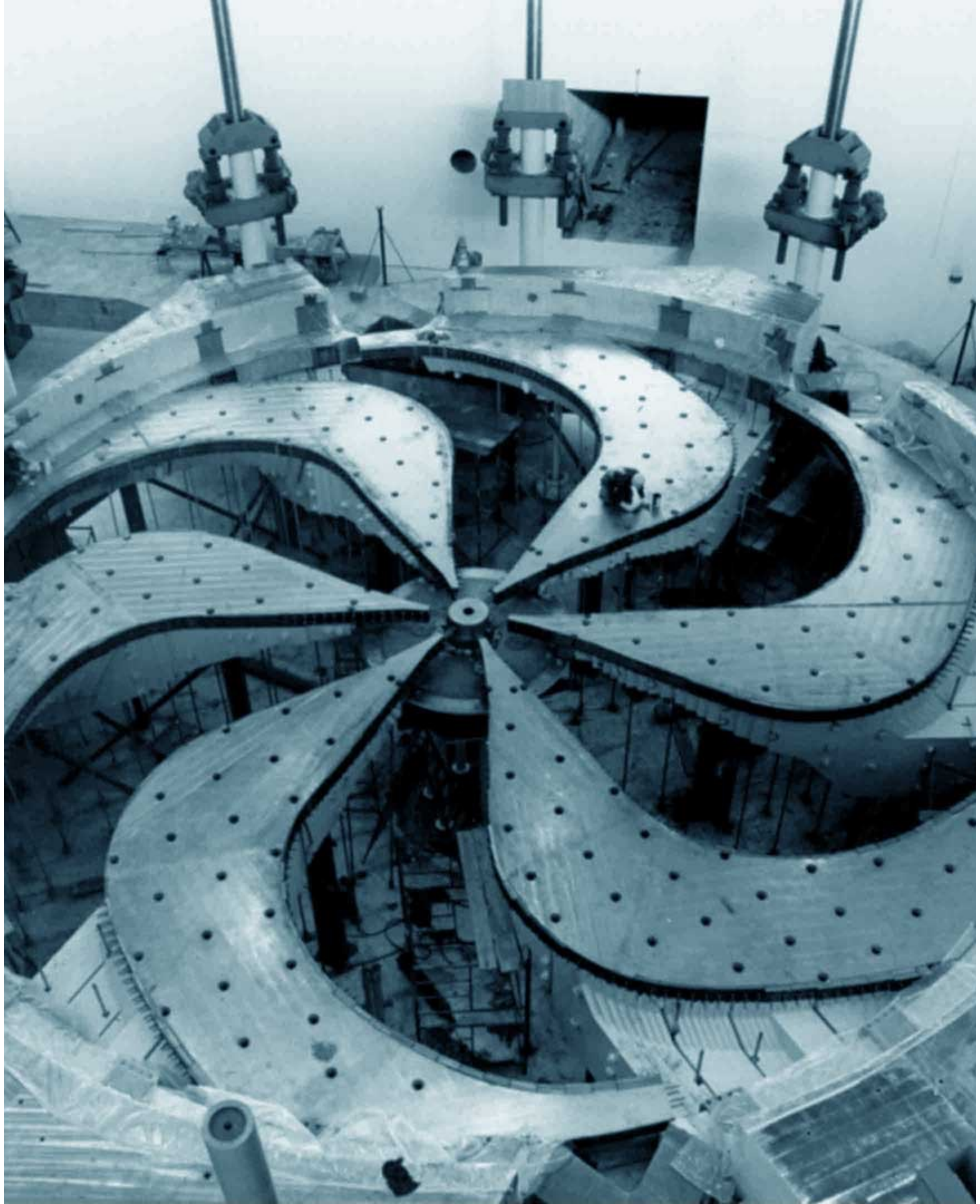
Towards reliable nuclear matrix elements for neutrinoless double beta decay

Antoine Belley

PAINT Workshop 2023

Collaborators: Jack Pitcher, Takayuki Miyagi, Ragnar Stroberg, Jason Holt

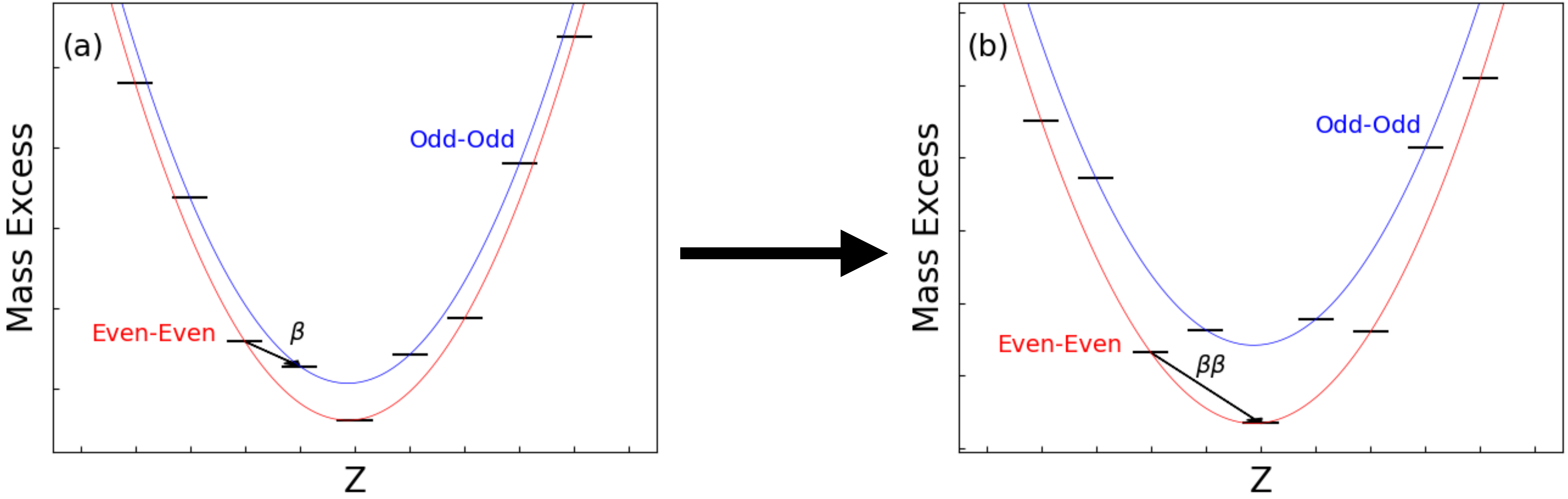


Discovery, accelerated

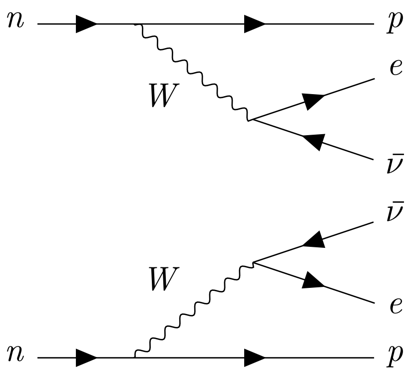
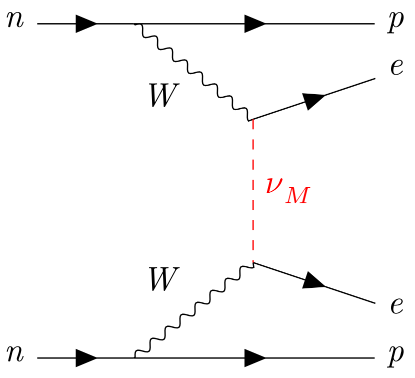
Double beta decays

Second order order weak process

Only possible when single beta decay is energetically forbidden (or strongly disadvantaged)



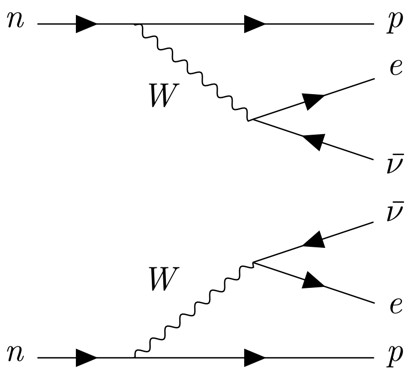
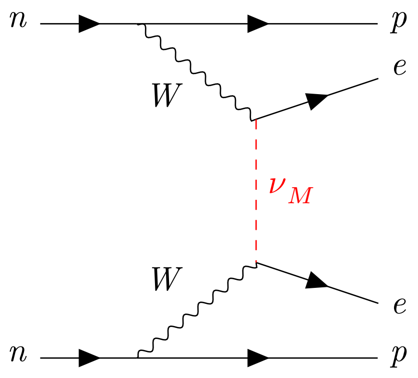
$2\nu\beta\beta$ vs $0\nu\beta\beta$

Decay	$2\nu\beta\beta$	$0\nu\beta\beta$
Diagram		
Half-life Formula	$[T^{2\nu}_{1/2}]^{-1} = G^{2\nu}\lvert M^{2\nu}\rvert^2$	$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}\lvert M^{0\nu}\rvert^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2$
NME Formula	$M^{2\nu} \approx M^{2\nu}_{GT}$	$M^{0\nu} = M^{0\nu}_{GT} - (\frac{g_v}{g_a})^2 M^{0\nu}_F + M^{0\nu}_T - 2g_{\nu\nu}M^{0\nu}_{CT}$
LNV	No	Yes!
Observed	Yes	No

*NME : Nuclear matrix elements

**LNV : Lepton number violation

$2\nu\beta\beta$ vs $0\nu\beta\beta$

<table>
<tr><th>Decay</th><th>$2\nu\beta\beta$</th><th>$0\nu\beta\beta$</th></tr>
<tr><td>Diagram</td><td>
</td><td>
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2νββ vs 0νββ

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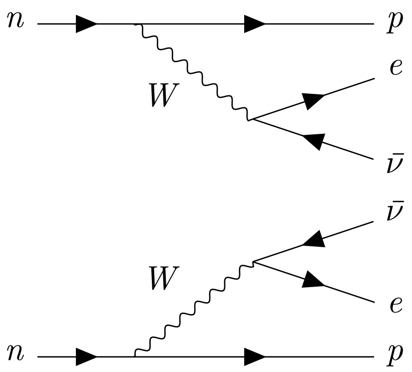
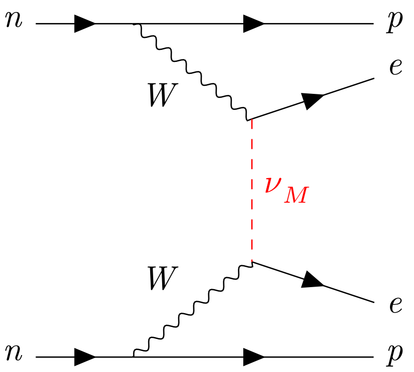
Diagram		
Half-life Formula	$[T^{2\nu}_{1/2}]^{-1} = G^{2\nu}\|M^{2\nu}\|^2$	$[T^{0\nu}_{1/2}]^{-1} = G^{0\nu}\|M^{0\nu}\|^2 \left(\frac{\langle m_{\beta\beta}\rangle}{m_e}\right)^2$
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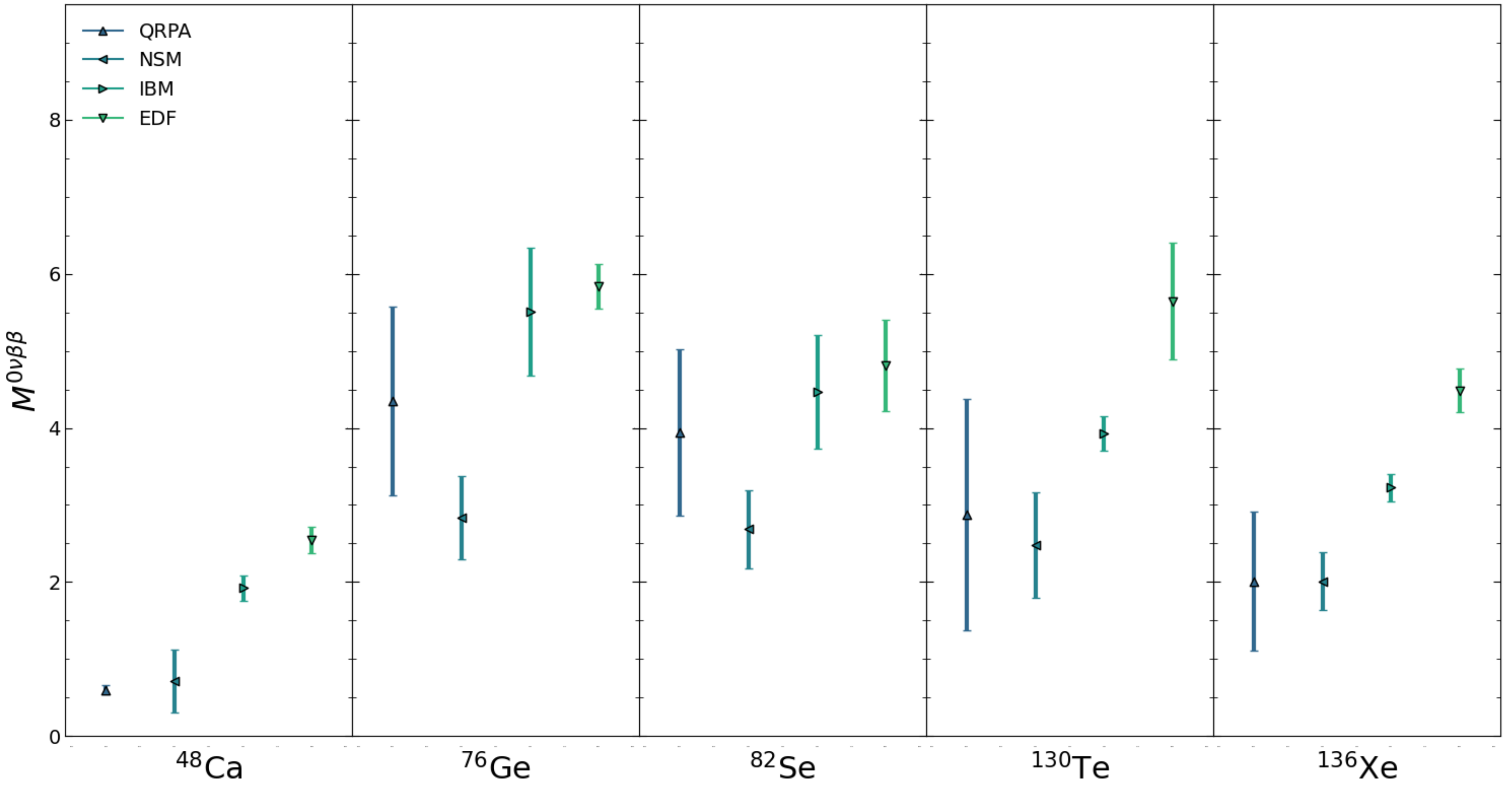
Status of 0νββ-decay Matrix Elements

Current calculations from phenomenological models have large spread in results.



