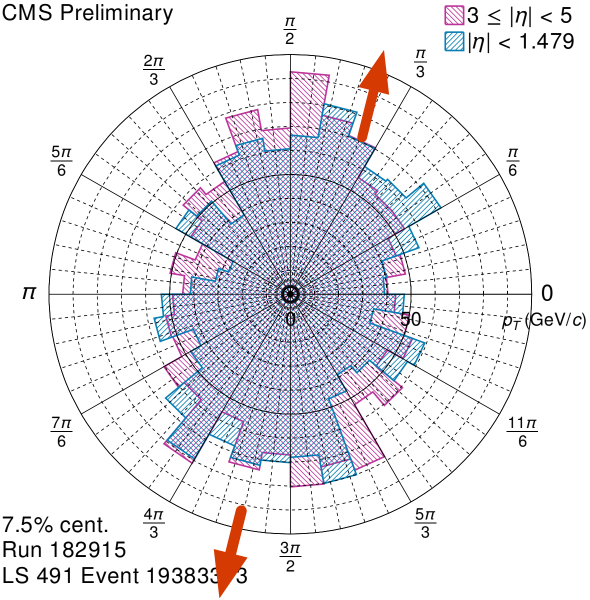
The QGP is Born

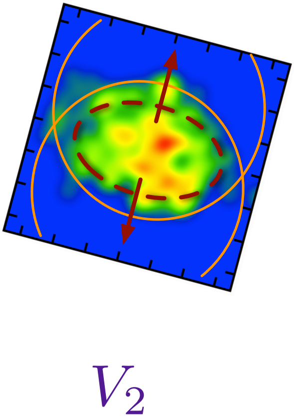




The nuclei *pass through* each other leaving QGP expanding rapidly

Measuring the hydrodynamics of the plasma





My path to Werner Israel: viscous hydro for elliptic flow

ANNALS OF PHYSICS **100**, 310–331 (1976)

Nonstationary Irreversible Thermodynamics: A Causal Relativistic Theory*

WERNER ISRAEL†

California Institute of Technology, Pasadena, California 91125

Received February 19, 1976

Remarkably modern discussion of hydro: equilibrium, frames, model...

From Black Holes to Hydrodynamics

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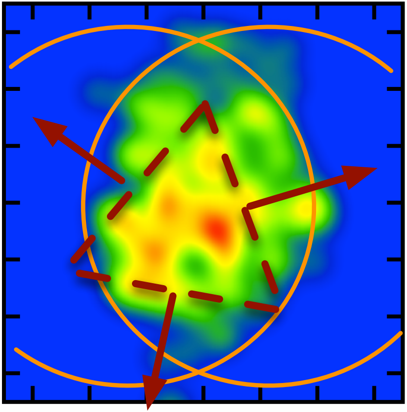
From Black Holes to Hydrodynamics

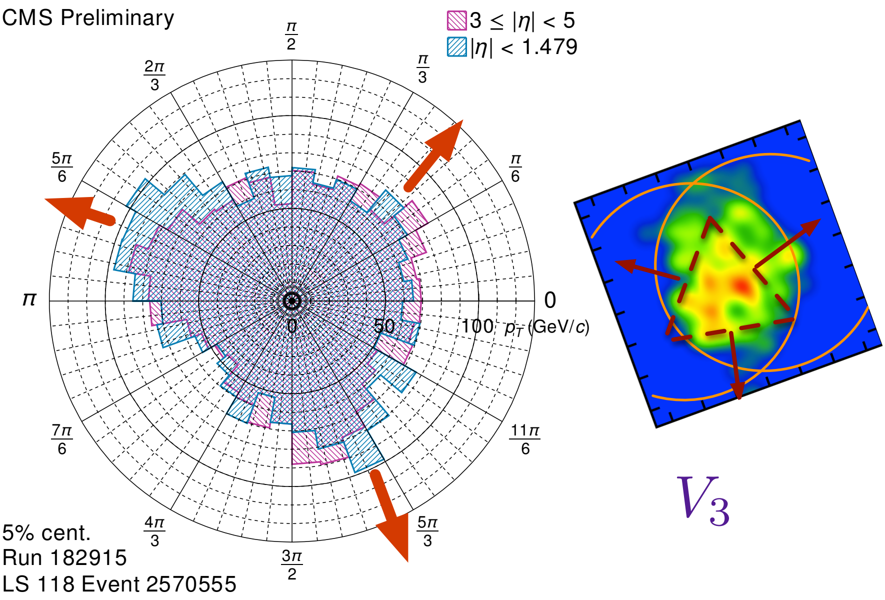
A symposium celebrating the 80th birthday of Werner Israel

