Higgs Boson Production via Bottom Quark mediated Gluon Fusion

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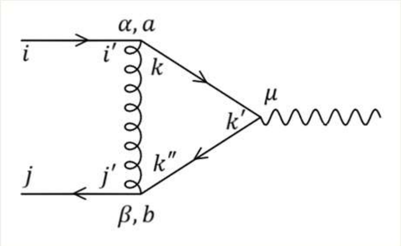
Motivation and Background

- Sudakov radiative corrections for the electron scattering amplitudes at high energies in QED ($\propto \alpha_e^n \ln^{2n}(m_e^2/q^2)$) [1]
- Extension to other processes, sub-leading logs, non-Abelian amplitudes as well as power suppressed amplitudes at high energies (different origin from Sudakov logs) [2]
- The leading deviation in the QFT amplitudes from the classical results in the asymptotic region [2]
- Necessary for accurate theoretical prediction for all major processes at LHC, Drell-Yan scattering, Deep Inelastic Scattering etc.
- Generalization to higher orders in small mass expansion is a challenging problem

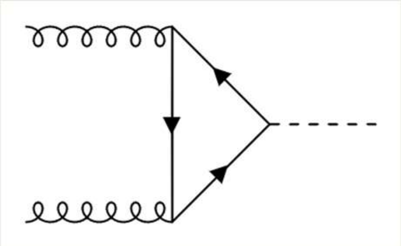
Motivation and Background

Example 1: Quark Scattering by color singlet gauge field



- Double log contribution to Dirac FF at all orders of α_s in leading order of small mass approximation ($\propto m_q^2$)
- Pauli FF for sub-leading order in mass $\left(\propto m_q^4\right)$ was not considered
- Important for $t\,\bar{t}$ pair production at future electron positron collider

Example 2: Higgs production via quark mediated gluon fusion

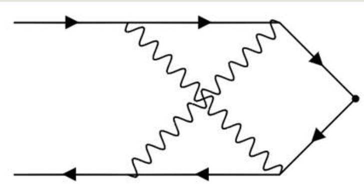


- top quark dominates ($\propto m_H^2$) while bottom quark amplitude suppressed by ($\propto m_b^2/m_H^2$)
- With logarithmic enhancement $\left(\ln^2\left(m_b^2/m_H^2\right)\alpha_s \approx 40\alpha_s\right)$
- $\mathcal{O}(m_b)$ amplitude already calculated, focus now on $\mathcal{O}\left(m_b^3\right)$

Origin of the double logs

Sudakov

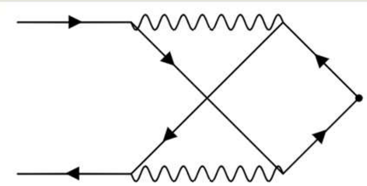
- Gauge boson(s) exchange
- Eikonal fermion and soft gauge boson propagators
- Not mass-suppressed
- Exponentiate



(a)

Non-Sudakov

- Fermion(s) exchange
- Soft fermion and eikonal gauge boson propagators
- Mass-suppressed
- Requires systematic Evaluation



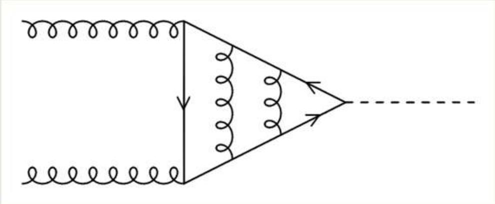
(b)