Compiled values from Engel and Menéndez, Rep. Prog. Phys. **80** 046301 (2017); Yao, arXiv:2008.13249 (2020); Brase et al, arXiv:2108.11805 (2021)

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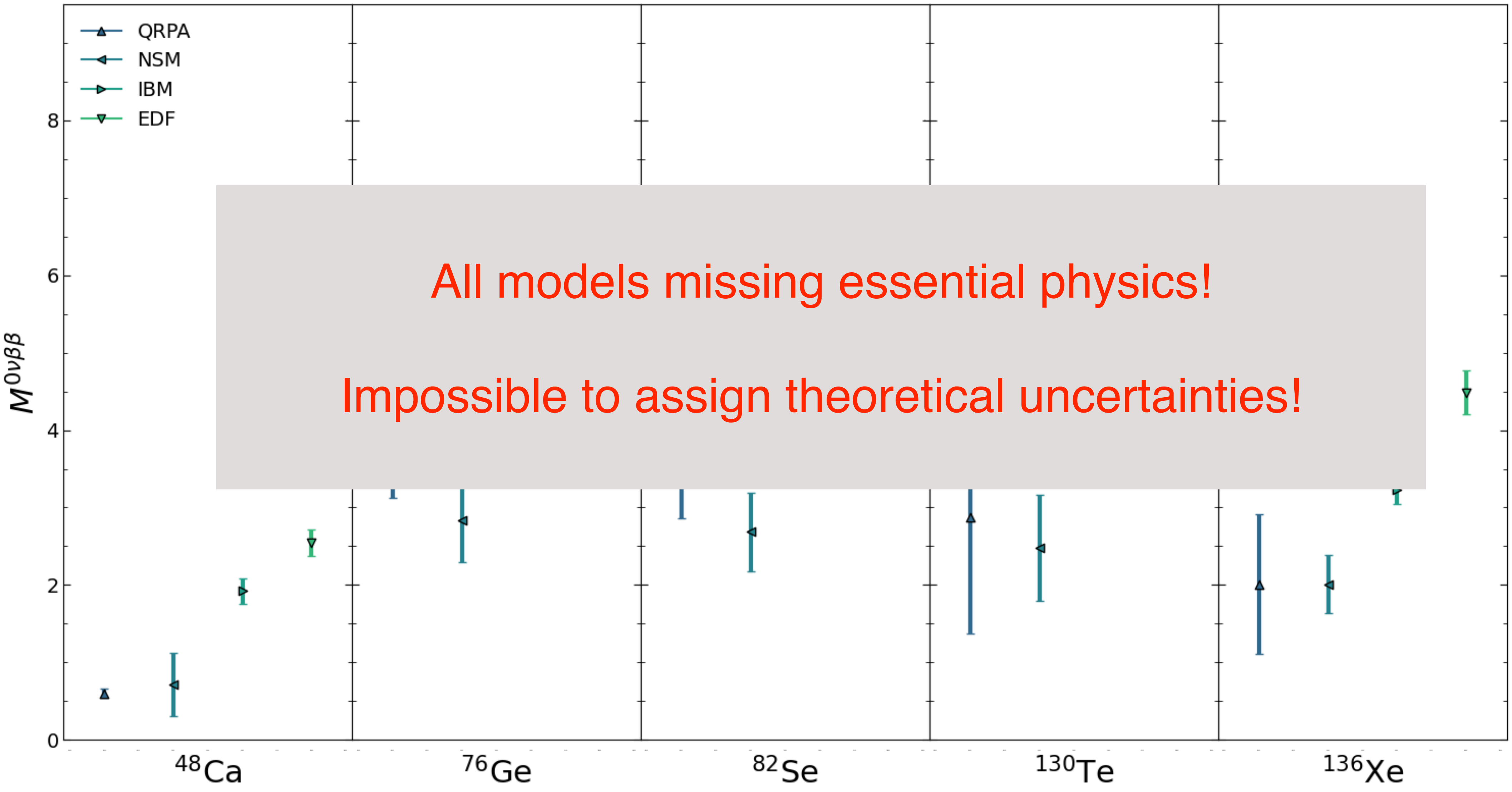

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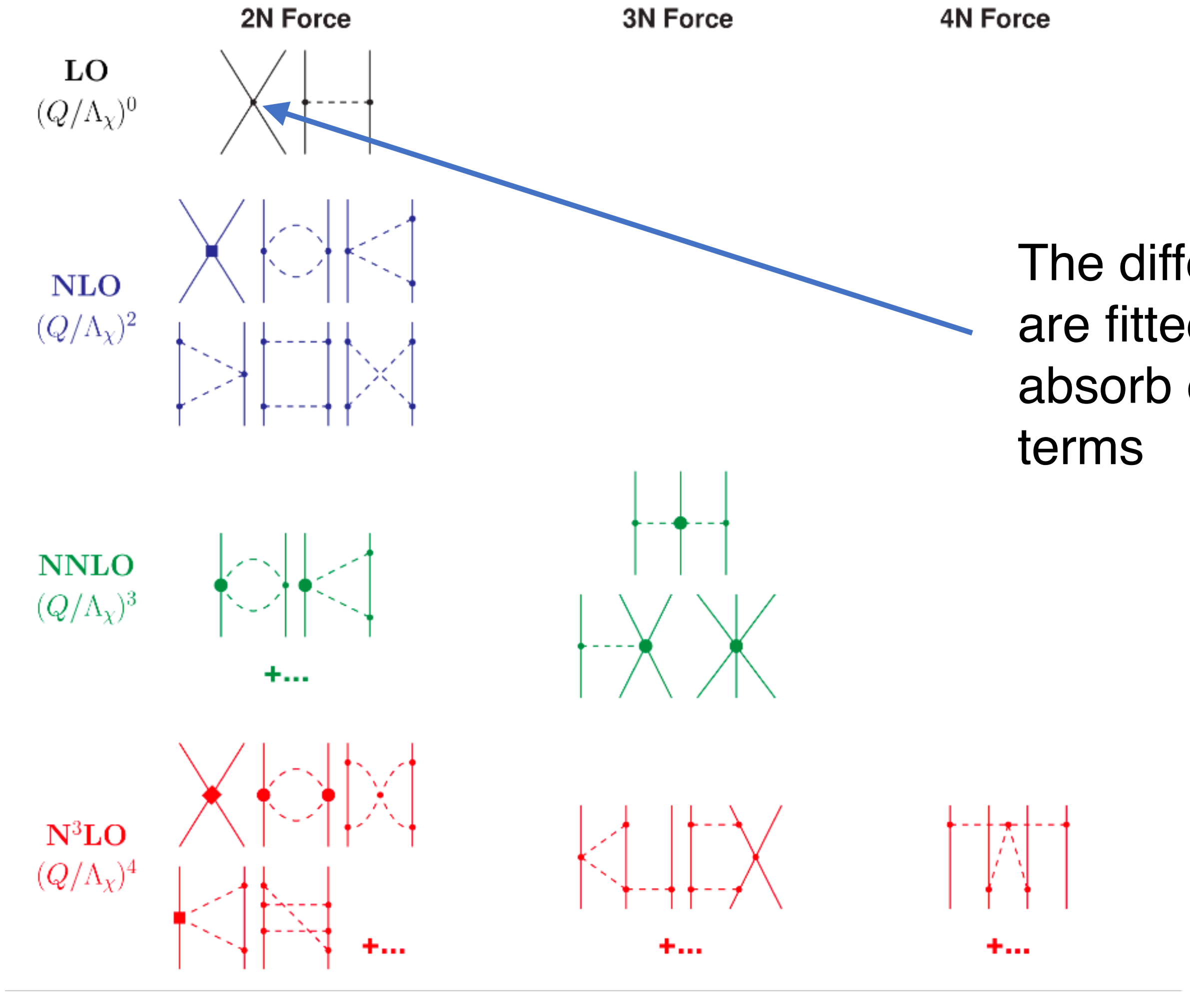
Ab initio nuclear theory

Expansion order by order of the nuclear forces

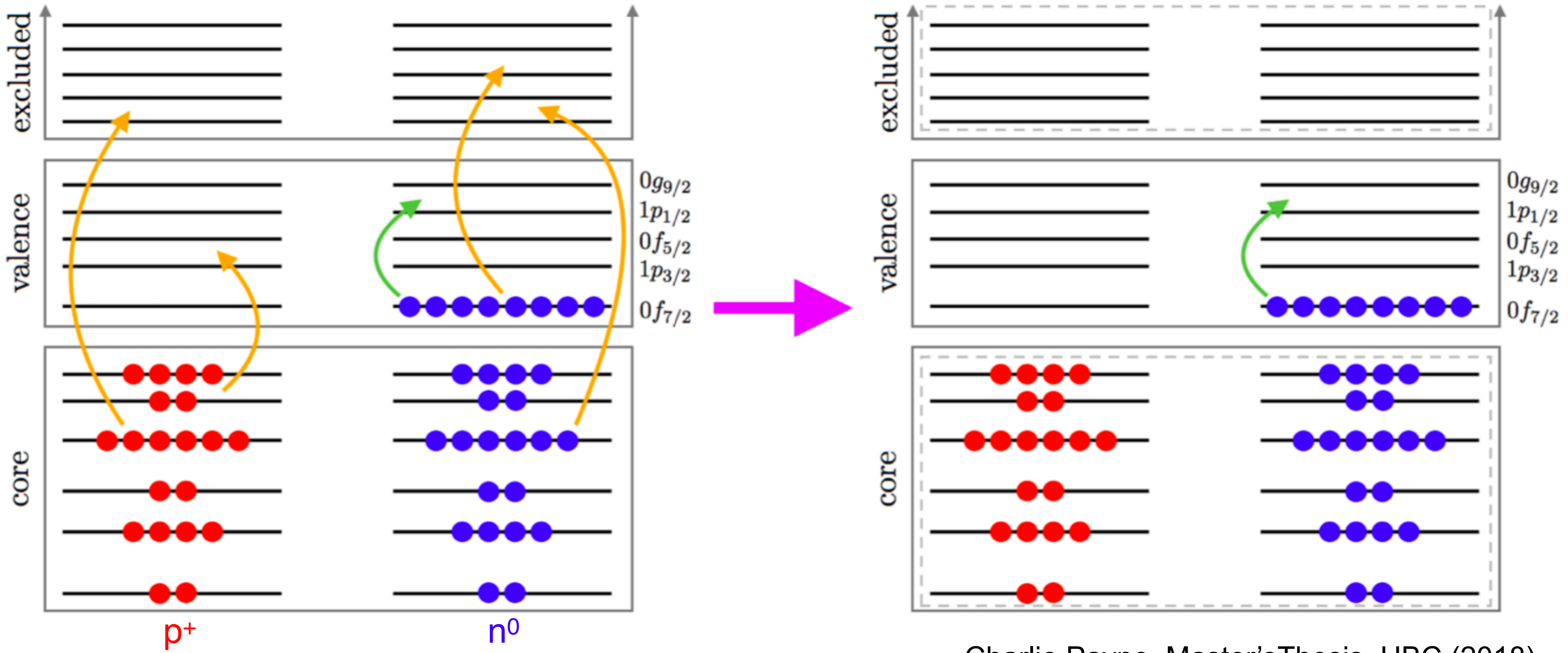
Reproduces symmetries of low-energy QCD using nucleons as fields and pions as force carriers.



The different coupling constants are fitted to few nucleons data to absorb effect of higher order terms

Machleidt and Entem, Phys. Rep., vol.503, no.1, pp.1–75 (2011)

Valence-Space In Medium Similarity Renormalization Group



Charlie Payne, Master'sThesis, UBC (2018)

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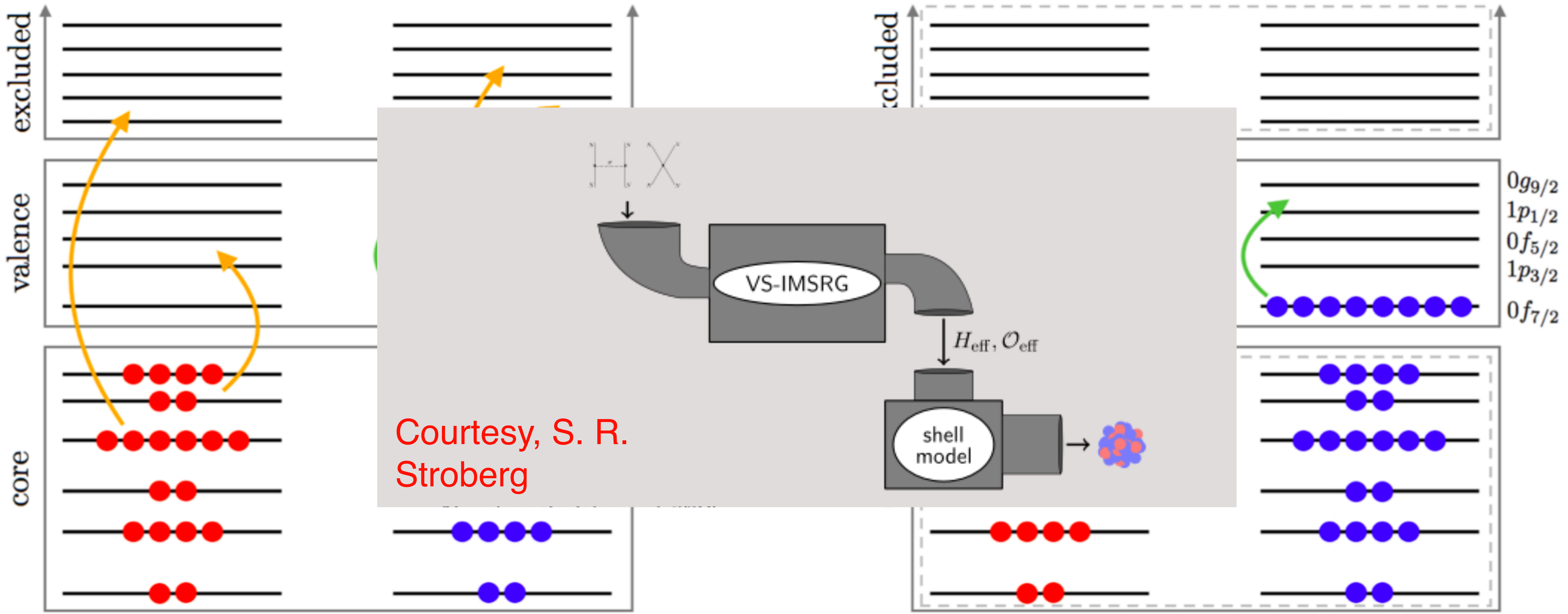

