Ab initio exploration of ^{12}Be

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Characteristics of ^{12}Be

- Breakdown of the $N = 8$ shell closure
  Intruder ground state
- 2α dumbell orbited by neutrons
- Shape isomer

Outline

No-core shell model predictions for ^{12}Be

- Rotational bands
- Level crossings and two-state mixing
- Shape observables
- $E2$ and $E0$ transitions
- Detangling the mixing problem
- Revisit shape observables

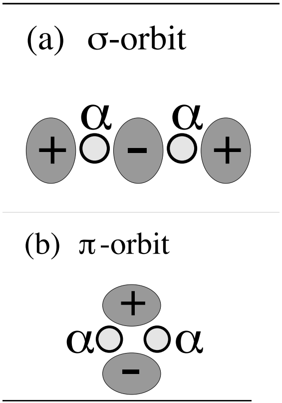
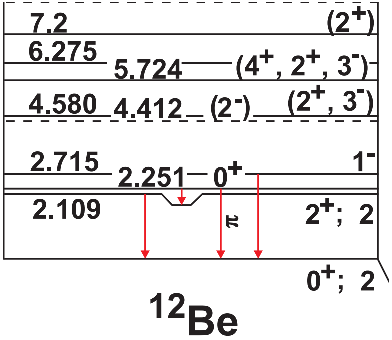
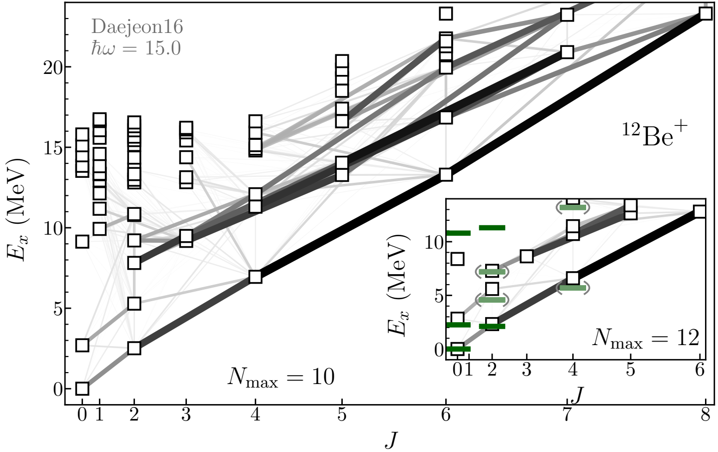


Figure: Y. Kanada-En'yo, H. Horiuchi, Phys. Rev. C **68**, 014319 (2003).

^{12}Be Spectrum





Level scheme: https://nucldata.tunl.duke.edu/

^{12}Be Rotational bands

Characterized by rotation of intrinsic state $|\phi_K\rangle$ by Euler angles ϑ $\quad (J = K, K+1, \ldots)$

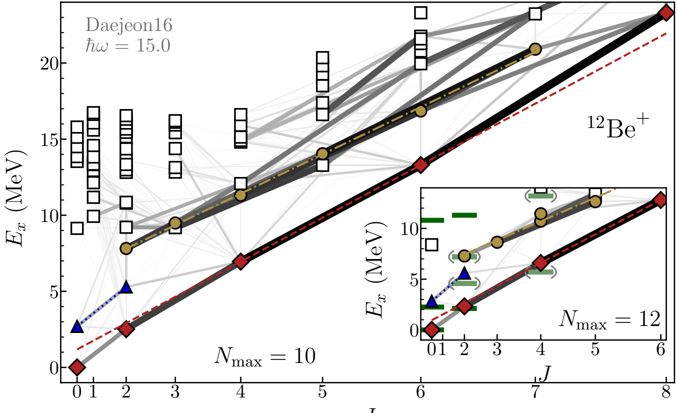
$$|\psi_{JKM}\rangle \propto \int d\vartheta \left[\mathscr{D}^J_{MK}(\vartheta)|\phi_K;\vartheta\rangle + (-)^{J+K}\mathscr{D}^J_{M-K}(\vartheta)|\phi_{\bar{K}};\vartheta\rangle \right]$$

Rotational energy:

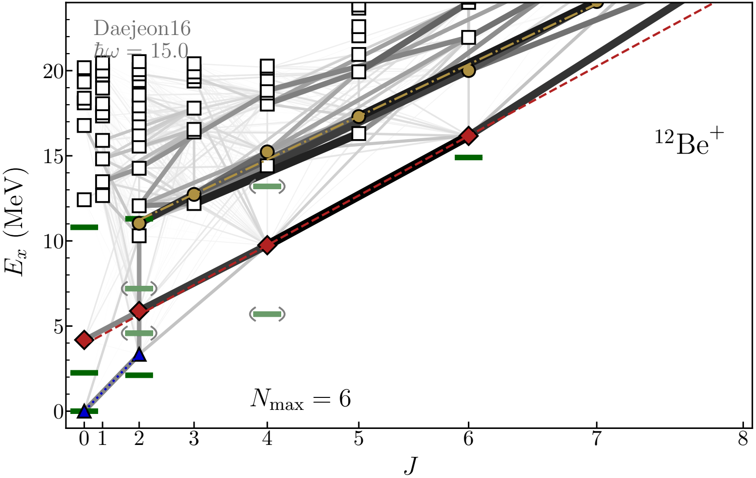
$$E(J) = E_0 + A[J(J+1)]$$

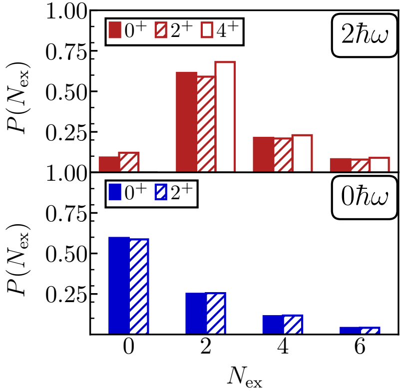
Rotational $E2$ transitions

$$B(E2; J_i \to J_f) = \frac{5}{16\pi}(J_i K; 20|J_f K)^2 (eQ_0)^2$$



^{12}Be Band Evolution





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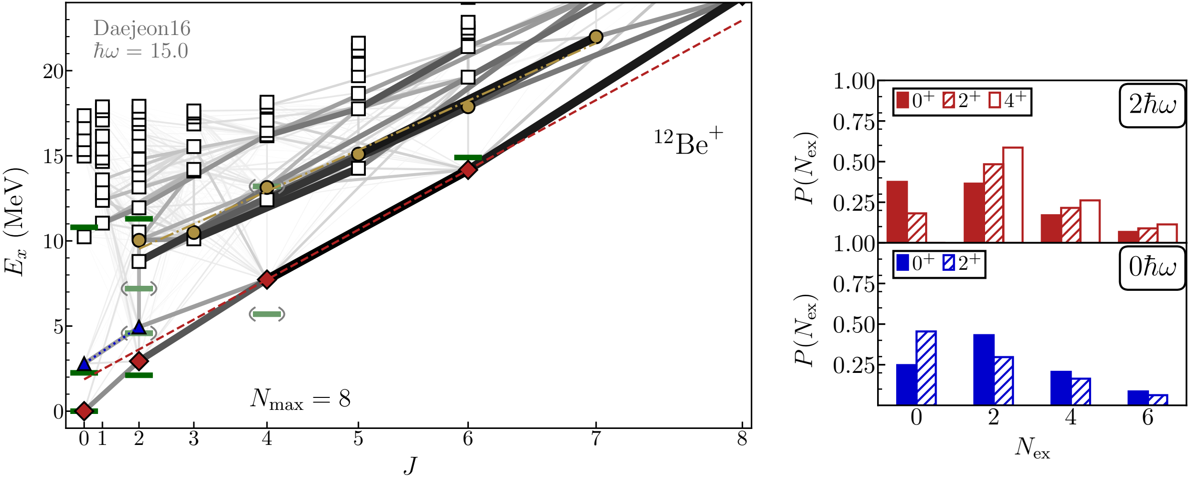

