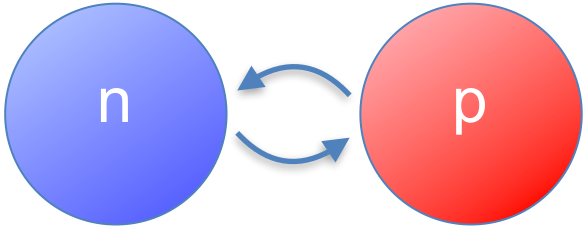
Isospin Breaking in the IMSRG

Ragnar Stroberg



Featuring work done in collaboration with:

Takayuki Miyagi, Antoine Belley, Jason Holt, Charlie Payne, Heiko Hergert, Jiangming Yao, Roland Wirth, Matt Martin, Kyle Leach, Gaute Hagen, Matthias Heinz, Emily Love

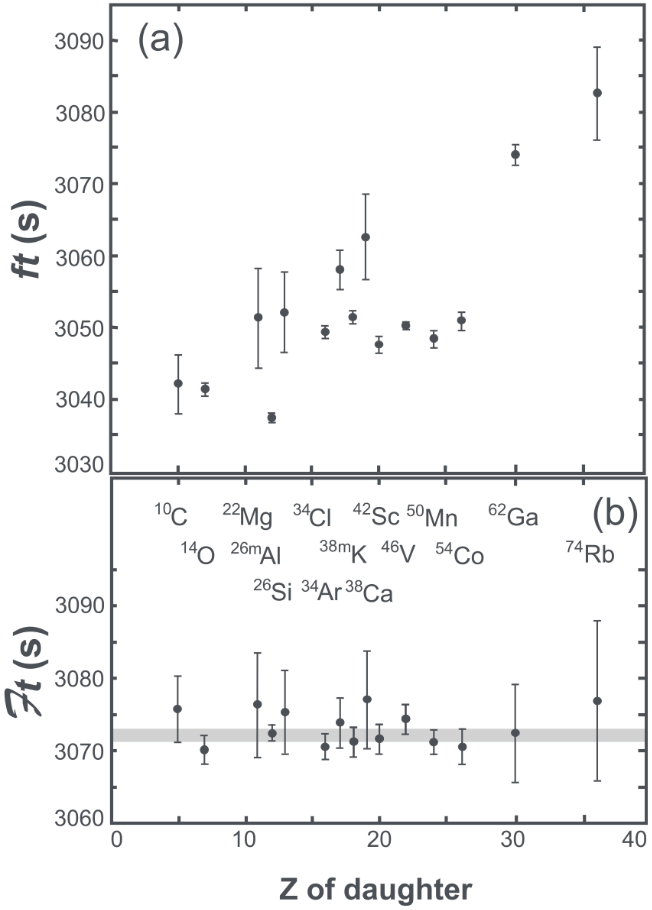
Progress in Ab Initio Nuclear Theory
Feb 28-March 3 2023
TRIUMF, Vancouver BC

Superallowed $0^+ \rightarrow 0^+ \, \beta$ decay

$$ft \approx \frac{K}{2G_F^2 V_{ud}^2}$$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ut} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Unitarity: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$



Towner & Hardy, PRC 102 045501 (2020)

$$\mathcal{F}t = ft(1 + \delta_{NS} - \delta_C)(1 + \delta'_R) = \frac{K}{2G_F V_{ud}(1 + \Delta_R^V)}$$

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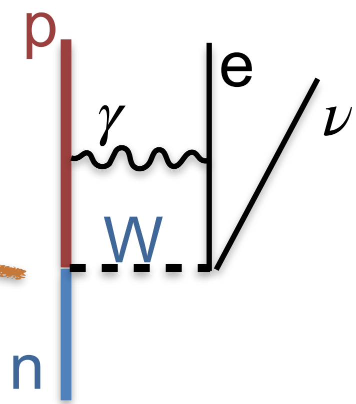

Radiative corrections (no nuclear structure needed)

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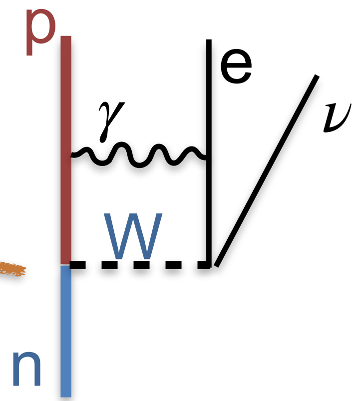

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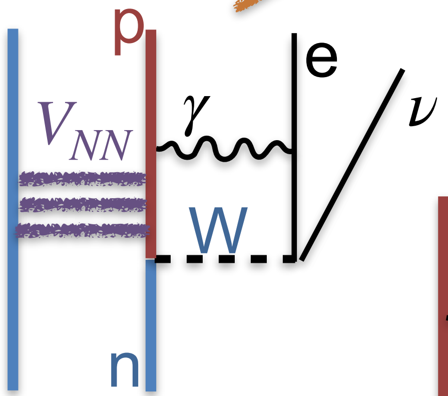
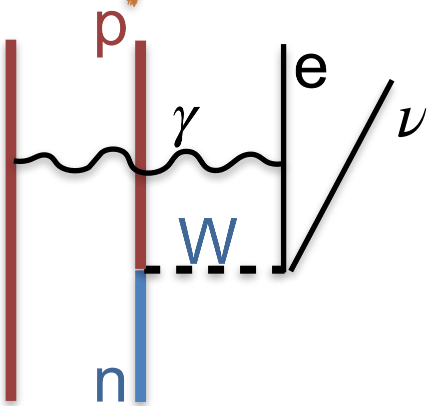


Nuclear-structure dependent radiative correction



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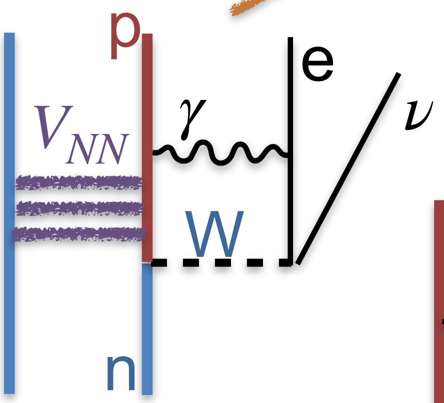
See poster by Michael Gennari