University of Victoria, April 26 & 27, 2011

Fluctuations: a mini revolution in heavy ions circa 2010-2011



The hydrodynamics take the input white spectrum in coordinate space, and *filters it* to generate all harmonics, $V_2, V_3, \ldots$


CMS Preliminary
3 ≤ |η| < 5
|η| < 1.479
π/2
π/3
2π/3
π/6
5π/6
π
0
0
50
100
p_T (GeV/c)
7π/6
11π/6
4π/3
5π/3
3π/2
5% cent.
Run 182915
LS 118 Event 2570555
V3

Amazing Success: the "Standard" Hydro Model

1. $V_1 \ldots V_6$, $\left\langle \delta p_T^2 \right\rangle$
2. Momentum dependence $V_n(p)$
3. Probabilities $P(|V_n|^2)$ and non-gaussianity.
4. Covariances between harmonics: $\langle V_2 V_3 V_5^* \rangle$ and $\left\langle |V_2^2| \delta p_T^2 \right\rangle$
5. Full covariance matrix: $\langle V_2(p_1) V_2^*(p_2) \rangle$

Uses the equation of state, fits viscosity, and solves:

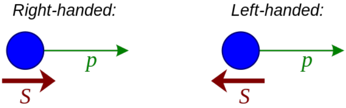
$$\partial_\mu T^{\mu\nu} = 0$$

$$\frac{\eta}{s} \simeq \frac{(1\leftrightarrow 3)}{4\pi} \frac{\hbar}{k_B}$$

JETSCAPE, 2011.01430

But, we want more . . .

QCD and Chiral Symmetry



u_R, d_R $\qquad$ u_L, d_L

$$\begin{pmatrix} u'_L \\ d'_L \end{pmatrix} = U_L \begin{pmatrix} u_L \\ d_L \end{pmatrix}$$

and ditto for right

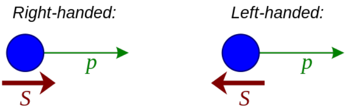
QCD is (almost) symmetric between, left and right, and up and down:

$$\mathcal{L}_{QCD} = \sum_{q=u,d} \bar{q}_L (i\not{D}) q_L + \bar{q}_R (i\not{D}) q_R \; - \underbrace{m_q \left(\bar{q}_L q_R + \bar{q}_R q_L \right)}_{\text{small}}$$

Then one would expect four approx. conservation laws, u_L, d_L, u_R, d_R:

n_B :	$(u_L + d_L) + (u_R + d_R)$	Baryon number
n_{anom} :	$(u_L - u_R) + (d_L - d_R)$	Anomalous: not consv.

QCD and Chiral Symmetry



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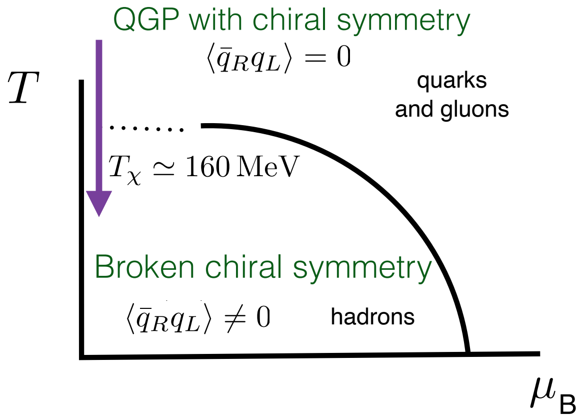
Then one would expect four approx. conservation laws, u_L, d_L, u_R, d_R:

$\vec{n}_V$:	$(u_L + u_R) - (d_L + d_R)$	Isovector charge
$\vec{n}_A$:	$(u_L - u_R) - (d_L - d_R)$	Isoaxial vect. charge

Chiral symmetry breaking and heavy ion collisions

Pisarski, Wilczek



For two massless quarks the chiral symmetry group is

$$SU_L(2) \times SU_R(2) \simeq O(4)$$

This is broken, and the transition is 2nd order.

The mass smooths the transition to a crossover, like a magnetic field in the Ising model

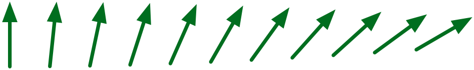
Chiral symmetry plays no role in the "Standard Hydro Model" ...

What is a poin?

