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Ab initio studies on ordinary muon capture

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PAINT 2023 Workshop, TRIUMF, Vancouver

Arthur B. McDonald Canadian Astroparticle Physics Research Institute

Universitat de Barcelona

ICCUB

Suomen Kulttuurirahasto

Discovery, accelerated

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Outline

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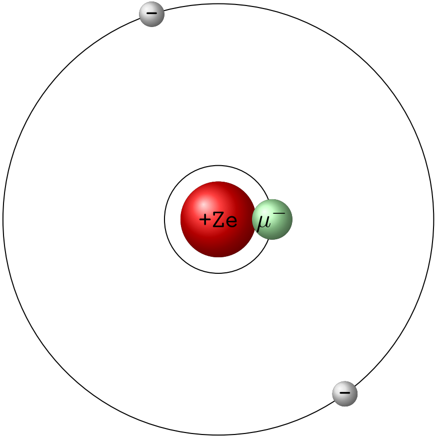
Ordinary Muon Capture

$$\mu^- + {}^A_Z\mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu + {}^{\;\;A}_{Z-1}\mathrm{Y}(J_f^{\pi_f})$$

- A negatively charged muon can replace an electron in an atom, forming a *muonic atom*



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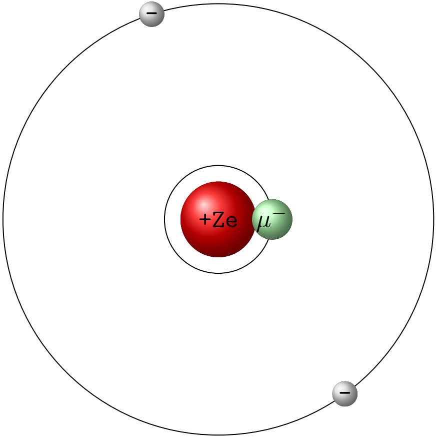
  - Eventually bound on **the $1s_{1/2}$ orbit**



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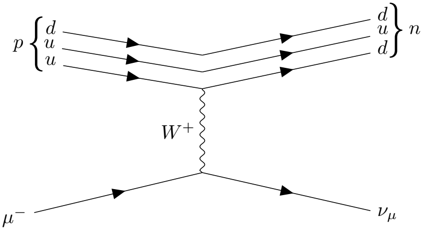

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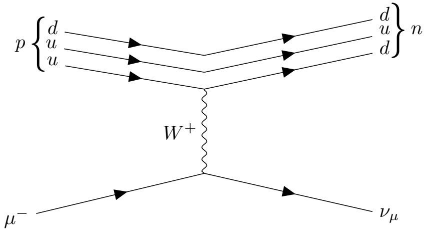
Ordinary = non-radiative

$$\left(\begin{array}{c} \text{Radiative muon capture (RMC):} \\ \mu^- +_Z^A \mathrm{X}(J_i^{\pi_i}) \rightarrow \nu_\mu +_{Z-1}^{A} \mathrm{Y}(J_f^{\pi_f}) + \gamma \end{array} \right)$$



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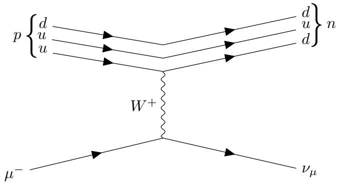
Ordinary Muon Capture (OMC) vs. $0\nu\beta\beta$



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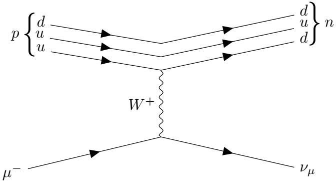

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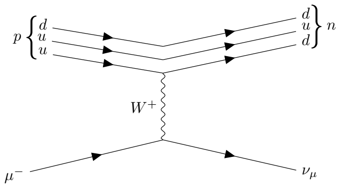

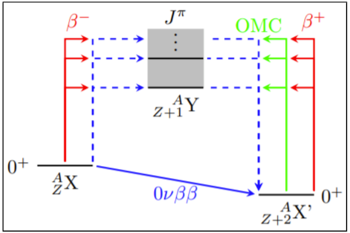

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- Large m_μ allows **transitions to all J^π states** up to high energies

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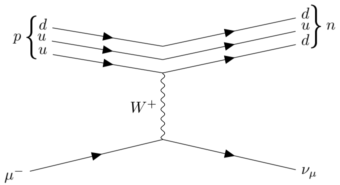

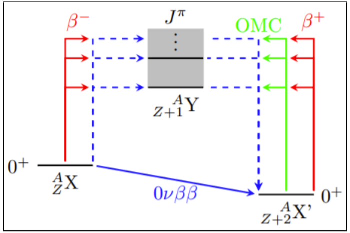

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- **Both the axial vector coupling g_A and the pseudoscalar coupling g_P involved**

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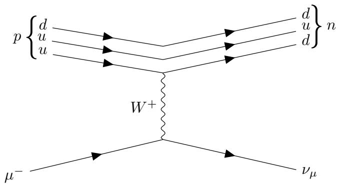

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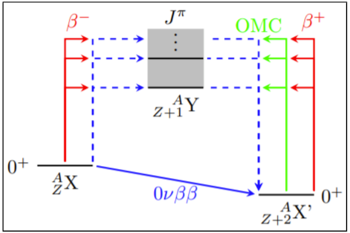

- Weak interaction process with momentum transfer $q \approx 100\ \mathrm{MeV}/c^2$
- Large m_μ allows **transitions to all J^π states** up to high energies
- **Both the axial vector coupling g_A and the pseudoscalar coupling g_P involved**
  - $\rightarrow$ Similar to $0\nu\beta\beta$ decay!

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g_A Quenching at High Momentum Exchange?