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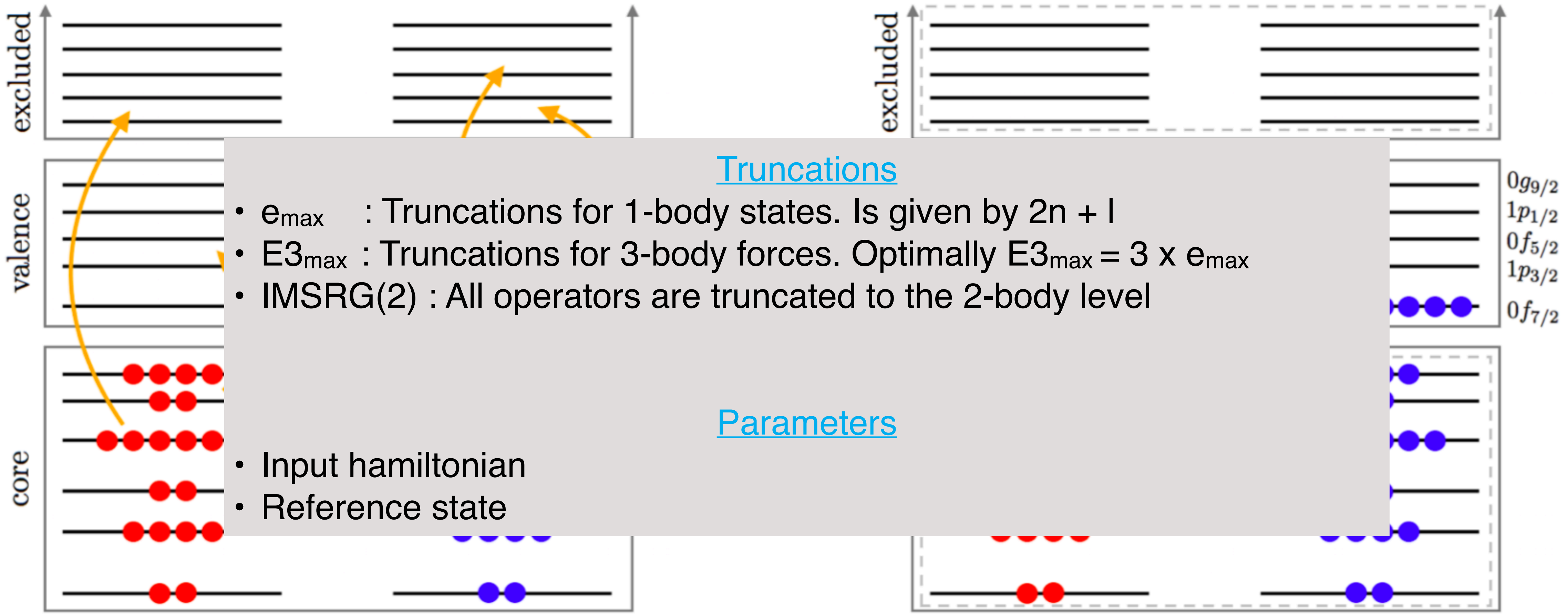


Truncations

- e_{max} : Truncations for 1-body states. Is given by 2n + l
- $E3_{max}$: Truncations for 3-body forces. Optimally $E3_{max} = 3 \times e_{max}$
- IMSRG(2) : All operators are truncated to the 2-body level

Parameters

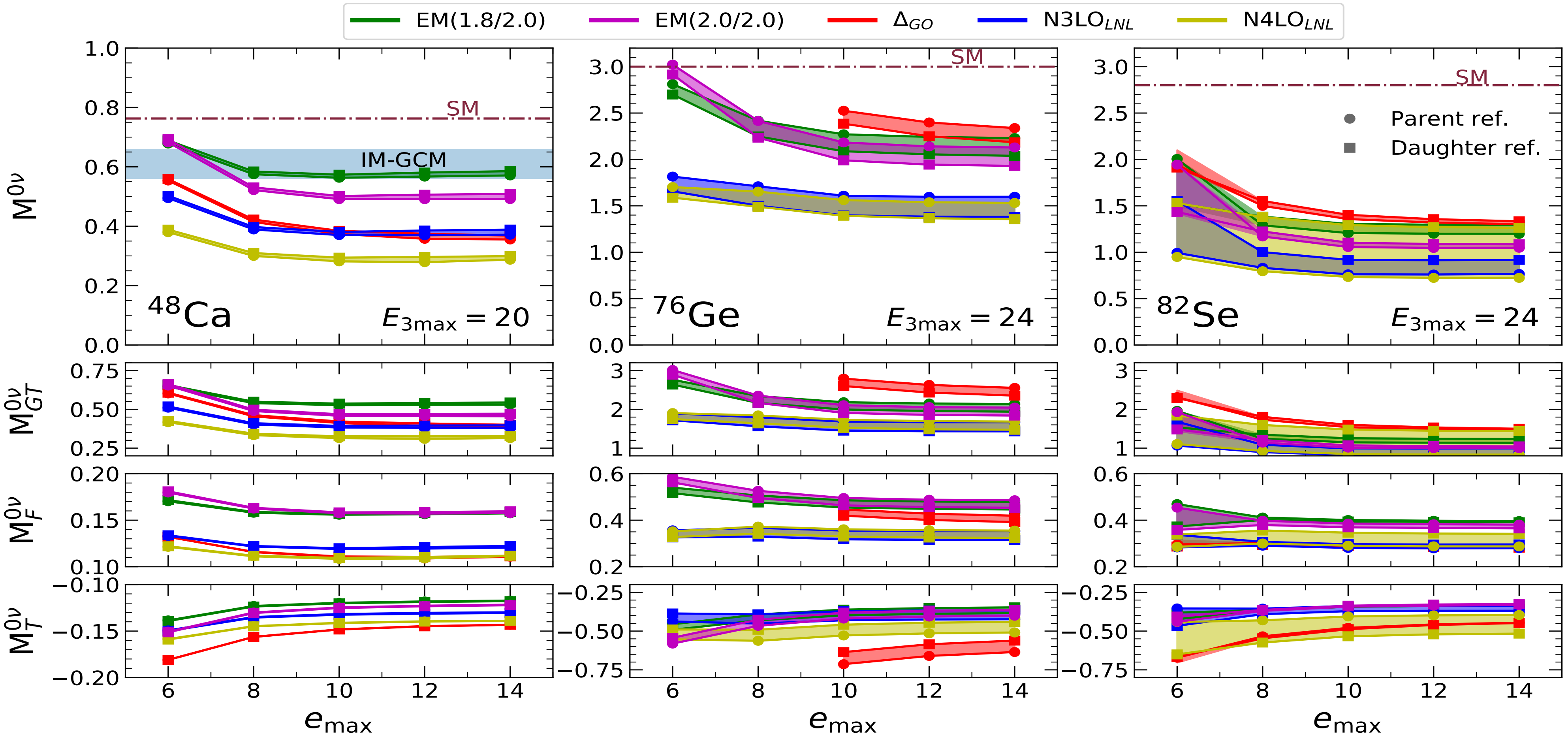
- Input hamiltonian
- Reference state

Results

TRIUMF

Ab Initio 0νββ Decay: ^{48}Ca, ^{76}Ge and ^{82}Se

Results with 5 different input hamiltonians to study uncertainty from interaction choice.