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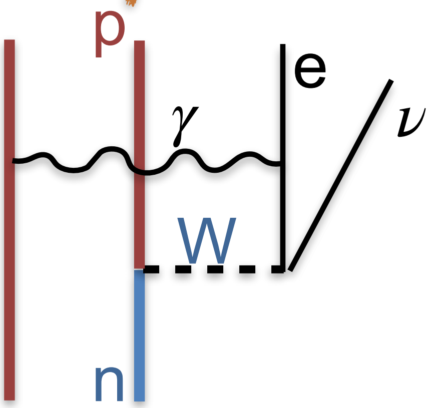
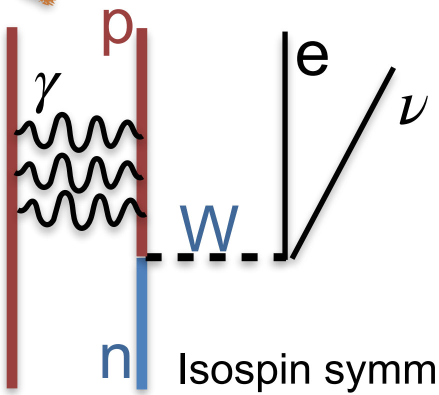
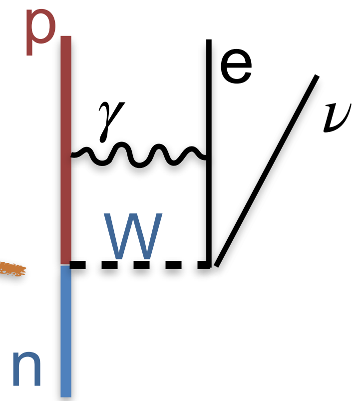




Isospin symmetry breaking correction



Radiative corrections (no nuclear structure needed)

$$|M_F|^2 = \langle \Psi_f | t_\pm | \Psi_f \rangle |^2 = (1 - \delta_C) \, | \langle TT_{z,f} | t_\pm | TT_{z,i} \rangle |^2$$

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=2 for most $0^+ \rightarrow 0^+$

$$\delta_C = \delta_{C1} + \delta_{C2} \sim 1\,\%$$

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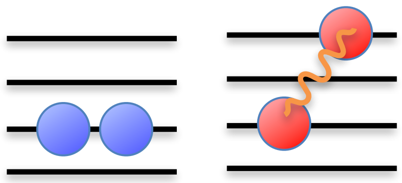
configuration mixing with phenomenological isospin-breaking interaction, adjusted case-by-case to IMME



$$|M_F|^2 = \langle \Psi_f | t_\pm | \Psi_f \rangle |^2 = (1-\delta_C)\, |\underbrace{\langle T T_{z,f} | t_\pm | T T_{z,i} \rangle}_{} |^2$$

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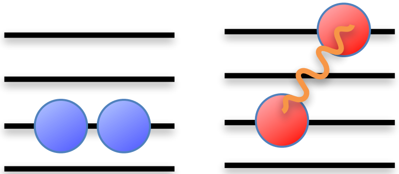
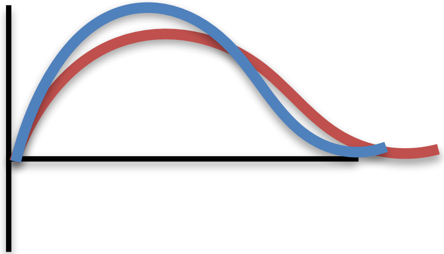


proton-neutron wave function mismatch, from Woods-Saxon adjusted case-by-case to S_p, S_n

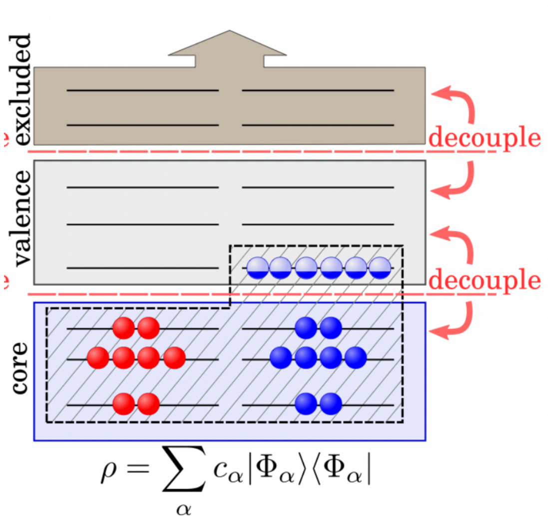


In-medium similarity renormalization group (IMSRG)

Normal-order w.r.t a reference $|\Phi_0\rangle$

Unitary transformation parameterized by flow parameter s

$$H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$



$$\langle \Psi_f | \mathcal{O}_F | \Psi_i \rangle = \langle \Phi_f^{\text{val}} | e^{\Omega} \mathcal{O}_F e^{-\Omega} | \Phi_i^{\text{val}} \rangle$$

$$H(s) = e^{\Omega(s)} H e^{-\Omega(s)}$$

$$= H + [\Omega, H] + \frac{1}{2}[\Omega, [\Omega, H]] + \dots$$

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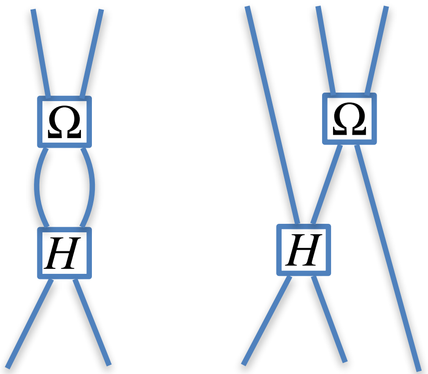
IMSRG(2): Truncate commutators at 2b level

IMSRG(3): Keep 3b terms (more painful)