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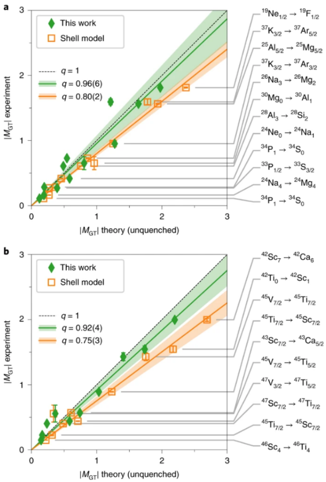
P. Gysbers *et al.*, *Nature Phys.* **15**, 428 (2019)



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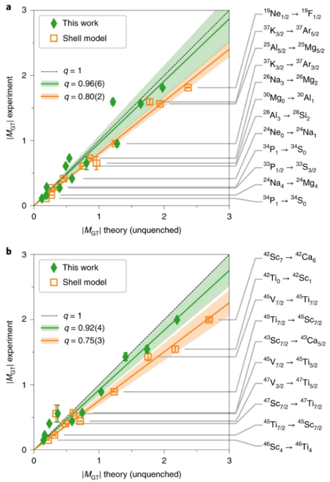

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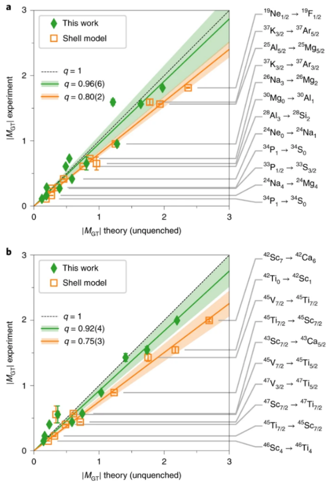

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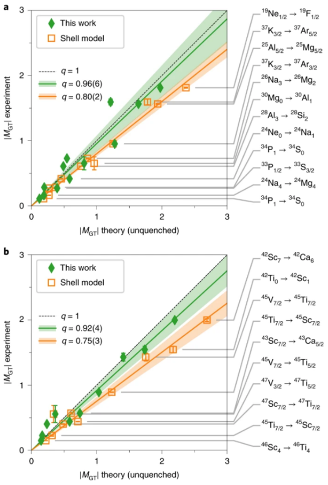
  - **OMC could provide a hint!**
- In principle, one could also access the pseudoscalar coupling g_P



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Muon-Capture Theory

- Interaction Hamiltonian $\rightarrow$ capture rate:

$$W(J_i \rightarrow J_f) = \frac{2J_f+1}{2J_i+1}\left(1-\frac{q}{m_\mu + AM}\right) q^2 \sum_{\kappa u} \left| g_{\mathrm{V}} M_{\mathrm{V}}(\kappa,u) + g_{\mathrm{A}} M_{\mathrm{A}}(\kappa,u) + g_{\mathrm{P}} M_{\mathrm{P}}(\kappa,u) \right|^2$$

PHYSICAL REVIEW VOLUME 118, NUMBER 2 APRIL 15, 1960

Theory of Allowed and Forbidden Transitions in Muon Capture Reactions*

MASATO MORITA
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AND

AKIHIKO FUJII†
Brookhaven National Laboratory, Upton, Long Island, New York

(Received November 9, 1959)

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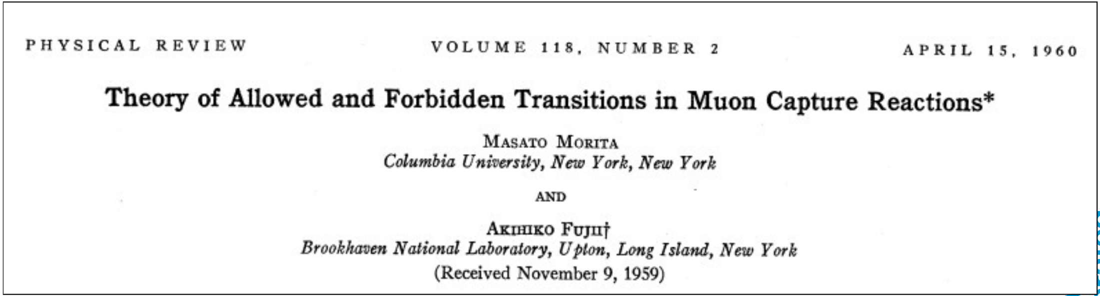
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- Use **realistic bound-muon wave functions**
- Add the effect of **two-body currents**

Bound-Muon Wave Functions

- Expand the muon wave function in terms of spherical spinors

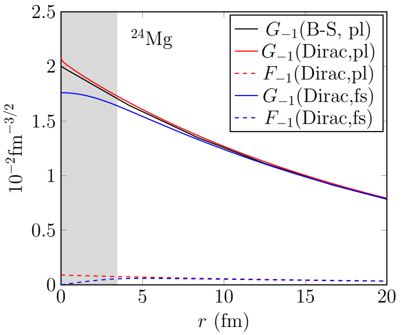
$$\psi_\mu(\kappa,\mu;\mathbf{r}) = \psi^{(\mu)}_{\kappa\mu} = \begin{bmatrix} -iF_\kappa(r)\chi_{-\kappa\mu} \\ G_\kappa(r)\chi_{\kappa\mu} \end{bmatrix},$$

where $\kappa = -j(j+1) + l(l+1) - \frac{1}{4}$
($\boldsymbol{\kappa = -1}$ for the $\mathbf{1}s_{1/2}$ orbit)

B-S = Bethe-Salpeter: $G_{-1} = 2(\alpha Z m'_\mu)^{\frac{3}{2}} e^{-\alpha Z m'_\mu r}$

pl = pointlike

fs = finite size nucleus



*LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen, Phys. Rev. C **107**, 014327 (2023)*

Discovery, accelerated

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($\boldsymbol{\kappa = -1}$ for the $\mathbf{1s_{1/2}}$ orbit)

- Solve the Dirac equations in the Coulomb $V(r)$:

$$\begin{cases} \frac{\mathrm{d}}{\mathrm{d}r}G_{-1} + \frac{1}{r}G_{-1} = \frac{1}{\hbar c}(mc^2 - E + V(r))F_{-1} \\ \frac{\mathrm{d}}{\mathrm{d}r}F_{-1} - \frac{1}{r}F_{-1} = \frac{1}{\hbar c}(mc^2 + E - V(r))G_{-1} \end{cases}$$

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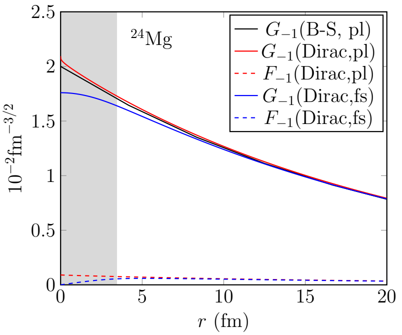