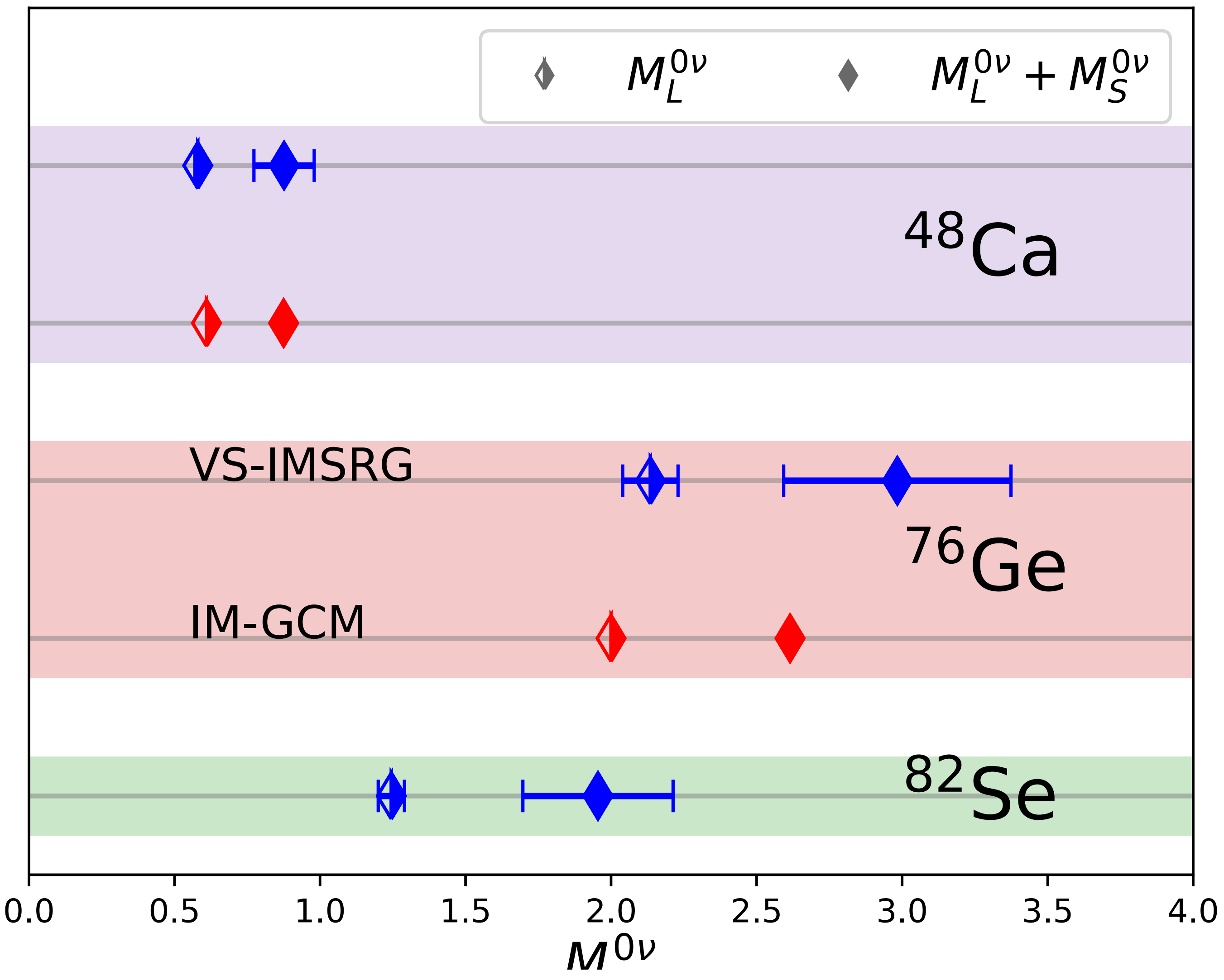


Things to add: valence space variation, two-body currents, IMSRG(3), …

Belley, et al., PRL126.042502

Belley, et al., in prep

Ab Initio 0νββ Decay: The contact term

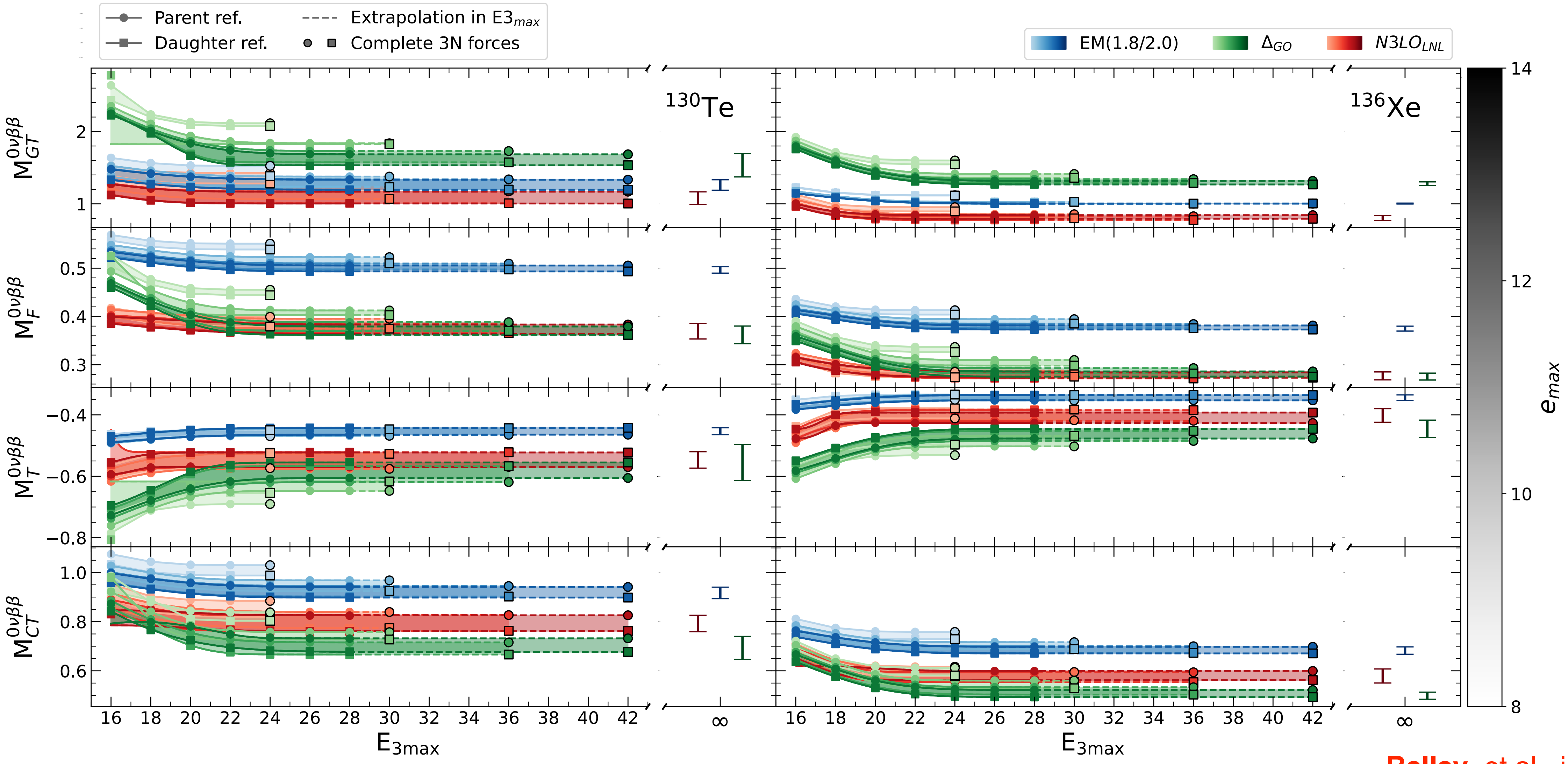


Belley, et al., in prep

Ab Initio $0\nu\beta\beta$ Decay: ^{130}Te, ^{136}Xe

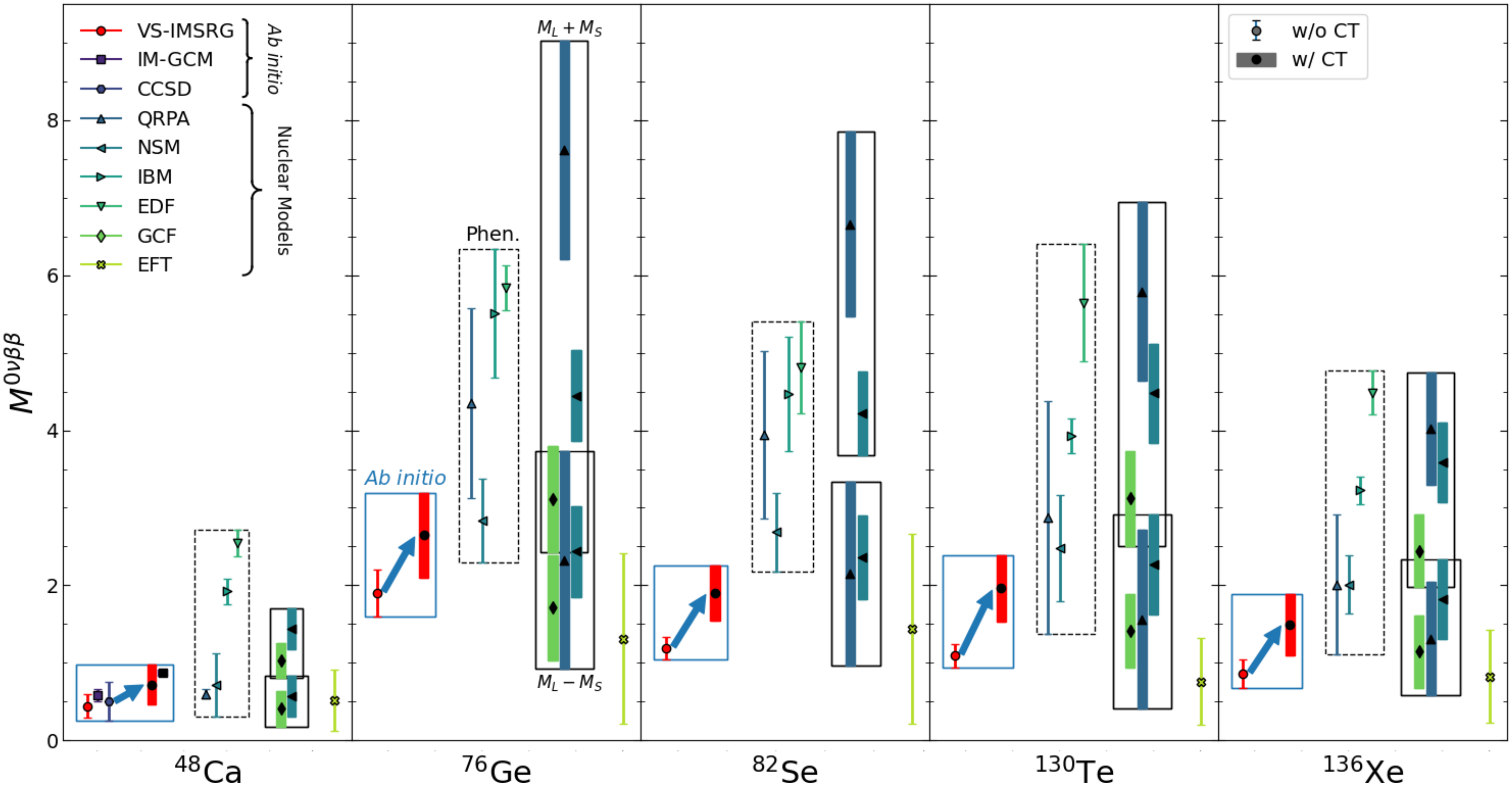
^{130}Te, ^{136}Xe major players in global searches with SNO+, CUORE and nEXO

Increased E_{3max} capabilities allow first converged ab initio calculations [EM1.8/2.0, Δ_{GO}, N3LO$_{LNL}$] 12



Belley, et al., in prep

0νββ-decay Matrix Elements: The new picture



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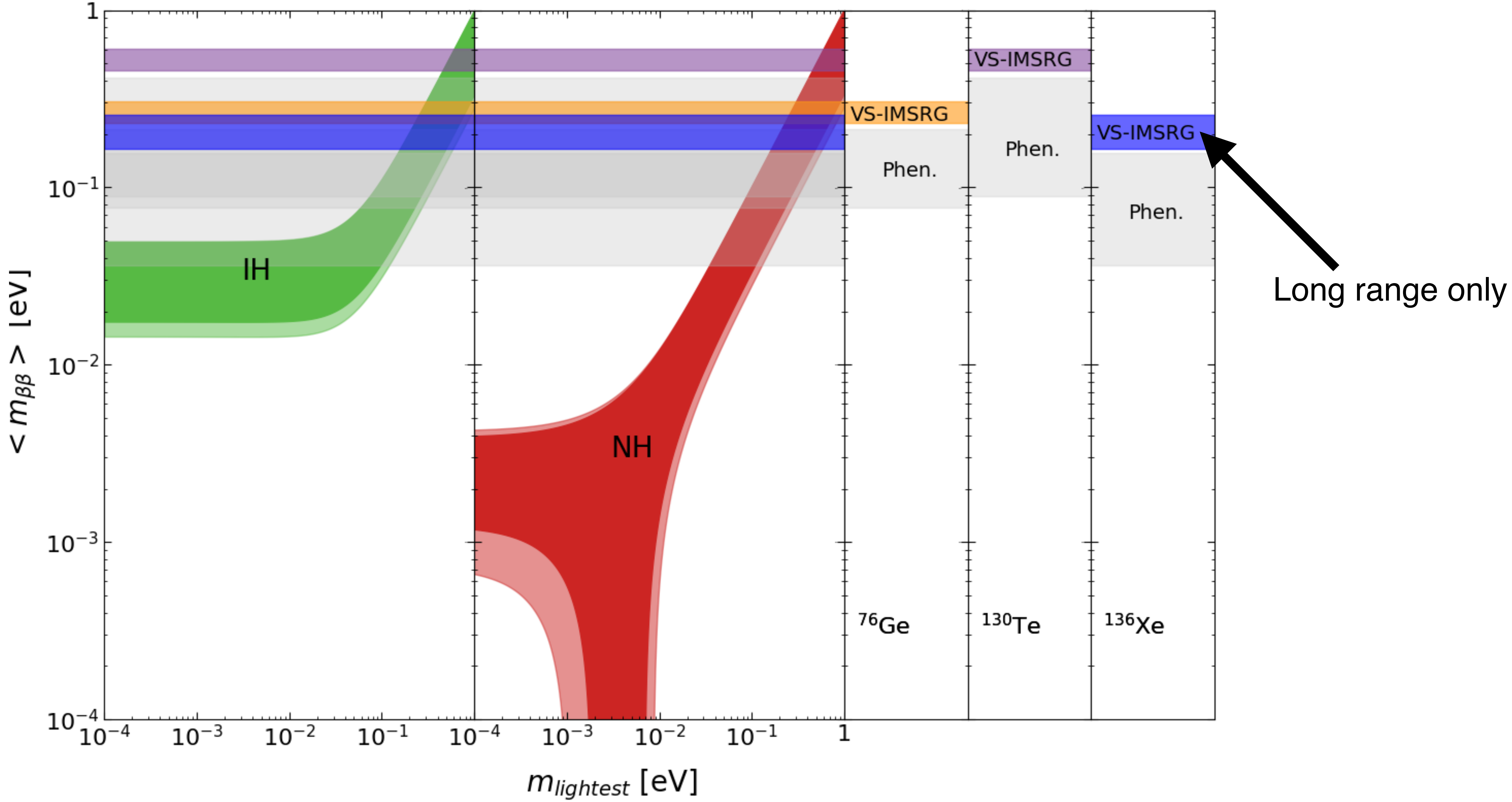
Ab Initio 0νββ Decay: Effect on experimental limits



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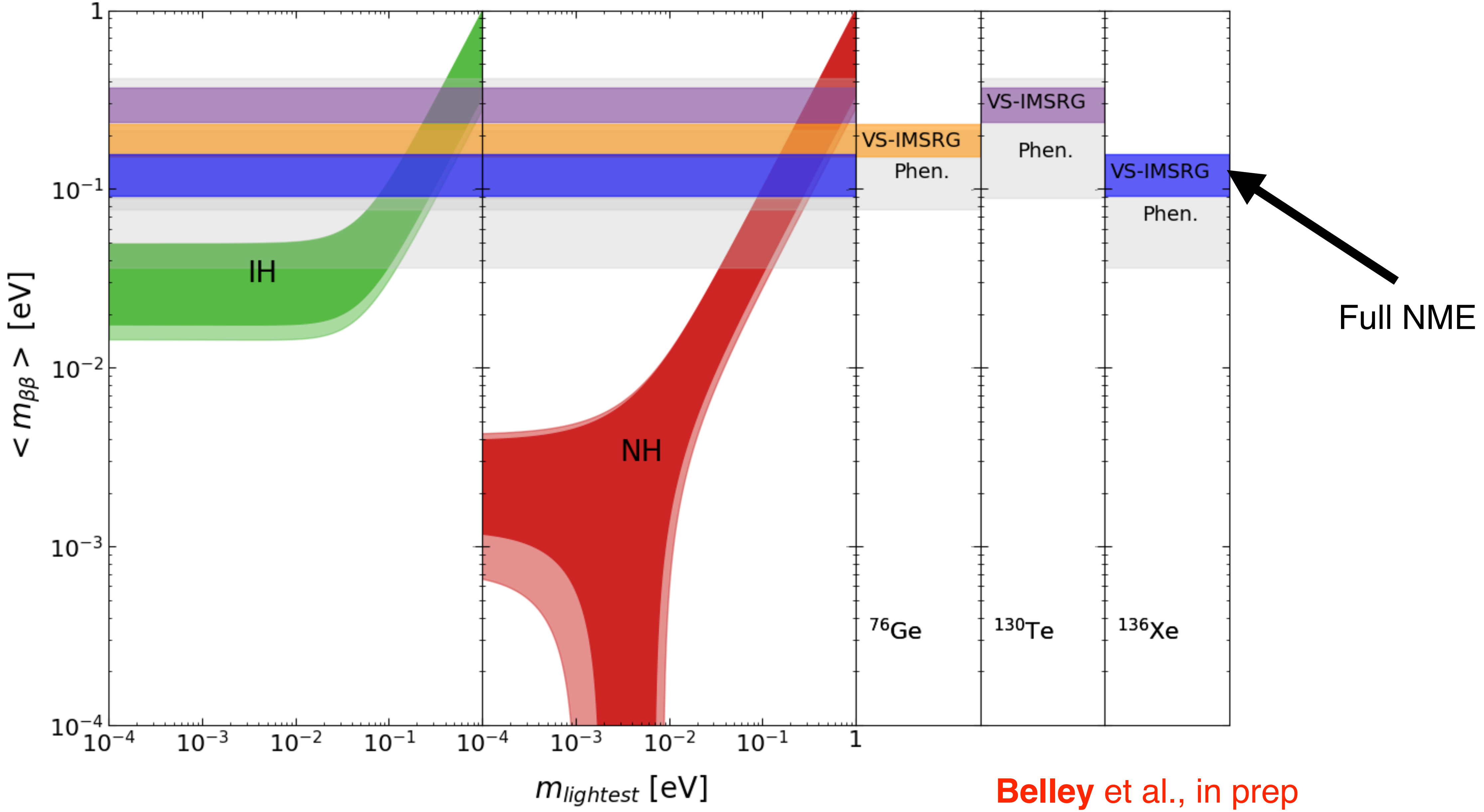

Belley et al., in prep

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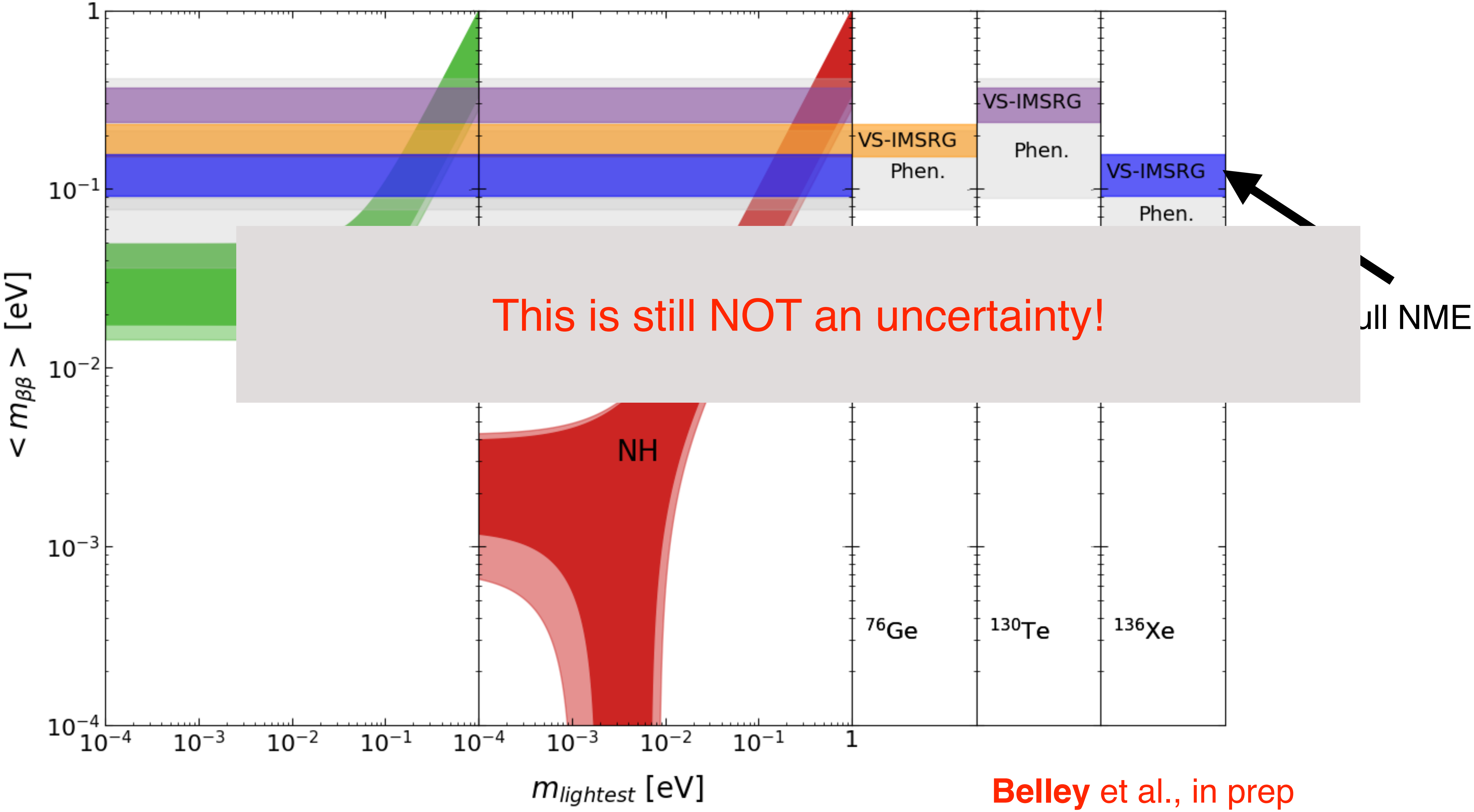

ull NME

This is still NOT an uncertainty!