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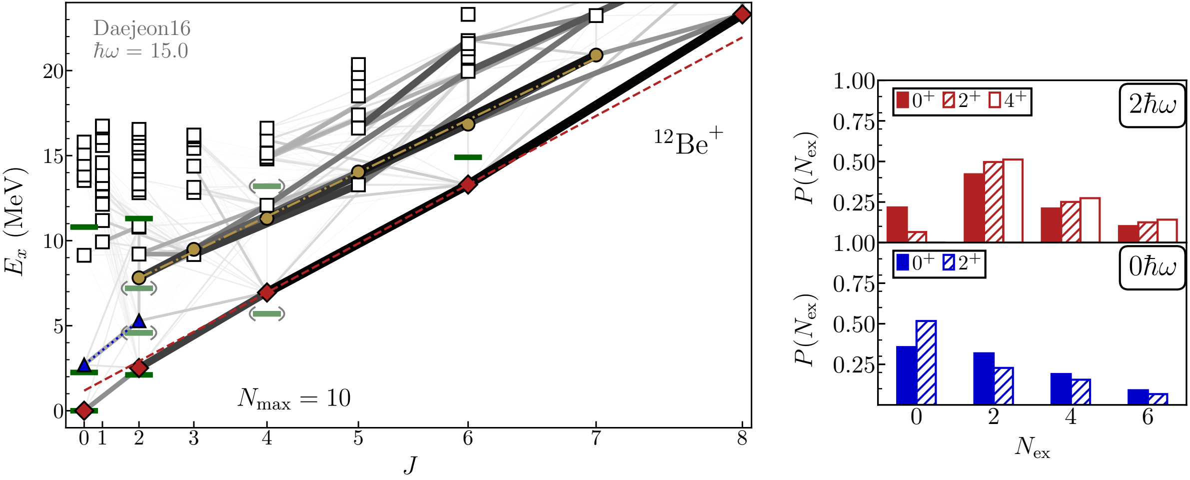

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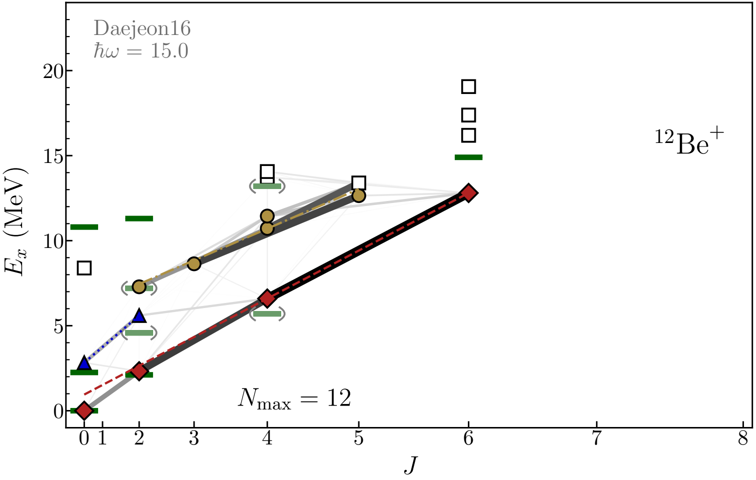

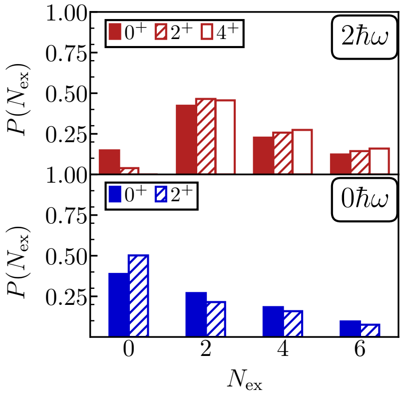

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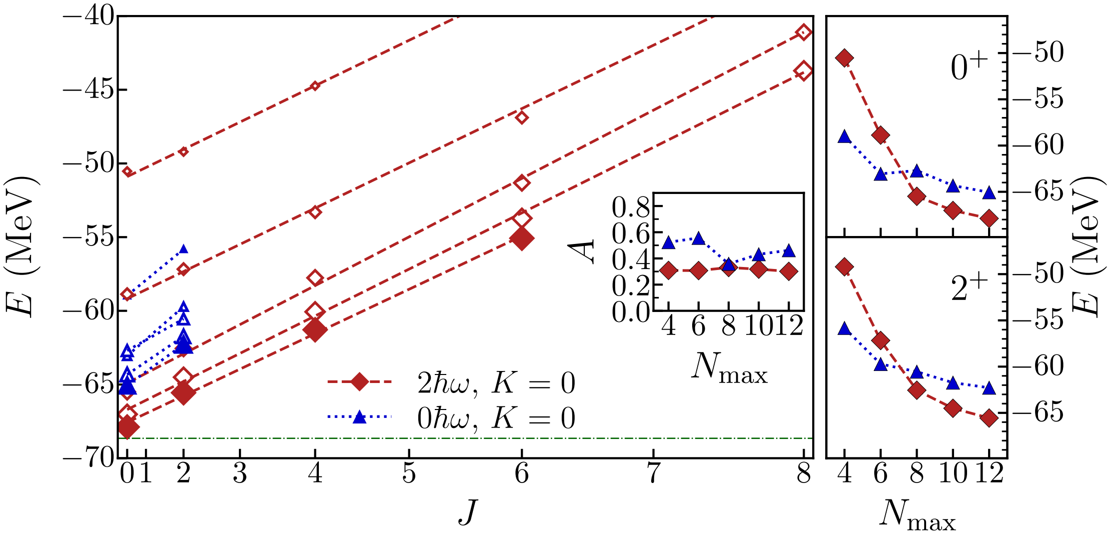

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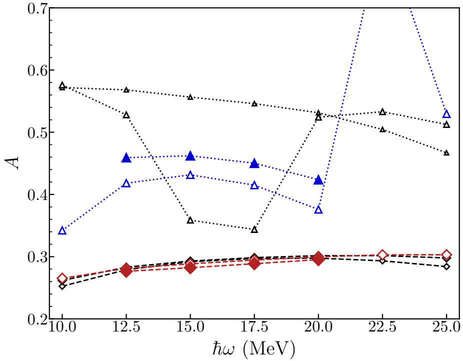

