Decay Rates of Positronium Species

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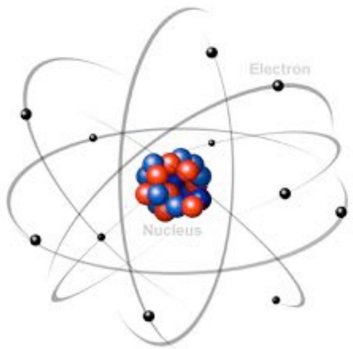
Outline

1. Exotic States
2. Polyelectrons (Ps, Ps^-, Ps_2)
3. Motivation
4. Spinor Matrix Method
5. Decay Rates of Di-positronium
6. Conclusion

Exotic Atoms

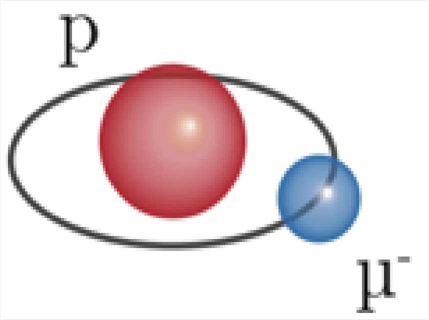
- We know an atom as :
  Positively charged NUCLEUS (made up of protons and neutrons)
  Negatively charged ELECTRONS (orbiting around nucleus)



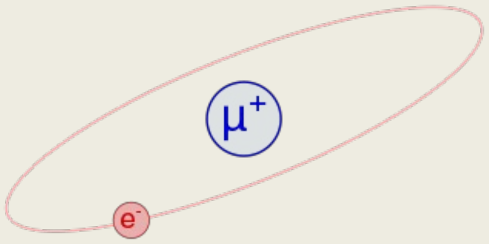
Is it possible to create atoms from subatomic particles other than electrons, protons and neutrons?

An exotic atom is an atom in which one or more sub-atomic particles have been replaced by other particles of the same charge.

A heavy negative particle (e.g muon) revolving around the nucleus.



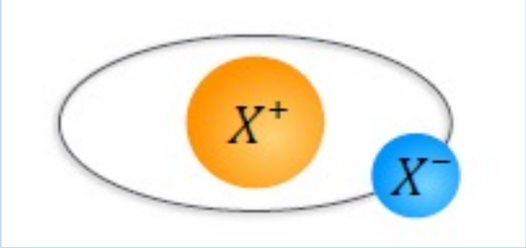
A heavier nuclear particle such as a muon or an antiproton.



Both Nucleons and electrons are replaced by heavier particles

Pionium

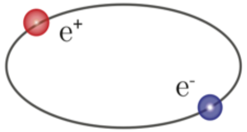
- hydrogen-like atom consisting of π^+ and π^- mesons.



Polyelectrons

Positronium Ps

- Bound state of e^+ *and* e^-
- Predicted in 1932 (Anderson) and 1934 Mohorovičič.
- confirmed by Martin Deutsch in 1951



S = 0 ; m = 0 p-Ps
S = 1 ; m = -1, 0, 1 o-Ps

$$\Gamma = \frac{m\alpha^5}{2} = \frac{1}{124\text{ps}}.$$

Positronium $Ps^{\pm}$

- 3-body Bound state consist of e^+ *and* e^-
- Observed in 1981 by A. P. Mills
- $Ps^- \rightarrow e^-\gamma$ in 1983 by Y. K. Ho
- $Ps^+ \rightarrow e^-\gamma$ in 1986 by M.C.Chu Corrected by S. I. Kryuchkov, in 1994.

Well-known 2 and 3 body states

Di-Positronium Ps_2

- 4-body Bound state of e^+'s *and* e^-'s
- Predicted in 1946 by Wheeler
- Observed in 2007 by David Cassidy and Allen Mills at the University of California.

Tree-level decays have not yet been correctly evaluated.

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(e^-, e^+)-pair annihilation in the positronium molecule Ps_2

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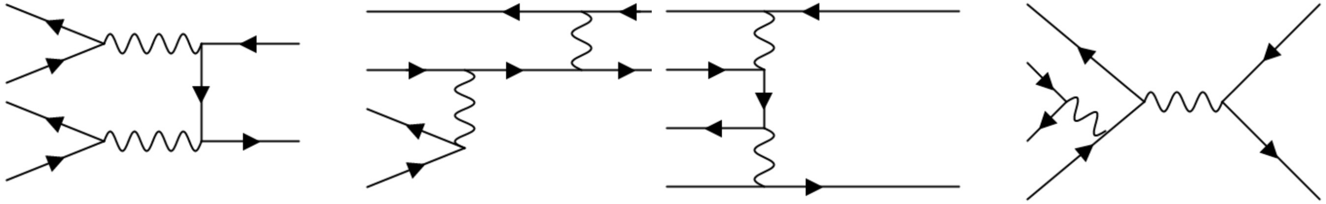
(Received 19 May 1994; revised manuscript received 28 November 1994)



$$\Gamma(e^-e^+) = \frac{147\sqrt{3}\,\pi^3\alpha^4}{2}|\Psi(0,0,0)|^2 \approx 2.32\times10^{-9}\ \text{sec}^{-1}$$