See poster by Matthias Heinz



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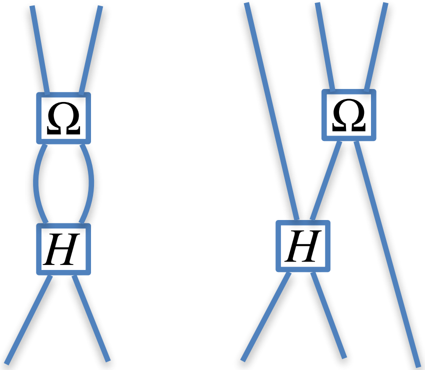
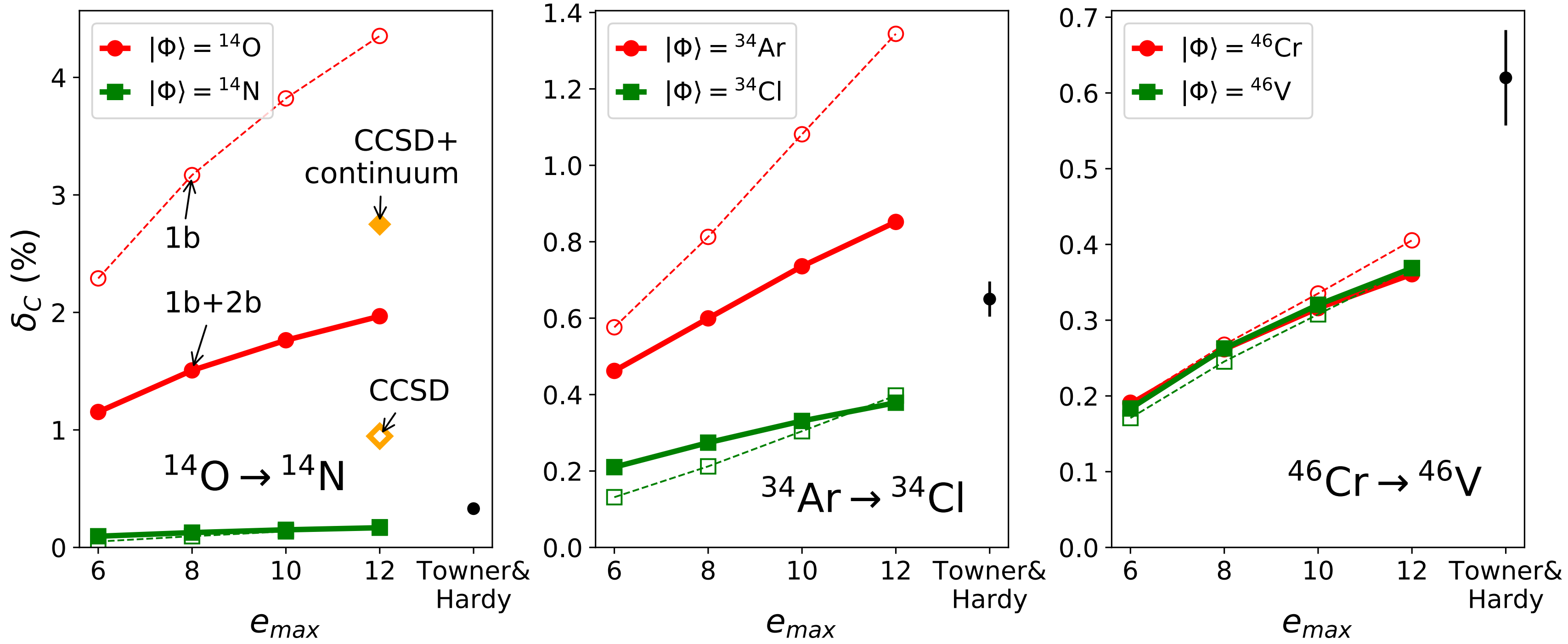


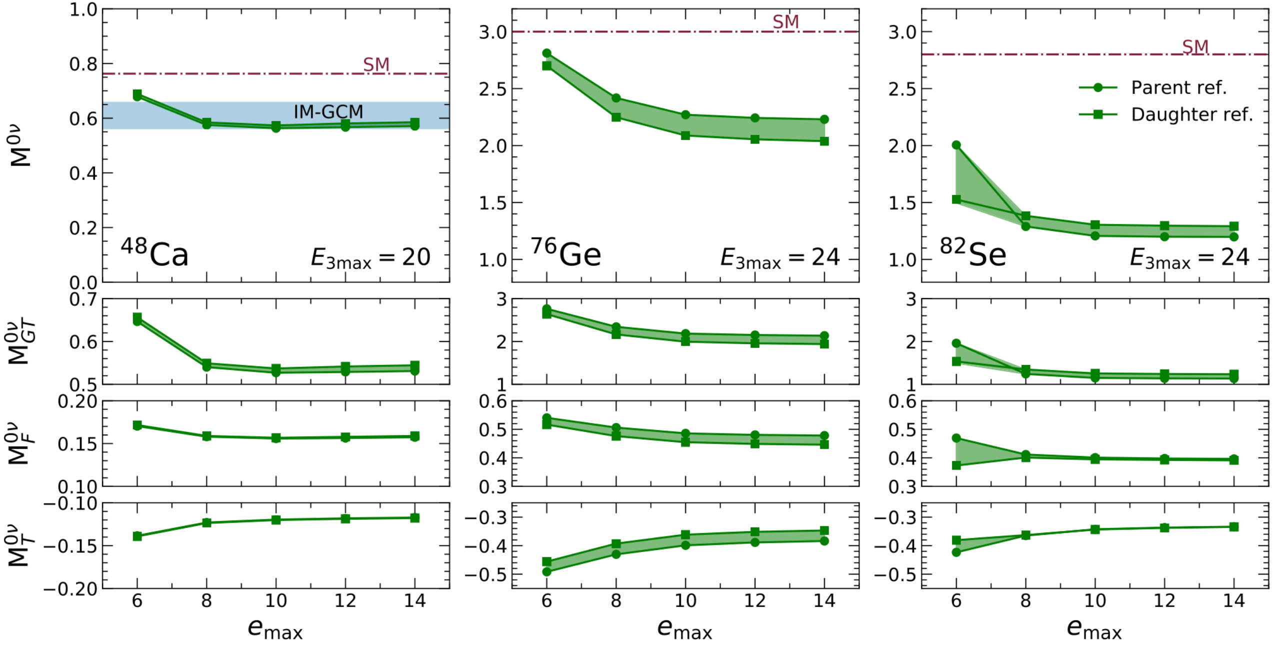
Due to normal ordering, truncation error depends on choice of reference $|\Phi_0\rangle$.

Should $|\Phi_0\rangle$ look like the parent? The daughter? Something in between?



SRS Particles 4, 521 (2021)

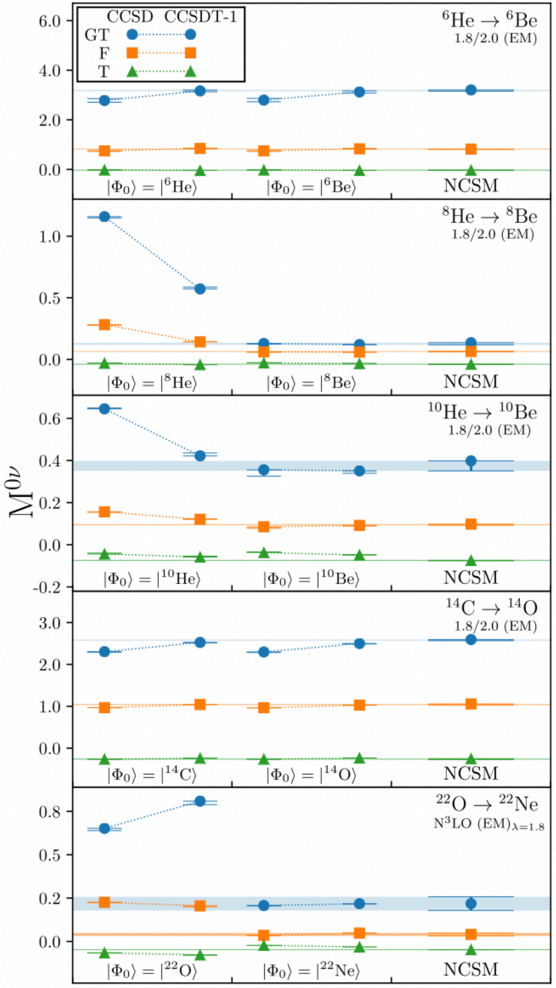
$0\nu\beta\beta$ decay



See talk by Antoine Belley

Belley+ PRL 126 042502 (2021)