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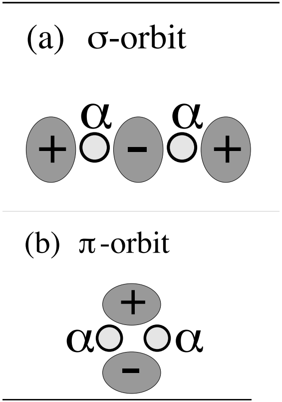
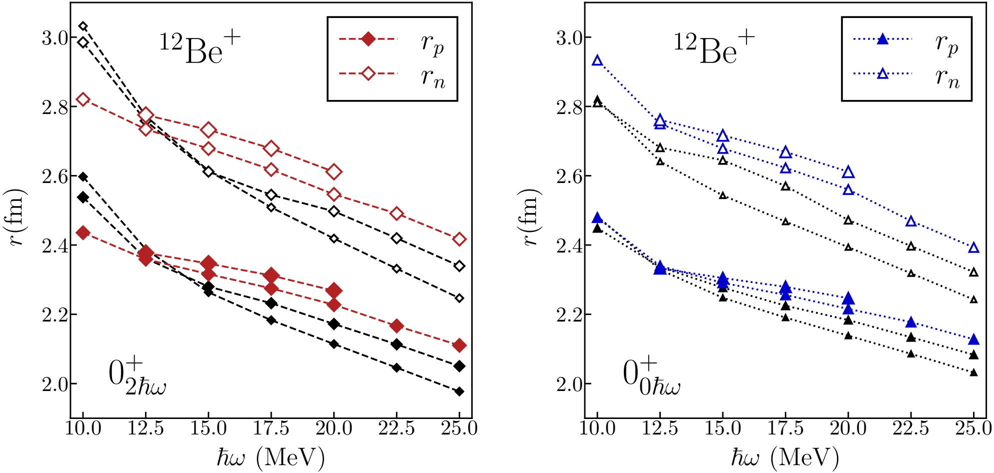
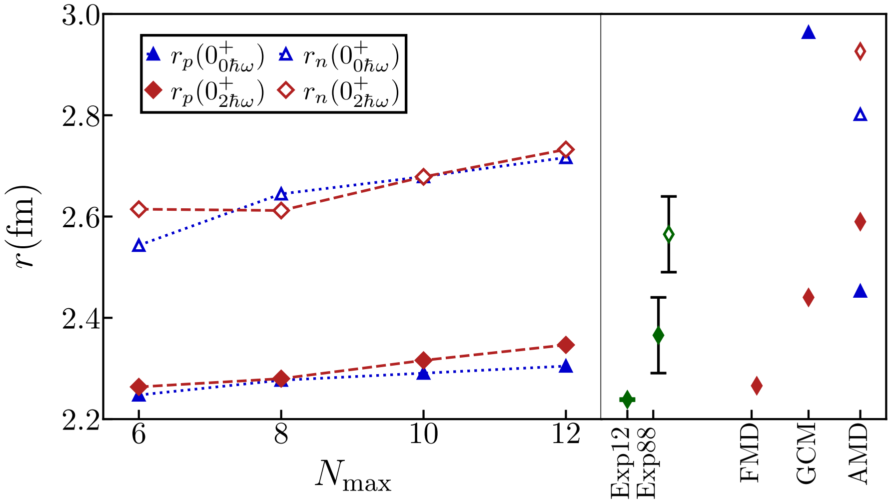


Figure: Y. Kanada-En'yo, H. Horiuchi, Phys. Rev. C **68**, 014319 (2003).

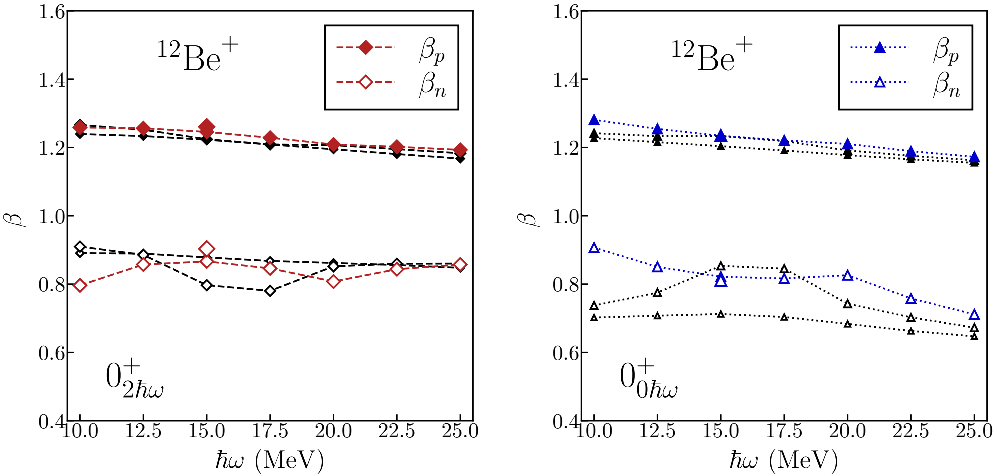
^{12}Be shape observables



^{12}Be shape observables



^{12}Be shape observables



^{12}Be shape observables

- r_p and β_p of $0^+_{2\hbar\omega}$ slightly larger than for $0^+_{0\hbar\omega}$

  $0^+_{2\hbar\omega}$ *observables less converged*

- r_n and β_n of $0^+_{2\hbar\omega}$ slightly larger than for $0^+_{0\hbar\omega}$

  $0^+_{2\hbar\omega}$ *observables less converged (maybe)*

- $r_p < r_n$

- $\beta_p > \beta_n$

To what extent are calculated observables impacted by transient mixing?

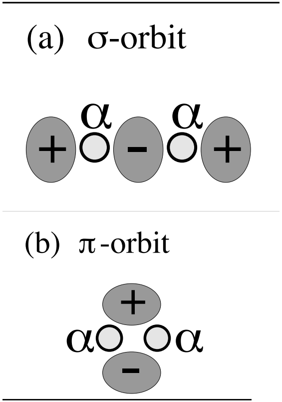
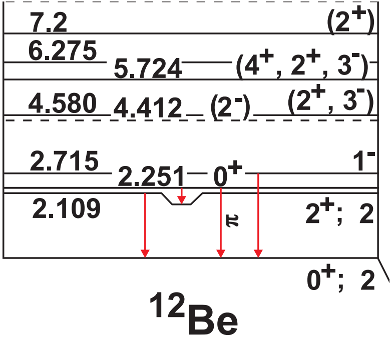


Figure: Y. Kanada-En'yo, H. Horiuchi, Phys. Rev. C **68**, 014319 (2003).

^{12}Be transitions

Do not expect inter-band transitions between bands with very different shape.