Two-photon total annihilation of molecular positronium

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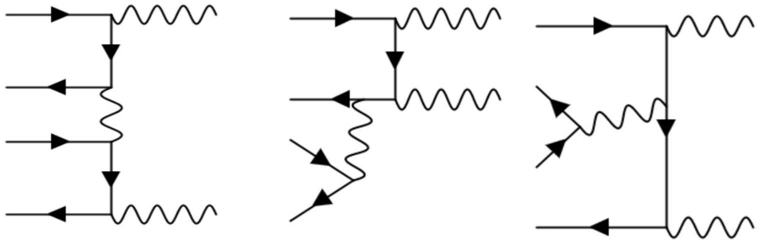
(Dated: December 18, 2014)

The rate for complete two-photon annihilation of molecular positronium Ps_2 is reported. This decay channel involves a four-body collision among the fermions forming Ps_2, and two photons of 1.022 MeV, each, as the final state. The quantum electrodynamics result for the rate of this process is found to be $\Gamma_{Ps_2\to\gamma\gamma} = 9.0\times10^{-12}\ \text{s}^{-1}$. This decay channel completes the most comprehensive decay chart for Ps_2 up to date.



$$\Gamma(\gamma\gamma) = \frac{521\,\pi^3\,\alpha^4}{1024}|\Psi(0,0,0)|^2 \approx 9\times10^{-12}\ \text{sec}^{-1}$$

$$\frac{\Gamma\left(Ps_2 \to e^+e^-\right)}{\Gamma\left(Ps_2 \to \gamma\gamma\right)} \simeq 250.$$

Puzzle

- Same order in α
- Two particles final state

Very large ratio, Why??

Love's Explanation
The zero-photon decay involves three vertices, whereas the two-photon decay channels require four vertices.

For $\Gamma(\gamma\gamma)$:
$Diagrams\ solved = 8$
$Total\ Diagrams = 40$

Spinor-Matrix Method

$Matrix\ Elements \sim Conjugate\ Spinor_{1\times4} \times Matrix_{4\times4} \times Spinor_{4\times1}$

$$\bar{u}Mv = \bar{u}_i M_{ij} v_j = v_j \bar{u}_i M_{ij} = (v\bar{u})_{ji} M_{ij} = \mathrm{Tr}\left[v\bar{u}M\right]$$

$$u = \sqrt{2m}\begin{pmatrix}1\\0\\0\\0\end{pmatrix} \qquad \bar{u} = \sqrt{2m}\begin{pmatrix}1 & 0 & 0 & 0\end{pmatrix} \qquad u\bar{u} = 2m\begin{pmatrix}1&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{pmatrix} = 2m\frac{\mathbf{1}+\gamma^0}{2}\frac{\gamma^5+\gamma^3}{2}\gamma^5$$

Advantages

- Amplitude level calculation.
- Gives amplitude for specific spins.
- Full knowledge on the amplitude values.
- Time efficient and very simple.