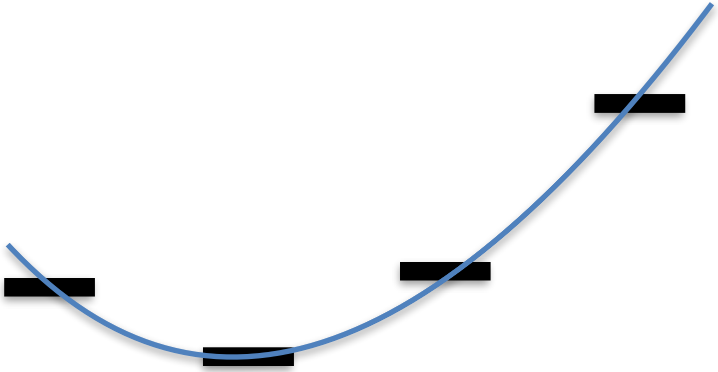
see also: JM Yao+ PRC 103 014315 (2021)

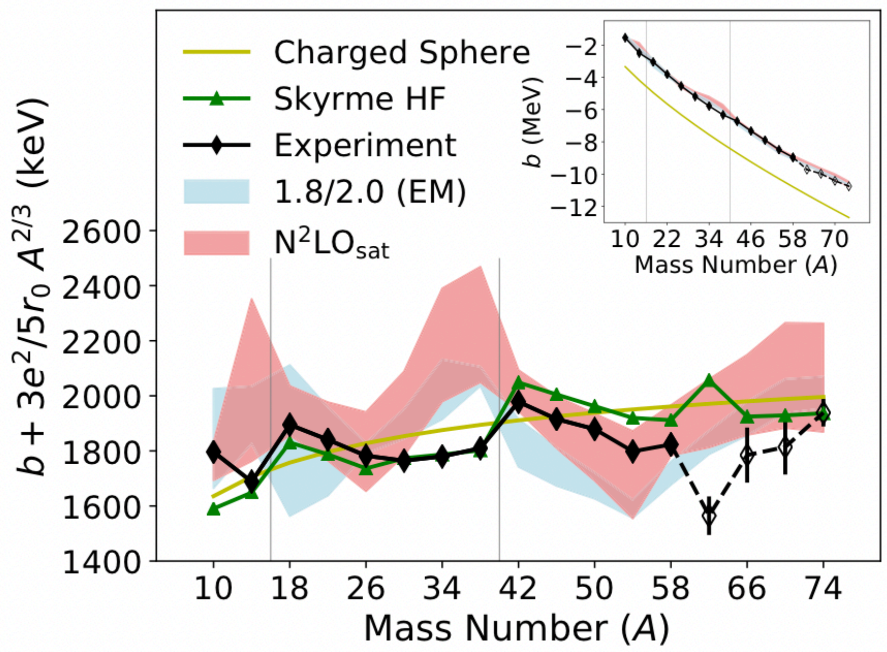


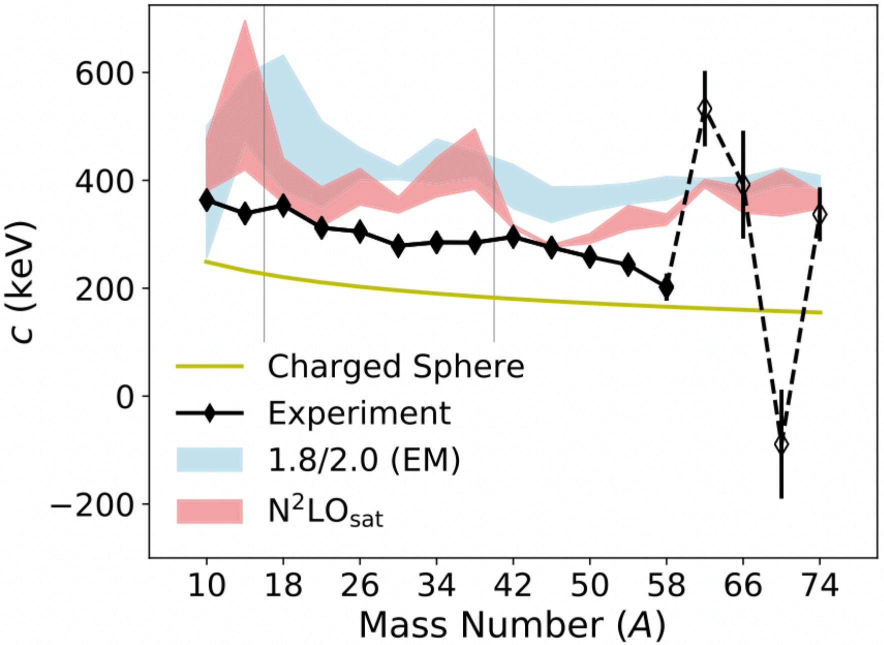
Novario+ PRL 126 182502 (2021)

Isobaric multiplet mass equation (IMME)