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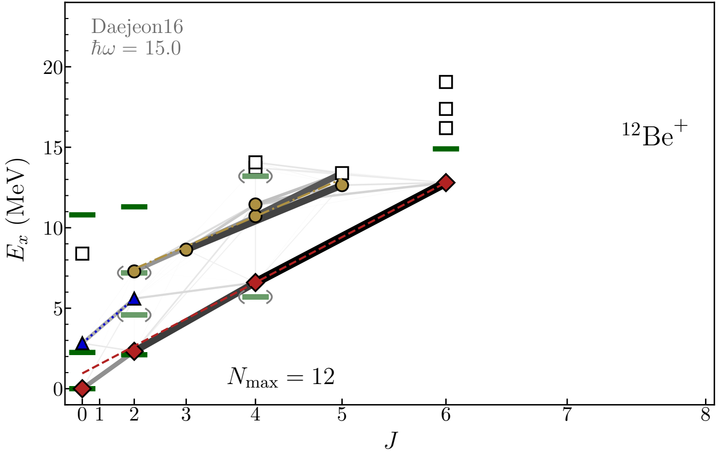
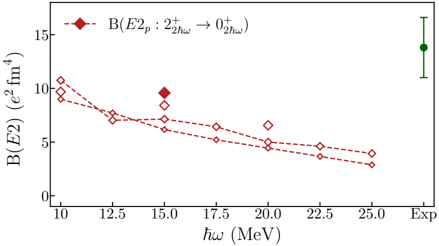
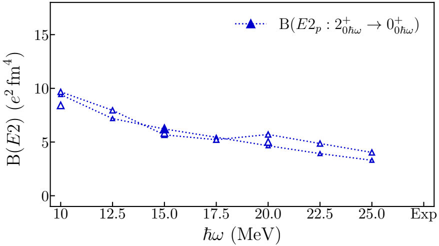
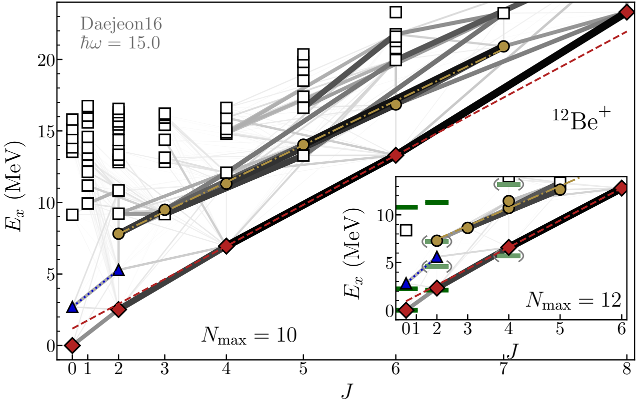


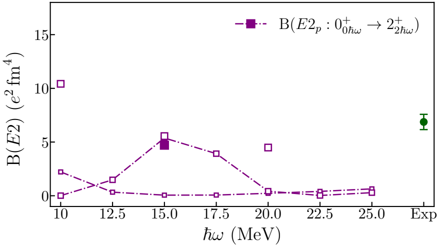
^{12}Be transitions

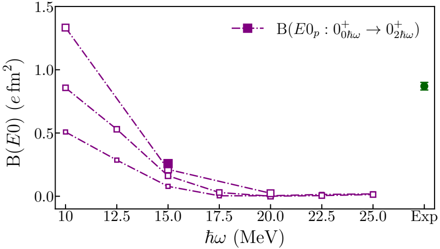


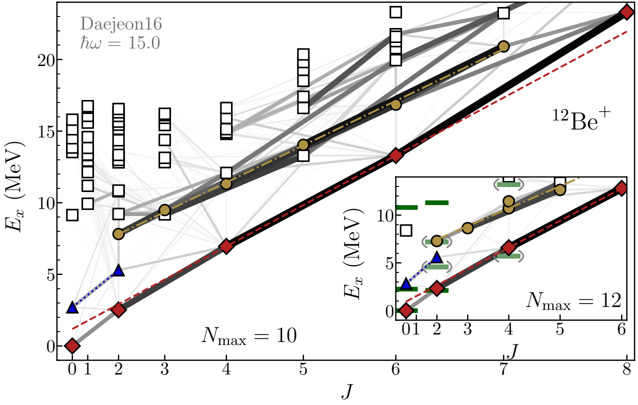




^{12}Be transitions





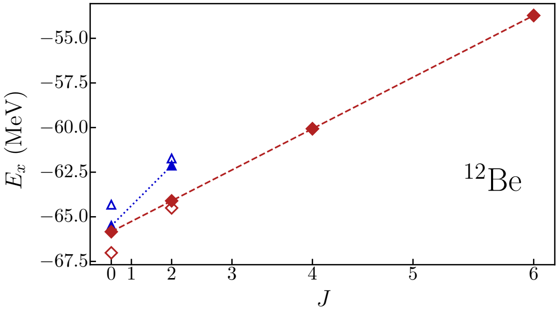


Two state mixing

$$H_{\text{mix}} = \underbrace{\begin{pmatrix} E_1 & v \\ v & E_2 \end{pmatrix}}_{\text{mixing Hamiltonian}} \quad \rightarrow \quad \underbrace{\begin{pmatrix} E_1' \\ E_2' \end{pmatrix}}_{\text{``mixed"}} = \underbrace{\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}}_{\text{mixing matrix}} \underbrace{\begin{pmatrix} E_1 \\ E_2 \end{pmatrix}}_{\text{``unmixed"}}$$

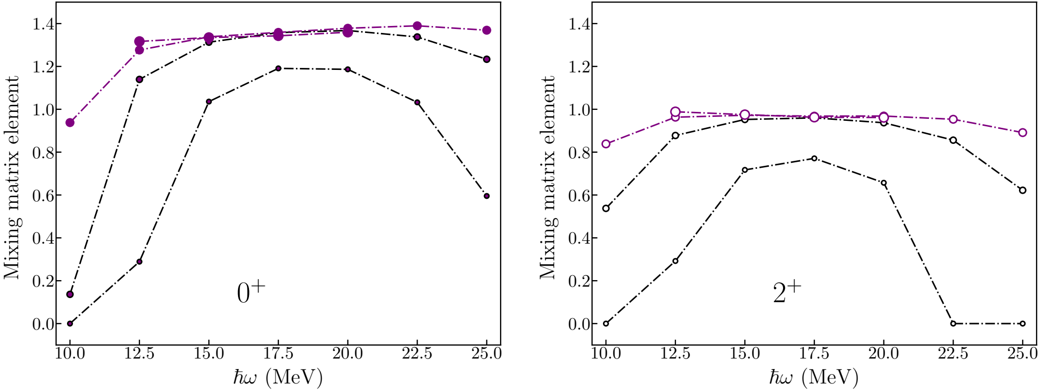
– Mixing angle θ depends on mixing matrix element v and $\Delta E = E_1 - E_2$