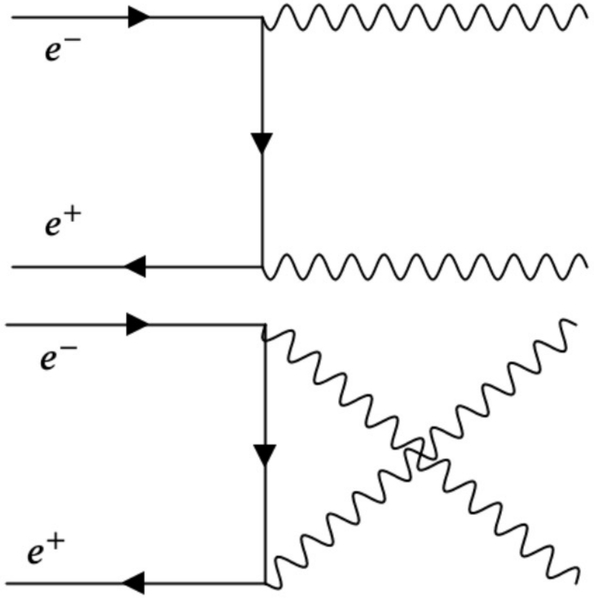
$Ps_2 \rightarrow e^+e^-$

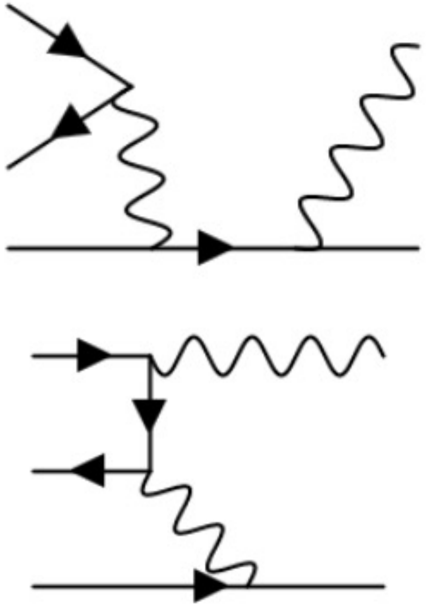
Frolov 1296 terms

Our Cal. 128 terms

Para-Positronium

Positronium Ion Ps^-





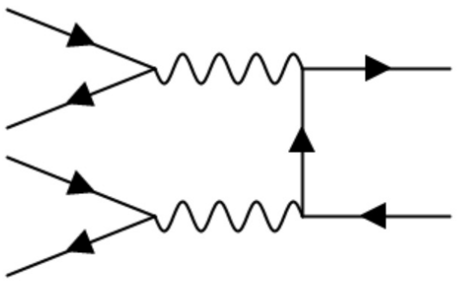
$$\Gamma\left(\text{p-Ps} \to \gamma\gamma\right) = \frac{m\alpha^5}{2} = \frac{1}{124\text{ps}}.$$

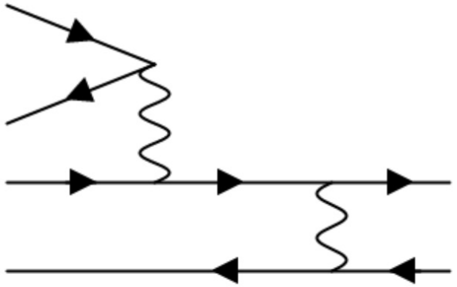
$$\Gamma\left(\text{Ps}^- \to e^-\gamma\right) = \frac{64}{27m^5}\pi^2\alpha^3\left|\Psi\left(0,0,0\right)\right|^2,$$

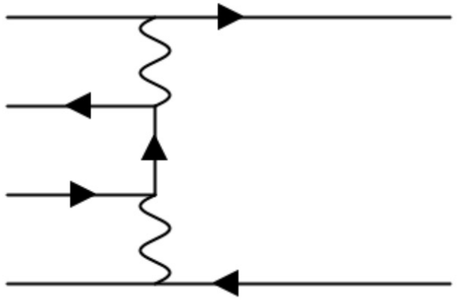
Tested Spinor Matrix method

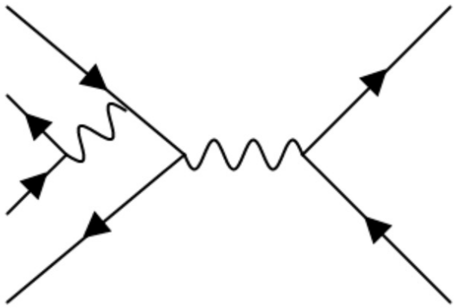
Radiation-less Decay of Ps_2

Possible Diagrams









=0

Ground State

Spatial part of wave function of $e^-(e^+)$ = Symmetric
Spin part of wave function of $e^-(e^+)$ = anti-symmetric

$$\mathcal{M} = \frac{1}{\sqrt{2}}\left(\mathcal{M}_{e^-_\uparrow e^-_\downarrow} - \mathcal{M}_{e^-_\downarrow e^-_\uparrow}\right)\cdot\frac{1}{\sqrt{2}}\left(\mathcal{M}_{e^+_\uparrow e^+_\downarrow} - \mathcal{M}_{e^+_\downarrow e^+_\uparrow}\right)$$
$$= \frac{1}{2}\left(\mathcal{M}_{e^-_\uparrow e^+_\uparrow e^-_\downarrow e^+_\downarrow} + \mathcal{M}_{e^-_\downarrow e^+_\downarrow e^-_\uparrow e^+_\uparrow} - \mathcal{M}_{e^-_\uparrow e^+_\downarrow e^-_\downarrow e^+_\uparrow} - \mathcal{M}_{e^-_\downarrow e^+_\uparrow e^-_\uparrow e^+_\downarrow}\right).$$

