Nuclear Forces with Symmetry Preserving Regulator

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Workshop on Progress in Ab Initio Nuclear Theory
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In collaboration with Evgeny Epelbaum & Patrick Reinert

LENPIC

Low Energy Nuclear Physics International Collaboration

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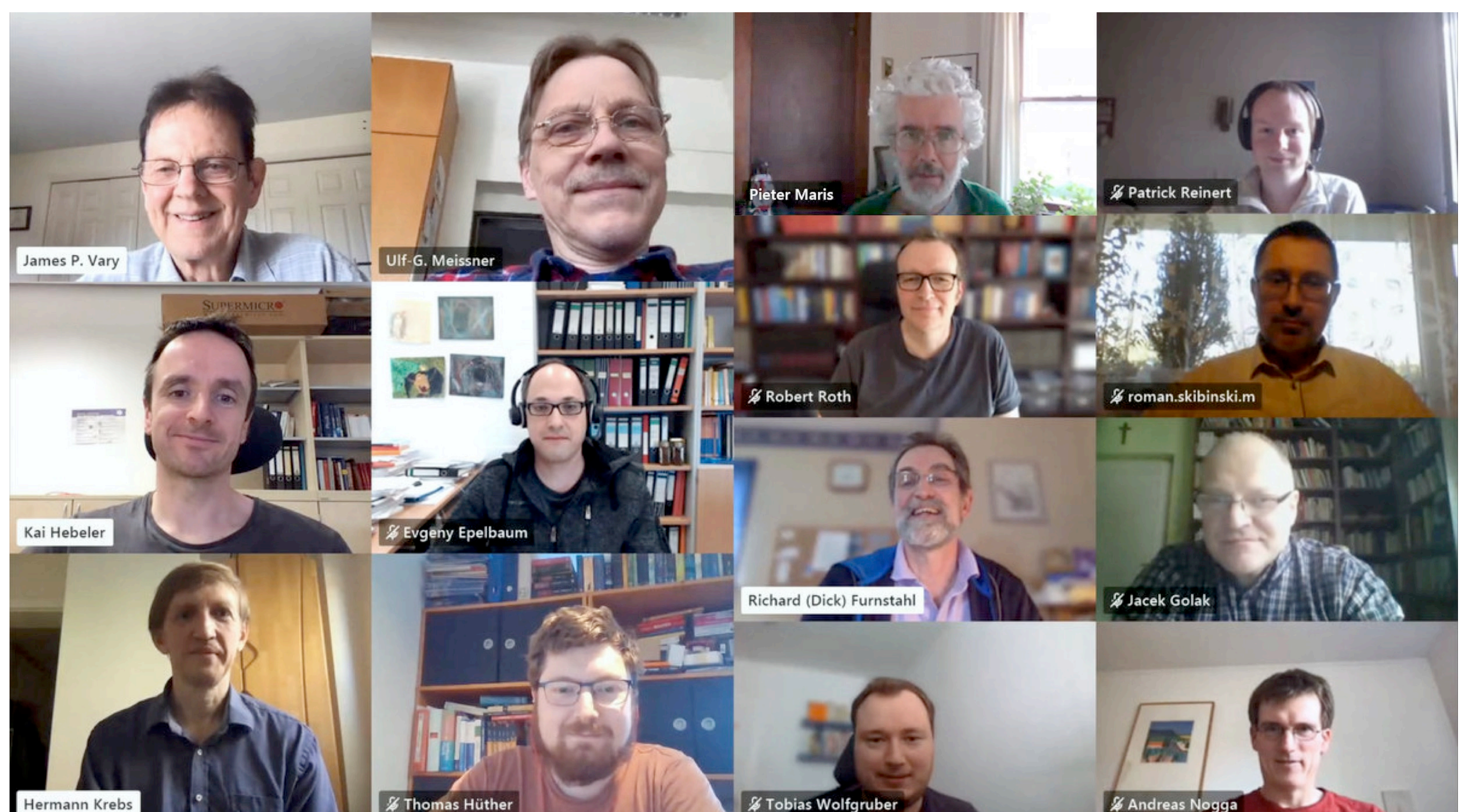
(LENPIC Collaboration)

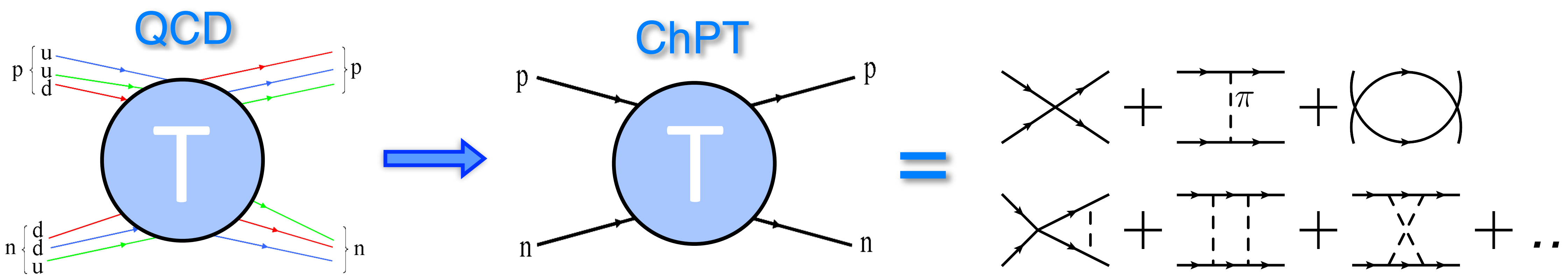
LENPIC aims to solve the structure and reactions of light nuclei including electroweak observables with consistent treatment of the corresponding exchange currents



Outline

- Nuclear forces in chiral EFT
- Selected observables in three-nucleon (3N) sector
- 3N forces (3NF) at N^3LO
  - Gradient flow regularization
  - Nuclear forces within path-integral formalism
  - Long range part of 3NF
  - Subtraction scheme

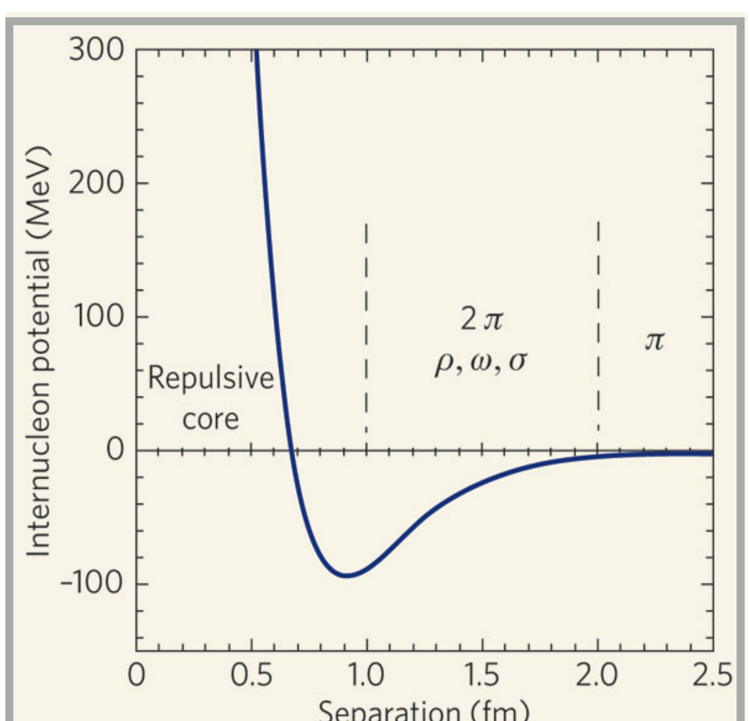
From QCD to nuclear physics

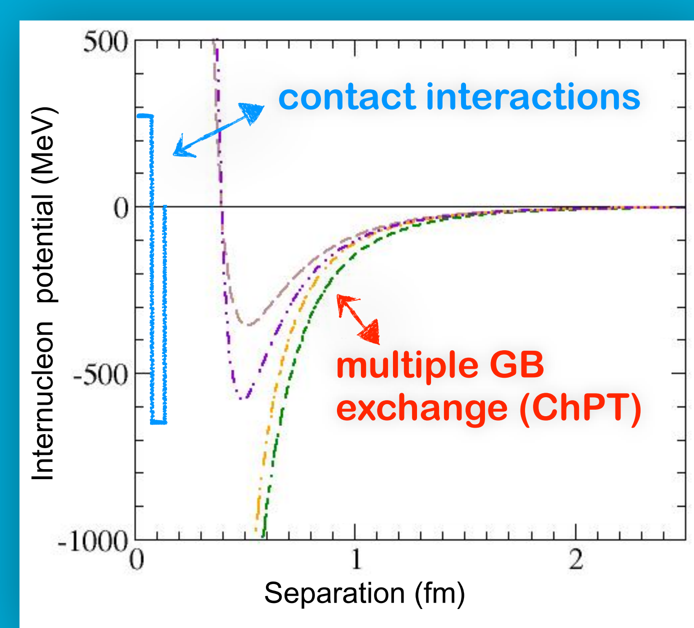


- **NN interaction is strong**: resummations/nonperturbative methods needed
- $1/m_N$ - expansion: nonrelativistic problem ($|\vec{p}_i| \sim M_\pi \ll m_N$) ⇨ the QM A-body problem

$$\left[\left(\sum_{i=1}^{A} \frac{-\vec{\nabla}_i^2}{2m_N} + \mathcal{O}(m_N^{-3})\right) + \underbrace{V_{2N} + V_{3N} + V_{4N} + \ldots}_{\textit{derived within ChPT}}\right] |\Psi\rangle = E|\Psi\rangle$$