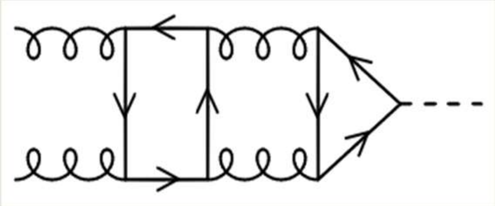
Higgs Production via Gluon Fusion

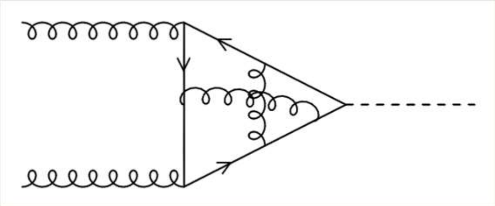
- Higgs production via gluon fusion process must have loops [3]
- $\mathcal{M}^{q}_{ggH} = T_F \frac{\alpha_s}{\pi} \frac{y_q m_q}{m_H^2} \left(p_1^\nu p_2^\mu - g^{\mu\nu} p_2 p_1\right) A^m_\mu(p_1)\, A^m_\nu(p_2)\, H\, M^q_{ggH}$
- Top loop dominates $\left(M^q_{ggH} = -\frac{2}{3\rho} = -\frac{2m_H^2}{3m_t^2}\right)$ but theoretical uncertainty comes from the bottom loop
- Asymptotic series expansion of the FF: $M^q_{ggH} = Z_g^2 \sum_{n=0}^{\infty} \rho^n M^{(n)}_{ggH}$ with universal Sudakov factor for external gluons $Z_g^2 = \exp\left(-\frac{C_A s^{-\epsilon}}{\epsilon^2} \frac{\alpha_s}{2\pi}\right)$ and $\rho = m_q^2/m_H^2$

Higgs Production via Gluon Fusion

- Leading coefficient $M_{ggH}^{(0)}$ (sub-leading in mass) was calculated [2], attention now on $M_{ggH}^{(1)}$
- 3 groups of diagrams contribute at this order
  - Single soft quark exchange
  - Triple soft quark exchange
  - Non-factorizing diagrams with single soft quark exchange
- On-shell conditions and double log variable:
  - $p_1^2 = p_2^2 = 0, \mathrm{q}^2 = (p_1 + p_2)^2 = 2p_2 p_1 = m_H^2$
  - $z = (C_A - C_F)x = (C_A - C_F)\frac{\alpha_s}{4\pi}\ln^2 \rho$

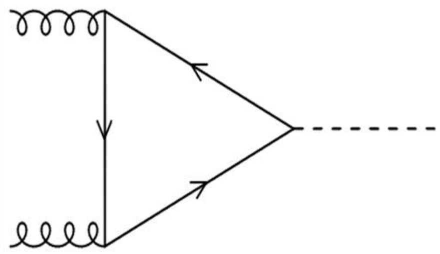






Higgs Production: Single Soft Quark Exchange

- $\mathcal{M}_{ggH,1L}^{\mu\nu,mn} = -g_s^2 y_q \int \frac{d^4 l_1}{(2\pi)^4} \frac{tr\, N_{ggH,1L}^{\mu\nu,mn}}{D_{1L}}$
- $D_{1L} = (l_1^2 - m^2)\{(p_1 - l_1)^2 - m^2\}\{(l_1 + p_2)^2 - m^2\}$
- Sudakov parameterization: $l_1 = u_1\, p_1 + v_1\, p_2 + l_{1_\perp}$
- Approximations in soft quark limit ($m_q \ll l \ll q$)
  - $l_1^2 - m^2 \approx \left[-2i\pi\, \delta\left(q^2 u_1 v_1 + l_{1_\perp}^{\,2} - m^2\right)\right]^{-1}$
  - $(p_1 - l_1)^2 - m^2 \approx -2p_1 l_1 \approx -2p_1 p_2 v_1 = -q^2 v_1$
  - $(p_2 + l_1)^2 - m^2 \approx 2p_2 l_1 \approx 2p_2 p_1 u_1 = q^2 u_1$
- Additional mass factors for $\mathcal{O}(m^3)$ related to $l_1^2 = m^2$, and not the chirality flip

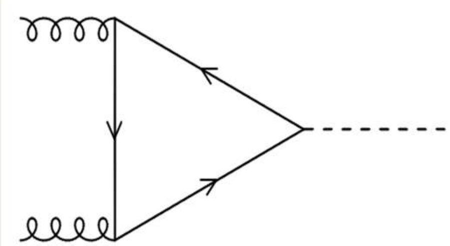


(a)

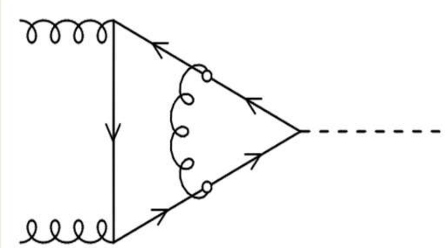
Higgs Production: Single Soft Quark Exchange

- 1-loop result:

$$M_{ggH}^{q,1L} = (1-4\rho)\ln^2\rho \Rightarrow \begin{cases} M_{ggH}^{(0),1L} = \ln^2\rho \\ \left[M_{ggH}^{(1),1L}\right]_{1q} = -4\ln^2\rho \end{cases}$$