Only 4 spin Configurations

$$\mathcal{M}_{e^-_\uparrow e^+_\uparrow e^-_\downarrow e^+_\downarrow} = 3\sqrt{3}\frac{ie^4}{m^2},\quad \mathcal{M}_{e^-_\downarrow e^+_\downarrow e^-_\uparrow e^+_\uparrow} = 3\sqrt{3}\frac{ie^4}{m^2},\quad \mathcal{M}_{e^-_\uparrow e^+_\downarrow e^-_\downarrow e^+_\uparrow} = -3\sqrt{3}\frac{ie^4}{m^2},\quad \mathcal{M}_{e^-_\downarrow e^+_\uparrow e^-_\uparrow e^+_\downarrow} = -3\sqrt{3}\frac{ie^4}{m^2}.$$

Ps_2 Decay Rate

$$\mathcal{M}\left(e^+e^-e^+e^- \to e^-e^+\right) = 96\sqrt{3}\frac{i\pi^2\alpha^2}{m^2},$$

Free and Bound State Amplitudes

$$\mathcal{M}\left(\mathrm{Ps}_2 \to e^-e^+\right) = \sqrt{2M}\Psi\left(0,0,0\right)\frac{\mathcal{M}_{\uparrow\downarrow}(e^+e^-e^+e^- \to e^-e^+)}{\sqrt{2E_1}\sqrt{2E_2}\sqrt{2E_2}\sqrt{2E_2}}$$

$$= 24\sqrt{6M}\frac{i\pi^2\alpha^2}{m^4}\Psi\left(0,0,0\right),$$

Rate

$$\Gamma\left(\mathrm{Ps}_2 \to e^-e^+\right) = \frac{1}{16}\cdot\frac{1}{4}\cdot\frac{1}{2M}\int d\Pi_{\mathrm{LIPS}}\left|\mathcal{M}\right|^2$$

$$= 4.27 \times 10^{-10}\ \mathrm{s}^{-1}$$

Comparison of Decay Rates

Frolov, Kryuchkov Result

$$\Gamma(e^+e^-) \approx 2.322\times 10^{-9} s^{-1}$$

$$\frac{\Gamma(e^+e^-)}{\Gamma(\gamma\gamma)} \approx 250$$

Our Result

$$\Gamma(e^+e^-) \approx 4.27\times 10^{-10} s^{-1}$$

$$\Gamma(\gamma\gamma) \approx 3.54\times 10^{-11} s^{-1}$$

$$\frac{\Gamma(e^+e^-)}{\Gamma(\gamma\gamma)} \approx 12$$

Perez, Love Result

$$\Gamma(\gamma\gamma) \approx 9\times 10^{-12} s^{-1}$$

Reasons for the large ratio ~ 250

- Overestimated the rate by a factor of 5.44.
- Summation over all the final state spins is taken, which includes contributions from triplet configurations of initial state electrons (and positrons).

- underestimated the rate by a factor of 3.93.
- Sum over all initial spin configurations
- sums all amplitudes without implementing anti-symmetrization.

Summary

Dipositonium Ps_2

- 4-body Bound state of e^{+}'s and e^{-} 's
- Can decay into $n\gamma$, $n = 0,1,2,3 \ldots$.

$$\Gamma(e^{+}e^{-}) \approx 2.322\times10^{-9}s^{-1}$$
$$\Gamma(\gamma\gamma) \approx 9\times10^{-12}s^{-1}$$
$$\frac{\Gamma(e^{+}e^{-})}{\Gamma(\gamma\gamma)} \approx 250$$

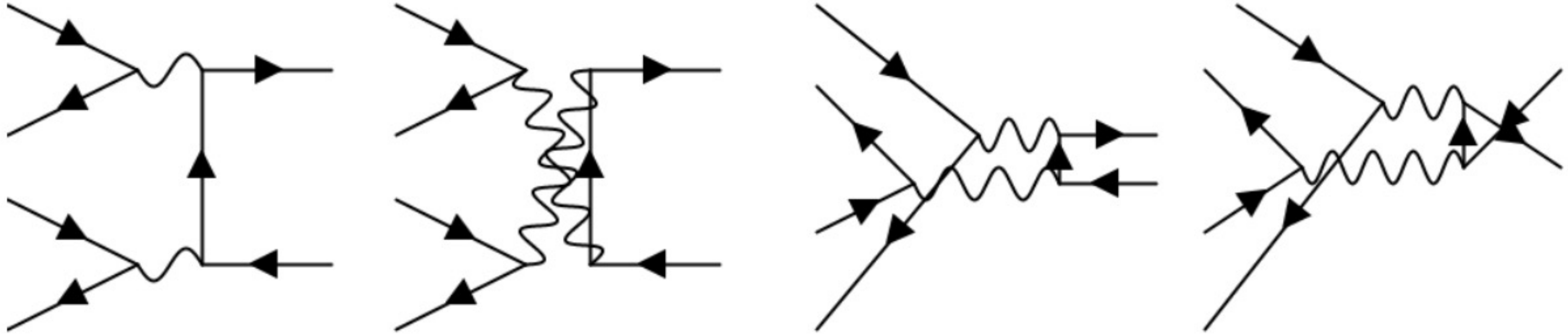
→

$$\Gamma(e^{+}e^{-}) \approx 4.27\times10^{-10}s^{-1}$$
$$\Gamma(\gamma\gamma) \approx 3.54\times10^{-11}s^{-1}$$
$$\frac{\Gamma(e^{+}e^{-})}{\Gamma(\gamma\gamma)} \approx 12$$