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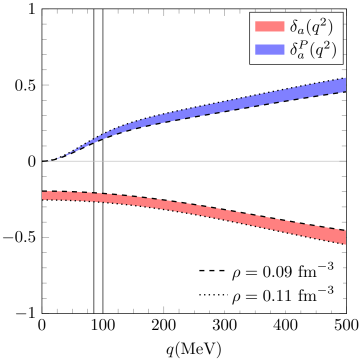
Hadronic Two-Body Currents (2BCs)

- The effect of the two-body currents can be approximated by

$$\begin{cases} g_A(q^2) \to g_A(q^2) + \delta_a(q^2), \\ g_P(q^2) \to \left(1 - \frac{q^2+m_\pi^2}{q^2}\delta_a^P(q^2)\right) g_P \end{cases}$$

Hoferichter, Menéndez Schwenk, *Phys. Rev. D* **102**,074018 (2020)



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Muon-Capture Studies at PSI, Switzerland

MONUMENT (OMC4DBD) collaboration aiming to measure:

- Partial muon-capture rates for OMC on $^{\mathbf{24}}$**Mg**, ^{32}S and ^{56}Fe

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- Partial muon-capture rates for OMC on $\mathbf{^{24}Mg}$, ^{32}S and ^{56}Fe
- Muon-capture strength functions in $\beta\beta$-decay triplets
- Potentially partial capture rates for $\mathbf{^{12}C}$, ^{13}C, ^{48}Ti

Discovery, accelerated

Outline

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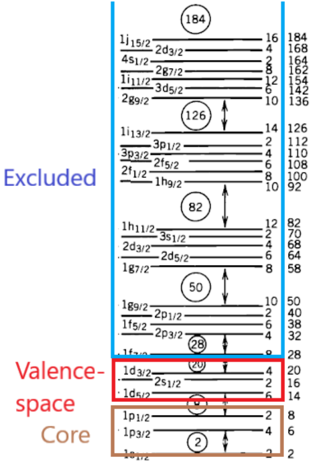
Valence-Space In-Medium Similarity Renormalization Group (VS-IMSRG)

- We choose a Hamiltonian based on the chiral EFT with EM 1.8/2.0 interaction



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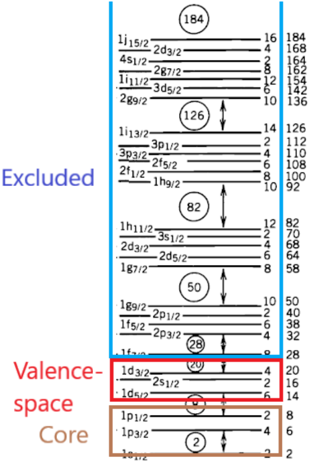
- **Valence-space Hamiltonian and OMC operators decoupled** with a unitary transformation



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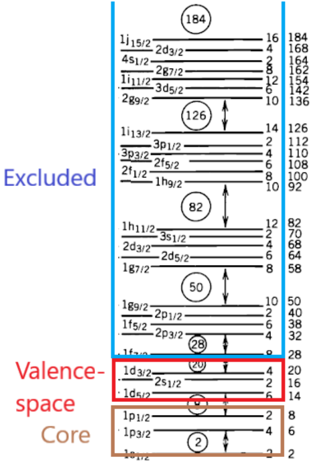

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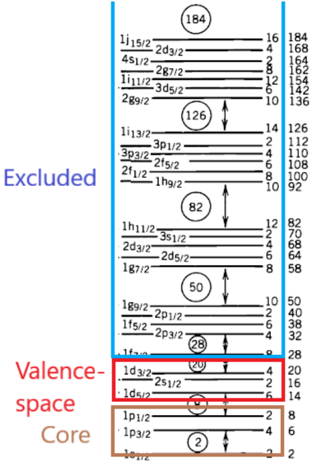

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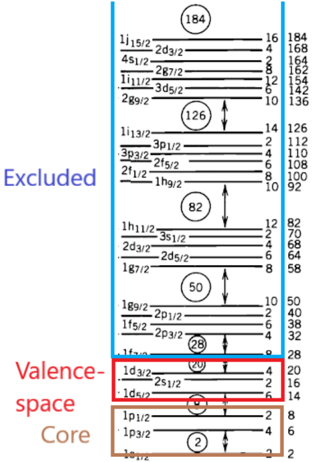
$\rightarrow$ **First case: OMC on ^{24}Mg**



Capture Rates to Low-Lying States in ^{24}Na

J_i^π	E_{exp} (MeV)	Rate (10^3 1/s)				
		Exp.[1]	NSM		IMSRG	
			1bc	1bc+2bc	1bc	1bc+2bc
1_1^+	0.472	(21.0 ± 6.6)	4.0	3.0	22.3	15.2
1_2^+	1.347	17.5 ± 2.3	32.7	21.3	7.7	4.9
Sum(1^+)		38.5 ± 8.9	36.7	24.5	30.0	20.0
2_1^+	0.563	17.5 ± 2.1	1.0	0.7	0.5	0.3
2_2^+	1.341	3.4 ± 0.5	3.1	2.5	1.0	0.9
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*LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen, Phys. Rev. C **107**, 014327 (2023)*

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$E_{\text{max}} = 12$
$E_{3\text{max}} = 24$

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- **1^+ states mixed**

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J_i^π	E_{exp} (MeV)	Rate (10^3 1/s)				
		Exp.[1]	NSM		IMSRG	
			1bc	1bc+2bc	1bc	1bc+2bc
1_1^+	0.472	(21.0 ± 6.6)	4.0	3.0	22.3	15.2
1_2^+	1.347	17.5 ± 2.3	32.7	21.3	7.7	4.9
Sum(1^+)		38.5 ± 8.9	36.7	24.5	30.0	20.0
2_1^+	0.563	17.5 ± 2.1	1.0	0.7	0.5	0.3
2_2^+	1.341	3.4 ± 0.5	3.1	2.5	1.0	0.9
Sum(2^+)		20.9 ± 2.6	4.1	3.2	1.5	1.2