Our cold world: T< Tcritical

$$\langle \bar{q}_R q_L \rangle = \bar{\sigma}\, \mathbb{I}_{2\times 2}$$

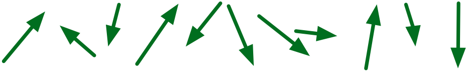
Order parameter $\langle \bar{q}_R q_L \rangle$ is like the magnetization. $q = u, d$



$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

The slow modulation of the $SU_A(2)$ phase of $\bar{q}_R q_L$ is a pion, $\vec{\pi}{=}\bar{\sigma}\vec{\varphi}$

The hot world: T> Tcritical



State is disordered: pion propagation is frustrated

The pion wave function is the $SU(2)$ phase $U(x) = e^{i\vec{\varphi}(x)\cdot\vec{\tau}}$

Ising O(4) Model		QCD	
magnetization	$\vec{M}$	$\bar{q}_L q_R = \sigma e^{i\vec{\tau}\cdot\vec{\varphi}}$	condensate
magnetic field	$\vec{H}$	m_q or H	quark mass

$$\mathcal{H} = \int d^3x\, \vec{H}\cdot\vec{M}$$

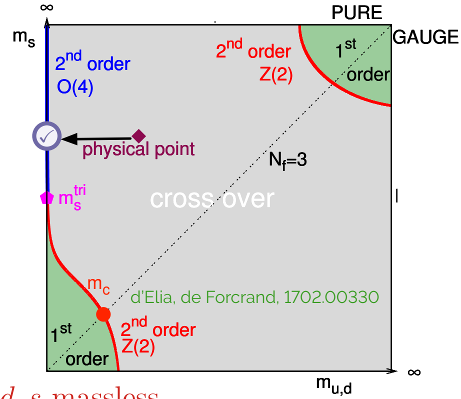
$$\mathcal{H} = \int d^3x\, m_q\left(\bar{q}_R q_L + \bar{q}_L q_R\right)$$

$\vec{\tau}$ are Pauli matrices for the SU(2) order parameter

Real World QCD

- There are three flavors of quarks u, d, s which are massive
  - This changes structure phase diagram
- We will assume the real world is "close" to the $O(4)$ critical point.

u, d massless



u, d, s massless

See review Phillipsen, 2021

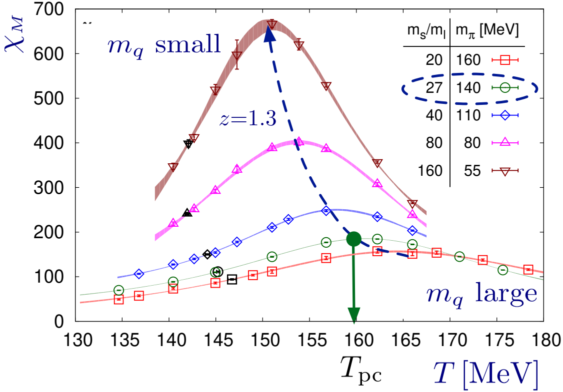
HotQCD 2019, 2020

Strong evidence of 2nd order phase transition at physical strange mass

Real world lattice QCD and the $O(4)$ critical point: Hot QCD, PRL 2019

Fluctuations of order parameter, $\sigma \propto \bar{u}u + \bar{d}d$, vs temperature and m_q

$$\chi_M = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$$



O(4) Scaling predictions

$$\chi_M = m_q^{1/\delta - 1} f_\chi(z)$$

$$z \equiv z_0 \frac{(T - T_c)}{T_c} m_q^{-1/\beta\delta}$$

The QCD lattice knows about the $O(4)$ critical point! Hydro should too!

Static Universality and the Chiral Phase Transition

- The $O(4)$ order parameter fluctuates in amplitude and phase:

$$\phi_a = (\phi_0, \phi_1, \phi_2, \phi_3) = (\sigma, \vec{\pi})$$

The quark condensate scales as

$$\bar{q}_R q_L \sim \sigma e^{i\vec{\tau}\cdot\vec{\varphi}} \simeq \sigma + i\vec{\tau}\cdot\vec{\pi}$$

- The Landau Ginzburg function for the $O(4)$ order parameter is: $\phi^2 \equiv \phi_a\phi_a$

$$\mathcal{H} = \int d^3x \, \frac{1}{2}\nabla\phi_a \cdot \nabla\phi_a + \frac{1}{2}m_0^2(T)\,\phi^2 + \frac{\lambda}{4}\,\phi^4 - \underbrace{H}_{\propto\, m_q}\,\sigma$$