– Get “unmixed" energy from $E(J) = E_0 + A[J(J+1)]$



Mixing matrix element

$$H = \begin{pmatrix} E_1 & v \\ v & E_2 \end{pmatrix}$$



Mixing angle

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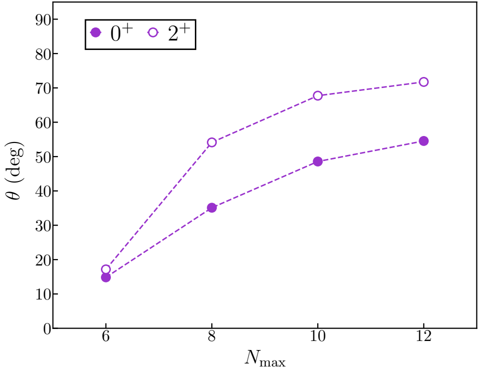
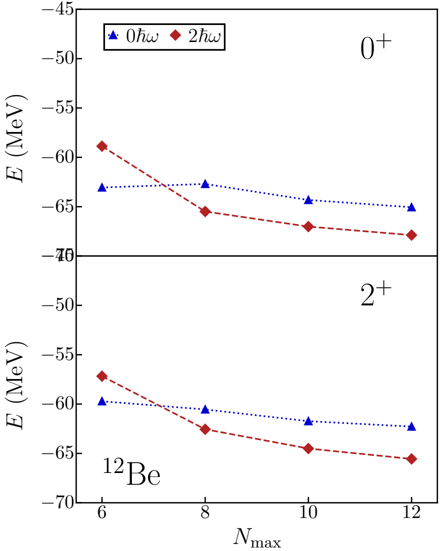
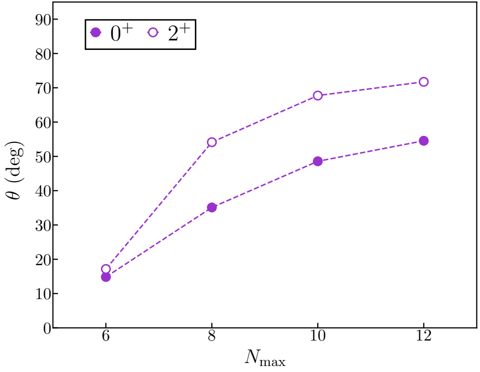
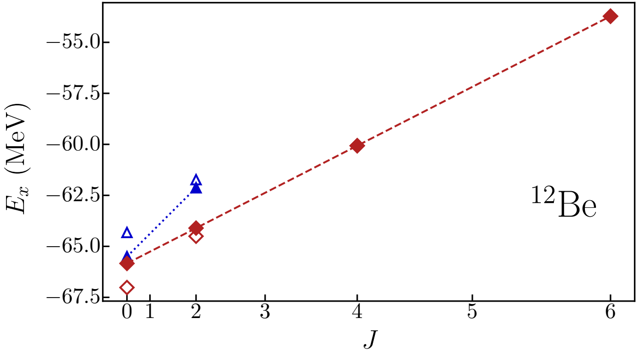




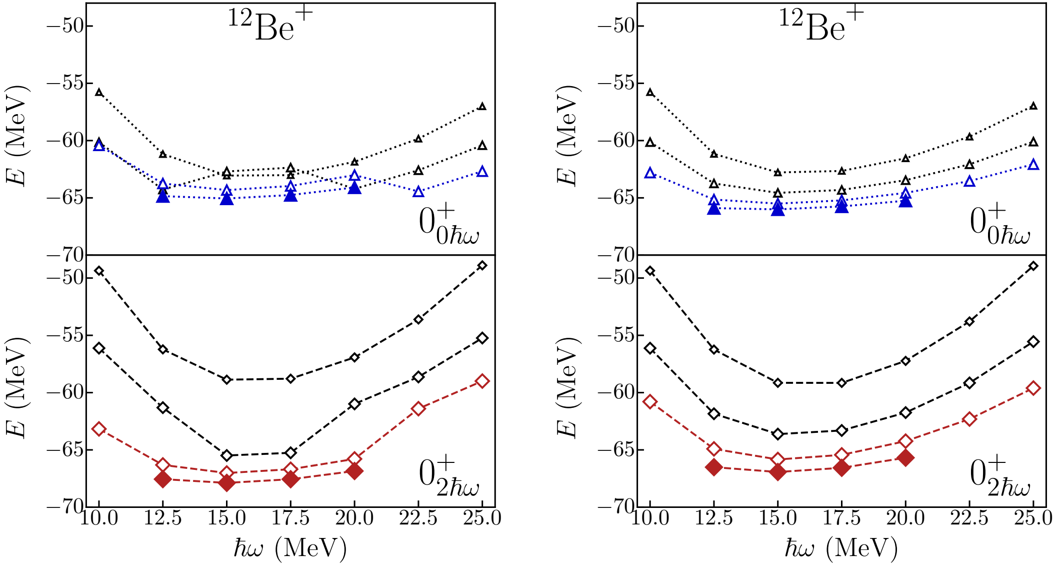
Mixing angle

$$\begin{pmatrix} E'_1 \\ E'_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}$$





Mixed vs. unmixed



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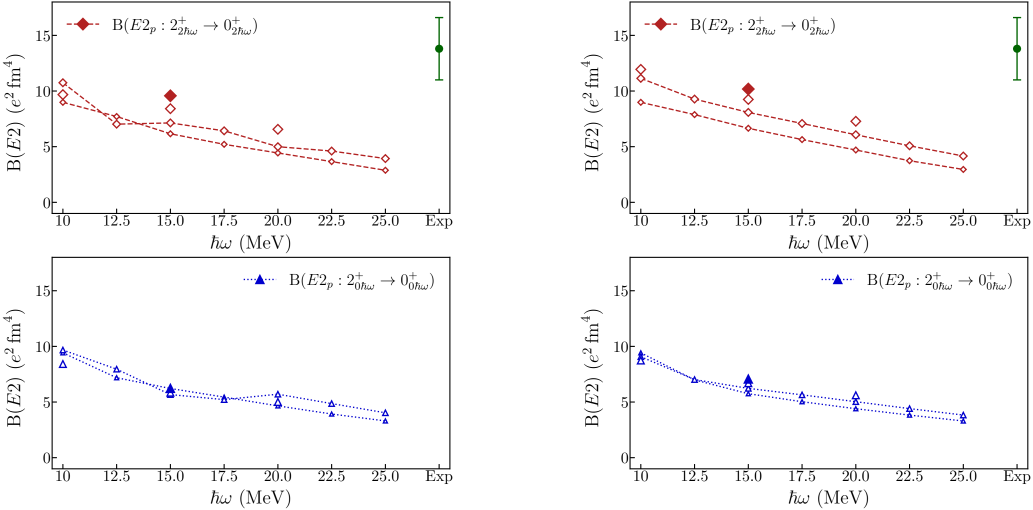