$\hbar\omega = 16$ MeV
$E_{\text{max}} = 12$
$E_{3\text{max}} = 24$

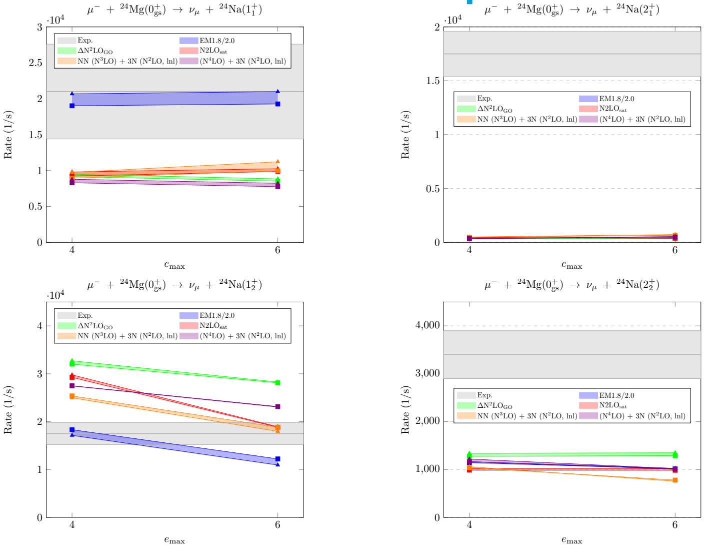
*LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen, Phys. Rev. C **107**, 014327 (2023)*

- **Rate to the lowest two 1^+ states agrees with experiment**
  - The effect of two-body currents may be overestimated
- **1^+ states mixed**
- **Both NSM and VS-IMSRG notably underestimate the rates to 2^+ states**

[1] Gorringe *et al.*, *Phys. Rev. C* **60**, 055501 (1999)

Interaction Dependence

- Rates are sensitive to the interaction

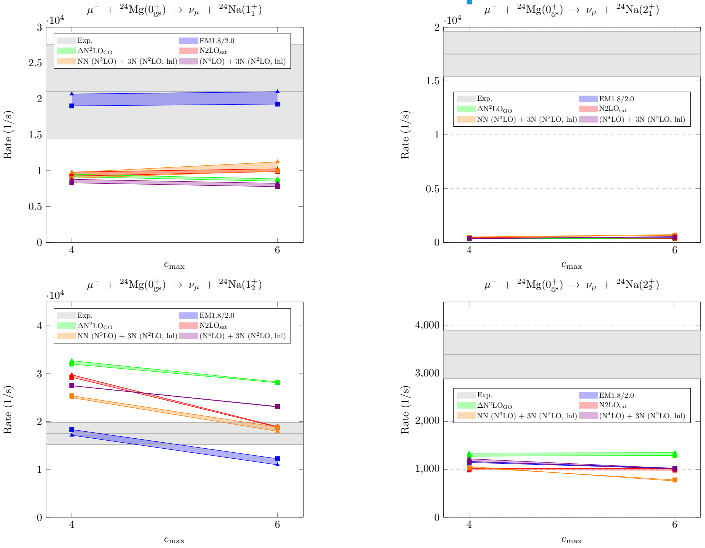


LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen, Phys. Rev. C ***107****, 014327 (2023)*

TRIUMF

Interaction Dependence

- Rates are sensitive to the interaction
- It does not explain the poor agreement with the measured rates to the 2^+ states (on the right)



*LJ, Miyagi, Stroberg, Holt, Kotila, Suhonen, Phys. Rev. C **107**, 014327 (2023)*

Outline

Introduction

VS-IMSRG Study on Muon Capture on ^{24}Mg

No-Core Shell-Model Studies on Muon Capture on Light Nuclei

Summary and Outlook

No-Core Shell Model (NCSM)

- OMC operators and one-body transition densities computed in large harmonic-oscillator (HO) basis

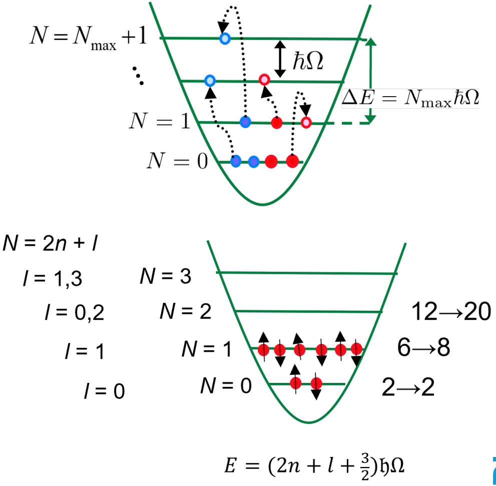


Figure courtesy of P. Navrátil

No-Core Shell Model (NCSM)

- OMC operators and one-body transition densities computed in large harmonic-oscillator (HO) basis
  - HO basis truncated with N_{max}

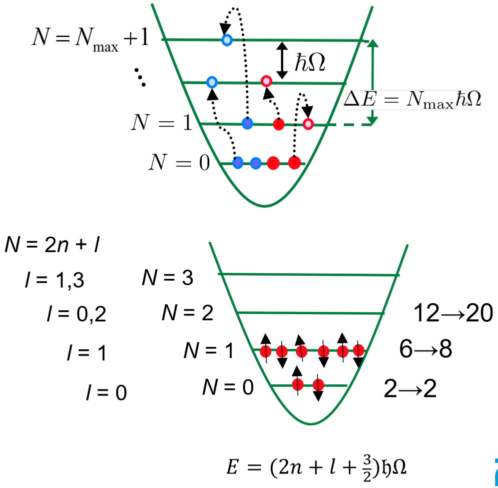


Figure courtesy of P. Navrátil

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- OMC operators and one-body transition densities computed in large harmonic-oscillator (HO) basis
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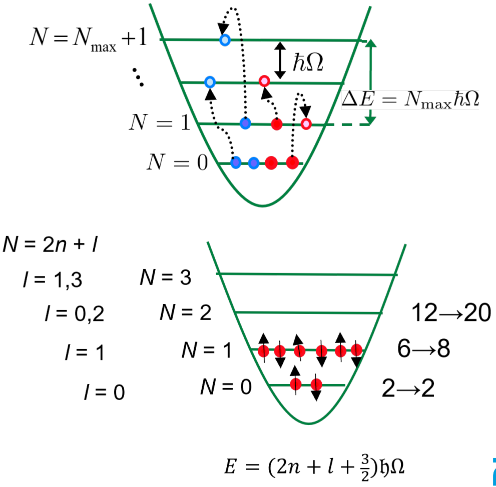