Belley et al., in prep

Assessing the uncertainty

Uncertainty can be split into 3 sources:

- The many-body method (VS-IMSRG)
- The χ-EFT interaction
- The operators

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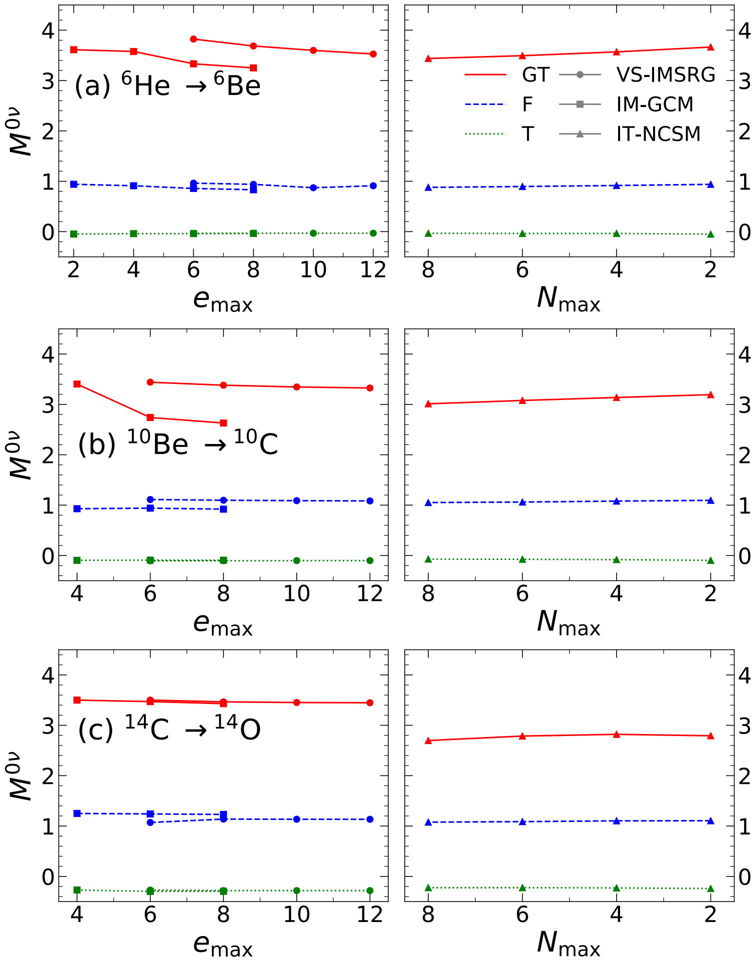
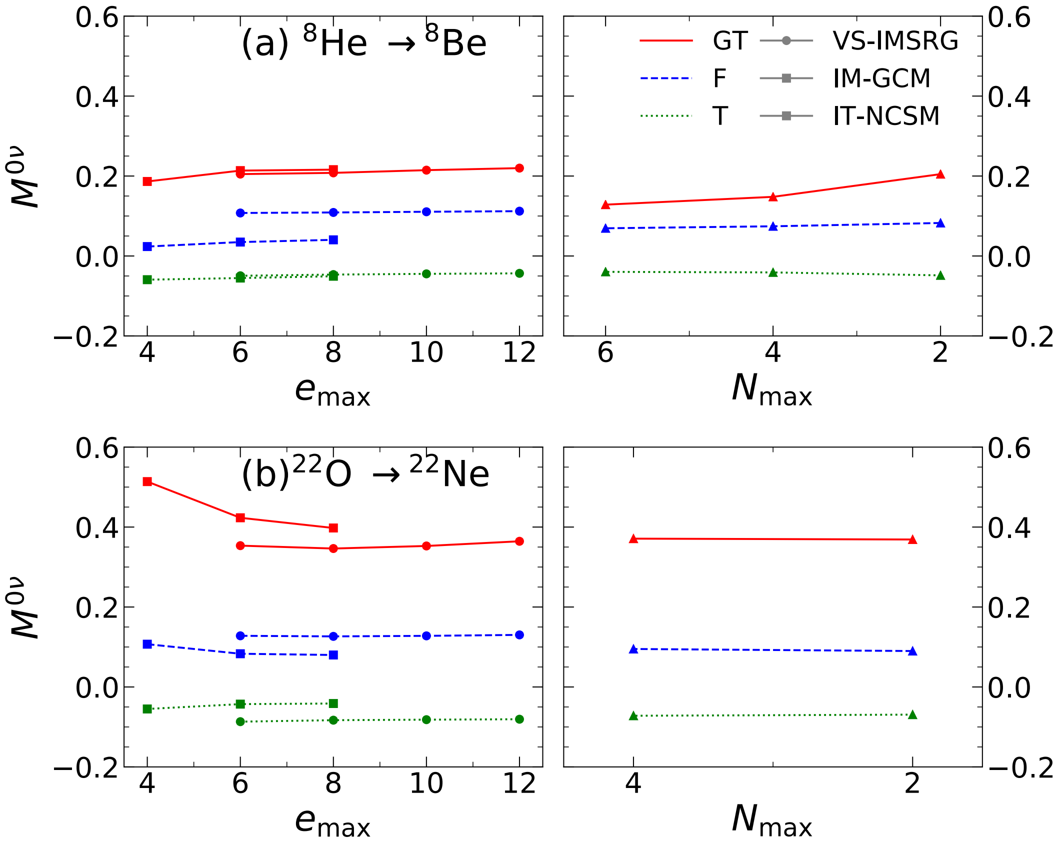
TRIUMF

Benchmarking 0νββ Decay in Light Nuclei: Summary

Benchmark with other ab initio method for fictitious decays in light nuclei

$\Delta T = 0$





$\Delta T = 2$

Yao, **Belley**, et al., PhysRevC.103.014315

Reasonable to good agreement in all cases

Assessing the uncertainty

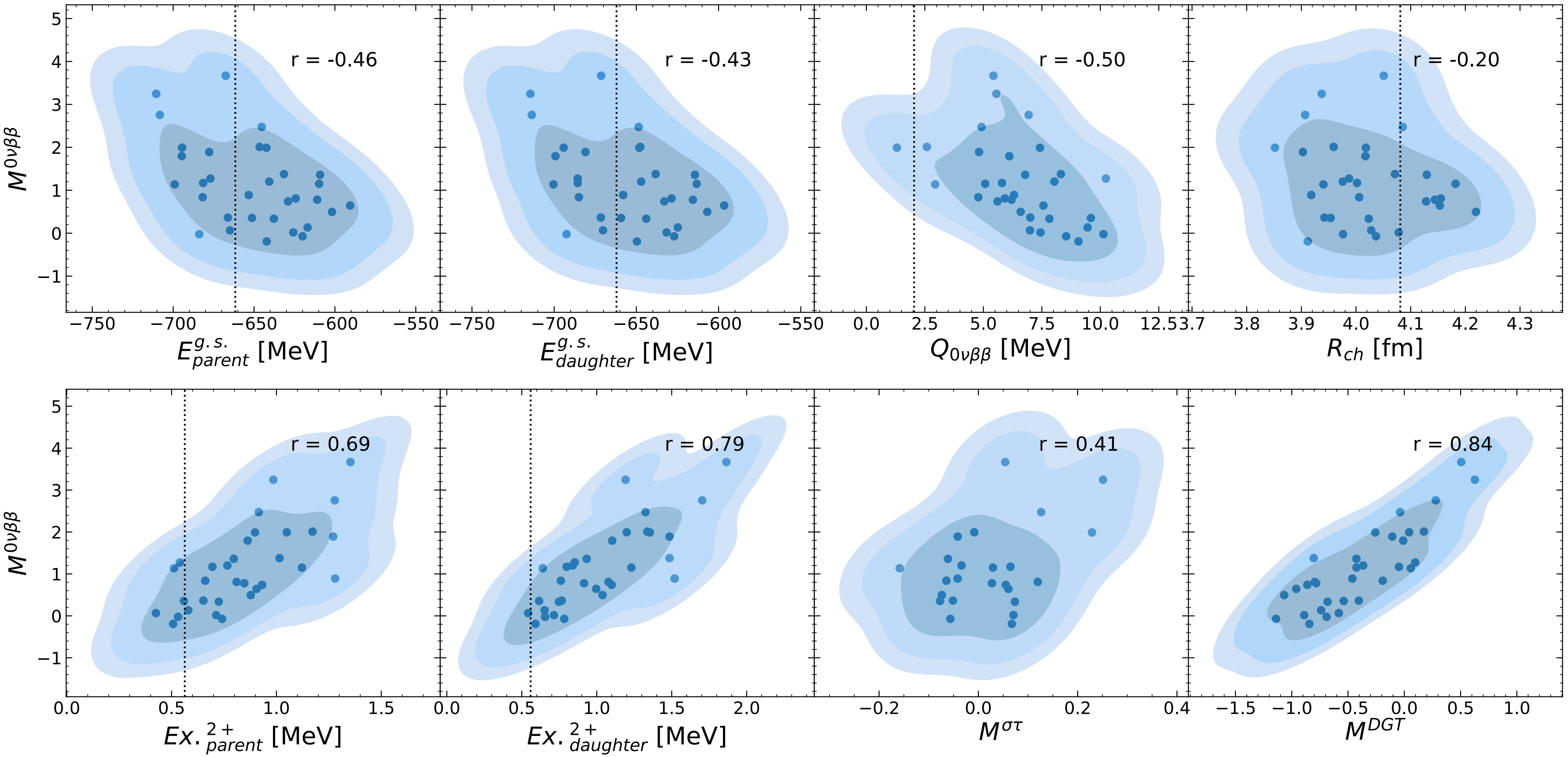
Uncertainty can be split into 3 sources:

- The many-body method (VS-IMSRG)
- The χ-EFT interaction
- The operators

Correlation between observables

In ^{76}Ge:

Belley et al., arXiv:2210.05809



Correlation between observables

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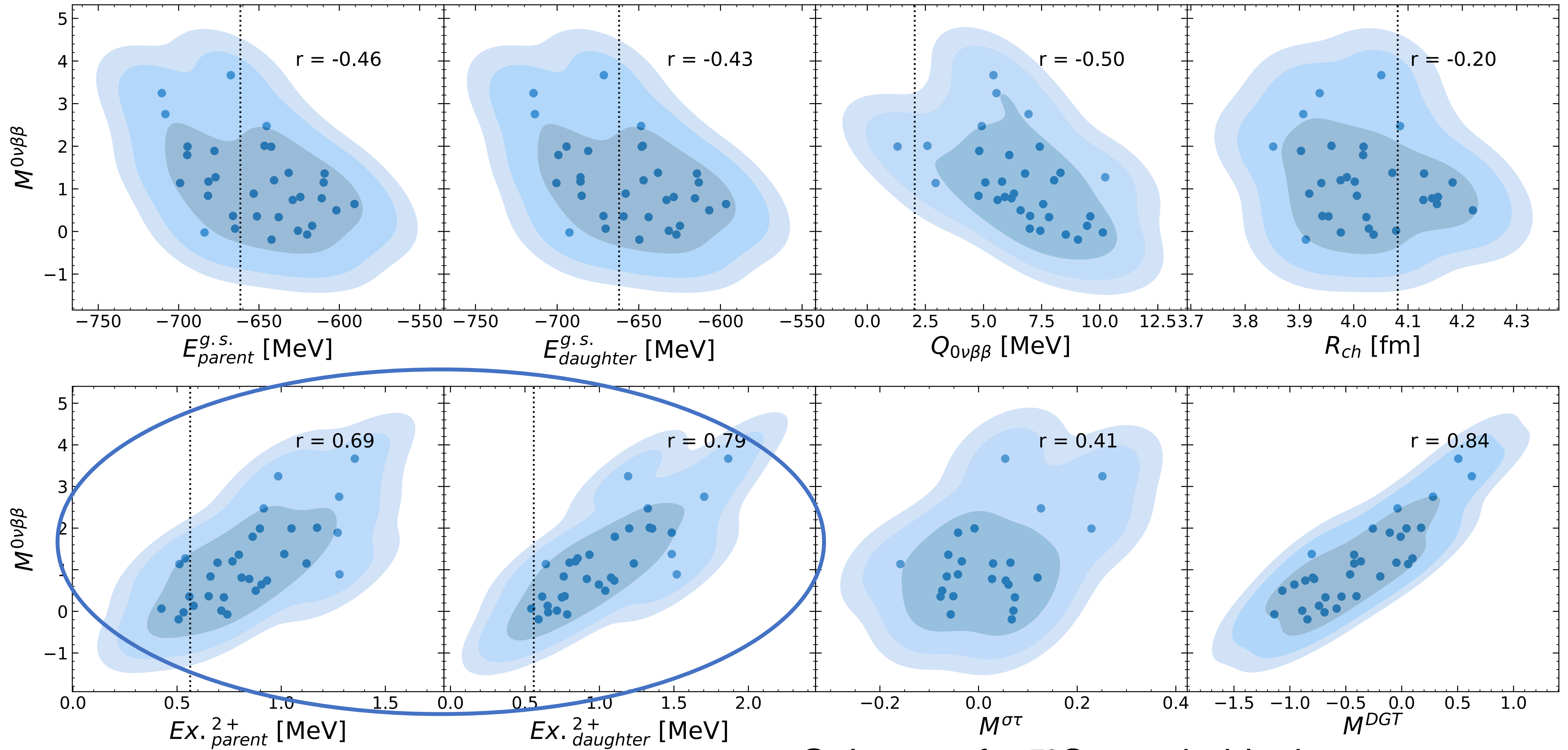
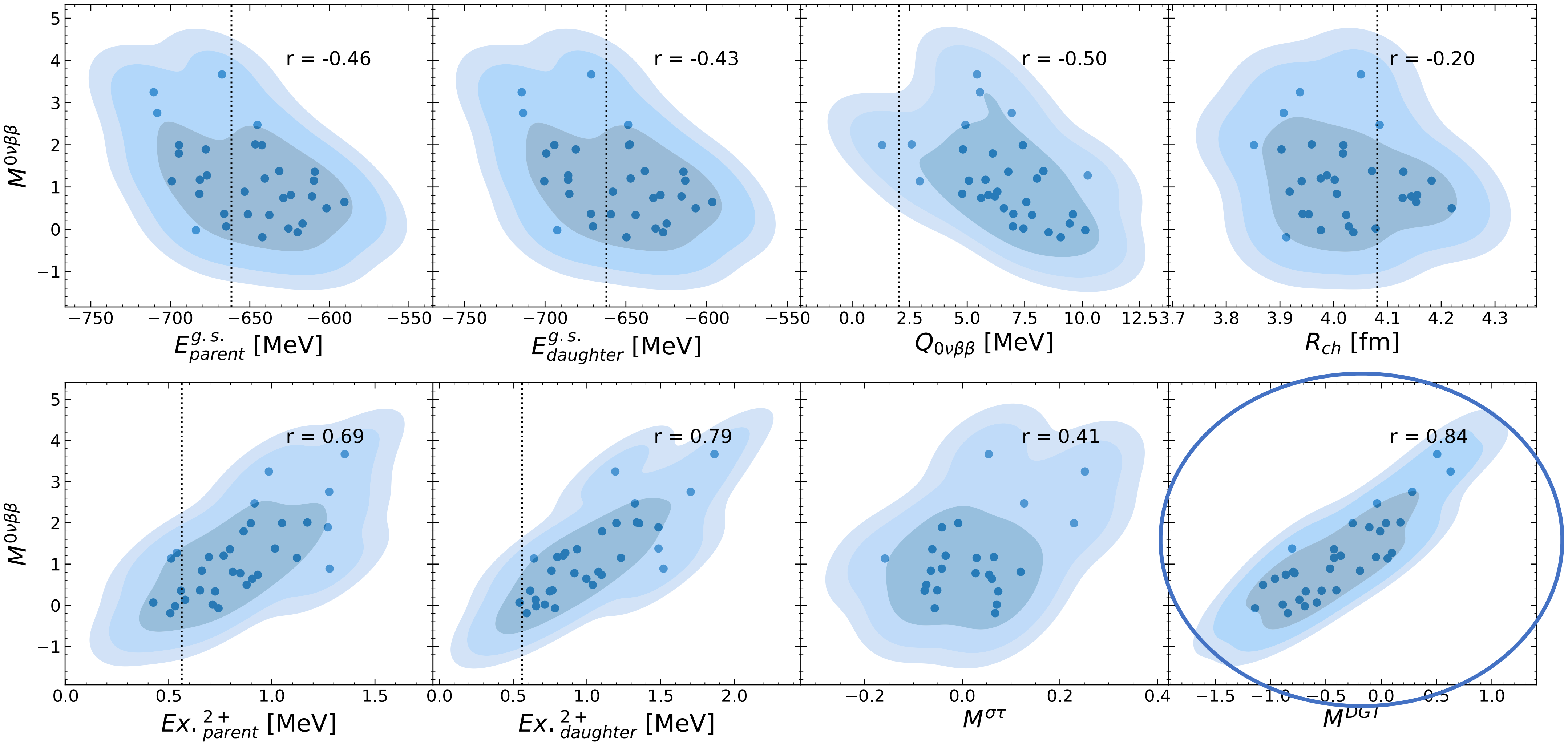


Only seen for ^{76}Ge, probably due to deformed nuclei involved.