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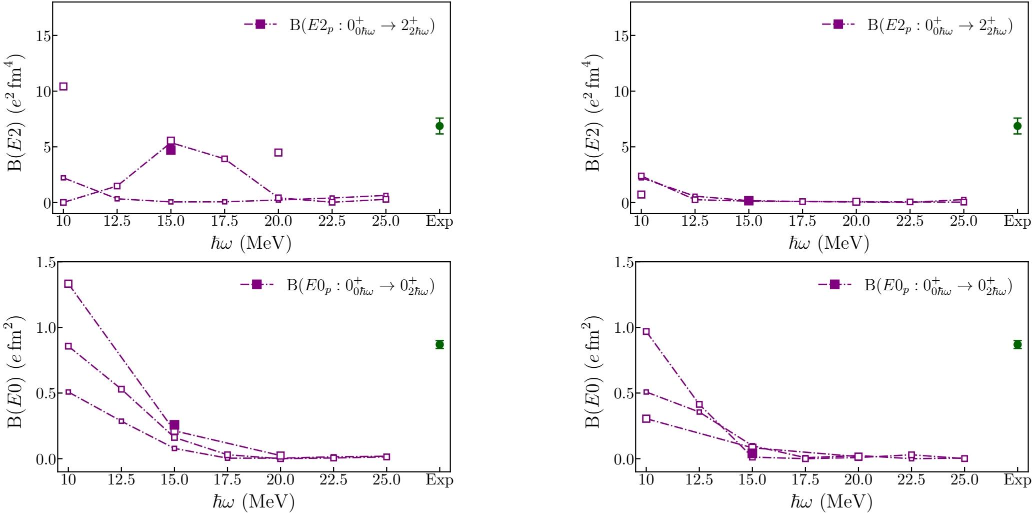

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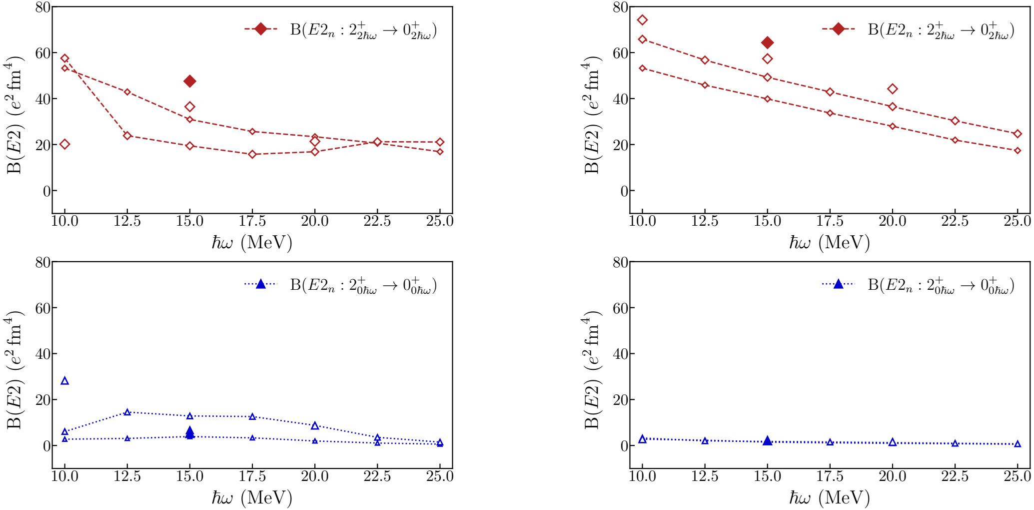

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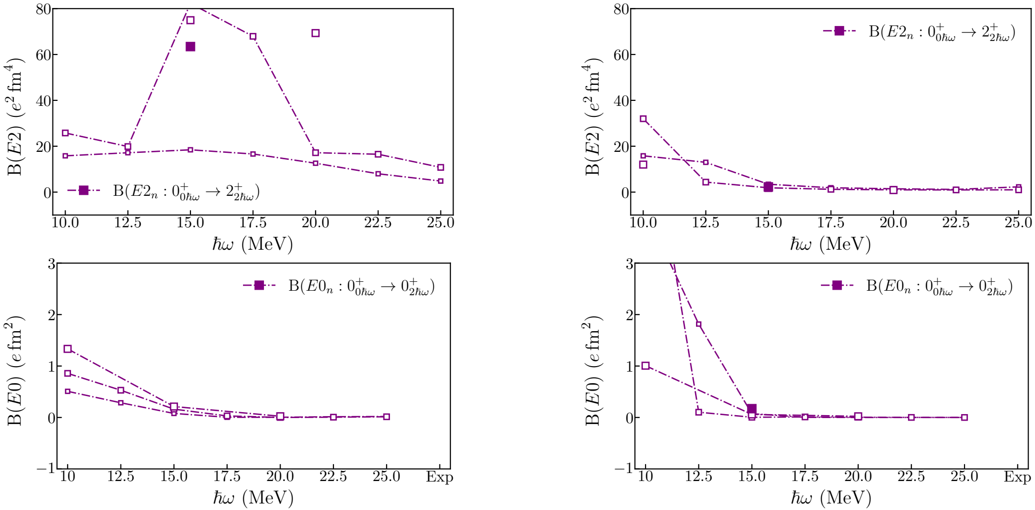
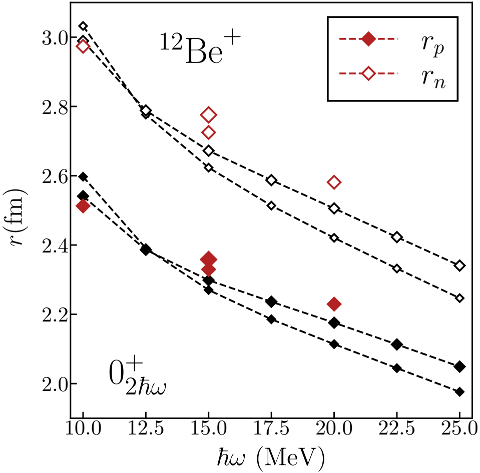
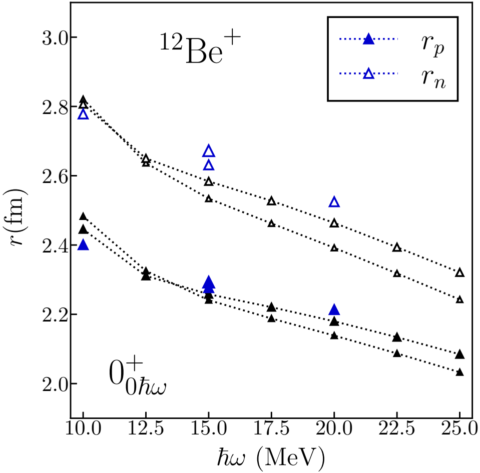