$$E(T_z) = a + bT_z + cT_z^2 + \ldots$$

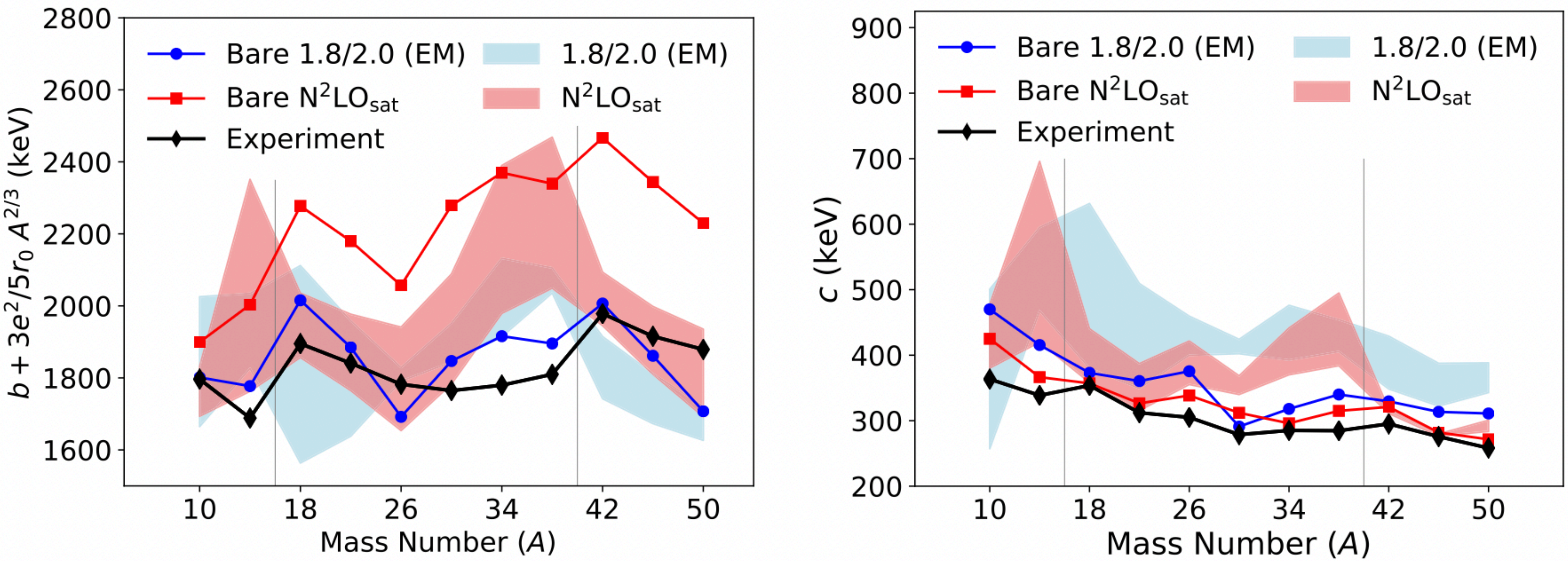






Shaded bands indicate sensitivity to reference $|\Phi_0\rangle$

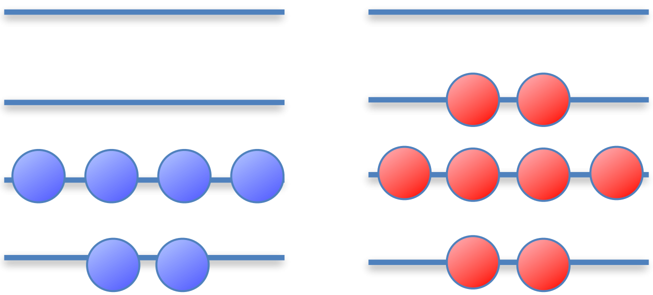
MS Martin+ PRC 104 014324 (2021)



"Bare" means no IMSRG evolution.

No obvious improvement from IMSRG

Spurious isospin breaking in Hartree-Fock for $N \neq Z$

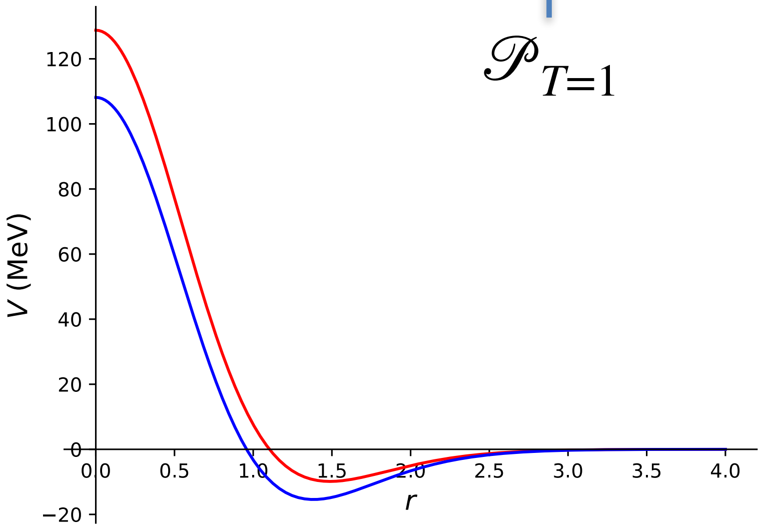


^{14}O

Even with an isospin-conserving interaction, $V_{pp} \neq V_{pn}$ so protons and neutrons see a different mean field. So $|HF\rangle$ is not an eigenstate of T^2.

Schematic isospin-conserving potential

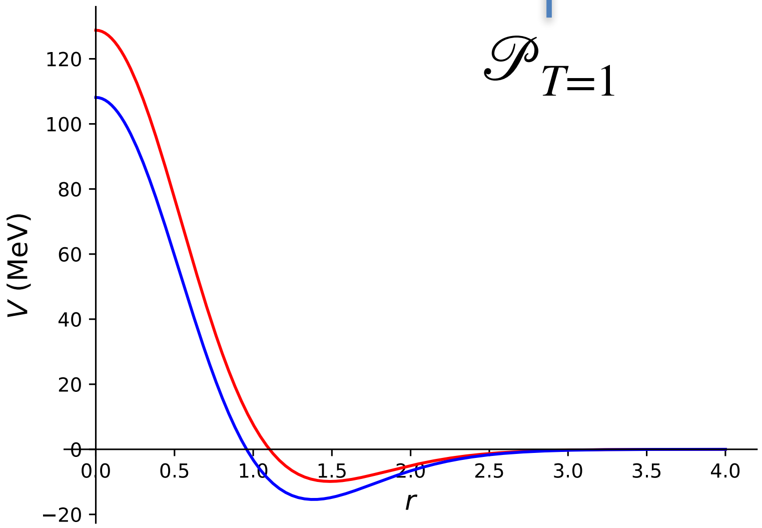
$$V = V_R\, e^{-k_R r^2} + V_S\, e^{-k_S r^2} \underbrace{\mathscr{P}_{S=0}}_{\mathscr{P}_{T=1}} + V_T\, e^{-k_T r^2} \underbrace{\mathscr{P}_{S=1}}_{\mathscr{P}_{T=0}} + V_{LS}\, \ell \cdot s$$



Schematic isospin-conserving potential

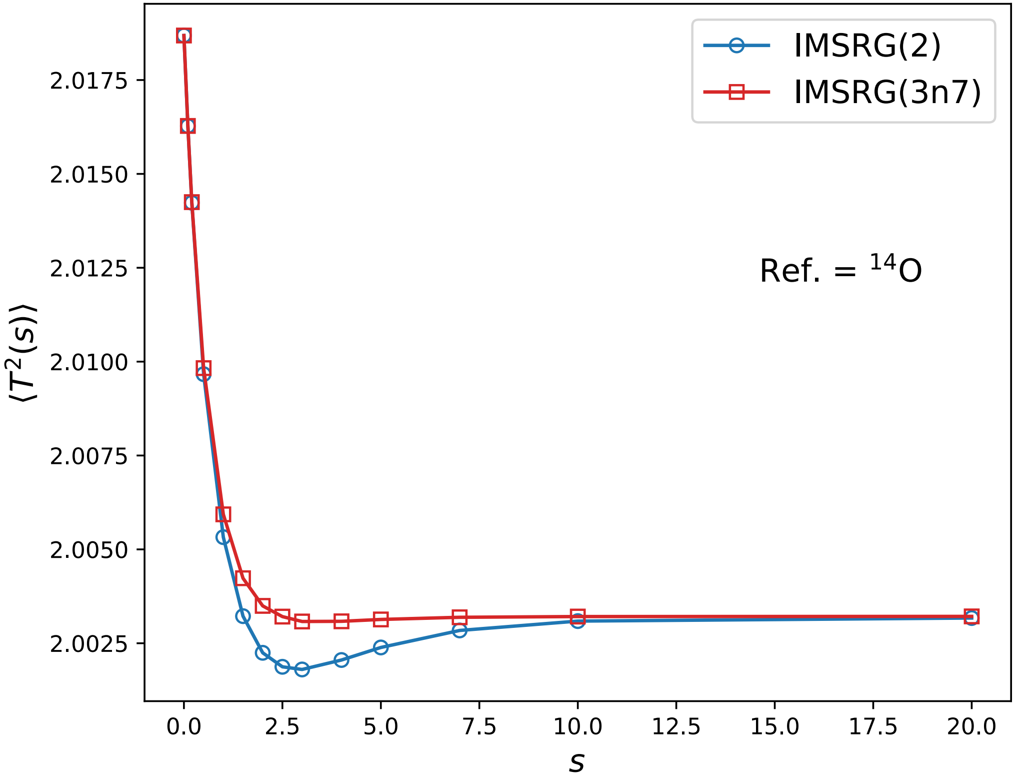
$$V = V_R\, e^{-k_R r^2} + V_S\, e^{-k_S r^2} \mathscr{P}_{S=0} + V_T\, e^{-k_T r^2} \mathscr{P}_{S=1} + V_{LS}\, \ell \cdot s$$