Correlation between observables

In ^{76}Ge:

Belley et al., arXiv:2210.05809



Only correlation seen in multiple nuclei is with the unobserved double Gamow-Teller transition NME.

Global sensitivity analysis

Global sensitivity analysis can probe how dependent the final result is to each input but require thousands of samples in order to do so.

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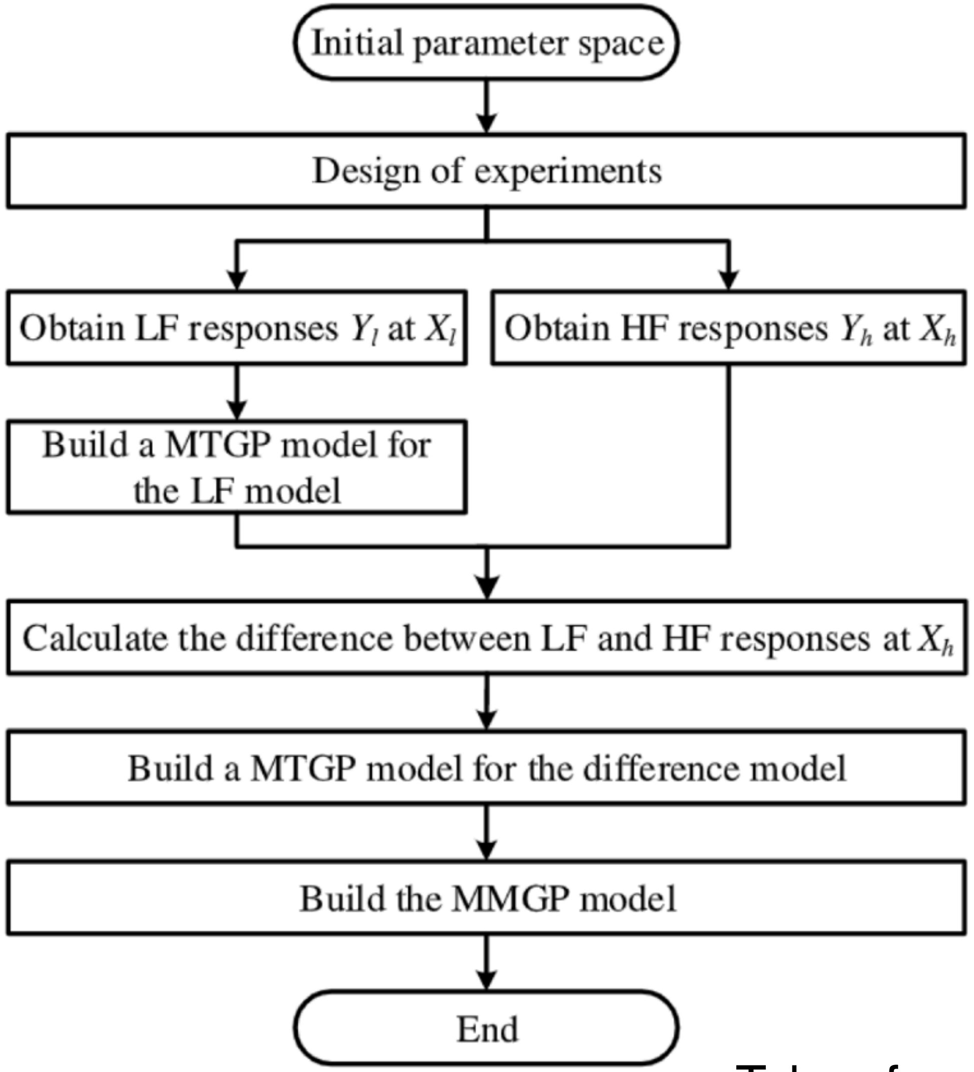
Global sensitivity analysis can probe how dependent the final result is to each input but require thousands of samples in order to do so.

$$\Rightarrow$$

Need emulators to speed up calculations.

Using Gaussian Process as an emulator

- Multi-output Multi-Fidelity Gaussian Process (MMGP) can be used to probe LEC space.
- Multi-Tasks Gaussian Process: Uses multiple correlated outputs from same inputs by defining the kernel as $k_{inputs} \otimes k_{outputs}$. This allows us to increase the number of data points without needing to do more expansive calculations.
- Multi-Fidelity Gaussian Process: Uses few data points of high fidelity (full IMSRG calculations) and many data points of low fidelity (e.g. Hartree-Fock results, lower e_{max}). The difference function is fitted by a Gaussian process in order to predict the value of full calculations using the low fidelity data points.

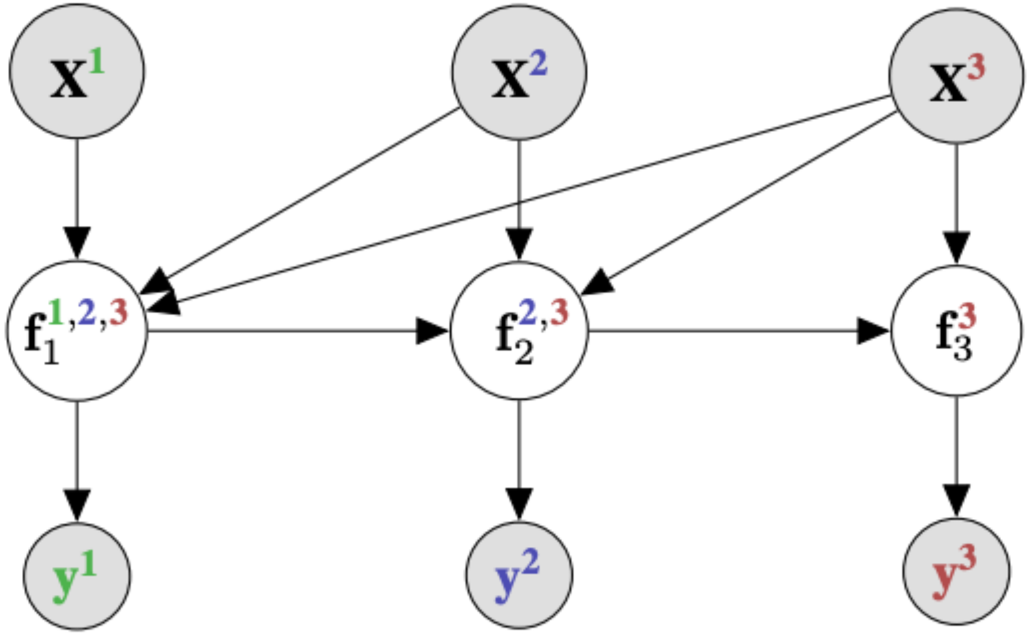


Taken from [1].

[1] Q. Lin, J. Hu, Q. Zhou, Y. Cheng, Z. Hu, I. Couckuyt, and T. Dhaene, Knowledge-Based Systems 227, 107151 (2021).

The MM-DGP algorithm

- When the relation between low-fidelity and high-fidelity data is complicated, the simple multi-fidelity approach does not produce good results.
- Deep gaussian process [1] link multiple gaussian processes inside a neural network to improve results.
- This can be used to model the difference function between the low-fidelity and high-fidelity by including outputs of the previous fidelity as an input of higher fidelity.
- This was developed for single-output gaussian processes and we have adapted it for multi-output case, creating the MM-DGP: Multi-output Multi-fidelity Deep Gaussian Process.
- Even if we use the same number of low- and high-fidelity data, using multiple-fidelities still improves the fit!



Taken from [1].

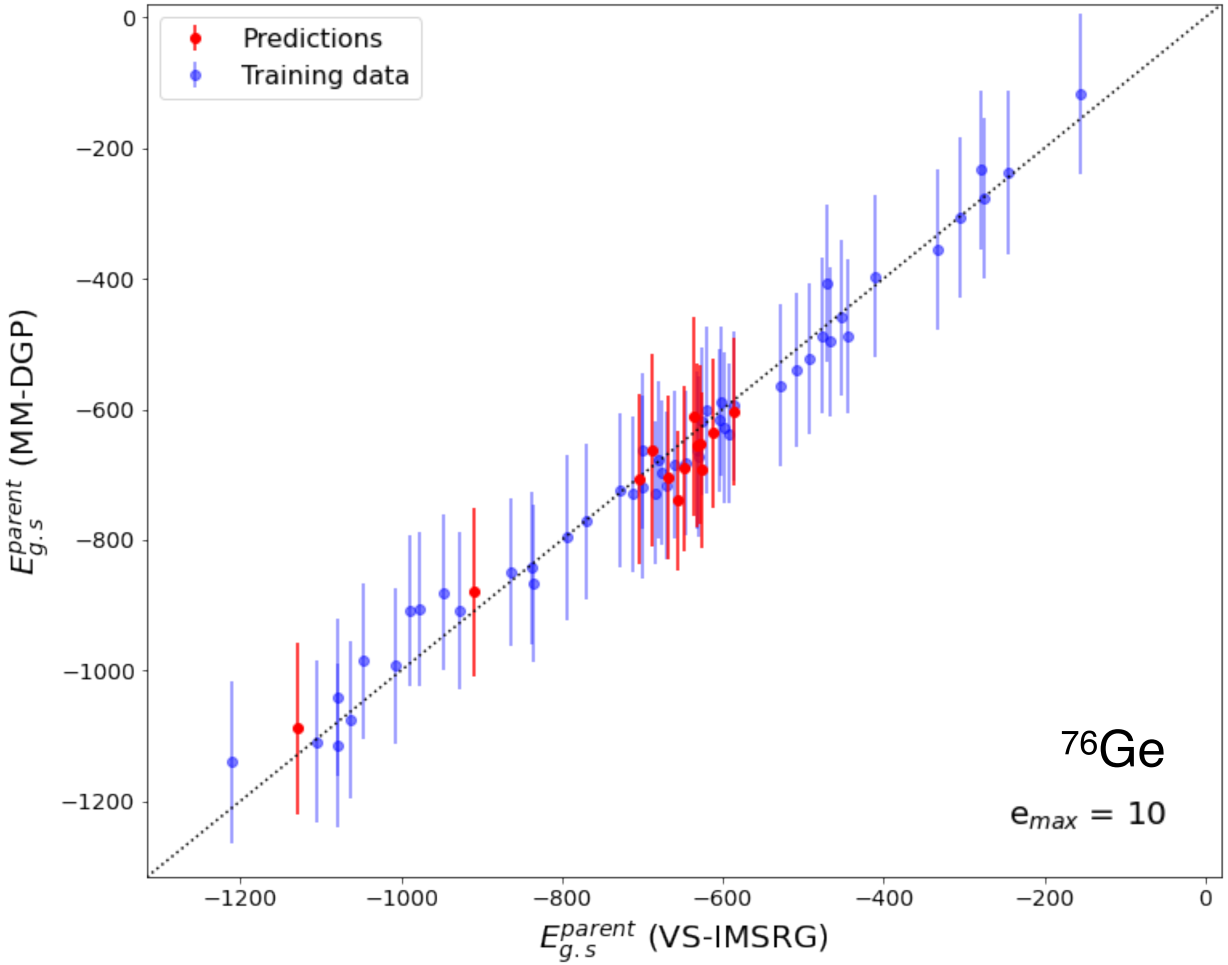
[1] Kurt Cutajar, Mark Pullin, Andreas Damianou, Neil Lawrence, Javier González arXiv:1903.07320 (2021).