(a)

- Resummation with the effective diagram:
  - $M_{ggH}^{(0)} = \ln^2\rho\, g(z)$
  - $\left[M_{ggH}^{(1)}\right]_{1q} = -4\ln^2\rho\, g(z) = -4\, M_{ggH}^{(0)}$
  - $g(z) = 2\int_0^1 d\eta_1 \int_0^{1-\eta_1} d\xi_1\, e^{2z\eta_1\xi_1}$

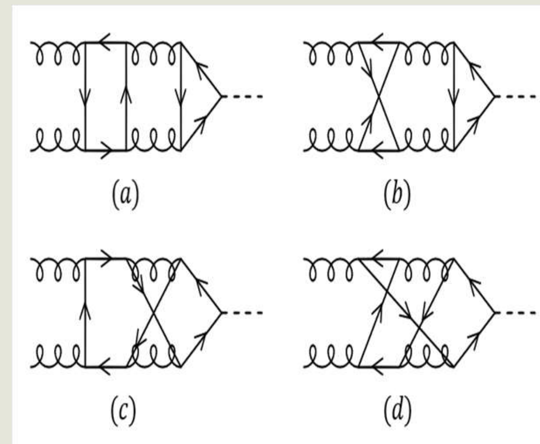

(b)

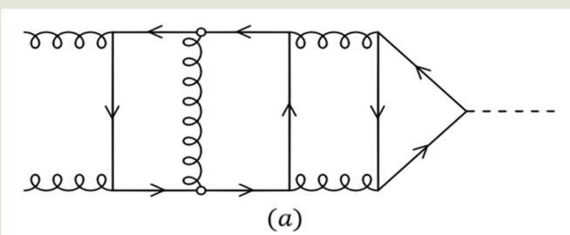
Higgs Production: Triple Soft Quark Exchange

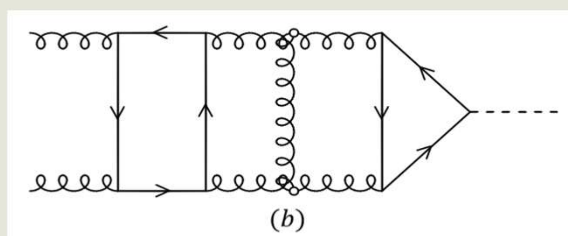
- Only (a) contributes: $\left[M_{ggH}^{(1),3L}\right]_{3q} = \frac{x^2 T_F C_F}{45} \ln^2 \rho$

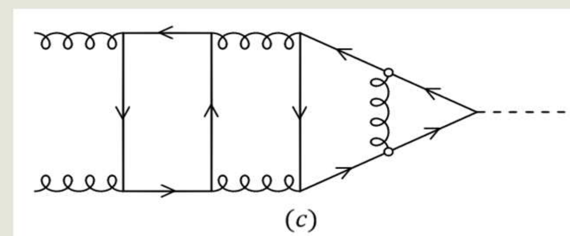
- Effective diagrams give $\left[M_{ggH}^{(1)}\right]_{3q} = \frac{x^2 T_F C_F}{45} \ln^2 \rho\, h(z)$

where $h(z) = 6! \int_0^1 d\eta_1 \int_0^{1-\eta_1} d\xi_1 \int_0^{\eta_1} d\eta_2 \int_0^{\xi_1} d\xi_2$
$\int_0^{\eta_2} d\eta_3 \int_0^{\xi_2} d\xi_3 \left(e^{2z\ _1\xi_1} e^{-2z\eta_2\xi_2} e^{2z\ _3\xi_3}\right)$