Figure courtesy of P. Navrátil

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    Entem, Machleidt, Nosyk, *Phys. Rev. C* **96**, 024004 (2017) (NN)
    Gysbers *et al.*, *Nature Phys.* **15**, 428 (2019) (3N)

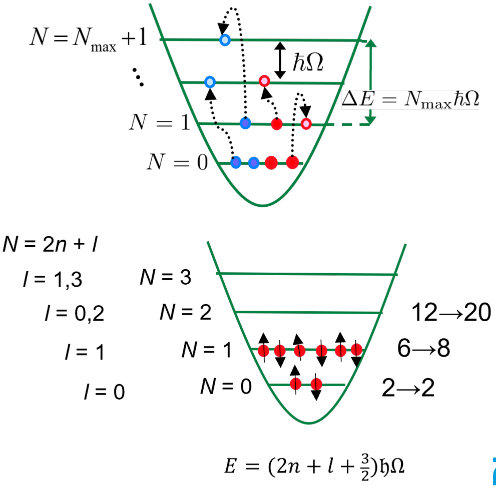

Figure courtesy of P. Navrátil

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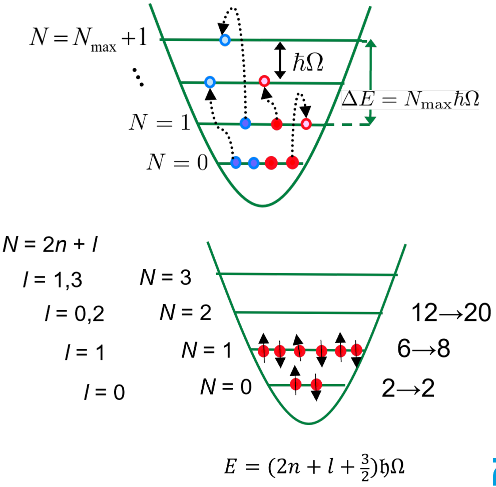

Figure courtesy of P. Navrátil

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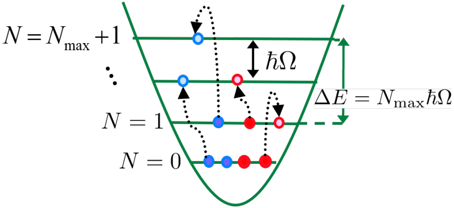

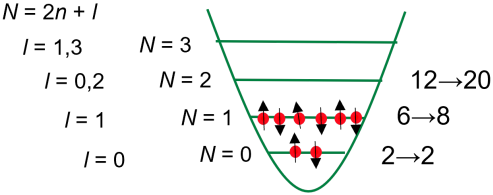

$$E = (2n + l + \tfrac{3}{2})\hbar\Omega$$

Figure courtesy of P. Navrátil

Discovery, accelerated

TRIUMF

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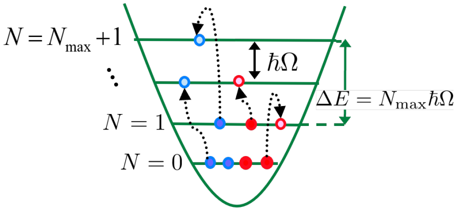
→ **OMC on ^{6}Li, ^{12}C and ^{16}O**



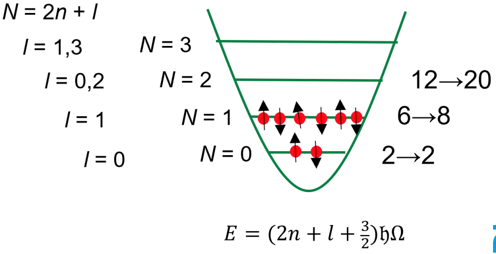
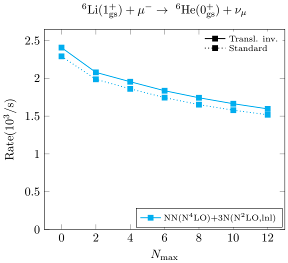


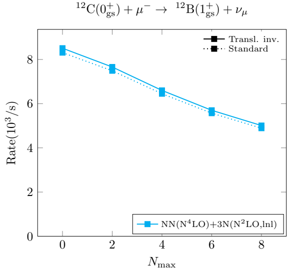
Figure courtesy of P. Navrátil

Discovery, accelerated

Spurious Center-of-Mass Motion

- OMC operators depend on single-particle coordinates $\mathbf{r_s}$ and $\mathbf{p_s}$ w. r. t. center of mass (CM) of the HO potential





LJ, Navrátil, Kotila and Kravvaris, in progress

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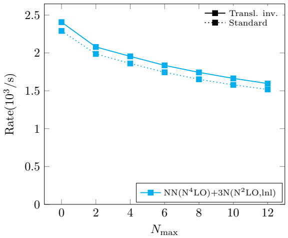
  Navrátil, *Phys. Rev. C* **104**, 064322 (2021)

$$(\Psi_f||\sum_{s=1}^{A}\hat{O}_s(\mathbf{r}_s-\mathbf{R}_{\rm CM},\mathbf{p}_s-\mathbf{P})||\Psi_i)$$
$$=\frac{1}{\sqrt{2J_f+1}}\times\sum_{pnp'n'}(n'||\hat{O}_s\left(-\sqrt{\frac{A-1}{A}}\boldsymbol{\xi}_s,-\sqrt{\frac{A-1}{A}}\boldsymbol{\pi}_s\right)||p')$$
$$\times(M^u)^{-1}_{n'p',np}\frac{1}{\sqrt{2u+1}}(\Psi_f||[a^\dagger_n\tilde{a}_p]_u||\Psi_i)\ ,$$

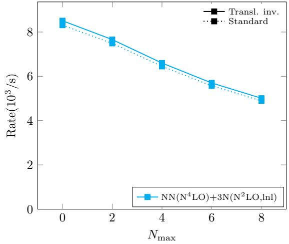
where

$$\boldsymbol{\xi}_s=-\sqrt{A/(A-1)}(\mathbf{r}_s-\mathbf{R}_{\rm CM})$$
$$\boldsymbol{\pi}_s=-\sqrt{A/(A-1)}(\mathbf{p}_s-\mathbf{P})$$