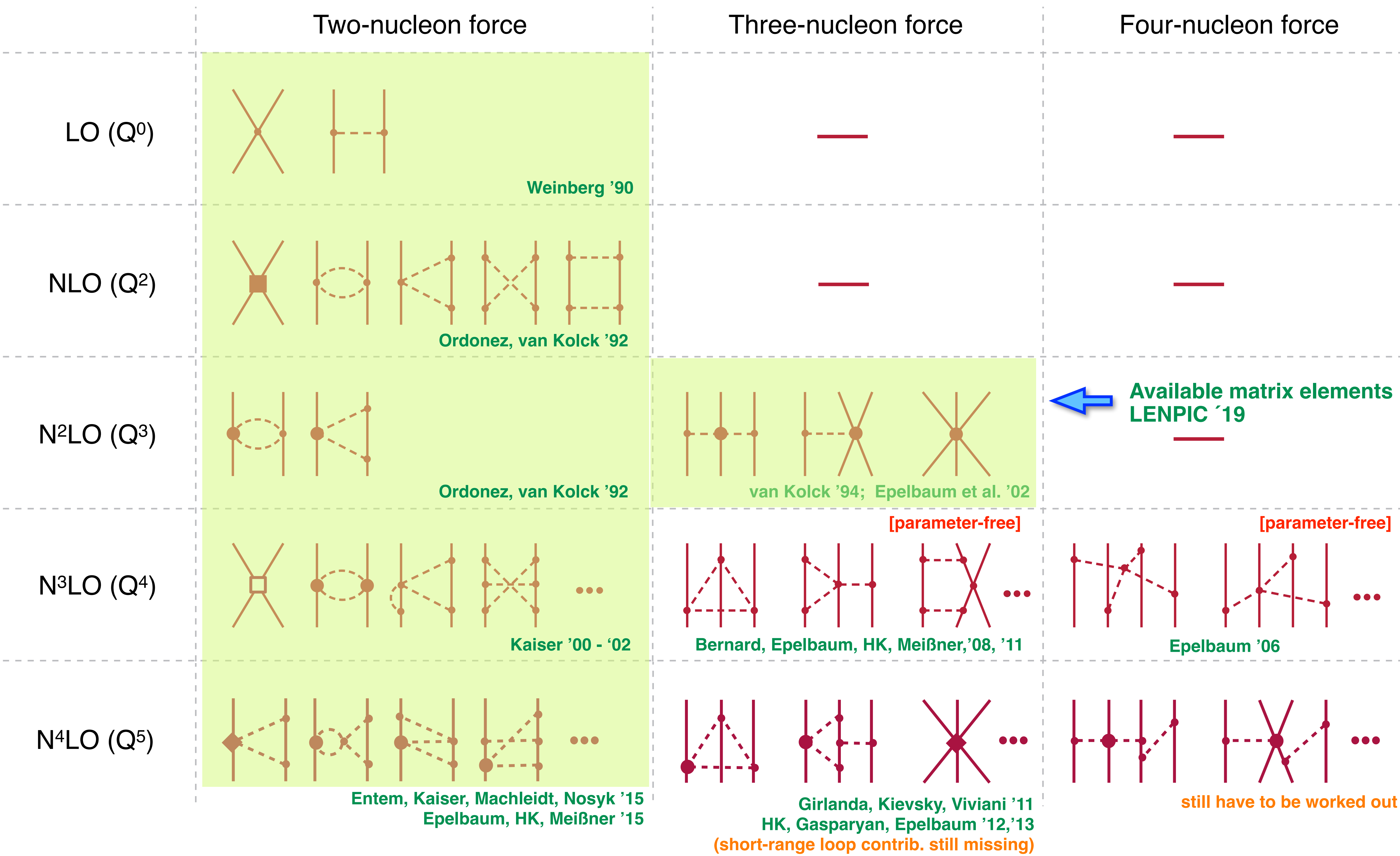
Weinberg '91





- unified description of $\pi\pi$, πN and NN
- consistent many-body forces and currents
- systematically improvable
- bridging different reactions (electroweak, π-prod., ...)
- precision physics with/from light nuclei

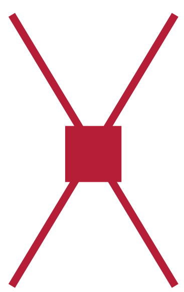
Chiral Expansion of the Nuclear Forces



Adjustable Parameters in NN

Reinert, HK, Epelbaum PRL126 (2021) 092501

Couplings of short-range interactions are fixed from NN - data



LO $[Q^0]$: 2 operators (S-waves)

NLO $[Q^2]$: + 7 operators (S-, P-waves and ε_1)

N^2LO $[Q^3]$: no new terms

N^3LO $[Q^4]$: + 12 operators (S-, P-, D-waves and ε_1, ε_2)

N^4LO $[Q^5]$: + 5 IB operators

N^4LO^+ $[Q^6]$: + 4 operators (F-waves)

# of adjustable LECs = 25 IC + 5 IB + 3 πN constants = 33 parameters

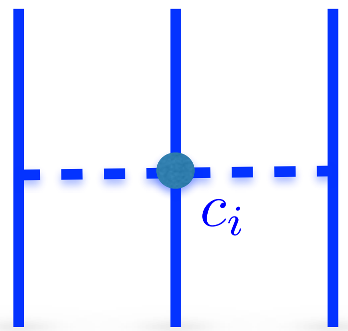
Summary on NN

- Employed a Bayesian approach to account for statistical and systematic uncertainties
- Extracted πN couplings from NN data within chiral EFT
- Achieved a statistically perfect description of NN data

χ^2/dat = 1.005 for ~5000 data in the energy range E_{lab} = 0 - 280 MeV

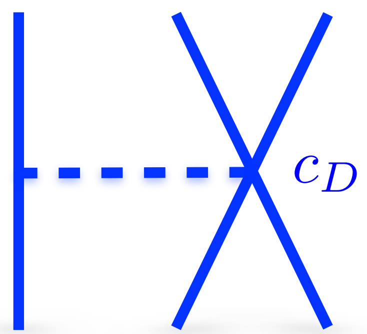
Three-Nucleon Force at N^{2}LO

Epelbaum et al. EPJA56 (2020) 92; Maris et al. PRC103 (2021) 054001



c_i´s are extracted from solutions of Roy-Steiner equation in pion-nucleon scattering: Hoferichter et al. PRL115 (2015) 192301

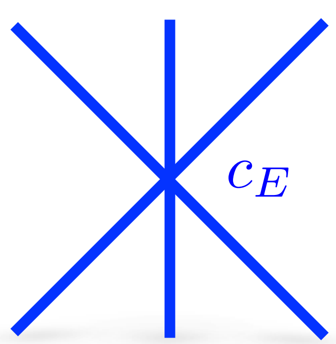
$$c_1 = -0.74\,\mathrm{GeV}^{-1} \qquad c_3 = -3.61\,\mathrm{GeV}^{-1} \qquad c_4 = 2.44\,\mathrm{GeV}^{-1}$$



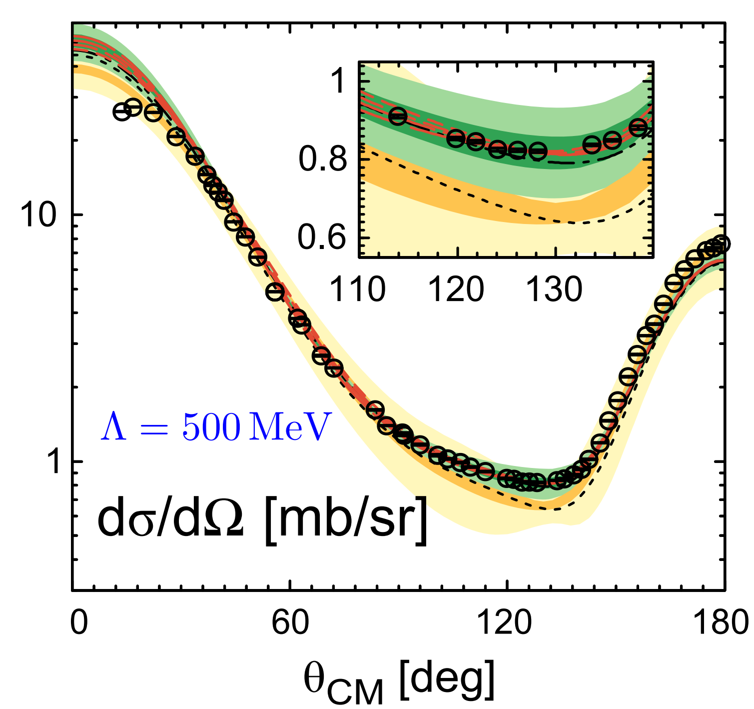
Requirement to reproduce ^{3}H correlates c_D & c_E