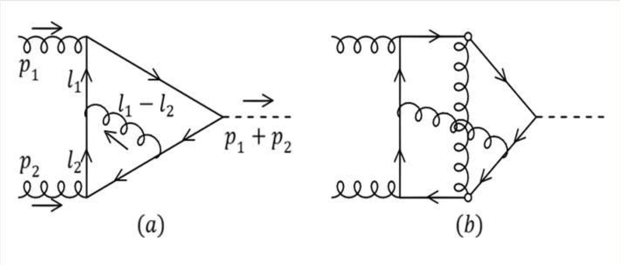


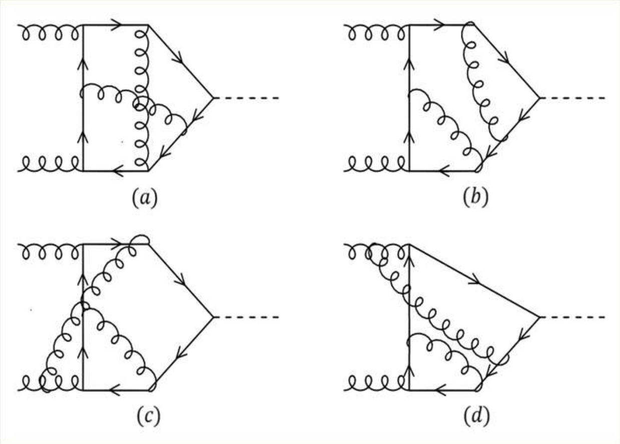
Higgs Production: Non-Factorizable

- Eikonal gluon, necessary structure not present at two loops
- At three loops:
  - (b) cancels specifically at this order
  - (c) and (d) factorize and don't contribute
  - Only (a) with effective vertices contributes

$$\left[M_{ggH}^{(1),3L}\right]_{NF} = -\frac{x^2(C_A - C_F)(C_A - 2C_F)}{9}\ln^2\rho$$

- Resummation requires systematic treatment of self-interacting gluons





Higgs Production : Complete NNLO Result

- Estimate $\left[M_{ggH}^{(1)}\right]_{NF} = -\frac{x^2(C_A-C_F)(C_A-2C_F)}{9}\ln^2\rho\, j(z)$ where $j(z) = ?$
- $M_{ggH}^{(1)} = \left[-4g(z) + x^2\left\{\frac{T_F C_F}{45}h(z) - \frac{(C_A-C_F)(C_A-2C_F)}{9}j(z)\right\}\right]\ln^2\rho$
- For QCD with $N_c \to \infty$, $M_{ggH}^{(1)} \approx -4g(z)\ln^2\rho$
- Opposite abelian limit $C_A = 0$, $z = -C_F x$

$$j^{ab}(z) = 72\int_0^1 d\eta_1 \int_0^{1-\eta_1} d\xi_1 \int_0^{1-\xi_1} d\eta_2 \int_0^{1-\eta_2-\xi_1} d\xi_2 \,\{\eta_1\xi_2\, e^{2z\eta_1(\xi_1+\xi_2)}\}\left[1 + \frac{e^{-2z\eta_1\xi_1-1}}{2} + \frac{e^{-2z\eta_1\xi_1-1+2z\eta_1\xi_1}}{4z\eta_1\xi_2}\right]$$

Higgs Production: Complete NNLO Result

- Asymptotic behavior as $z \to \infty$, $g(z) \sim \left(\frac{2\pi e^z}{z^3}\right)^{1/2}$ and $g(-z\,) \sim \frac{\gamma_E + \ln 2z}{z}$
- Large N_c limit, $M_{ggH}^{(1)} = -4\ln^2\rho\, g\left(\frac{N_c x}{2}\right)$ i.e., exponential enhancement
- Abelian limit: $g(-z) \propto \frac{1}{z}$, $h(-z) \propto \frac{1}{z^3}$, $j^{ab}(-z) \propto \frac{1}{z^2} \Rightarrow$ reduces to $g(-z)$
- $1 + \rho\left[-4 + \frac{x^2}{g(z)}\left\{\frac{T_F C_F}{45} h(z) - \frac{(C_A - C_F)(C_A - 2C_F)}{9} j(z)\right\}\right]$
- For b-quark $\rho \approx 1.6 \times 10^{-3} \Rightarrow -0.64\%$ universal (same for QCD and QED) correction with respect to leading order [3]

Thank you!

Questions

References

[1] V.V. Sudakov. Vertex parts at very high-energies in quantum electrodynamics. Sov.Phys. JETP 3 (1956), pp. 65–71.

[2] T. Liu and A. Penin, “High-energy limit of mass-suppressed amplitudes in gauge theories”, Journal of High Energy Physics, vol. 2018, Nov 2018.

[3] T. Liu, S. Modi, and A. A. Penin, “Higgs boson production and quark scattering amplitudes at high energy through the next-to-next-to-leading power in quark mass,” Journal of High Energy Physics, vol. 2022, p. 170, Feb 2022.