Thank You
For Your Attention

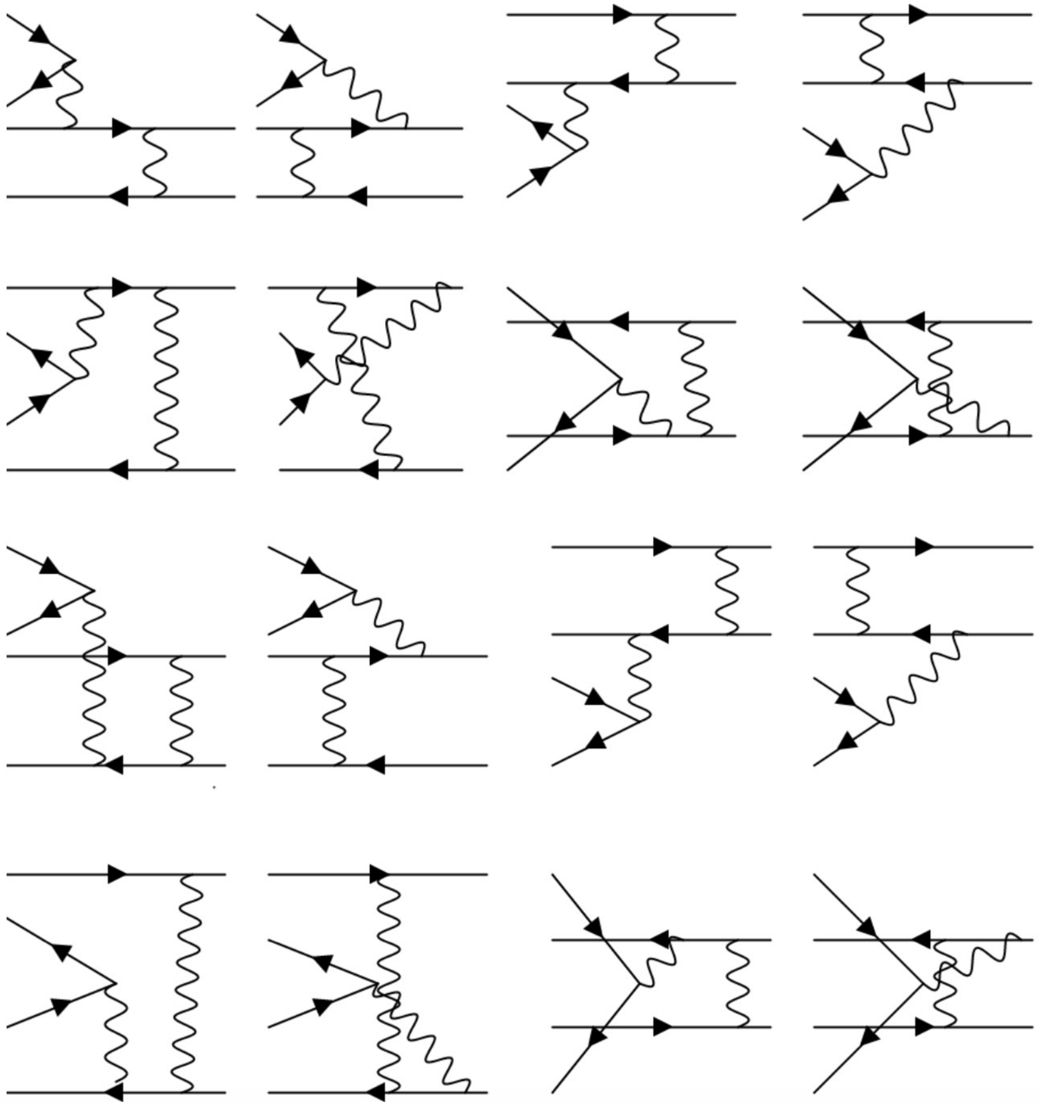
Backup Slides

Amplitudes-I



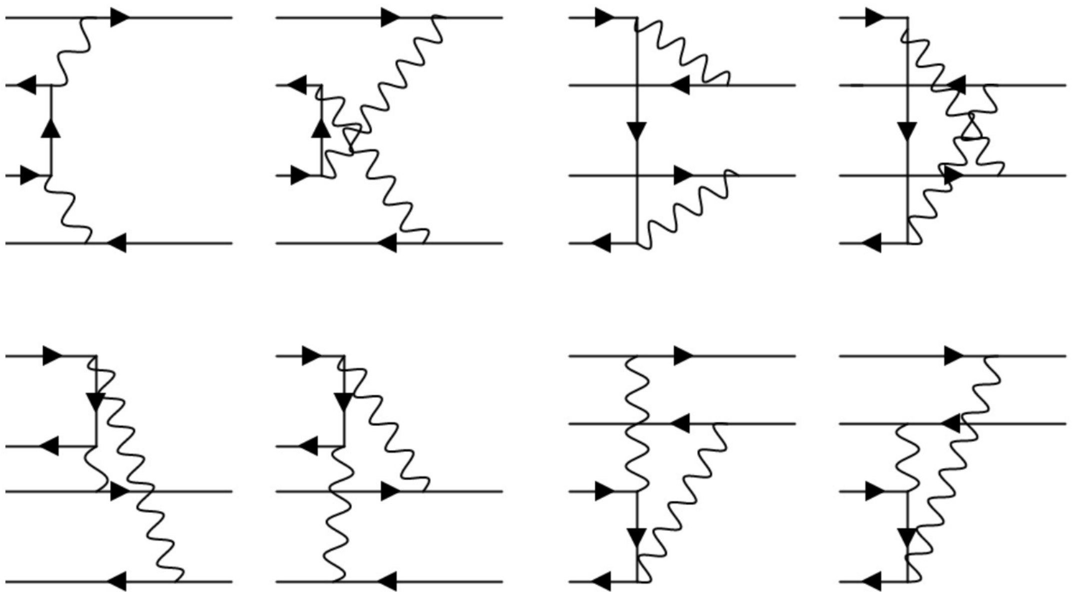
$\times 16\sqrt{3}\frac{ig_e^4}{m^5}$	$\mathcal{M}_{e_\uparrow^- e_\uparrow^+ e_\downarrow^- e_\downarrow^+}$	$\mathcal{M}_{e_\downarrow^- e_\downarrow^+ e_\uparrow^- e_\uparrow^+}$	$\mathcal{M}_{e_\uparrow^- e_\downarrow^+ e_\downarrow^- e_\uparrow^+}$	$\mathcal{M}_{e_\downarrow^- e_\uparrow^+ e_\uparrow^- e_\downarrow^+}$
$\mathcal{M}_{01}$	0	0	$-\frac{1}{32}$	$-\frac{1}{32}$
$\mathcal{M}_{02}$	0	0	$-\frac{1}{32}$	$-\frac{1}{32}$
$\mathcal{M}_{03}$	$\frac{1}{32}$	$\frac{1}{32}$	0	0
$\mathcal{M}_{04}$	$\frac{1}{32}$	$\frac{1}{32}$	0	0
	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{16}$	$-\frac{1}{16}$

Amplitudes-II



$\times 16\sqrt{3}\frac{ig_e^4}{m^5}$	$\mathcal{M}_{e^-_\uparrow e^+_\uparrow e^-_\downarrow e^+_\downarrow}$	$\mathcal{M}_{e^-_\downarrow e^+_\downarrow e^-_\uparrow e^+_\uparrow}$	$\mathcal{M}_{e^-_\uparrow e^+_\downarrow e^-_\downarrow e^+_\uparrow}$	$\mathcal{M}_{e^-_\downarrow e^+_\uparrow e^-_\uparrow e^+_\downarrow}$