^{6}Li$(1^+_{gs})+\mu^-\rightarrow{}^6He(0^+_{gs})+\nu_\mu$



^{12}C$(0^+_{gs})+\mu^-\rightarrow{}^{12}B(1^+_{gs})+\nu_\mu$



LJ, Navrátil, Kotila and Kravvaris, in progress

TRIUMF

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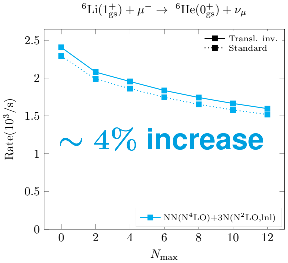
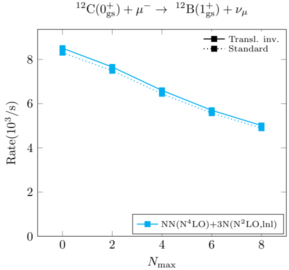
$$\times (M^u)^{-1}_{n'p',np} \frac{1}{\sqrt{2u+1}} (\Psi_f || [a_n^\dagger \tilde{a}_p]_u || \Psi_i) ,$$

where

$$\boldsymbol{\xi}_s = -\sqrt{A/(A-1)}(\mathbf{r}_s - \mathbf{R}_{\mathrm{CM}})$$
$$\boldsymbol{\pi}_s = -\sqrt{A/(A-1)}(\mathbf{p}_s - \mathbf{P})$$



~ 4% increase



LJ, Navrátil, Kotila and Kravvaris, in progress

Discovery, accelerated

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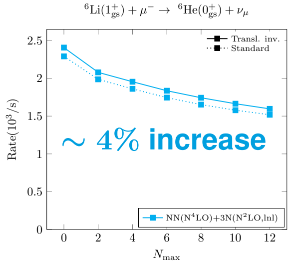
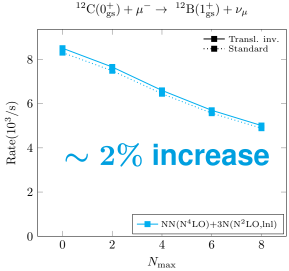




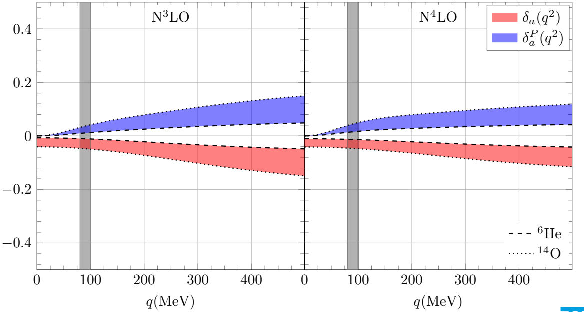
LJ, Navrátil, Kotila and Kravvaris, in progress

TRIUMF

Two-Body Currents

- Fermi-gas density ρ adjusted so that $\delta_a(0)$ reproduces the effect of exact two-body currents in

  P. Gysbers *et al.*, *Nature Phys.* **15**, 428 (2019)



LJ, Navrátil, Kotila and Kravvaris, work in progress

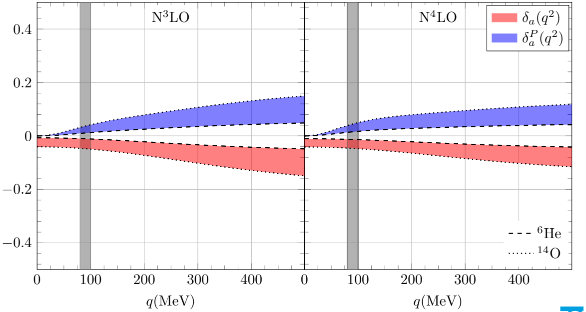
Discovery, accelerated

Two-Body Currents

- Fermi-gas density ρ adjusted so that $\delta_a(0)$ reproduces the effect of exact two-body currents in

  P. Gysbers *et al.*, *Nature Phys.* **15**, 428 (2019)

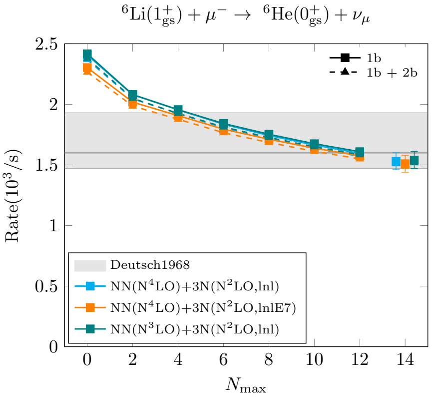
- Two-body currents typically **reduce** the OMC rates by $\sim 1-2\%$ **in ^{6}Li** and by $\lesssim 10\%$ **in ^{12}C and ^{16}O**



LJ, Navrátil, Kotila and Kravvaris, work in progress

Capture Rates to the Ground State of ^{6}He

- NCSM in keeping with experiment



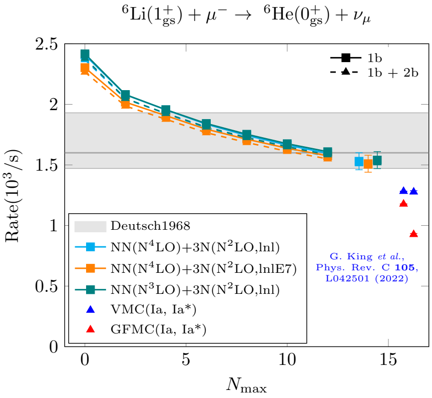
LJ, Navrátil, Kotila, Kravvaris,

work in progress

Capture Rates to the Ground State of ^{6}He

- NCSM in keeping with experiment
- The rates can be compared with the variational (VMC) and Green's function Monte-Carlo (GFMC) calculations

King *et al.*, Phys. Rev. C **105**, L042501 (2022)



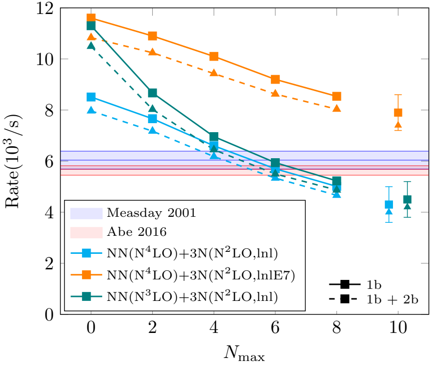
LJ, Navrátil, Kotila, Kravvaris, work in progress