- The model has a critical mass, $m_0 - m_c \propto (T - T_c)$

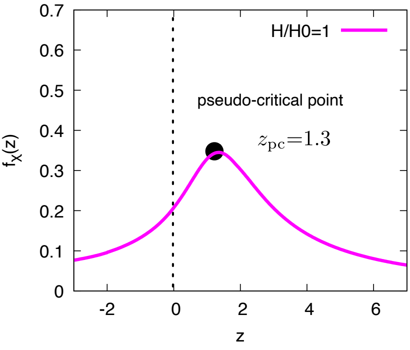
The critical model makes a definite prediction for the susceptibility:

Scaling predictions from the $O(4)$ model

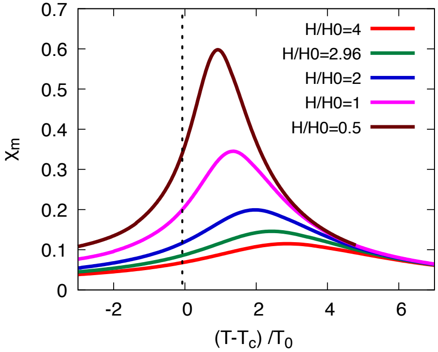
Simulations at different magnetic field are related to each other

$$\chi_M = h^{1/\delta - 1} f_\chi(z) \qquad z = z_0\, t_r h^{-1/\beta\delta}$$

Here $h \propto H$ and $t_r \propto (T - T_C)$ are the reduced field and temperature

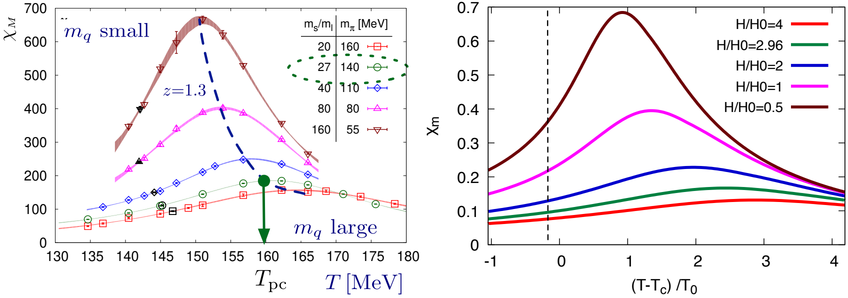


numerical data from
Engels, Seniuch, Fromme, Karsch



Scaling predictions and QCD

$$\chi_M = \langle \sigma^2 \rangle - \langle \sigma \rangle^2$$



Scaling predictions reasonably describe how the peak rises and shifts.

$$\chi_M \propto m_q^{1/\delta - 1} f_\chi(z) \qquad z = z_0 \left(\frac{T - T_C}{T_C}\right) m_q^{-1/\beta\delta}$$

From Thermodynamics to Hydrodynamics

Hydrodynamics of the $O(4)$ transition:

Rajagopal and Wilczek '92, Son '99, Son and Stephanov '01, and finally us, arxiv:2101.10847.

1. The order parameter

$$\phi_a = (\sigma, \vec{\pi})$$

2. The approximately conserved charges quantities:

$$\vec{n}_V = \underbrace{\bar{\psi}\gamma^0\vec{\tau}\psi}_{\text{isovect chrg}} \quad \textit{and} \quad \vec{n}_A = \underbrace{\bar{\psi}\gamma^0\gamma^5\vec{\tau}\psi}_{\text{isoaxial-vect chrg}}$$

which are combined into an anti-symmetric $O(4)$ tensor n_{ab}

$$n_{ab} = (\vec{n}_A, \vec{n}_V)$$

The charge n_{ab} generates $O(4)$ rotations, $\phi \to \phi_c + \frac{i}{\hbar}\theta_{ab}[n_{ab}, \phi_c]$, implying a Poisson bracket between the hydrodynamic fields:

$$\{n_{ab}(\boldsymbol{x}), \phi_c(\boldsymbol{y})\} = \epsilon_{abcd}\, \phi_d(\boldsymbol{x})\, \delta(\boldsymbol{x} - \boldsymbol{y})$$

The Landau-Ginzburg Hamiltonian for the $O(4)$ transition:

The Hamiltonian is tuned to the crit. point with $m_0^2(T) < 0$ and $H \propto m_q$:

$$\mathcal{H} = \int d^3x \, \frac{1}{2}\nabla\phi_a \cdot \nabla\phi_a + \frac{1}{2}m_0^2(T)\phi^2 + \frac{\lambda}{4}\phi^4 - H\sigma + \frac{n_{ab}^2}{4\chi_0}$$

and gives the equilibrium distribution with the correct critical EOS:

$$Z = \int D\phi \, Dn \, e^{-\mathcal{H}[\phi,n]/T_c}$$

The hydro equations of motion take the form

$$\frac{\partial \phi}{\partial t} + \{\phi, \mathcal{H}\} = 0 + \text{visc. corrections} + \text{noise}$$

$$\frac{\partial n_{ab}}{\partial t} + \{n_{ab}, \mathcal{H}\} = 0 + \text{visc. corrections} + \text{noise}$$

The Landau-Ginzburg Hamiltonian for the $O(4)$ transition:

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The hydro equations of motion take the form

$$\frac{\partial \phi}{\partial t} + \{\phi, \mathcal{H}\} = -\Gamma \frac{\delta \mathcal{H}}{\delta \phi_a} + \xi_a$$

$$\frac{\partial n_{ab}}{\partial t} + \{n_{ab}, \mathcal{H}\} = \underbrace{\sigma_0 \nabla^2 \frac{\delta \mathcal{H}}{\delta n_{ab}}}_{\text{dissipation}} + \underbrace{\nabla \cdot \xi_{ab}}_{\text{noise}}$$

The equations and the simulations:

see also Schlichting, Smekal

We have a charge diffusion equation coupled to order parameter:

$$\partial_t n_{ab} + \underbrace{\nabla \cdot (\nabla \phi_{[a}\phi_{b]})}_{\text{poisson bracket}} + H_{[a}\phi_{b]} = \underbrace{D_0 \nabla^2 n_{ab}}_{\text{diffusion}} + \underbrace{\nabla \cdot \xi_{ab}}_{\text{noise}}$$

and a rotation of the order parameter induced by the charge:

$$\partial_t \phi_a + \underbrace{\frac{n_{ab}}{\chi_0}\phi_b}_{\text{poisson bracket}} = \underbrace{\Gamma_0 \frac{\delta H}{\delta \phi_a}}_{\text{dissipation}} + \underbrace{\xi_a}_{\text{noise}}$$

Numerical scheme based operator splitting:

1. Evolve the Hamiltonian evolution with a position Verlet type stepper
2. Treat the dissipative Langevin steps as Metropolis-Hastings updates

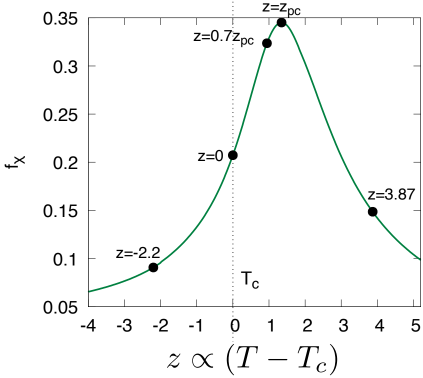
Scan the phase transition:

Engels, Fromme, Seniuch, Karsch; Hasenbuch

First measure mean order parameter, susceptibility, etc:

$$\langle \sigma \rangle = h^{1/\delta} f_G(z) \qquad z = t_{\rm r} h^{-1/\beta\delta}$$

with scaling parameters, $h = H/H_0$, and $t_{\rm r} = (m_0^2 - m_c^2)/\mathfrak{m}^2$

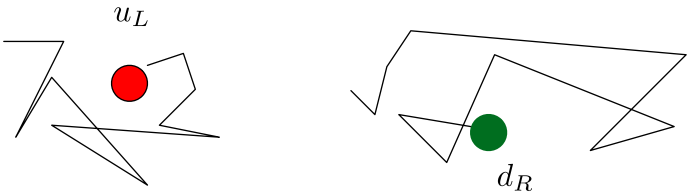


$$\chi_M = \frac{\partial \bar{\sigma}}{\partial H} = \frac{h^{1/\delta - 1}}{H_0} f_\chi(z)$$

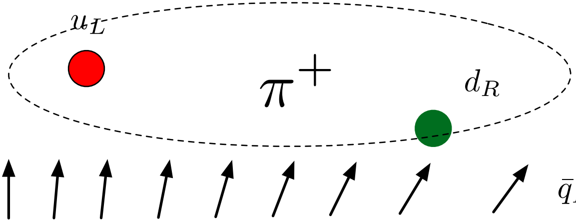
“Artists” conception of the phase transition dynamics