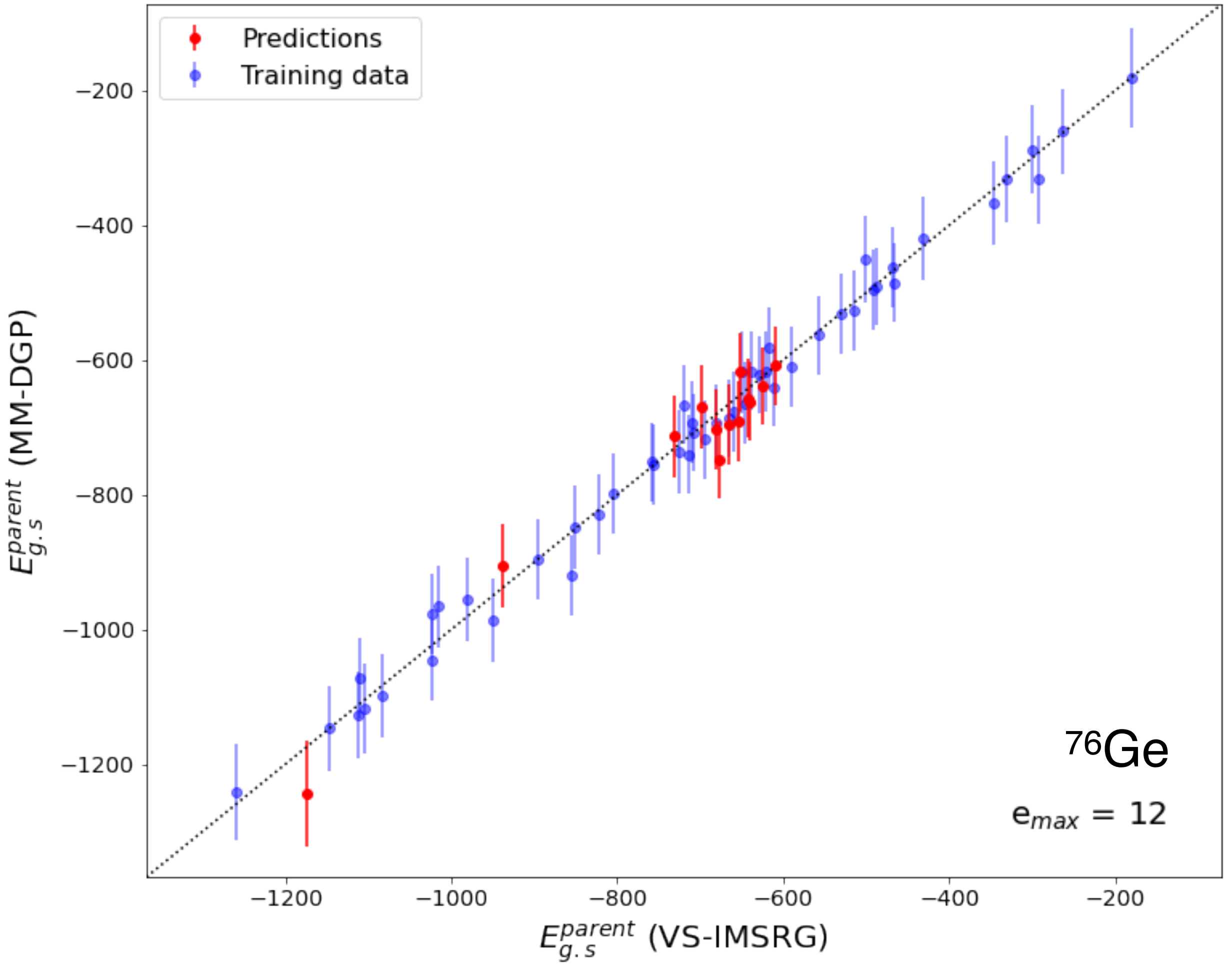
TRIUMF

The MM-DGP algorithm: Energies

Using Δ-full chiral EFT interactions at N2LO:

Low-Fidelity → High-Fidelity





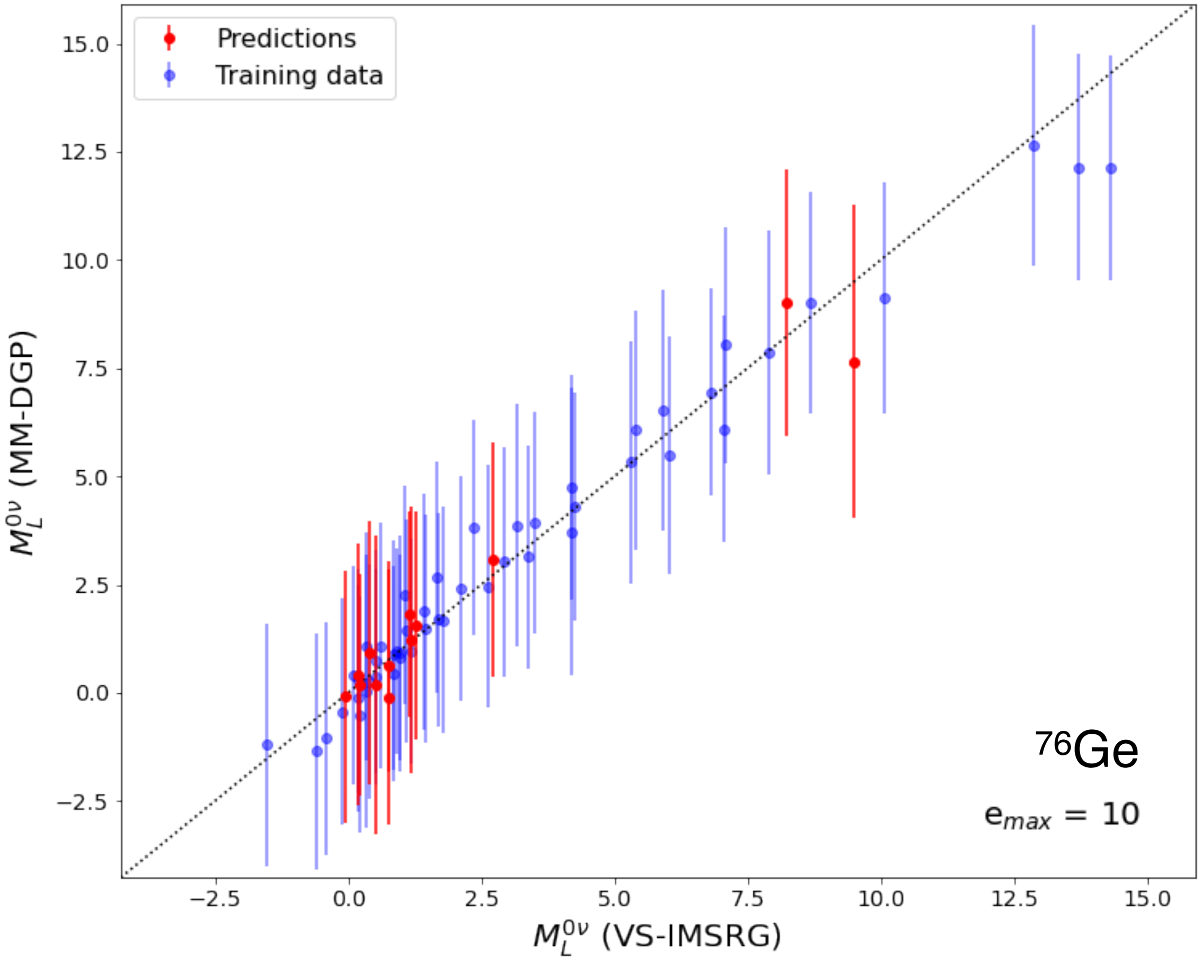
Belley, Pitcher et al. in prep.

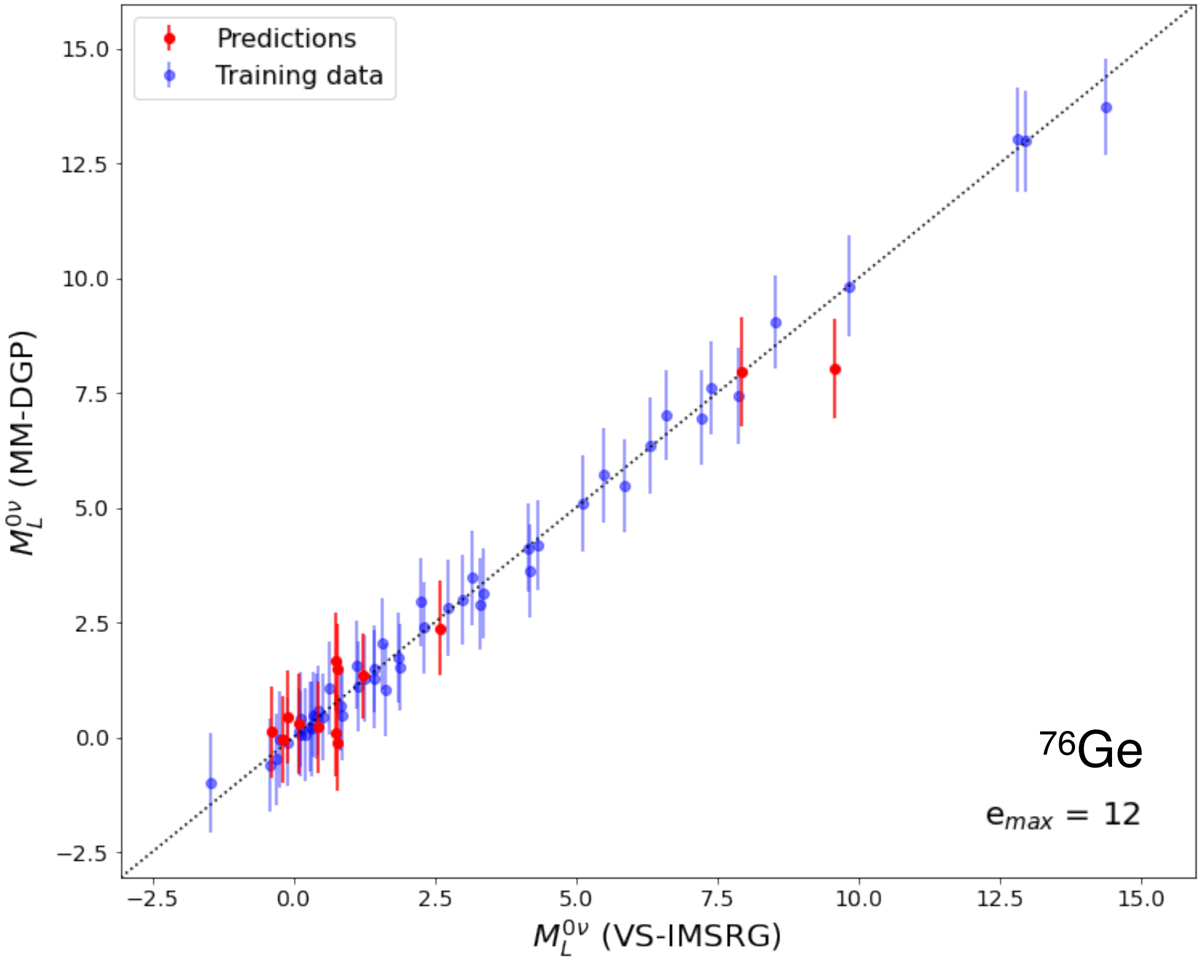
The MM-DGP algorithm: 0νββ NMEs

TRIUMF

Using Δ-full chiral EFT interactions at N2LO:

Low-Fidelity → High-Fidelity

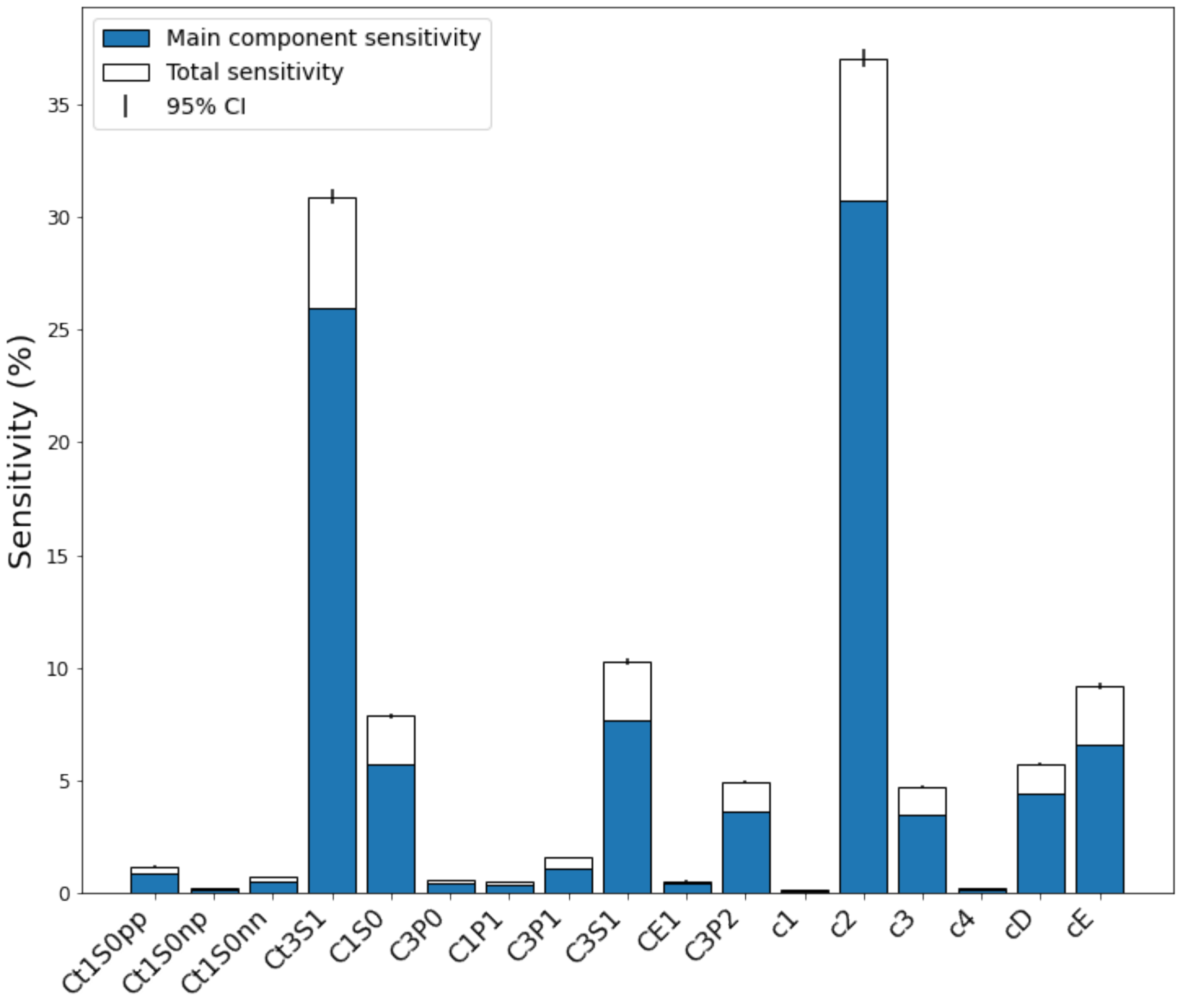




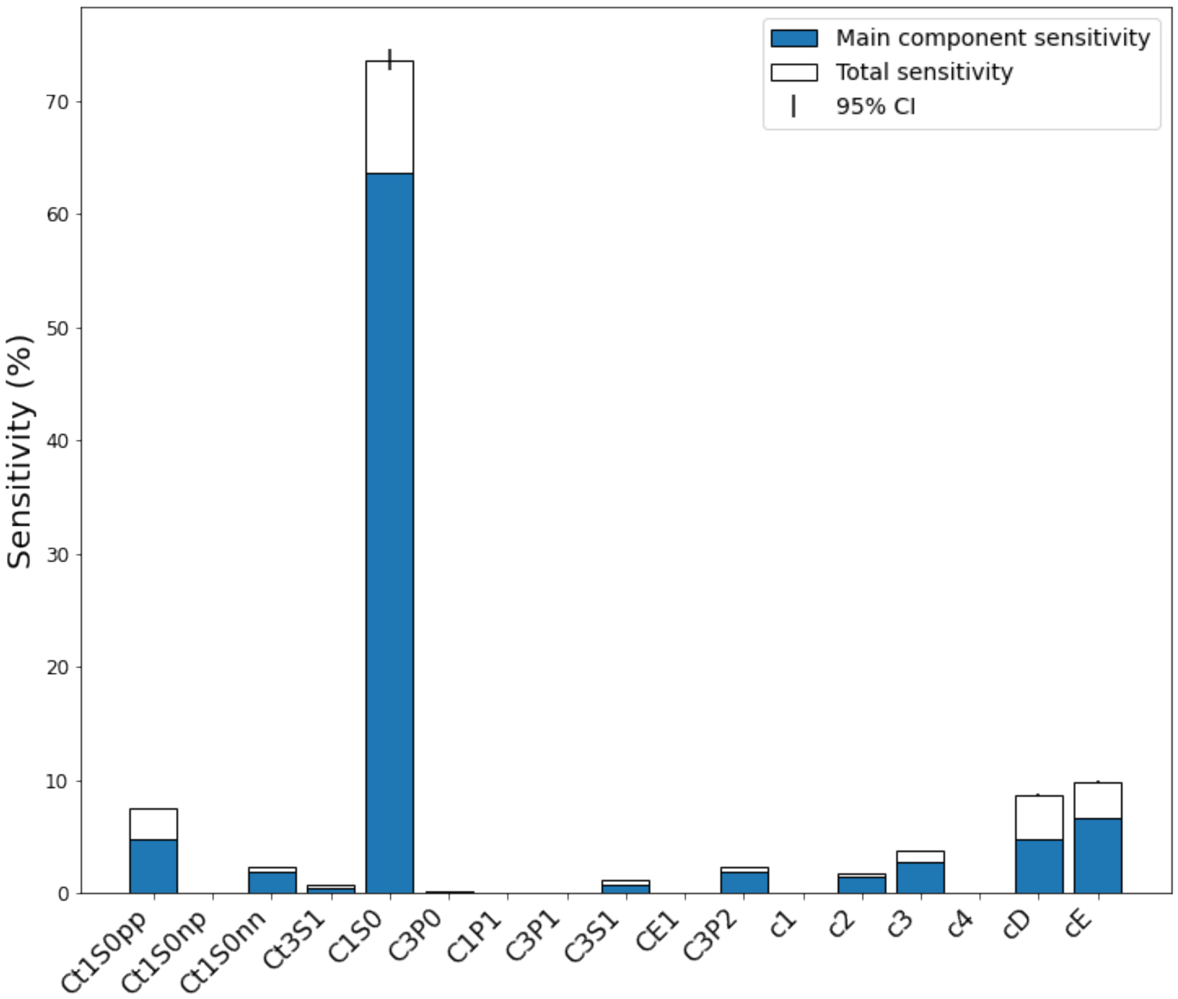
Belley, Pitcher et al. in prep.