Discovery, accelerated

Capture Rates to the Ground State of ^{12}B

- Interaction dependence

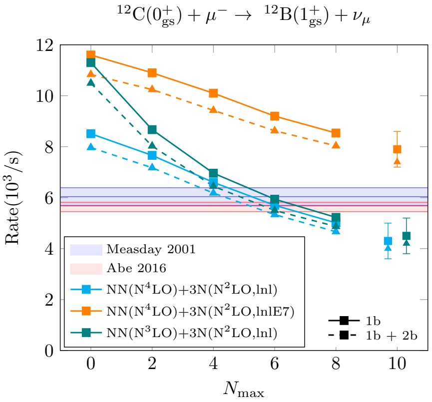
$^{12}C(0^+_{gs}) + \mu^- \to \, ^{12}B(1^+_{gs}) + \nu_\mu$



LJ, Navrátil, Kotila, Kravvaris, work in progress

Capture Rates to the Ground State of ^{12}B

- Interaction dependence
- Adding the E_7 spin-orbit term improves agreement with experiment



LJ, Navrátil, Kotila, Kravvaris,

work in progress

Discovery, accelerated

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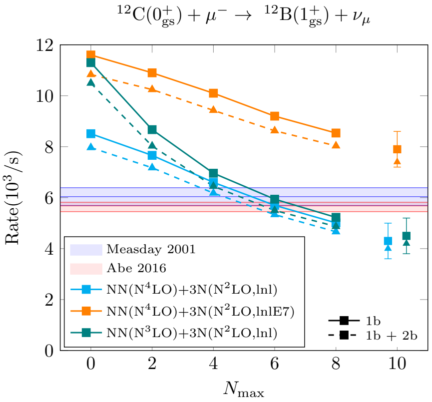

LJ, Navrátil, Kotila, Kravvaris, work in progress

Discovery, accelerated

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- The results can be compared against earlier NCSM ones obtained with NN(CD-Bonn) and NN(AV'8)+3N(TM'(99)) interactions

Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)

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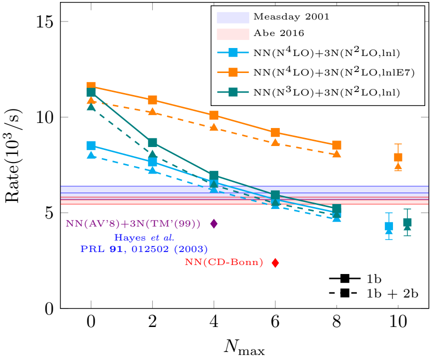

LJ, Navrátil, Kotila, Kravvaris, work in progress

Discovery, accelerated

TRIUMF

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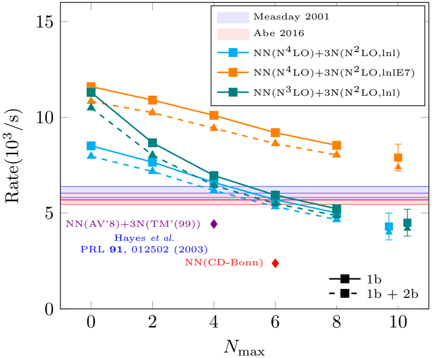
Hayes *et al.*, *Phys. Rev. Lett.* **91**, 012502 (2003)

- 3-body forces essential to reproduce the measured rate

$$^{12}\mathrm{C}(0^+_{\mathrm{gs}}) + \mu^- \rightarrow \, ^{12}\mathrm{B}(1^+_{\mathrm{gs}}) + \nu_\mu$$



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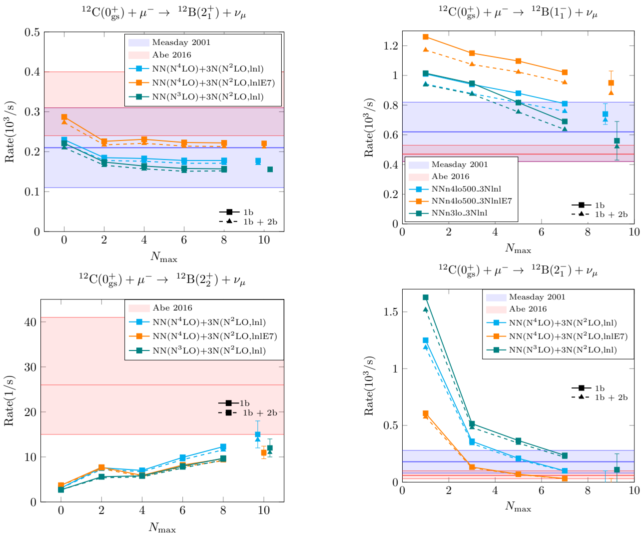
Discovery, accelerated

Capture Rates to Low-Lying States in ^{12}B

- Interaction dependence



LJ, Navrátil, Kotila, Kravvaris, work in progress

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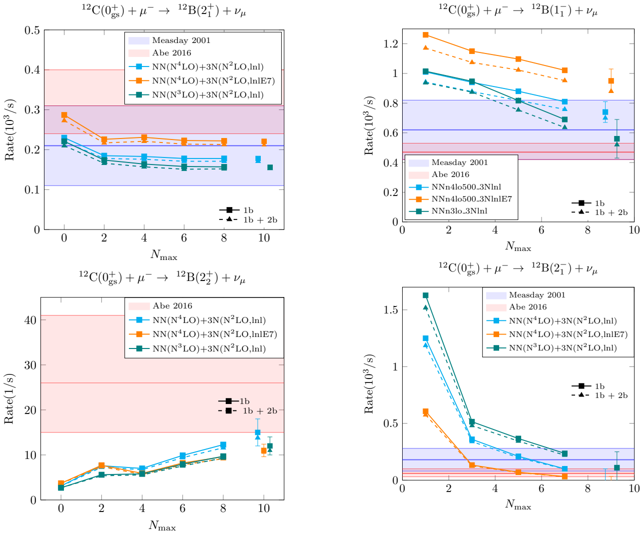