Unmixed ^{12}Be shape observables





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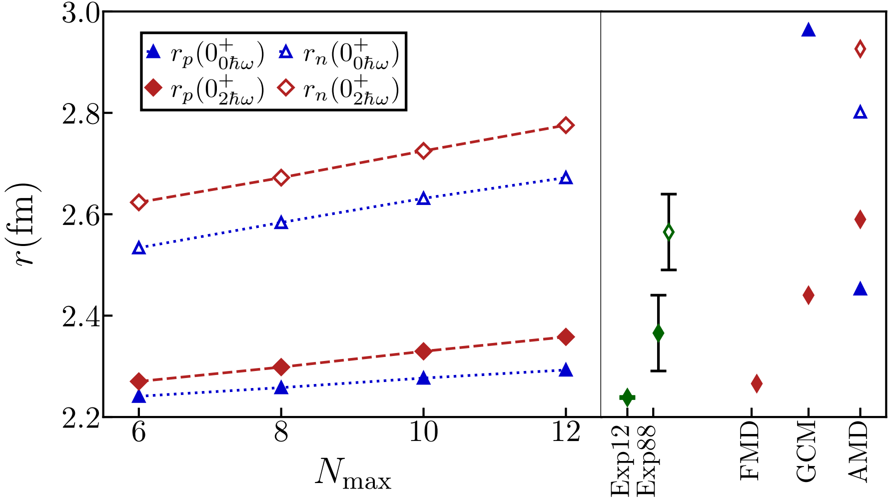

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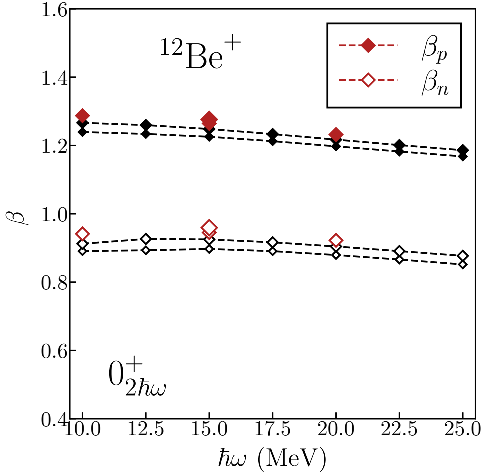
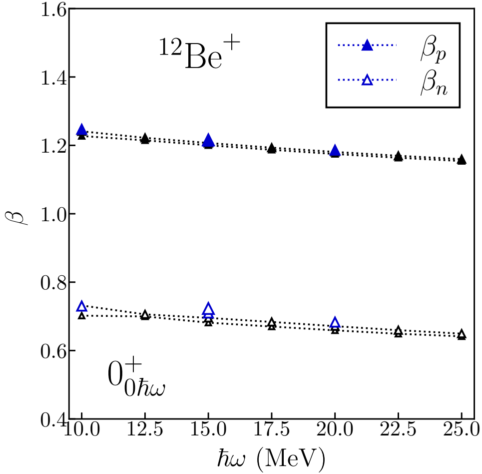




Shape summary

- Predictions for radii r and deformation β indicate for both 0^+ states:
  - Neutron radius is larger than proton radius *and still growing*
  - Protons are more deformed than neutrons *Approaching convergence*

- Radii of 0_1^+ larger than radii 0_2^+ (and is less converged)

- 0_1^+ has larger radii and is more deformed than 0_2^+

- Consistent with 2α dumbbell surrounded by neutron cloud

Probing underlying symmetries

- *Ab initio* calculations provides access to underlying wave functions of the collective states

- Using the "Lanczos trick" we can decompose the wave functions according to different symmetries

  C. W. Johnson. Phys. Rev. C **91** *(2015) 034313.*

- Elliott's SU(3): In limit of large quantum numbers, labels (λ, μ) are associated with deformation parameters

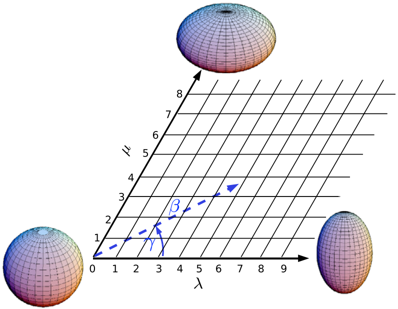
  O. Castanos, J. P. Draayer, Y. Leschber, Z. Phys. A **329** *(1988) 3*

$$\beta^2 \propto r^{-4}(\lambda^2 + \lambda\mu + \mu^2 + 3\lambda + 3\mu + 3)$$

$$\gamma = \tan^{-1}[\sqrt{3}(\mu + 1)/(2\lambda + \mu + 3)]$$

SU(3) generators

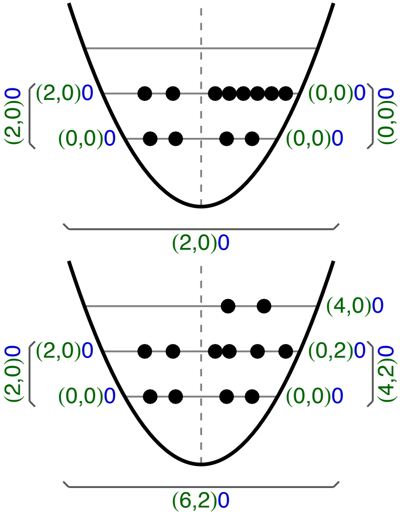
Q_{2M}	*Algebraic quadrupole*
L_{1M}	*Orbital angular momentum*



Elliott U(3)

SU(3) symmetry of a configuration

- SU(3) coupling particles within major shells *Each particle has SU(3) symmetry* $(N,0)$, $N = 2n+\ell$.
- SU(3) coupling successive shells
- SU(3) coupling protons and neutrons



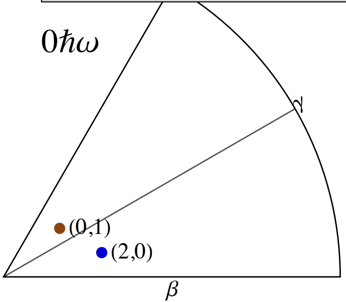
- Different configurations lead to different $N_{\text{ex}}(\lambda,\mu)S$
- Lowest energies correspond to most deformed intrinsic state $\langle Q\cdot Q\rangle / r^4 \propto \beta^2$

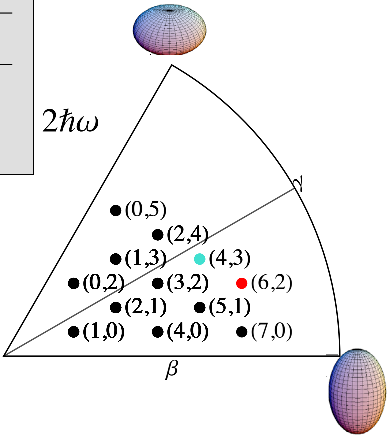
$$H \propto -Q\cdot Q + E(N_{\text{ex}})$$
$$= -6C_{\text{SU(3)}}(\lambda,\mu) + 3\mathbf{L}^2 + E(N_{\text{ex}})$$

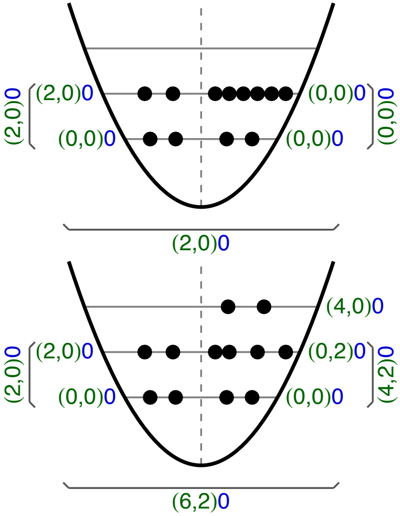
Elliott rotational bands: ^{12}Be

$$H \propto -Q\cdot Q = -6C_{\mathrm{SU(3)}} + 3\mathbf{L}^2 + E(N_{\mathrm{ex}})$$