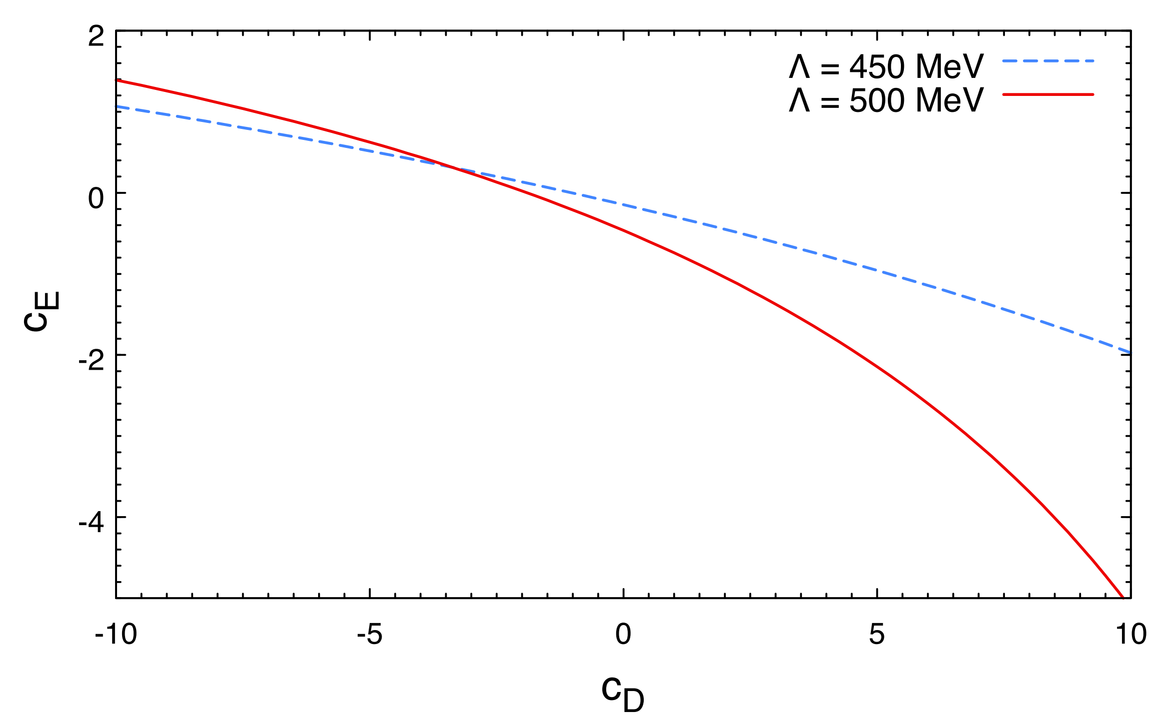
c_D is fitted to the minimum of Nd-scattering cross section at $E^N_{\mathrm{lab}} = 70\,\mathrm{MeV}$

Sekiguchi et al. PRC65 (2002) 034003



$$c_D = 2.485, \quad c_E = -0.528 \quad \text{for} \quad \Lambda = 450\,\mathrm{MeV}$$
$$c_D = -1.626, \quad c_E = -0.063 \quad \text{for} \quad \Lambda = 500\,\mathrm{MeV}$$





Neutron-Deuteron Scattering at N⁴LO⁺

Maris et al. PRC106 (2022) 6; Maris et al. PRC103 (2021) 054001

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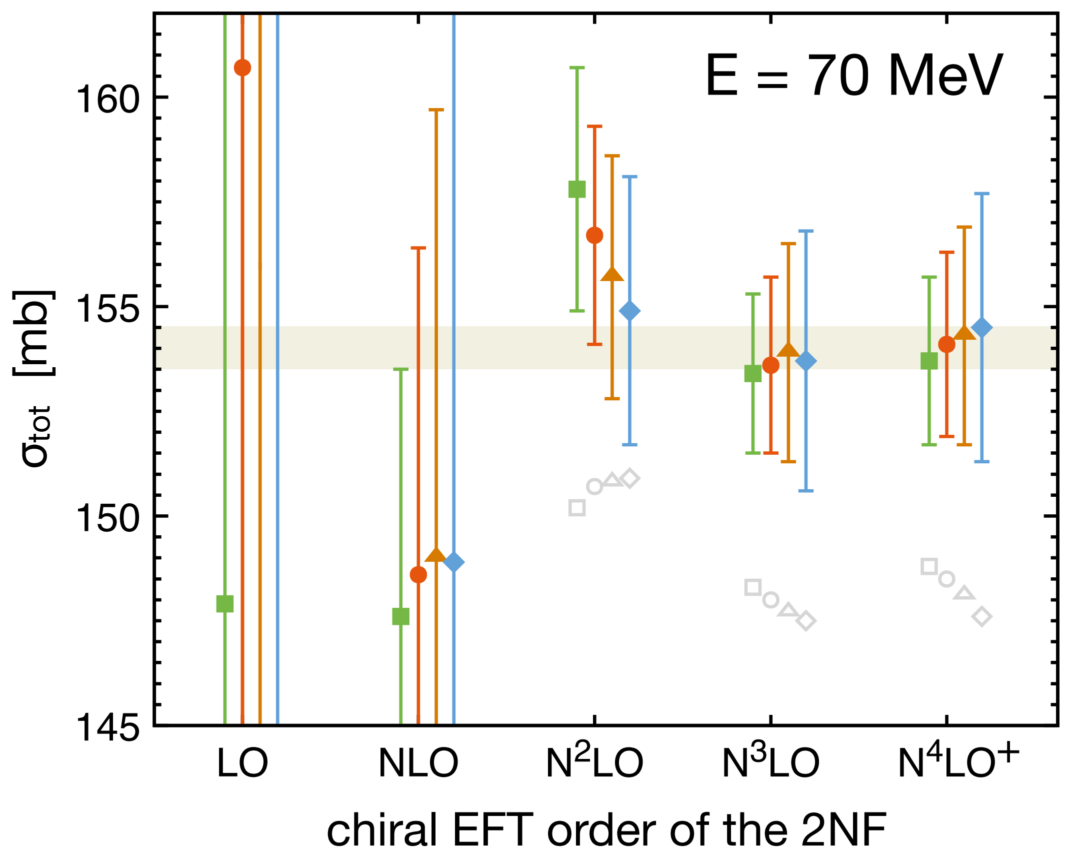
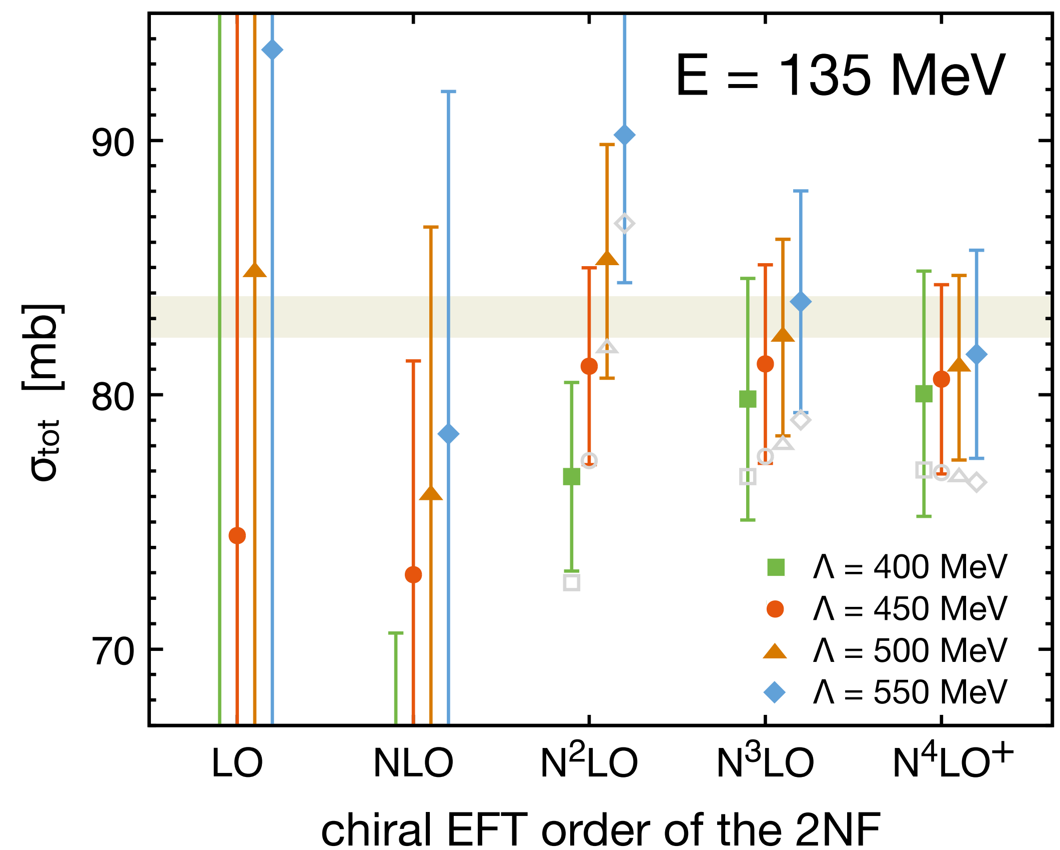