High Temperature: Diffusion of axial charge

$n_A = u_L - d_R$



Low Temperature: pion propagation

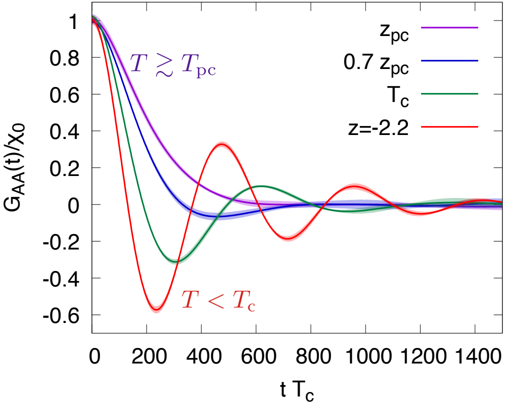


$\bar{q}_R q_L = \bar{\sigma} e^{i\vec{\tau}\cdot\vec{\varphi}}$

The phase transition and axial charge correlations:

$$G_{AA}(t) = \int \mathrm{d}^3x \; \langle \vec{n}_A(t, \boldsymbol{x}) \cdot \vec{n}_A(0, \mathbf{0}) \rangle$$

See a change in the dynamics across $T_{\rm pc}$:

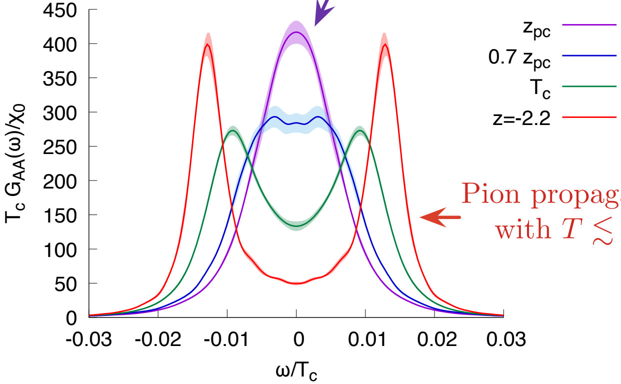


Let's take a fourier transform and analyze the transition

Features of the phase transition in the axial charge correlations:

$$G_{AA}(\omega) = \int \mathrm{d}t\,\mathrm{d}^3x\, e^{i\omega t}\ \langle \vec{n}_A(t,\boldsymbol{x}) \cdot \vec{n}_A(0,\mathbf{0}) \rangle$$

Diffusion of quarks with $T \gtrsim T_{\rm pc}$

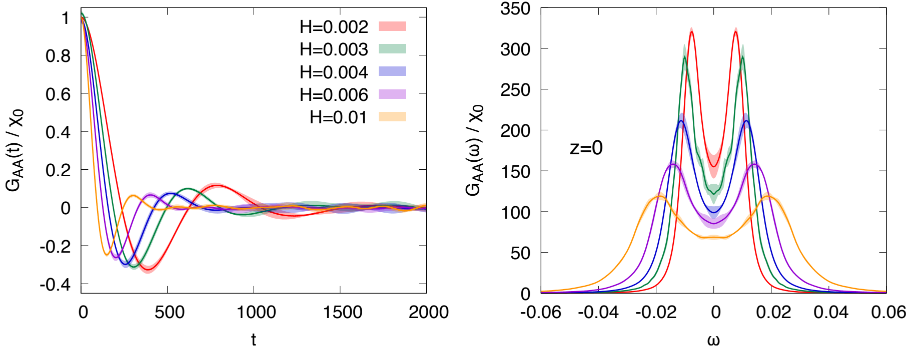


Pion propagation with $T \lesssim T_c$

Can see the transition from diffusion of quarks to propagation of pions!

Scaling of simulations at T_c:

At $T = T_c$, we varied the magnetic field, finding the response functions:



See a scaling behavior of the real time correlations, with quark mass, which tunes the correlation length

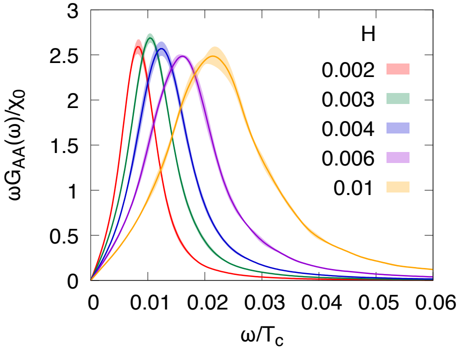
Dynamical critical exponent of the $O(4)$ transition:

Rajagopal, Wilczek

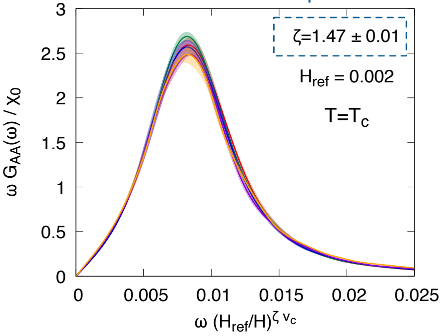
The relaxation time and correlations *scale* with the correlation length ξ:

$$\omega\, G_{AA}(\omega, \xi) = \underbrace{f(\omega\, \tau_R)}_{\text{universal fcn}} \quad \text{with} \quad \underbrace{\tau_R \propto \xi^{\zeta}}_{\text{relaxation time}}$$

The correlation length scales as $\xi \propto H^{-\nu_c}$ and the time as $\tau_R \propto H^{-\zeta\nu_c}$:

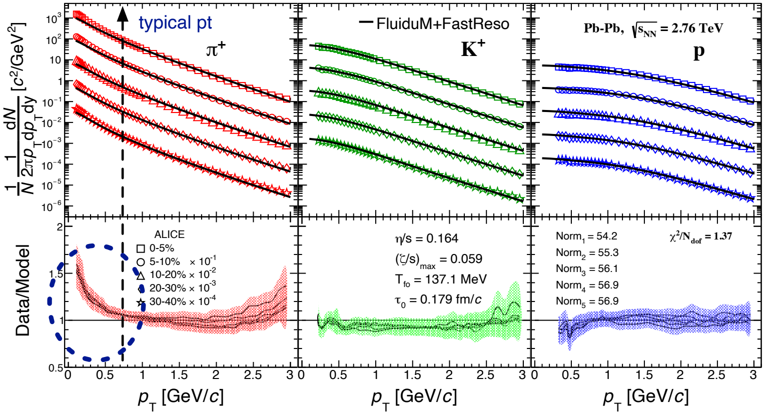


compare to 3/2



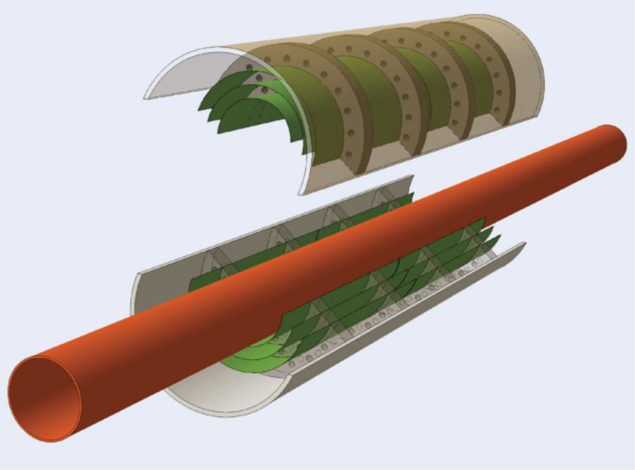
Evidence for the chiral crossover in the heavy ion data?

A recent ordinary hydro fit from Devetak et al 1909.10485



See also, Guillen&Ollitrault arXiv:2012.07898; Schee, Gürsoy, Snellings: arXiv:2010.15134

New Detector: ALICE ITS3



Summary and Outlook:

1. We are simulating the real-time dynamics of the chiral critical point
   - The numerical method may be useful for stochastic hydro generally
2. We reproduced the expected dynamical scaling laws:

$$\tau_R \propto \xi^{\zeta} \qquad \zeta = \frac{d}{2} \simeq 1.47 \pm 0.01$$

3. The pion waves are well calibrated.
4. The next step is to study the expanding case:
   - This will predict soft pions and their correlations with expansion for heavy ion collisions

The hadronization of the pion is the (only) hadronization process that can be studied rigorously, *and only with hydrodynamics!*

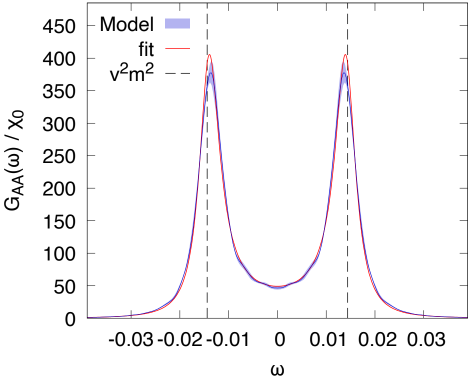
Backup

Quantitative analysis of a pion EFT well below T_c, $z = -2.2$:

The predicted pole position m_p^2 of pion waves is given by static quantities:

$$m_p^2 = v^2 m^2 = \frac{H\bar{\sigma}}{\chi_0}$$

This is the finite temperature Gell-Mann Oakes Rener relation:



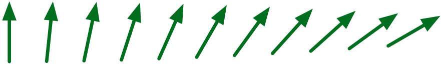
These are static inputs to fit:

$$v^2 = \frac{f^2}{\chi_0}$$

$$m^2 = \text{screening mass}$$

The Pion EFT

- Below T_C the condensate is frozen up to phase fluctuations



$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

- The ideal equations of motion the phase is:

$$\partial_t \varphi = \mu_A \qquad \text{Josephson Constraint}$$

while the axial charge is

$$\partial_t n_A + \nabla \cdot \boldsymbol{J}_A = f^2 m^2 \varphi \qquad \text{Axial Current}$$

where the current is the gradient of the phase: $\boldsymbol{J}_A = f^2 \nabla \varphi$

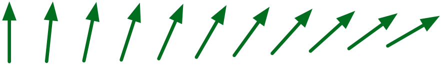
- The pion EFT is written with $f^2 \simeq \bar{\sigma}^2$ and $f^2 m^2 = H\bar{\sigma}$

We can use the EFT to find the dispersion curve of soft pions, including dissipative corrections

The Pion EFT

Son and Stephanov '01+bit from us

- Below T_C the condensate is frozen up to phase fluctuations



$$\bar{q}_R q_L = \bar{\sigma}\, e^{i\vec{\tau}\cdot\vec{\varphi}(x)}$$

- The ideal equations of motion the phase is:

$$\partial_t \varphi = \mu_A + \mathcal{O}(\Gamma\nabla^2\varphi) \qquad \text{Josephson Constraint}$$

while the axial charge is

$$\partial_t n_A + \nabla\cdot\boldsymbol{J}_A = f^2 m^2 \varphi + \mathcal{O}(D\nabla^2 n_A) \qquad \text{Axial Current}$$

where the current is the gradient of the phase: $\boldsymbol{J}_A = f^2\nabla\varphi$

- The pion EFT is written with $f^2 \simeq \bar{\sigma}^2$ and $f^2 m^2 = H\bar{\sigma}$

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Long wavelength pion (superfluid) modes:

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- Linearizing the equations, the quasi particle energy is

$$\omega_q^2 \equiv v_0^2(q^2 + m^2) \qquad\qquad v_0^2(T) \equiv \frac{f^2}{\chi_0} \quad \Leftarrow \text{pion velocity}$$

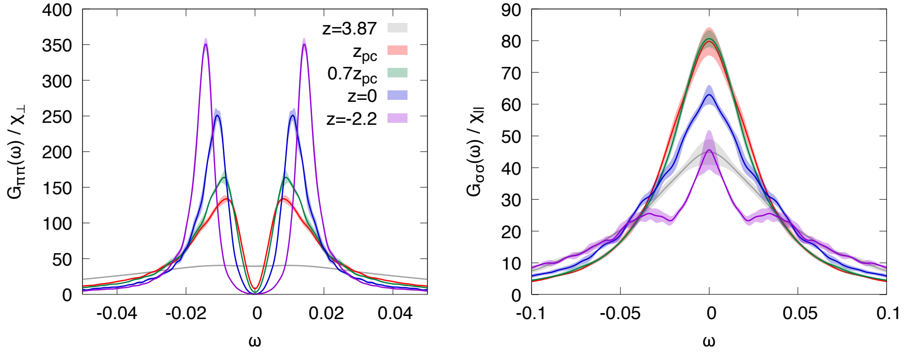
Both v_0 and m scale with the condensate:

$$v_0^2 \propto \underbrace{\bar{\sigma}^2}_{\text{condensate}}$$

$$v_0^2 m^2 \propto \underbrace{\bar{\sigma}}_{\text{condensate}}$$

which vanishes at the critical point, $\bar{\sigma} \propto (-t)^{\beta}$

Comparison of π and σ



Dynamical scaling of σ correlation functions:

$$G_{\sigma\sigma}(\omega) = \int \mathrm{d}t\, \mathrm{d}^3x\, e^{i\omega t}\ \langle \sigma(t, \boldsymbol{x}) \cdot \sigma(0, \mathbf{0}) \rangle$$