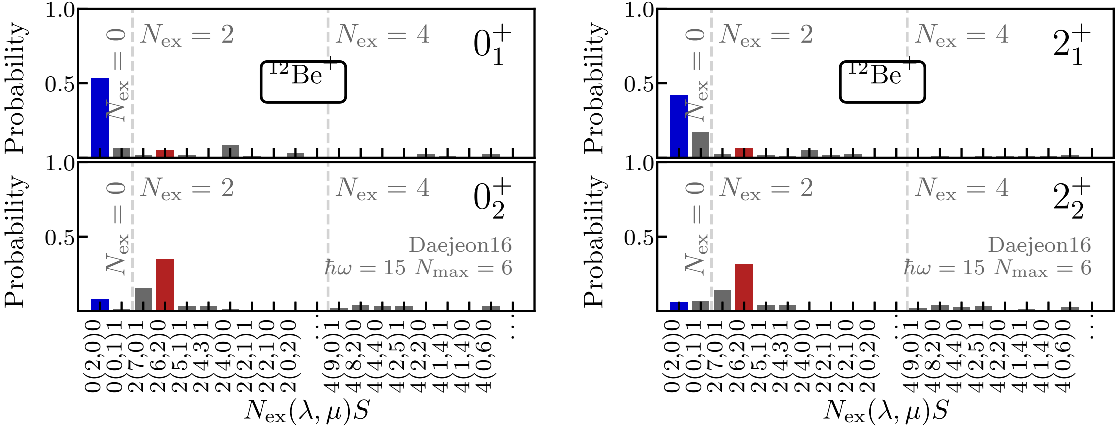
$N_{\mathrm{ex}}(\lambda,\mu)$	S	$\Rightarrow$	$\langle C_{\mathrm{SU(3)}}\rangle$
0(0,1)	1	$\Rightarrow$	4
0(2,0)	0	$\Rightarrow$	10
…			
2(4,3)	0,1	$\Rightarrow$	58
2(7,0)	1	$\Rightarrow$	70
2(6,2)	0	$\Rightarrow$	76







Decomposition by Elliott U(3)



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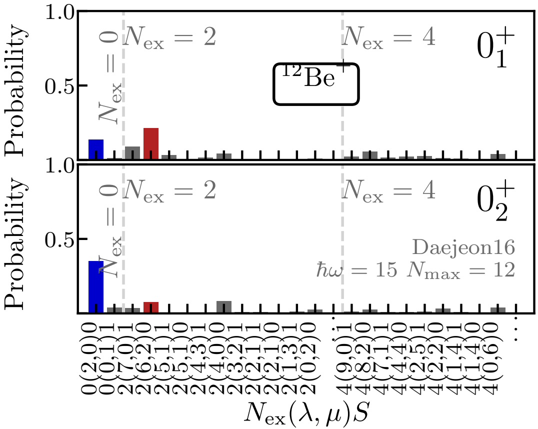

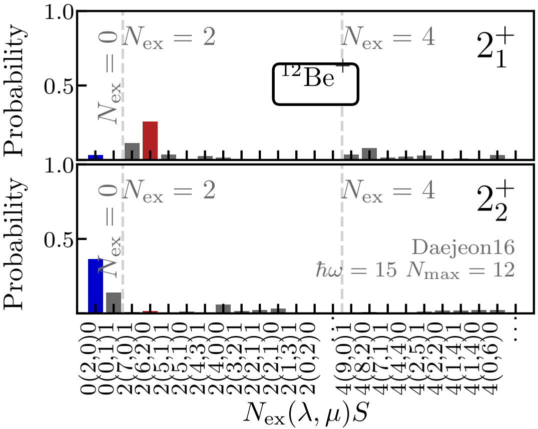

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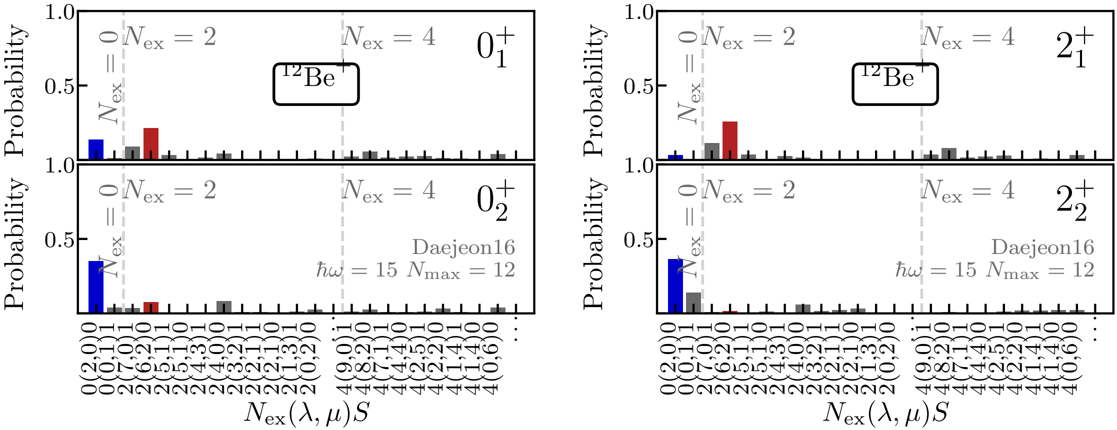


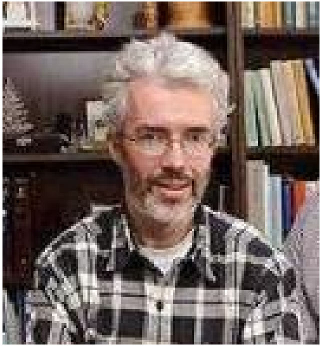
Unmixed states have very different SU(3) *fingerprint (different "shapes")*

Acknowledgements

In collaboration with...

Mark Caprio
University of Notre Dame

Pieter Maris
Iowa State University



Summary

- *Ab initio* NCSM with Daejeon16 predicts an intruder ground state
- 0^+ and 2^+ states mix as energies cross. *Making analysis of convergence hard!*
  - State mixing appears to be well modeled by two-state mixing problem.
  - States do not fully un-mix (non-zero interband $E0$ and $E2$ transitions).
- Predictions for radii r and deformation β indicate for 0^+ states:
  - Neutron radius is larger than proton radius
  - Protons are more deformed than neutrons
  - 0_1^+ has larger radius and is more deformed than 0_2^+
- 0_1^+ and 0_2^+ states have very different SU(3) (*different "shapes"*)
- Next steps:
  - Known exp. energies + mixing matrix element v, fixes "physical" mixing angle
  - Use physical mixing angle to re-mix observables (or rather ratios of observables)