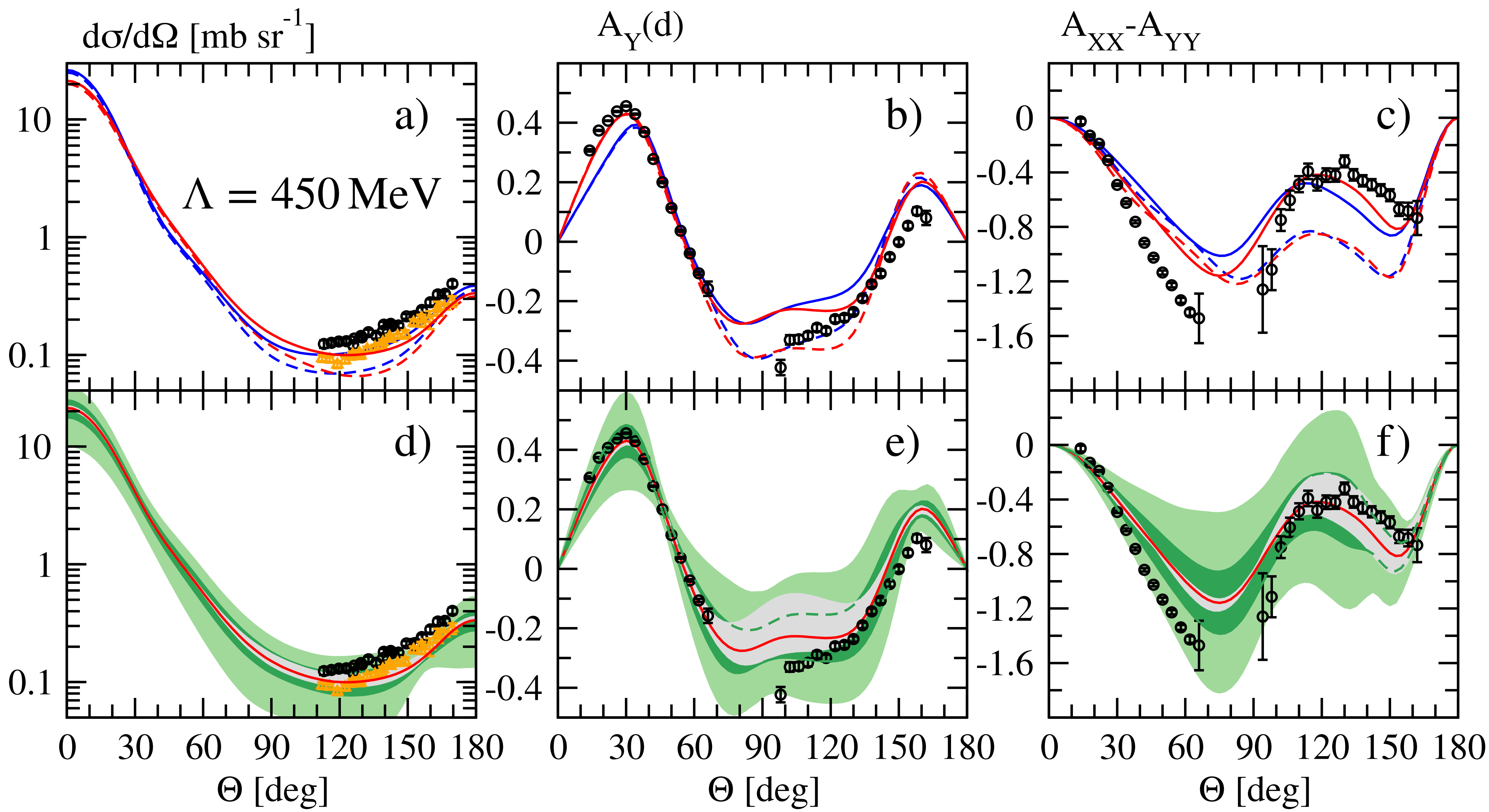
Error bar from Bayesian analysis: 68% DoB **Epelbaum et al. EPJA56 (2020) 92**
Furnstahl et el. PRC92 (2015) 2, 024005

$$X = X_{\rm ref}\left(c_0 + c_2 Q^2 + c_3 Q^3 + \dots\right) \qquad Q = \max\left(\frac{p}{\Lambda_b}, \frac{M_\pi^{\rm eff}}{\Lambda_b}\right) \qquad \Lambda_b = 650\,\mathrm{MeV}$$

- Similar to phenomenological potentials NN only from N^2LO on underestimate $\sigma_{\rm tot}$
- N^2LO 3NF increases the total cross section bringing the calculations in agreement with the data

Nd Scattering

Differential cross section and selected analyzing powers of elastic Nd scattering at E_N = 200 MeV



NN (N^2LO) NN (N^4LO^+) NN (N^2LO) +3NF NN (N^4LO+) +3NF

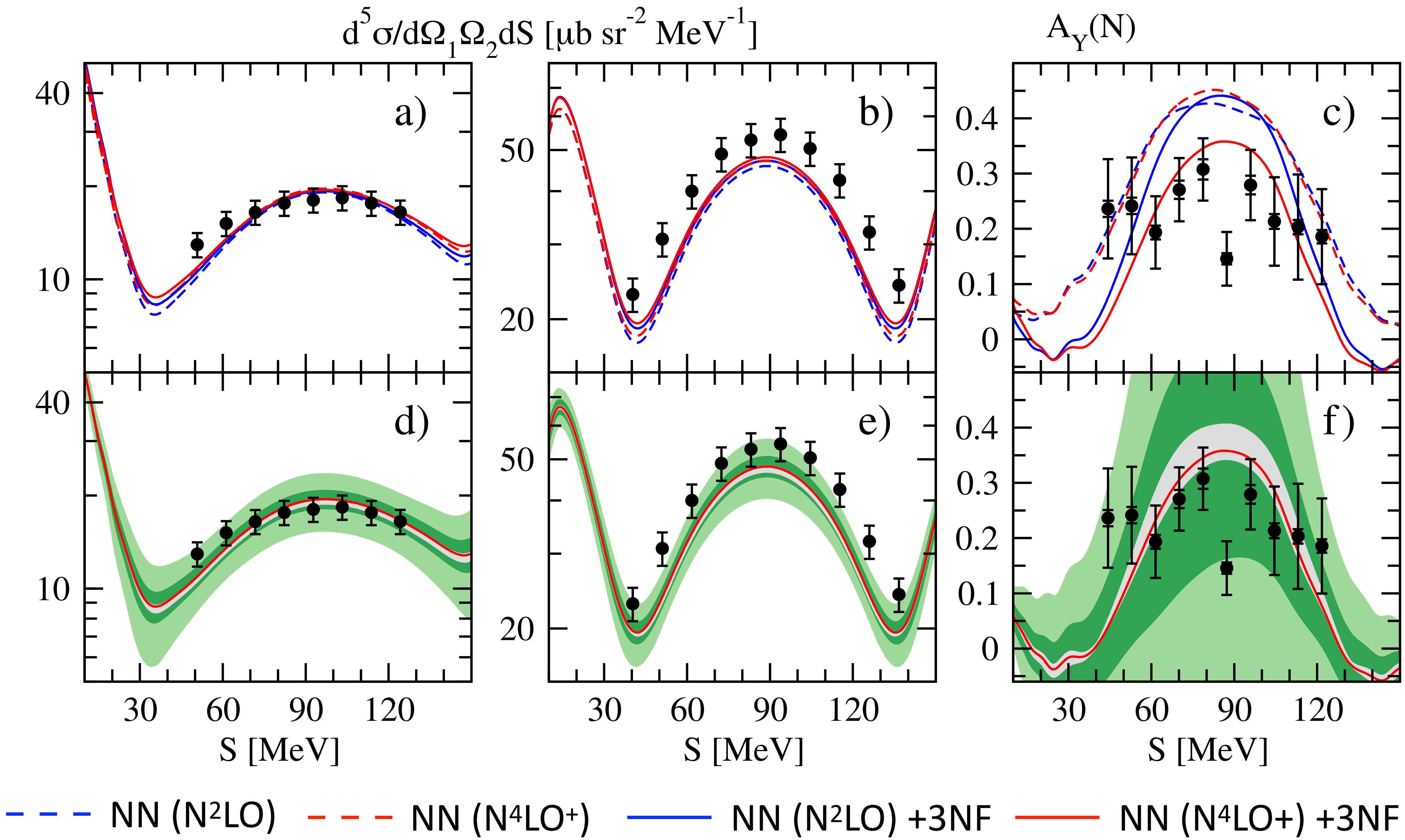
Light (dark) green band correspond to truncation errors at 95% (68%) DoB

- Similar to phenomenological forces $d\sigma/d\Omega$ is slightly underestimated

Nd Breakup Scattering

Maris et al. PRC106 (2022) 6

Differential cross section and selected analyzing powers of Nd breakup at E_N = 135 MeV