The MM-DGP algorithm: GSA

Ground state energies



$M_L^{0\nu}$



Belley, Pitcher et al. in prep.

TRIUMF

Summary…

1) Computed first ever ab initio NMEs of isotopes of experimental interest, which is a first step towards computing NME with reliable theoretical uncertainties.

2) Computed NME with multiple interactions for ^{48}Ca, ^{76}Ge, ^{82}Se, ^{130}Te and ^{136}Xe.

3) Study of effect of the contact term on the NMEs.

4) Studied correlations between multiple operators using a wide range of interactions.

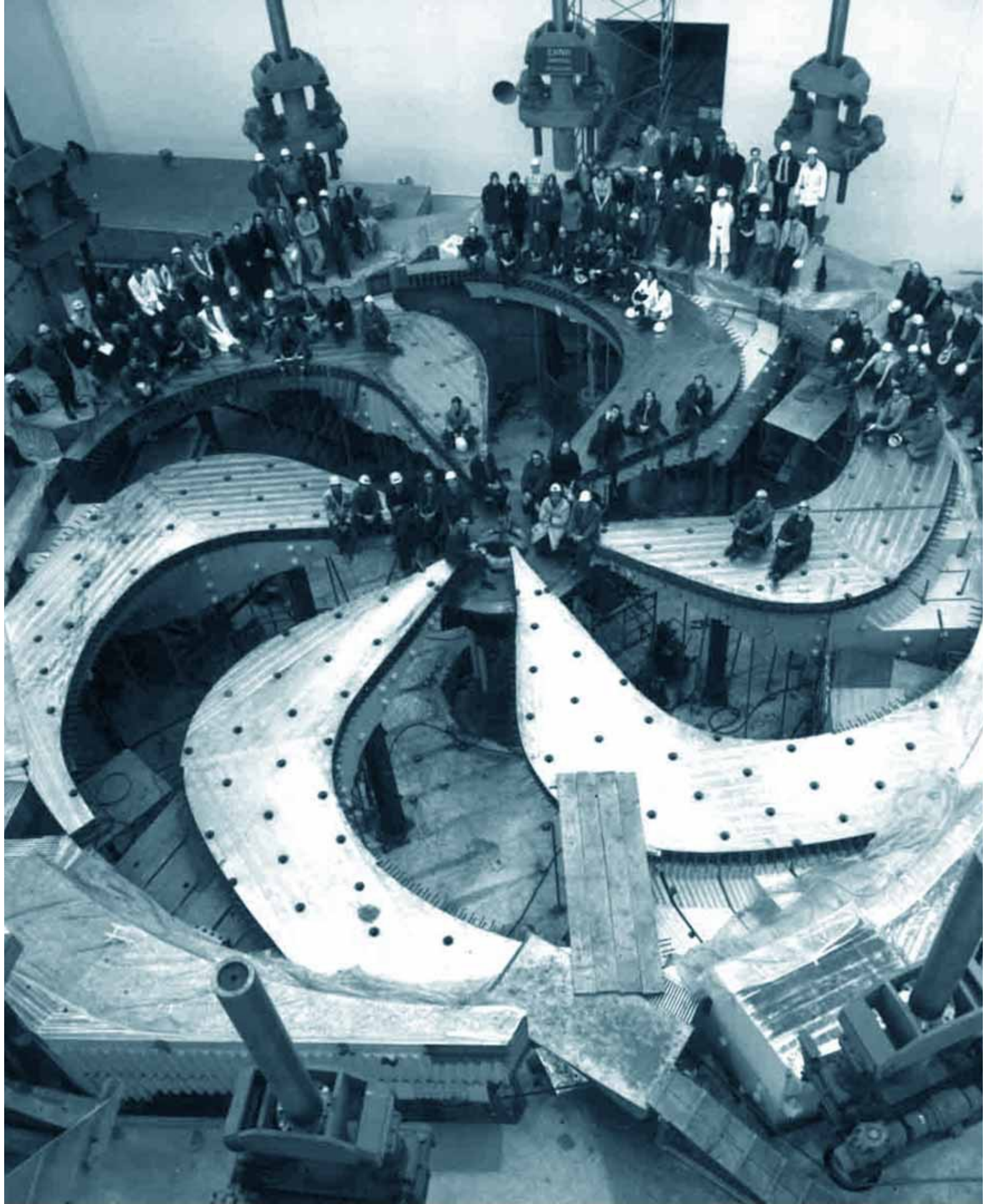
5) Developed an emulator for the VS-IMSRG based on Gaussian processes.

… and outlook

1) Include finite momentum 2-body currents and other higher order effects.

2) Large scale ab initio uncertainty analysis with other methods for “final” NMEs.

3) Study other exotic mechanism proposed for $0\nu\beta\beta$.



Discovery, accelerated

TRIUMF

Questions?

abelley@triumf.ca

Discovery,
accelerated

Long-Range Matrix Elements

$$M_L^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}$$

Long-Range Matrix Elements

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(under closure approximation)

$$M_\alpha^{0\nu} = \langle 0_f^+ | V_\alpha(\boldsymbol{q}) S_\alpha(\boldsymbol{q}) \tau_1^+ \tau_2^+ | 0_i^+ \rangle$$

Long-Range Matrix Elements

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$$M_\alpha^{0\nu} = \langle 0_f^+ | V_\alpha(\boldsymbol{q}) S_\alpha(\boldsymbol{q}) \tau_1^+ \tau_2^+ | 0_i^+ \rangle$$

Scalar potential

$$V_\alpha(q) = \frac{R_{Nucl}}{2\pi^2} \frac{h_\alpha(q)}{q(q + E_{cl})}$$

Long-Range Matrix Elements

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$$V_\alpha(q) = \frac{R_{Nucl}}{2\pi^2} \frac{h_\alpha(q)}{q(q+E_{cl})}$$

E_{cl} → Closure energy

Long-Range Matrix Elements

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Neutrino Potential

$$h_F(q) = \frac{g_V^2(q)}{g_V^2}$$

$$h_{GT}(q) = \frac{1}{g_A^2}\left[g_A^2(q) - \frac{g_A(q)g_P(q)q^2}{3m_N} + \frac{g_P^2(q)q^4}{12m_N^2} + \frac{g_M^2(q)q^2}{6m_N^2}\right]$$

$$h_T(q) = \frac{1}{g_A^2}\left[\frac{g_A(q)g_P(q)q^2}{3m_N} - \frac{g_P^2(q)q^4}{12m_N^2} + \frac{g_M^2(q)q^2}{12m_N^2}\right].$$

Long-Range Matrix Elements

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$$M_\alpha^{0\nu} = \langle 0_f^+ | V_\alpha(\boldsymbol{q}) S_\alpha(\boldsymbol{q}) \tau_1^+ \tau_2^+ | 0_i^+ \rangle$$

Operator acting on spin

$$V_\alpha(q) = \frac{R_{Nucl}}{2\pi^2} \frac{h_\alpha(q)}{q(q+E_{cl})}$$

$$h_F(q) = \frac{g_V^2(q)}{g_V^2}$$

$$h_{GT}(q) = \frac{1}{g_A^2}\left[g_A^2(q) - \frac{g_A(q)g_P(q)q^2}{3m_N} + \frac{g_P^2(q)q^4}{12m_N^2} + \frac{g_M^2(q)q^2}{6m_N^2}\right]$$

$$h_T(q) = \frac{1}{g_A^2}\left[\frac{g_A(q)g_P(q)q^2}{3m_N} - \frac{g_P^2(q)q^4}{12m_N^2} + \frac{g_M^2(q)q^2}{12m_N^2}\right].$$

$$S_F = 1$$

$$S_{GT} = \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}$$

$$S_T = -3[(\boldsymbol{\sigma_1} \cdot \hat{\boldsymbol{q}})(\boldsymbol{\sigma_2} \cdot \hat{\boldsymbol{q}}) - (\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2})].$$

Long-Range Matrix Elements

$$M_L^{0\nu} = M_{GT}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_F^{0\nu} + M_T^{0\nu}$$

$$M_\alpha^{0\nu} = \langle 0_f^+ | V_\alpha(\boldsymbol{q}) S_\alpha(\boldsymbol{q}) \tau_1^+ \tau_2^+ | 0_i^+ \rangle$$

$$V_\alpha(q) = \frac{R_{Nucl}}{2\pi^2} \frac{h_\alpha(q)}{q(q + E_{cl})}$$

$$h_F(q) = \frac{g_V^2(q)}{g_V^2}$$

$$h_{GT}(q) = \frac{1}{g_A^2}\left[g_A^2(q) - \frac{g_A(q)g_P(q)q^2}{3m_N} + \frac{g_P^2(q)q^4}{12m_N^2} + \frac{g_M^2(q)q^2}{6m_N^2}\right]$$

$$h_T(q) = \frac{1}{g_A^2}\left[\frac{g_A(q)g_P(q)q^2}{3m_N} - \frac{g_P^2(q)q^4}{12m_N^2} + \frac{g_M^2(q)q^2}{12m_N^2}\right].$$

$$S_F = 1$$

$$S_{GT} = \boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2}$$

$$S_T = -3[(\boldsymbol{\sigma_1} \cdot \hat{\boldsymbol{q}})(\boldsymbol{\sigma_2} \cdot \hat{\boldsymbol{q}}) - (\boldsymbol{\sigma_1} \cdot \boldsymbol{\sigma_2})].$$

Short-Range Matrix Elements

$$M_S^{0\nu} = -2 g_{\nu\nu} M_{CT}^{0\nu}$$

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Contact operator regularized with non-local regulator matching the nuclear interaction used:

$$M_{CT}^{0\nu} = \langle 0_f^+ | \frac{R_{Nucl}}{8\pi^3} \left(\frac{m_N g_A^2}{4f_\pi^2} \right)^2 \exp(-(\frac{p}{\Lambda_{int}})^{2n_{int}}) \exp(-(\frac{p'}{\Lambda_{int}})^{2n_{int}}) | 0_i^+ \rangle$$

The IMSRG:NO2B Hamiltonian

Considering the nuclear Hamiltonian:

$$\hat{H} = \left(1 - \frac{1}{\hat{A}}\right) \sum_i \frac{\hat{\boldsymbol{p}}_i^2}{2m} + \frac{1}{\hat{A}}\left(- \frac{1}{m} \sum_{i<j} \hat{\boldsymbol{p}}_i \hat{\boldsymbol{p}}_j \right) + \hat{V}^{[2]} + \hat{V}^{[3]}$$

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One-body kinetic energy $\hat{T}^{[1]}$

The IMSRG:NO2B Hamiltonian

Considering the nuclear Hamiltonian:

Two-body kinetic energy $\hat{T}^{[2]}$

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The IMSRG:NO2B Approximation

Considering the nuclear Hamiltonian:

3N forces

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We can rewrite the Hamiltonian in terms of normal ordered operators as:

$$\hat{H} = E + \sum_{ij} f_{ij} \{a_i^\dagger a_j\} + \frac{1}{4} \sum_{ijkl} \Gamma_{ijkl} \{a_i^\dagger a_j^\dagger a_l a_k\} + \frac{1}{36} \sum_{ijklmn} W_{ijklmn} \{a_i^\dagger a_j^\dagger a_k^\dagger a_n a_m a_l\}$$

The IMSRG:NO2B Approximation

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The IMSRG:NO2B Approximation

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$$f_{ij} = \left(1 - \frac{1}{A}\right)\langle i|\hat{T}^{[1]}|j\rangle + \sum_a \langle ia|\frac{1}{A}\hat{T}^{[2]} + \hat{V}^{[2]}|ja\rangle n_a + \frac{1}{2}\sum_{abc}\langle iab|\hat{V}^{[3]}|jab\rangle n_a n_b$$

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$$\Gamma_{ijkl} = \langle ij|\frac{1}{A}\hat{T}^{[2]} + \hat{V}^{[2]}|kl\rangle + \sum_a \langle ija|\hat{V}^{[3]}|kla\rangle n_a$$

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The VS-IMSRG

Choose generator in order to decouple the valence-space from the excluded space:

$$\eta = \sum_{ij} \eta_{ij}\{a_i^\dagger a_j\} + \sum_{ijkl} \eta_{ijkl}\{a_i^\dagger a_j^\dagger a_l a_k\}$$

for $ij \in [pc, ov]$ and $ijkl \in [pp'cc', pp'vc, opvv']$ for c in the core, v in the valence-space, o outside the valence-space and p not in the core.

$$\eta_{ij} = \frac{1}{2}\arctan\left(\frac{2f_{ij}}{f_{ii} - f_{jj} + \Gamma_{ijij}}\right)$$

$$\eta_{ijkl} = \frac{1}{2}\arctan\left(\frac{2\Gamma_{ijkl}}{f_{ii} + f_{jj} - f_{kk} - f_{ll} + \Gamma_{ijij} + \Gamma_{klkl} - \Gamma_{ikik} - \Gamma_{ilil} - \Gamma_{jkjk} - \Gamma_{jljl}}\right)$$