$\mathcal{M}_{05}$	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{32}$	$-\frac{1}{32}$
$\mathcal{M}_{06}$	0	0	0	$\frac{1}{16}$
$\mathcal{M}_{07}$	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{32}$	$-\frac{1}{32}$
$\mathcal{M}_{08}$	0	0	$\frac{1}{16}$	0
$\mathcal{M}_{09}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{16}$	$-\frac{1}{16}$
$\mathcal{M}_{10}$	$-\frac{1}{16}$	0	0	0
$\mathcal{M}_{11}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{16}$	$-\frac{1}{16}$
$\mathcal{M}_{12}$	0	$-\frac{1}{16}$	0	0
$\mathcal{M}_{13}$	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{32}$	$-\frac{1}{32}$
$\mathcal{M}_{14}$	0	0	0	$\frac{1}{16}$
$\mathcal{M}_{15}$	$\frac{1}{16}$	$\frac{1}{16}$	$-\frac{1}{32}$	$-\frac{1}{32}$
$\mathcal{M}_{16}$	0	0	$\frac{1}{16}$	0
$\mathcal{M}_{17}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{16}$	$-\frac{1}{16}$
$\mathcal{M}_{18}$	$-\frac{1}{16}$	0	0	0
$\mathcal{M}_{19}$	$\frac{1}{32}$	$\frac{1}{32}$	$-\frac{1}{16}$	$-\frac{1}{16}$
$\mathcal{M}_{20}$	0	$-\frac{1}{16}$	0	0
	$\frac{1}{4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$