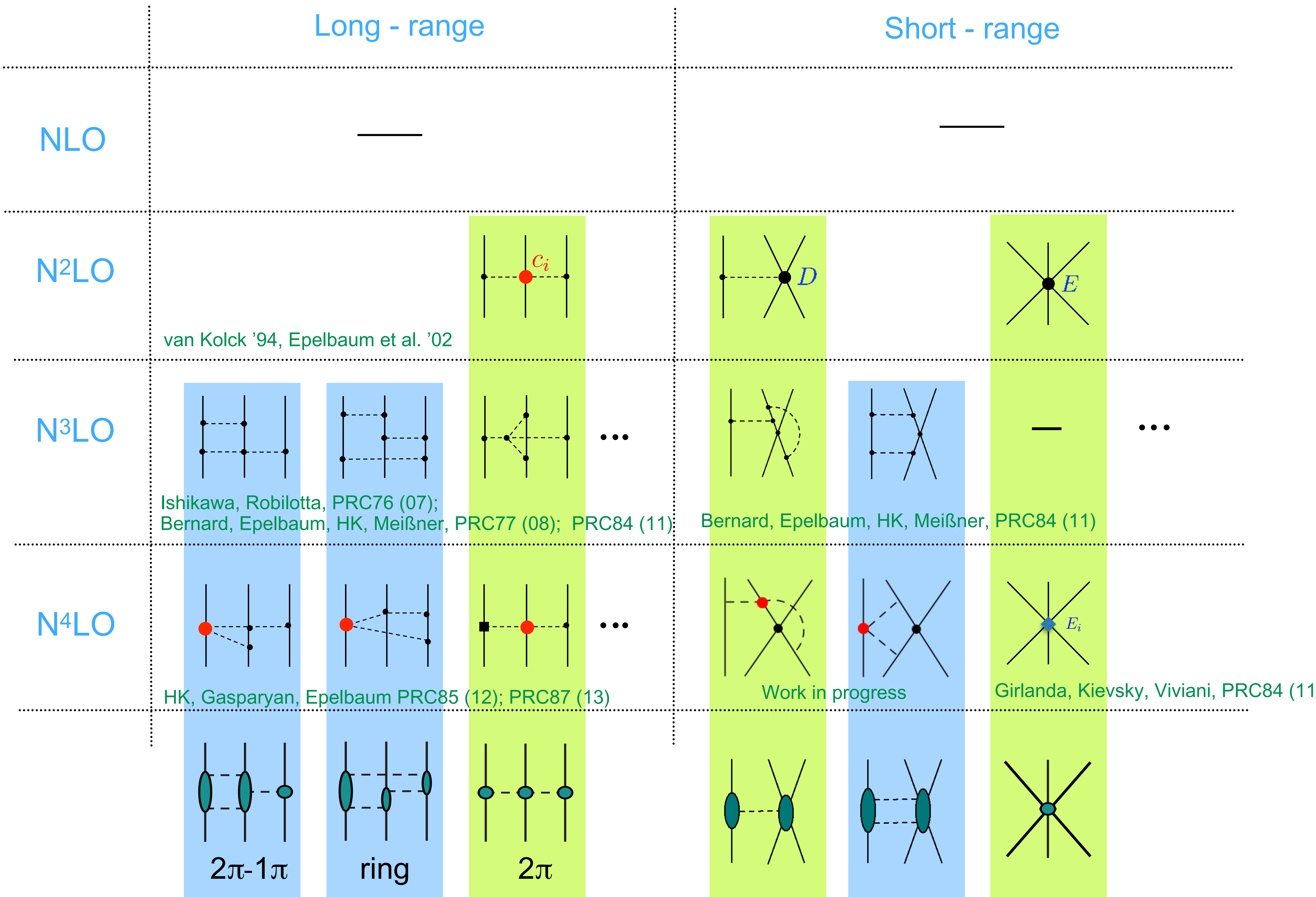


Light (dark) green band correspond to truncation errors at 95% (68%) DoB

Satisfactory description of 3N data leaving room for corrections from higher order

Three-Nucleon Forces at N^3LO

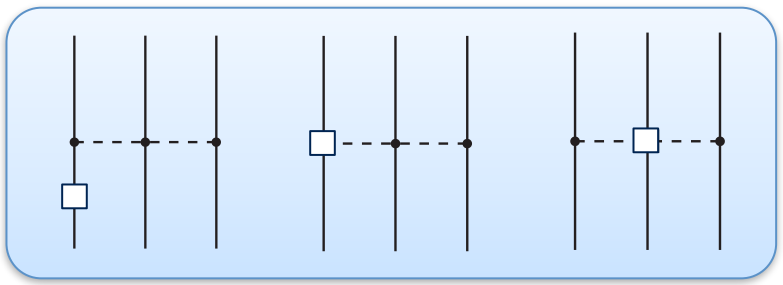
3NF up to N^{4}LO



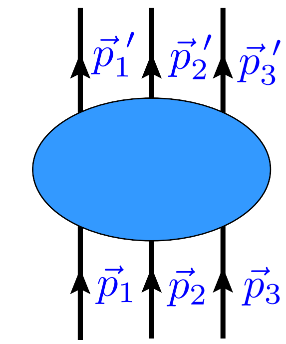
Call for Consistent Regularization

Violation of chiral symmetry due to different regularizations: Dim. reg. vs cutoff reg.

Epelbaum, HK, Reinert, Front. in Phys. 8 (2020) 98



1/m - corrections to TPE 3NF $\sim g_A^2$



$$V_{2\pi,1/m}^{g_A^2} = i\frac{g_A^2}{32mF_\pi^4}\frac{\vec{\sigma}_1\cdot\vec{q}_1\,\vec{\sigma}_3\cdot\vec{q}_3}{(q_1^2+M_\pi^2)(q_3^2+M_\pi^2)}\tau_1\cdot(\tau_2\times\tau_3)(2\vec{k}_1\cdot\vec{q}_3+4\vec{k}_3\cdot\vec{q}_3+i\,[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2)$$

$$\vec{q}_i = \vec{p}_i{}' - \vec{p}_i$$

$$\vec{k}_i = \frac{1}{2}(\vec{p}_i{}' + \vec{p}_i)$$

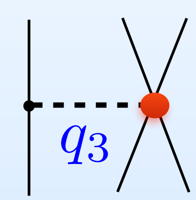
Naive local cut-off regularization of the current and potential

$$V_{2\pi,1/m}^{g_A^2,\Lambda} = V_{2\pi,1/m}^{g_A^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)\exp\left(-\frac{q_3^2+M_\pi^2}{\Lambda^2}\right) \quad \& \quad V_{1\pi}^{Q^0,\Lambda} = -\frac{g_A^2}{4F_\pi^2}\tau_1\cdot\tau_2\frac{\vec{q}_1\cdot\vec{\sigma}_1\vec{q}_1\cdot\vec{\sigma}_2}{q_1^2+M_\pi^2}\exp\left(-\frac{q_1^2+M_\pi^2}{\Lambda^2}\right)$$

First iteration with OPE NN potential

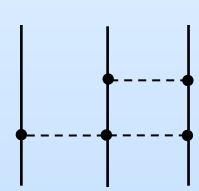
$$V_{2\pi,1/m}^{g_A^2,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{1\pi}^{Q^0,\Lambda} + V_{1\pi}^{Q^0,\Lambda}\frac{1}{E-H_0+i\epsilon}V_{2\pi,1/m}^{g_A^2,\Lambda} = \Lambda\frac{g_A^4}{128\sqrt{2}\pi^{3/2}F_\pi^6}(\tau_2\cdot\tau_3-\tau_1\cdot\tau_3)\frac{\vec{q}_2\cdot\vec{\sigma}_2\vec{q}_3\cdot\vec{\sigma}_3}{q_3^2+M_\pi^2}+\ldots$$

No such D-like term in chiral Lagrangian



The problematic divergence is canceled by the one $V_{2\pi-1\pi}$ if calculated via cutoff regularization

In dim. reg. $V_{2\pi-1\pi}$ =



+ … is finite

Higher Derivative Lagrangian

To construct a parity-conserving regulator it is convenient to work with building-blocks

$$u_\mu = i\, u^\dagger \nabla_\mu U u^\dagger, \quad D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\left[u^\dagger, \partial_\mu u\right] - \frac{i}{2} u^\dagger r_\mu u - \frac{i}{2} u\, l_\mu u^\dagger$$

$$\chi_\pm = u^\dagger \chi\, u^\dagger \pm u \chi^\dagger u, \quad \chi = 2B(s + i\,p), \quad u = \sqrt{U}, \quad \mathrm{ad}_A B = [A, B]$$

Possible ansatz for higher derivative pion Lagrangian

$$\mathcal{L}^{(2)}_{\pi,\Lambda} = \mathcal{L}^{(2)}_{\pi} + \frac{F^2}{4} \mathrm{Tr}\left[\mathrm{EOM} \frac{1 - \exp\left(\frac{\mathrm{ad}_{D_\mu}\mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+}{\Lambda^2}\right)}{\mathrm{ad}_{D_\mu}\mathrm{ad}_{D^\mu} + \frac{1}{2}\chi_+} \mathrm{EOM}\right]$$

$$\mathcal{L}^{(2)}_{\pi} = \frac{F^2}{4} \mathrm{Tr}\left[u_\mu u^\mu + \chi_+\right] \qquad \mathrm{EOM} = -\left[D_\mu, u^\mu\right] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}\left(\chi_-\right)$$

✓ Leads to regularized nuclear forces up to N^{4}LO

✖ Leads to unregularized nuclear currents starting from N^{3}LO

→ We need a better formalism

Gradient-Flow Equation

Yang-Mills gradient flow in QCD: **Lüscher, JHEP 04 (2013) 123**

$\partial_\tau B_\mu = D_\nu G_{\nu\mu}$ with $B_\mu|_{\tau=0} = A_\mu$ & $G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$

B_μ is a regularized gluon field

- Apply this idea to ChPT **(Proposed in various talks by D. Kaplan for nuclear forces)**

Introduce a smoothed pion field W with $W|_{\tau=0} = U$ satisfying gradient-flow equation

$$\partial_\tau W = i\, w\, \mathrm{EOM}(\tau)\, w \text{ with } w = \sqrt{W} \text{ and } \mathrm{EOM} = [D_\mu, w_\mu] + \frac{i}{2}\chi_- - \frac{i}{4}\mathrm{Tr}(\chi_-)$$

$$w_\mu = i(w^\dagger(\partial_\mu - i\, r_\mu)w - w(\partial_\mu - i\, l_\mu)w^\dagger), \quad \chi_- = w^\dagger \chi w^\dagger - w\chi^\dagger w, \quad \chi = 2B(s + ip)$$

Solution of $1/F$ - expanded gradient flow equation: $W = \exp\left(i\dfrac{\tau \cdot \phi}{F}\right)$

$$\phi^c = e^{-\tau(-\partial^2 + M^2)}\pi^c + \int_0^\tau ds\, e^{-(\tau - s)(-\partial^2 + M^2)}\left(2BFp^c - F\partial_\mu a_\mu^c\right) + \mathcal{O}(\pi^2, \pi\, \text{source})$$