$\mathscr{P}_{T=1}$ $\mathscr{P}_{T=0}$



Caveat: what follows is **very** preliminary

Correlations on top of HF approximately restore good isospin

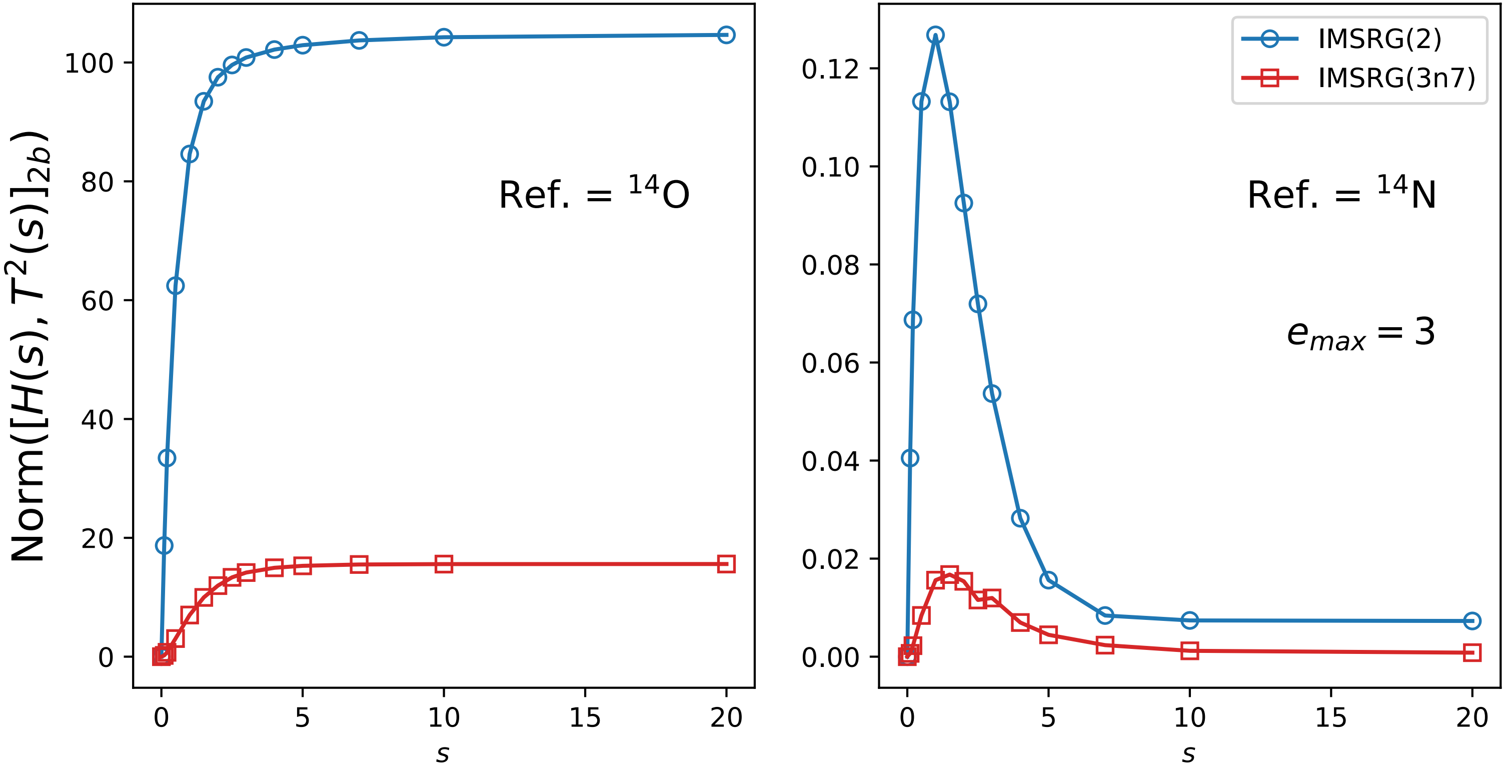


$$\frac{d}{ds}H(s) = [\eta(s), H(s)]$$

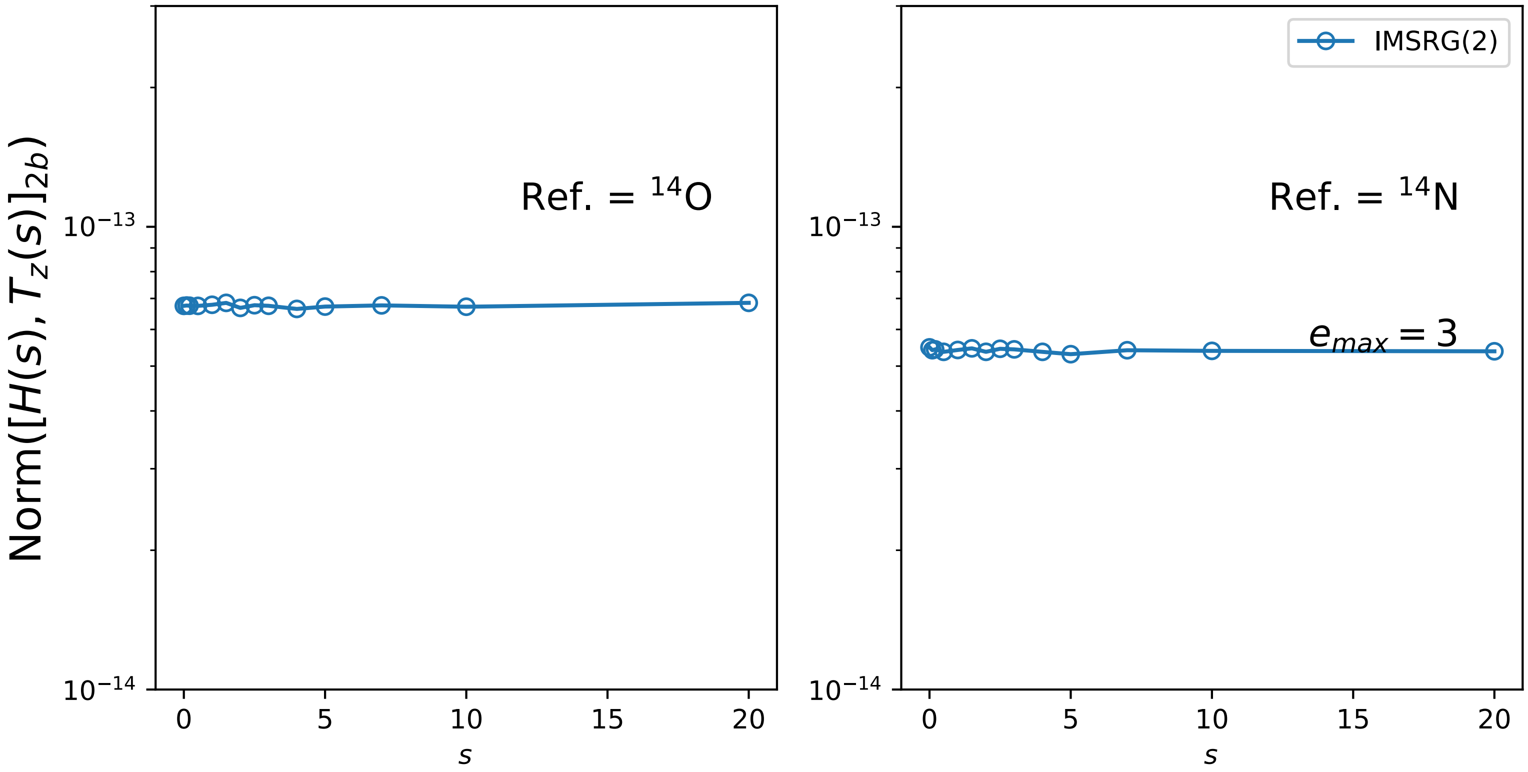
$$\frac{d}{ds}\mathcal{O}(s) = [\eta(s), \mathcal{O}(s)]$$