Amplitude-III



$\times 16\sqrt{3}\frac{ig_e^4}{m^5}$	$\mathcal{M}_{e_\uparrow^- e_\uparrow^+ e_\downarrow^- e_\downarrow^+}$	$\mathcal{M}_{e_\downarrow^- e_\downarrow^+ e_\uparrow^- e_\uparrow^+}$	$\mathcal{M}_{e_\uparrow^- e_\downarrow^+ e_\downarrow^- e_\uparrow^+}$	$\mathcal{M}_{e_\downarrow^- e_\uparrow^+ e_\uparrow^- e_\downarrow^+}$
$\mathcal{M}_{21}$	0	0	$-\frac{1}{8}$	$-\frac{1}{8}$
$\mathcal{M}_{22}$	0	$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\mathcal{M}_{23}$	0	0	$-\frac{1}{8}$	$-\frac{1}{8}$
$\mathcal{M}_{24}$	$-\frac{1}{8}$	0	$\frac{1}{8}$	$\frac{1}{8}$
$\mathcal{M}_{25}$	$\frac{1}{8}$	$\frac{1}{8}$	0	0
$\mathcal{M}_{26}$	$-\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$	0
$\mathcal{M}_{27}$	$\frac{1}{8}$	$\frac{1}{8}$	0	0
$\mathcal{M}_{28}$	$-\frac{1}{8}$	$-\frac{1}{8}$	0	$\frac{1}{8}$
	$-\frac{1}{8}$	$-\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

Spinor Products

Table 1.3.1: Gamma matrix representation of the product of spinors.

Spinors product	$\gamma-$Matrix representation
$u_\uparrow(p)\,\bar{v}_\uparrow(p)$	$m\left(1+\gamma^0\right)\frac{\gamma^1+i\gamma^2}{2}$
$u_\uparrow(p)\,\bar{v}_\downarrow(p)$	$-m\left(1+\gamma^0\right)\frac{\gamma^5+\gamma^3}{2}$
$u_\downarrow(p)\,\bar{v}_\uparrow(p)$	$m\left(1+\gamma^0\right)\frac{\gamma^5-\gamma^3}{2}$
$u_\downarrow(p)\,\bar{v}_\downarrow(p)$	$-m\left(1+\gamma^0\right)\frac{\gamma^1-i\gamma^2}{2}$
$v_\downarrow(k_2)\,\bar{u}_\uparrow(k_1)$	$-m\left[\sqrt{3}\left(1-i\gamma^2\gamma^1\right)-3\left(\gamma^5-\gamma^3\right)\frac{1+\gamma^0}{2}-\left(\gamma^5+\gamma^3\right)\frac{1-\gamma^0}{2}\right]$
$u_\uparrow(p)\,\bar{u}_\uparrow(k_1)$	$m\frac{1+\gamma^0}{\sqrt{2}}\left[\sqrt{3}\left(1-i\gamma^2\gamma^1\right)-\gamma^5-\gamma^3\right]$
$u_\downarrow(p)\,\bar{u}_\uparrow(k_1)$	$-m\frac{1+\gamma^0}{\sqrt{2}}\left[1+3\gamma^5\right]\left(\gamma^1-i\gamma^2\right)$
$v_\downarrow(k_2)\,\bar{v}_\uparrow(p)$	$-\frac{m}{\sqrt{2}}\left[1-\sqrt{3}\gamma^5\right]\left(1+\gamma^0\right)\left(\gamma^1+i\gamma^2\right)$
$u_\downarrow(k_2)\,\bar{u}_\downarrow(p)$	$\frac{m}{\sqrt{2}}\left[1-\sqrt{3}\gamma^5\right]\left(1+\gamma^0\right)\left(\gamma^3+\gamma^5\right)$

Free and Bound State Amplitudes(Ps)

- The force of attraction between e^- and e^+ is only Coulomb force.
- Solving Schrödinger equation will give us Ψ(r).
- The bound state is linear superposition of free states with definite r or k.
- It is convenient to express superposition in momentum space

$$\Psi(\boldsymbol{k}) = \int d^3x e^{i\mathbf{k}.\mathbf{r}} \Psi(\boldsymbol{r}),$$