Regularization of Forces and Currents

To regularize long-range part of the nuclear forces and currents

- Leave pionic Lagrangians $\mathscr{L}_\pi^{(2)}$ & $\mathscr{L}_\pi^{(4)}$ unregularized
- Replace all pion fields in pion-nucleon Lagrangians $\mathscr{L}_{\pi N}^{(1)}, \ldots, \mathscr{L}_{\pi N}^{(4)}$: $U \to W$

For $\tau = \dfrac{1}{2\Lambda^2}$ this regulator reproduces SMS regularization of OPE

Check of chiral symmetry:

- We checked infinitesimal chiral transformation property of W up to four pions and one external source: $W \to RWL^\dagger$ if $U \to RUL^\dagger$

The check requires construction of W - field up to six pions and two sources

Conceptional Challenge

$$\phi^c = e^{-\tau(-\partial^2+M^2)}\pi^c + \ldots = e^{-\tau(-\partial_0^2-\vec{\nabla}^2+M^2)}\pi^c + \ldots$$

Appearance of second and higher order in time-derivatives of pion fields

→ Canonical quantization of the regularized theory becomes difficult (Ostrogradski - approach, Constrains, …)

- Unitary transformation (UT) approach can not be used any more

→ Use path-integral (PI) quantization

Canonical Quantization of QFT		**Path-Integral Quantization of QFT**
Hamiltonian & Hilbert space	⟷	Lagrangian & action
Creation/annihilation operators		Summation over all classical paths
Time-ordered perturbation theory		Loop expansion & Feynman rules

- PI approach was a natural choice in pionic and single-nucleon sector

Gasser, Leutwyler, Annals Phys. 158 (1984) 142;
Bernard, Kaiser, Kambor, Meißner, Nucl. Phys. B 388 (1992) 315

Nuclear Forces in PI Formulation

In two - and more - nucleon sector Weinberg used canonical quantization language

Weinberg Nucl. Phys. B 362 (1991) 3

In using old-fashioned perturbation theory we must work with the Hamiltonian rather than the Lagrangian. The application of the usual rules of canonical quantization to the leading terms in (1) and (9) yields the total

Can we choose a formulation where we can work with the Lagrangian?

Derivation steps of nuclear forces in PI formulation

Inspired by: Friar et al. Phys. Rev. C 70 (2004) 044001, Borasoy et al. EPJA 31 (2007)105

- Perform a perturbative loop expansion in pion fields

  → Non-instant NN, 3N & 4N interactions

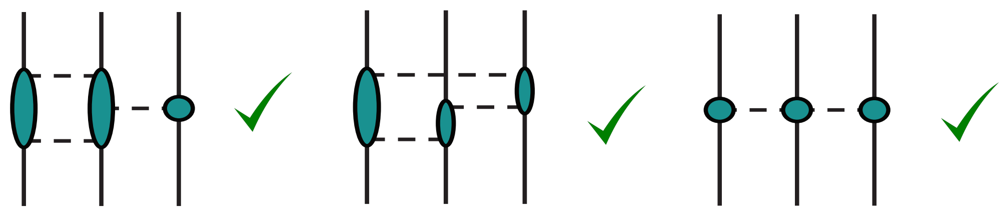
- Perform non-local nucleon field redefinitions to bring all non-instant interactions into instant form

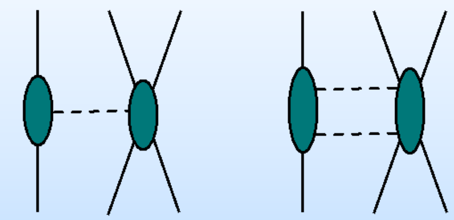
- Due to non-locality of nucleon-field redefinitions we get functional determinants

  → $\det\left(\frac{\delta(N'^{\dagger}, N')}{\delta(N^{\dagger}, N)}\right)$ include loop corrections to nuclear forces

Status Report on 3N at N³LO

- UT and PI approaches lead to the same 3NF & 4NF up to N⁴LO within dim. reg.
- We calculated all long-range contributions to 3NF & 4NF at N³LO



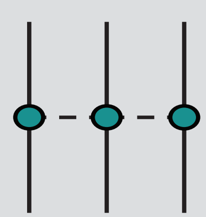
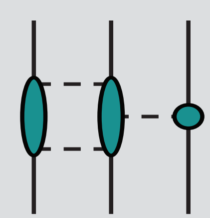
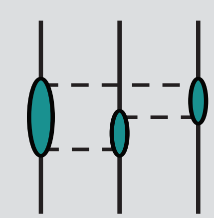


to be done

3NF's are given in terms of integrals over Schwinger parameters

$$V_{3N}^{2\pi-1\pi} = \tau_1\cdot\tau_2\times\tau_3\,\vec q_1\cdot\vec\sigma_1\times\vec\sigma_2\,\vec q_3\cdot\vec\sigma_3\,\frac{e^{-\frac{q_3^2+M_\pi^2}{\Lambda^2}}}{q_3^2+M_\pi^2}\left(-\frac{g_A^4}{F_\pi^6}\frac{q_1}{2048\pi}\int_0^\infty d\lambda\,\mathrm{erfi}\left(\frac{q_1\lambda}{2\Lambda\sqrt{2+\lambda}}\right)\frac{\exp\left(-\frac{q_1^2+4M_\pi^2}{4\Lambda^2}(2+\lambda)\right)}{2+\lambda}+\dots\right)+\dots$$

Dimension of integrals over Schwinger parameters depends on topology

Space			
Momentum	2	1	3
Coordinate	4	1	0

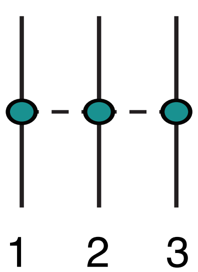
Subtraction Scheme

Choice of the short-range scheme

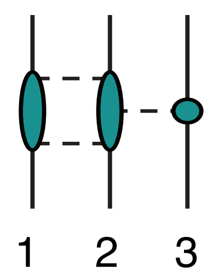
- NN case: local part of NN force vanishes if distance between nucleons vanishes

  → leads to natural size of LECs

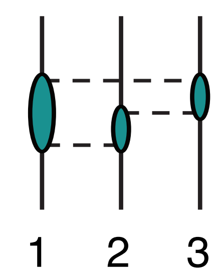
- 3N case: vanishing of the local part of 3NF is topology dependent



$= 0$ if $r_{12} = 0$ or $r_{23} = 0$

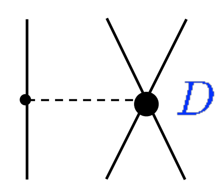


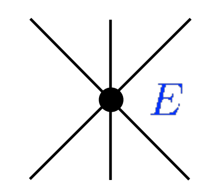
$= 0$ if $r_{12} = 0$



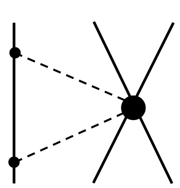
$= 0$ if $r_{12} = r_{23} = 0$

Can be achieved by adjustment of D- and E-like terms:





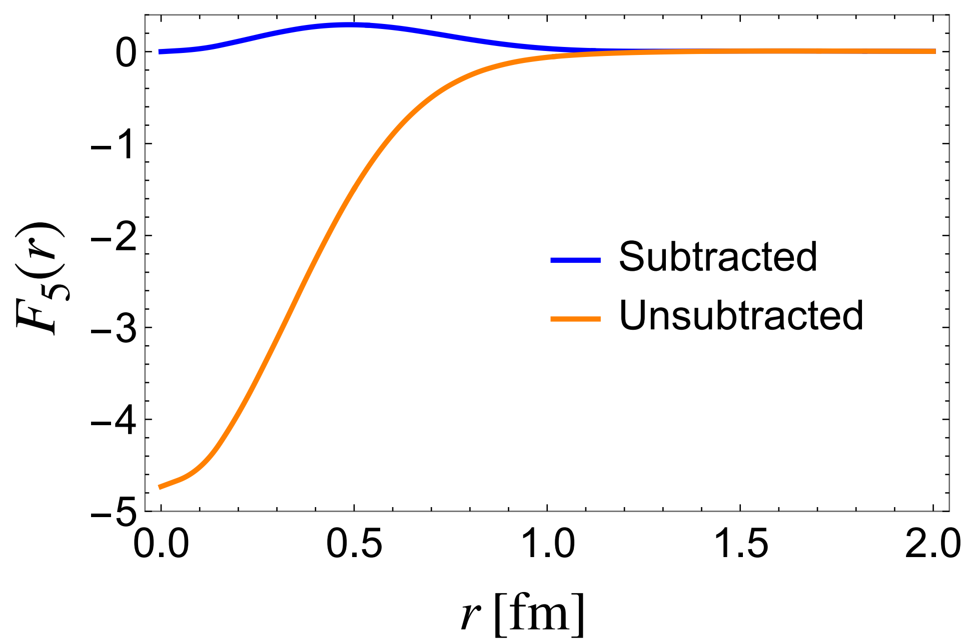
Vanishing of 3NF for any $r_{ij} = 0$ would require inclusion of two-pion-contact terms