Isospin conservation during IMSRG flow

$[H(s), T^2(s)] = 0$ @ s=0

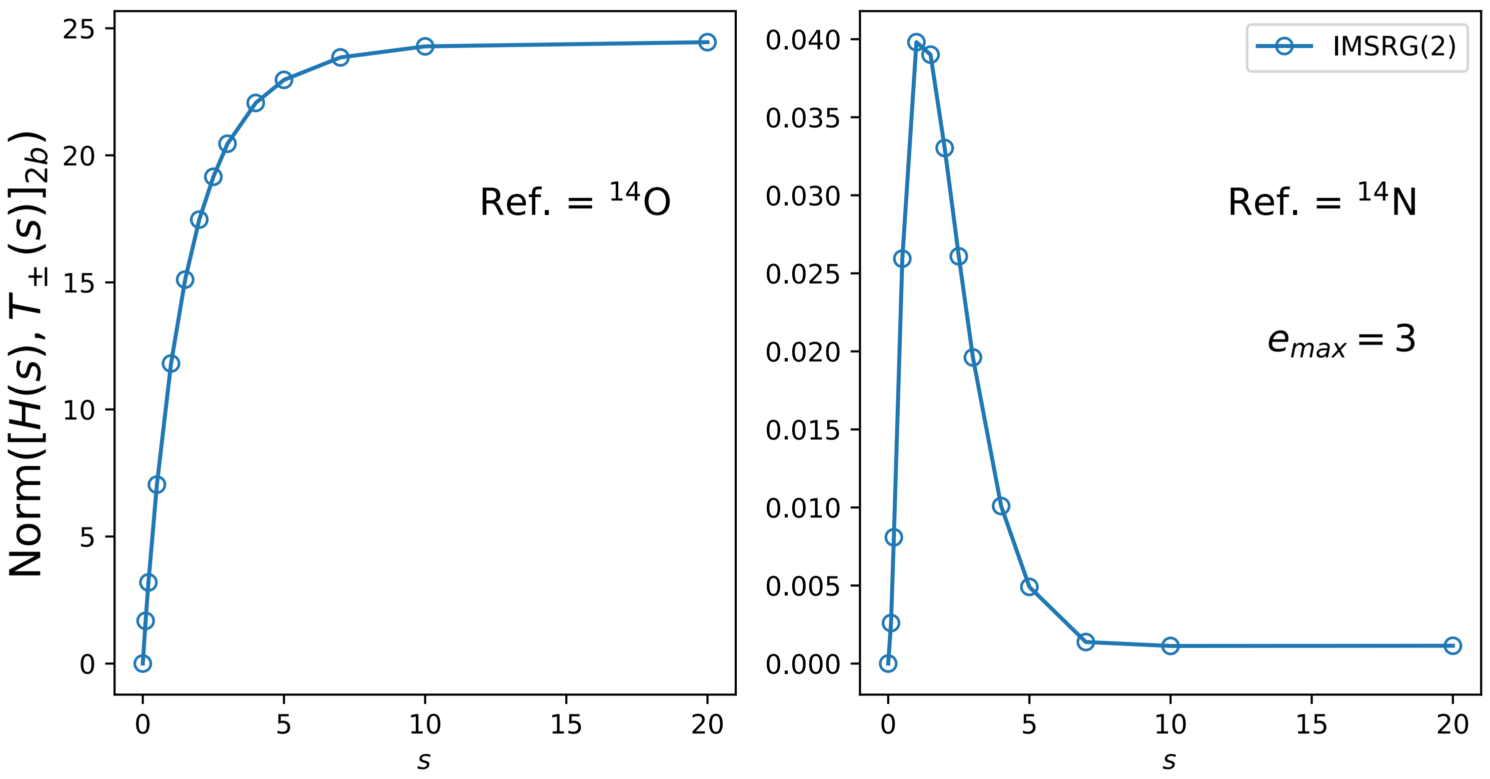


$[H(s), T_z(s)]$



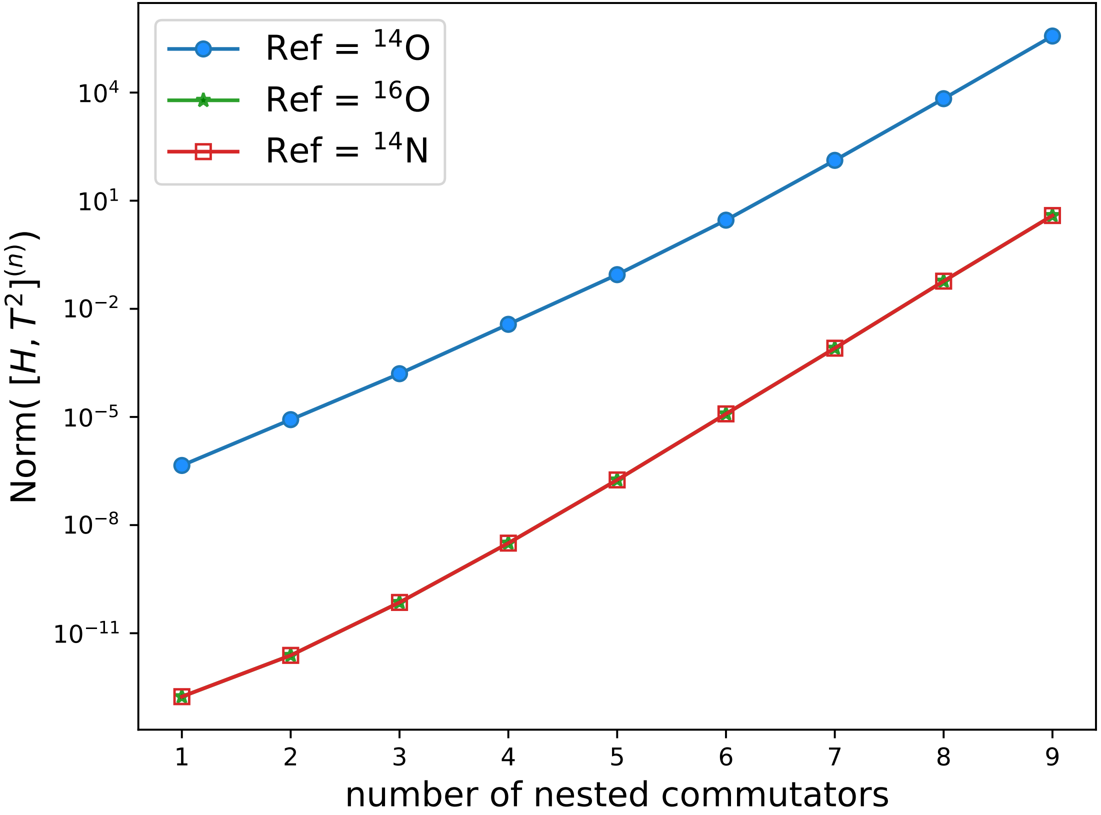
Norm([H(s), T_z(s)]_2b)
10^−13
10^−14
Ref. = ^14O
0
5
10
15
20
s
IMSRG(2)
Ref. = ^14N
e_max = 3

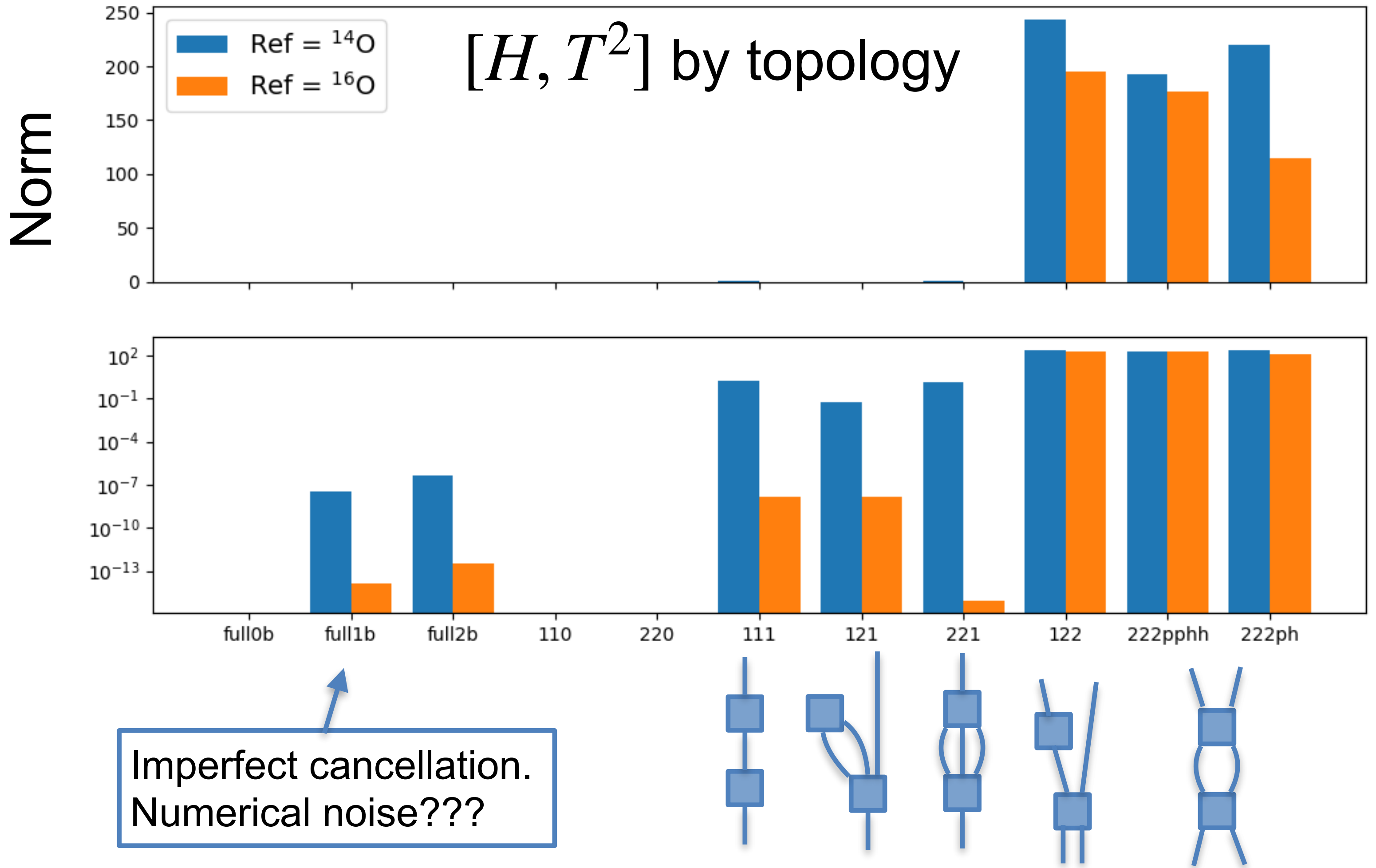
$[H(s), T_{\pm}(s)]$



Norm($[H(s), T_{\pm}(s)]_{2b}$)
25
20
15
10
5
0
0
5
10
15
20
s
Ref. = ^{14}O
0.040
0.035
0.030
0.025
0.020
0.015
0.010
0.005
0.000
IMSRG(2)
Ref. = ^{14}N
$e_{max} = 3$

$$[H,T^2]^{(n)} = \Big[H,\big[H\ldots,[H,T^2]\big]\Big]$$





Imperfect cancellation.
Numerical noise???

- If $|\Phi_0\rangle$ is not an eigenstate of T^2, this appears to lead to spurious isospin symmetry breaking in the IMSRG via the normal ordering.
- Analogous to m-scheme HF with $M_J \neq 0$.
- Unlike J^2, T^2 is not an exact symmetry, so we shouldn't simply project onto good T. (That would give $\delta_C = 0$).
- When searching for $O(1\%)$ corrections, spurious isospin breaking needs to be carefully controlled.