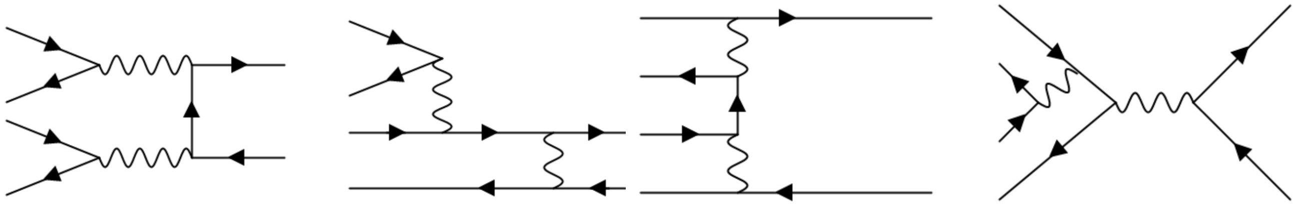
- The bound state is

$$|B\rangle = \sqrt{2M} \int \frac{d^3k}{(2\pi)^3} \Psi(\boldsymbol{k}) \frac{1}{\sqrt{2E_1}} \frac{1}{\sqrt{2E_2}} |\boldsymbol{k}_1 \uparrow, \boldsymbol{k}_2 \downarrow\rangle.$$

- The amplitude will be

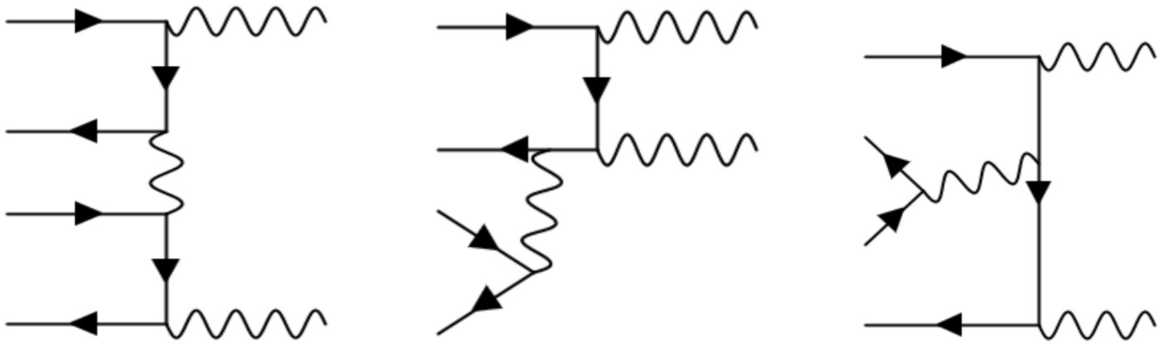
$$\mathcal{M}_{\uparrow\downarrow}(\text{p-P}_s \to \gamma\gamma) = \sqrt{2M} \int \frac{d^3k}{(2\pi)^3} \Psi\left(\boldsymbol{k}\right) \frac{1}{\sqrt{2E_1}} \frac{1}{\sqrt{2E_2}} \mathcal{M}_{\uparrow\downarrow}(e^+e^- \to \gamma\gamma)$$

Comparison



$$\mathcal{M}_{e^-_\uparrow e^+_\uparrow e^-_\downarrow e^+_\downarrow} = 3\sqrt{3}\frac{ie^4}{m^2}, \qquad \mathcal{M}_{e^-_\downarrow e^+_\downarrow e^-_\uparrow e^+_\uparrow} = 3\sqrt{3}\frac{ie^4}{m^2},$$

$$\mathcal{M}_{e^-_\uparrow e^+_\downarrow e^-_\downarrow e^+_\uparrow} = -3\sqrt{3}\frac{ie^4}{m^2}, \qquad \mathcal{M}_{e^-_\downarrow e^+_\uparrow e^-_\uparrow e^+_\downarrow} = -3\sqrt{3}\frac{ie^4}{m^2}.$$



$$\mathcal{M}_{e^-_\uparrow e^+_\uparrow e^-_\downarrow e^+_\downarrow} = -\frac{ie^4}{4m^4}, \qquad \mathcal{M}_{e^-_\downarrow e^+_\downarrow e^-_\uparrow e^+_\uparrow} = -\frac{ie^4}{4m^4},$$

$$\mathcal{M}_{e^-_\uparrow e^+_\downarrow e^-_\downarrow e^+_\uparrow} = +\frac{ie^4}{4m^4}, \qquad \mathcal{M}_{e^-_\downarrow e^+_\uparrow e^-_\uparrow e^+_\downarrow} = +\frac{ie^4}{4m^4}.$$

$$\frac{\int d\Pi_{\text{LIPS}}\,(e^-e^+)}{\int d\Pi_{\text{LIPS}}\,(\gamma\gamma)} = \sqrt{3}.$$

$$|\text{Amplitudes Ratio}|^2 \sim 6.75.$$