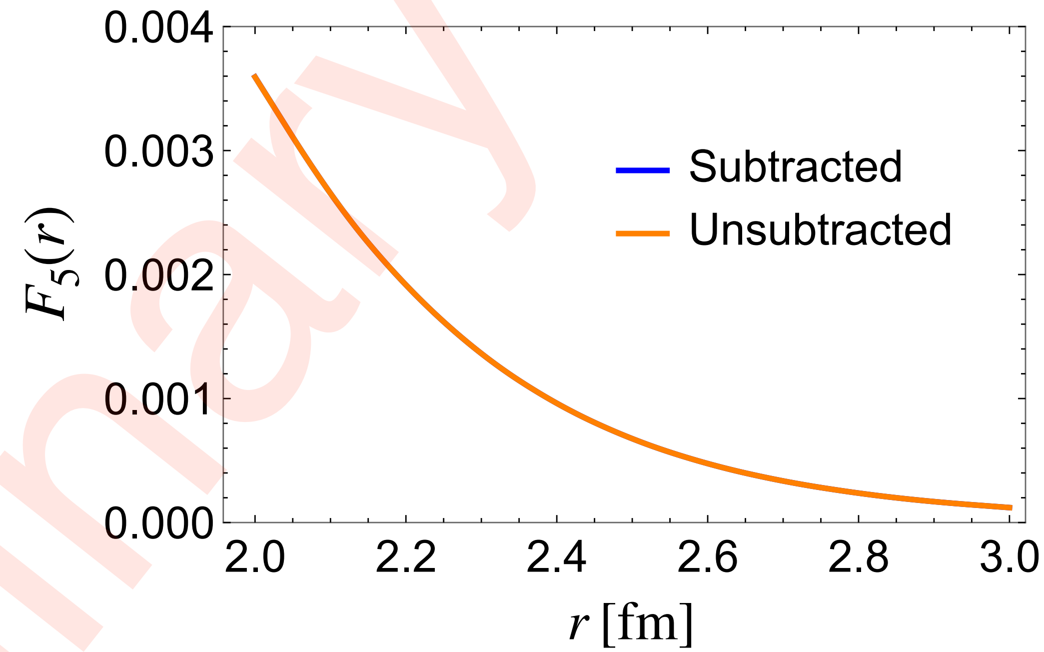


Appear first at N^5LO and are expected to be small

Selected Profile Functions

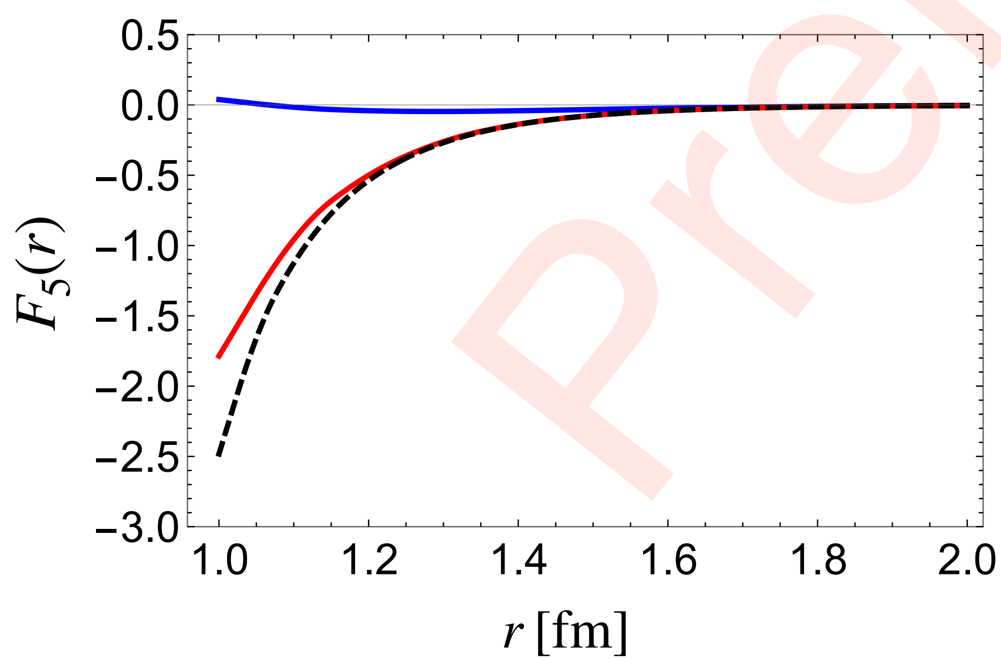
$V_{3N}^{\text{ring}} = F_1(r_{12}, r_{23}, r_{13}) + \ldots + \tau_2 \cdot \tau_3 \vec{\sigma}_1 \cdot \vec{\sigma}_2 F_5(r_{12}, r_{23}, r_{13}) + \ldots$ $F_5(r) = F_5(r, r, r)$ [MeV]

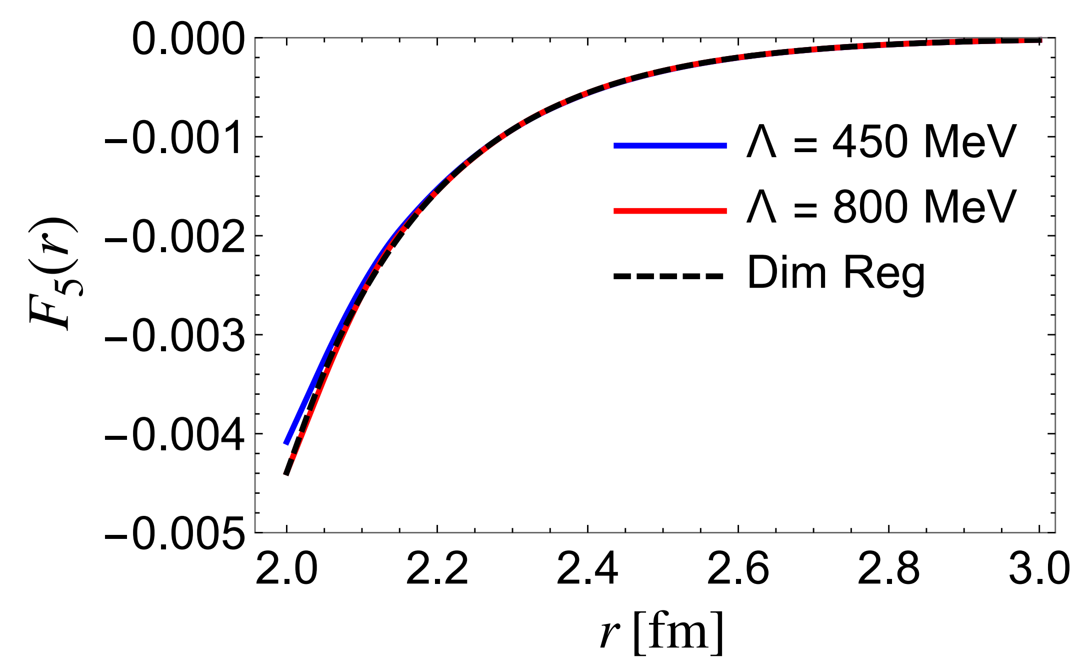




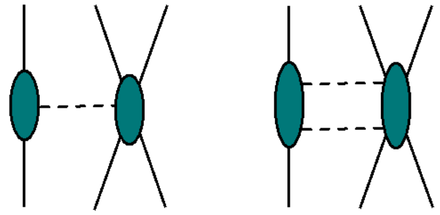
By construction: subtracted & unsubtracted forces differ in the short-range region

At $\Lambda \to \infty$ regularized 3NF reproduce dim. reg. results from **Bernard et al. PRC77 (08)**





Homework

- Calculation of short-range contributions 

  Due to non-locality of the short-range regulator, expressions get more complicated

- c_i's LECs in 3NF at N^{2}LO have to be refitted

  Calculation of pion-nucleon scattering within gradient-flow regularization required

- Partial wave decomposition (PWD): K. Hebeler, A. Nogga & R. Skibinski

  PWD is computationally more expensive, due to higher dimension of integrals over Schwinger parameters

Summary on 3N

- Gradient flow regularization preserves chiral symmetry
- Path-integral approach for derivation of nuclear forces
- Long-range part of 3NF at N^3LO has been calculated

Outlook

- Short-range part of 3NF at N^3LO
- Partial wave decomposition
- Symmetry preserving regularized nuclear currents

A = 3 & 4 Nuclei

Faddeev and Yakubovsky equations in momentum space: **Nogga et al. PRC65 (2002) 054003**

All angular momenta ≤ 5 of the two-body subsystem are taken into account

Numerical accuracy $\sim 1\,\mathrm{keV}$ reached for $A = 3$ binding energies and expectation values

Maris et al. arXiv: 2206.13303v1

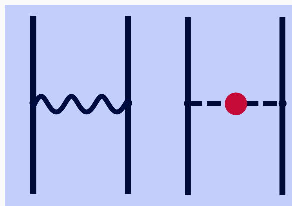
		Λ	E	$\langle V_{3NF} \rangle$
^{3}H	LO	450	-12.22	—
	NLO		-8.515	—
	N^2LO		-8.483	-0.459
	N^3LO		-8.483	-0.520
	N^4LO		-8.483	-0.430
	N^4LO^+		-8.483	-0.460
^{3}H	LO	500	-12.52	—
	NLO		-8.325	—
	N^2LO		-8.482	-0.660
	N^3LO		-8.483	-0.724
	N^4LO		-8.483	-0.628
	N^4LO^+		-8.484	-0.672
Expt. ^{3}H			-8.482	—

		Λ	E	$\langle V_{3NF} \rangle$
^{3}He	LO	450	-11.34	—
	NLO		-7.751	—
	N^2LO		-7.734	-0.452
	N^3LO		-7.737	-0.509
	N^4LO		-7.739	-0.423
	N^4LO^+		-7.740	-0.452
^{3}He	LO	500	-11.63	—
	NLO		-7.574	—
	N^2LO		-7.739	-0.641
	N^3LO		-7.738	-0.705
	N^4LO		-7.743	-0.615
	N^4LO^+		-7.744	-0.658
Expt. ^{3}He			-7.718	—

Attractive contribution of 3NF brings E to its physical value

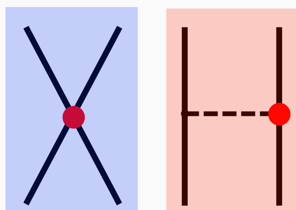
Point Coulomb interaction has been included for pp system in ^{3}He calc

Isospin-breaking in the Nuclear Force

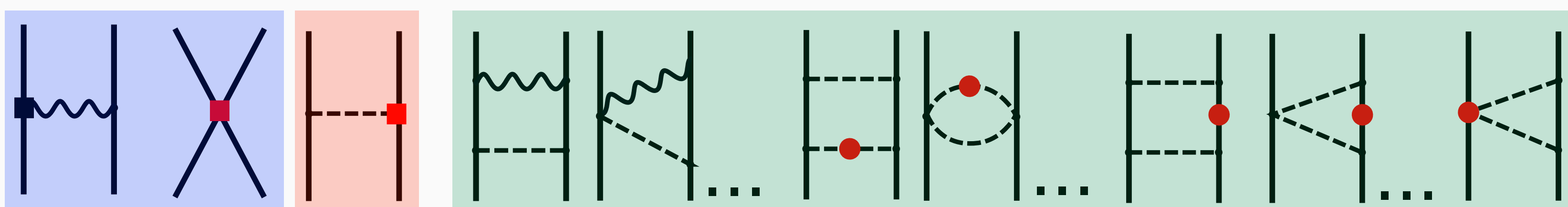
NLO 

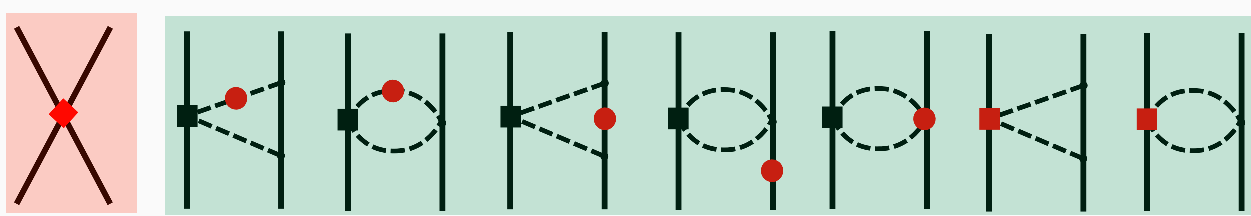
Have been employed in **Reinert, HK, Epelbaum, EPJA 54 (2018) 88**

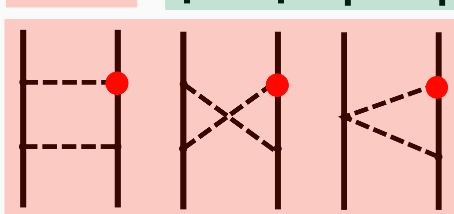
Parameter-free: depend on δM_π, δm = 1.29 MeV and $(\delta m)^{QCD}$ = -1.87(16) MeV **[Gasser, Leutwyler, Rusetsky '21]**

N^2LO 

Depend on 3 πN coupling constants + 3 IB contact terms in p-waves

N^3LO 

N^4LO 



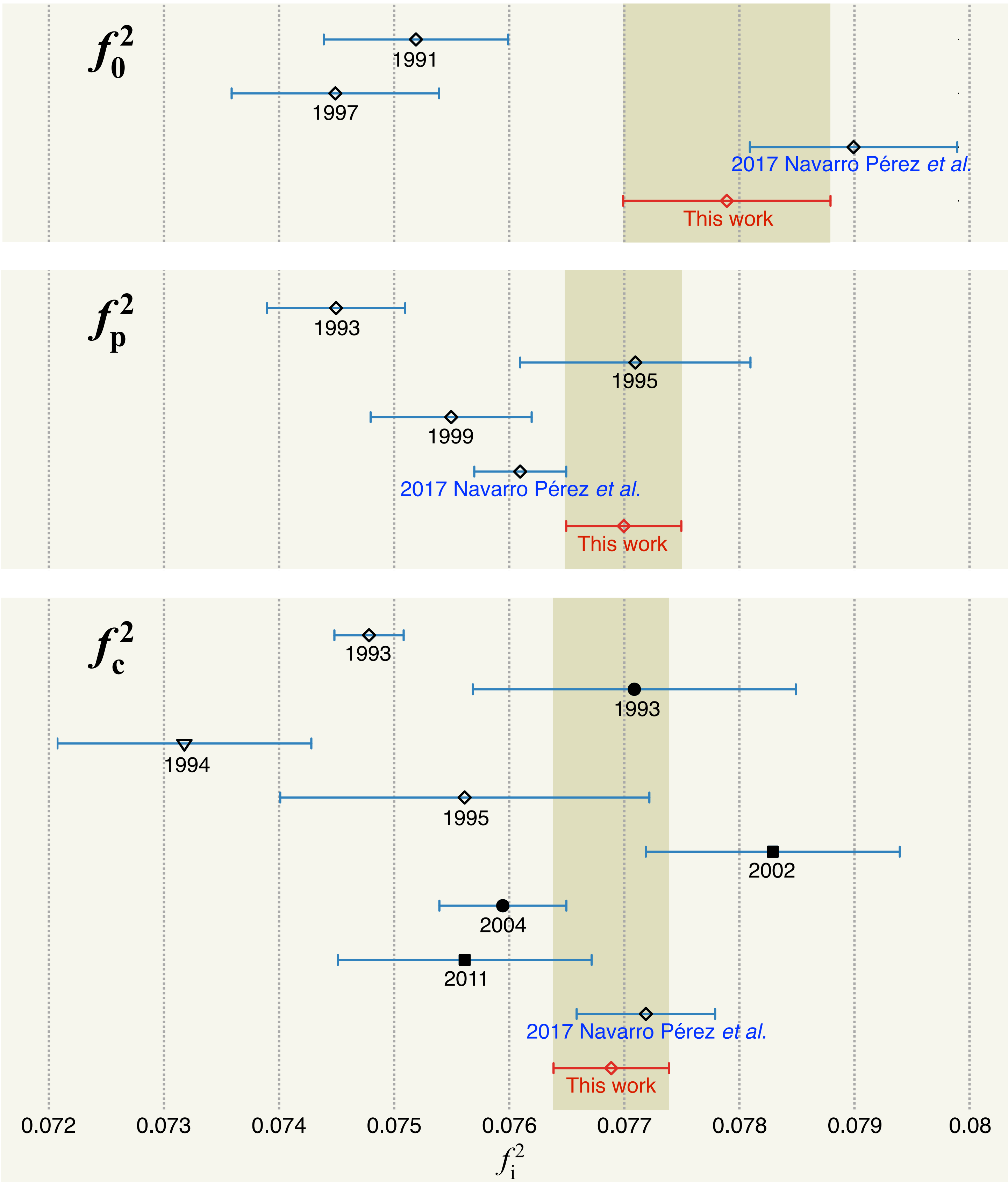
van Kolck et al.'98; Friar et al. '99,'03,'04; Niskanen '02; Epelbaum, Meißner '05

Away from the isospin limit, one introduces 3 **πN coupling constants**:

$$f_{\pi^0 \rm pp} = \frac{M_{\pi^\pm}\, g_{\pi^0 \rm pp}}{2\sqrt{4\pi} m_{\rm p}} \qquad f_{\pi^0 \rm nn} = \frac{M_{\pi^\pm}\, g_{\pi^0 \rm nn}}{2\sqrt{4\pi} m_{\rm n}} \qquad f_{\pi^\pm \rm pn} = \frac{M_{\pi^\pm}\, g_{\pi^\pm \rm pn}}{\sqrt{4\pi}(m_{\rm p} + m_{\rm n})}$$

$$\begin{aligned} f_0^2 &= -f_{\pi^0 \rm nn}\, f_{\pi^0 \rm pp} \\ f_p^2 &= f_{\pi^0 \rm pp}\, f_{\pi^0 \rm pp} \\ 2 f_c^2 &= f_{\pi^\pm \rm pn}\, f_{\pi^\pm \rm pn} \end{aligned}$$

Determination of πN constants



Reinert, HK, Epelbaum PRL126 (2021) 092501

$$f_0^2 = 0.0779(9)(1.3)$$
$$f_p^2 = 0.0770(5)(0.8)$$
$$f_c^2 = 0.0769(5)(0.9)$$

statistical **and systematic** errors due to the EFT truncation, choice of E_{max} and data selection

uncertainty in the subleading πN LECs

No evidence for charge dependence of the πN coupling constants

Our f_c^2 value is consistent with the extractions from the πN system

f_c^2 corresponds to $g_{\pi NN} = 13.23 \pm 0.04$

Pionic hydrogen exp. at PSI **Hirtl et al.´21**

$\epsilon_{1s}^{\pi H} + \epsilon_{1s}^{\pi D}: \quad g_{\pi NN} = 13.10 \pm 0.10$

$\Gamma_{1s}^{\pi H}: \quad g_{\pi NN} = 13.24 \pm 0.10$