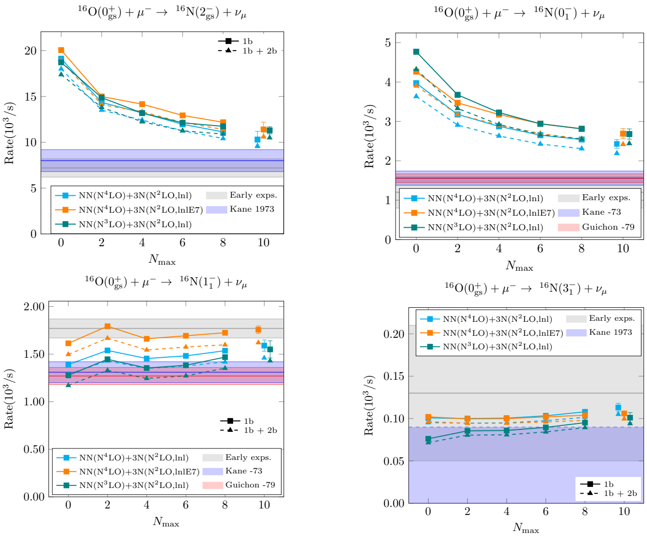
LJ, Navrátil, Kotila, Kravvaris, work in progress

Discovery, accelerated

TRIUMF

Capture Rates to Low-Lying States in ^{16}N

- NCSM describes well the complex systems ^{16}O and ^{16}N

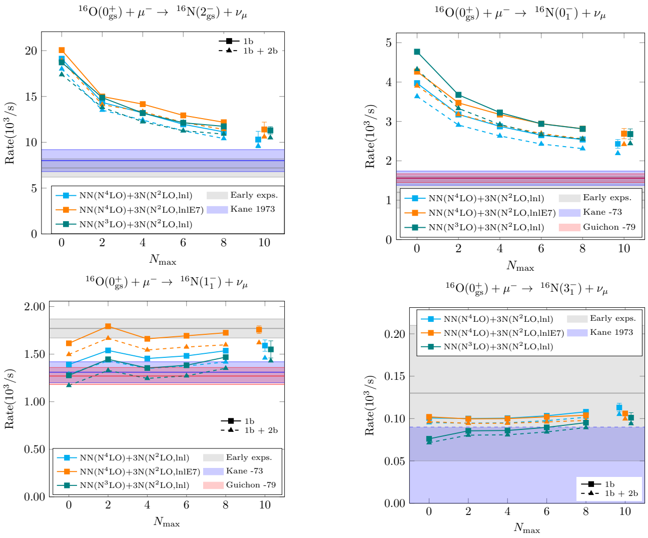


LJ, Navrátil, Kotila, Kravvaris, work in progress

Discovery, accelerated

Capture Rates to Low-Lying States in ^{16}N

- NCSM describes well the complex systems ^{16}O and ^{16}N
- Less sensitive to the interaction than $^{12}\mathrm{C}(\mu^-,\nu_\mu)^{12}\mathrm{B}$



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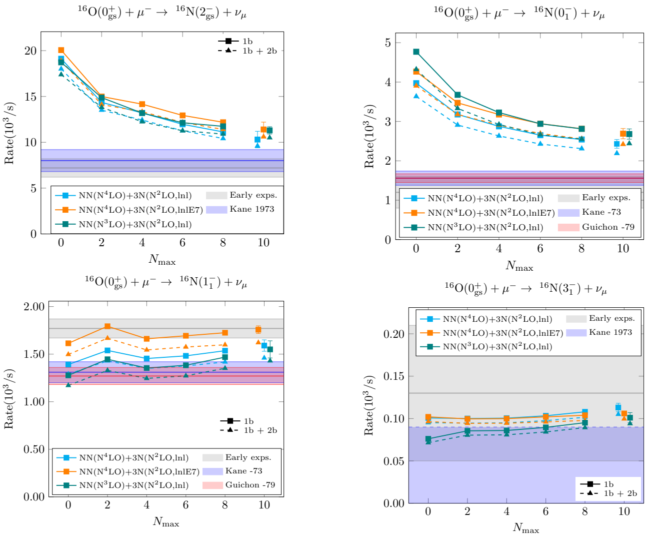

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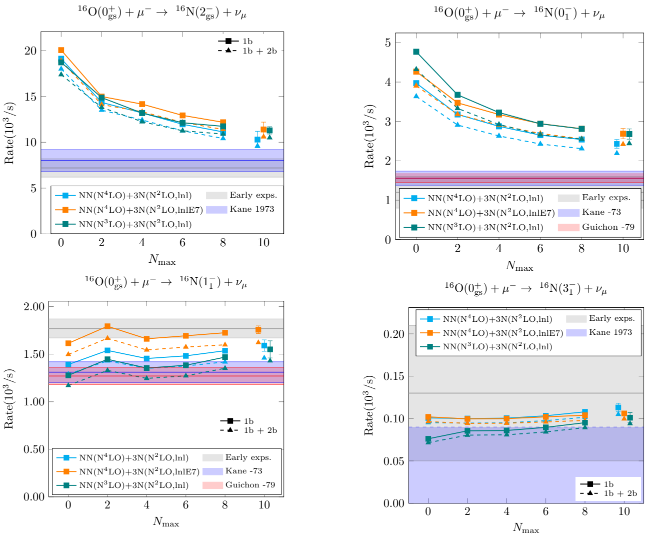
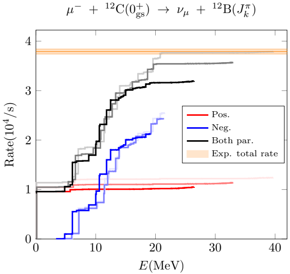
  - *See the talks by D. Gazit and A. Glick-Magid!*

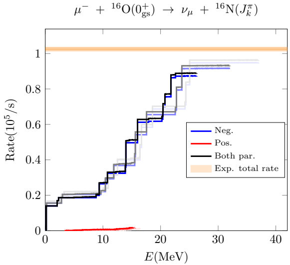


LJ, Navrátil, Kotila, Kravvaris, work in progress

Total Muon-Capture Rates in ^{12}B and ^{16}N

- Color gradient: increasing $N_{\max}$ (3,5,7 for ^{12}C and 2,4,6 for ^{16}O)





LJ, Navrátil, Kotila, Kravvaris, work in progress

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Total Muon-Capture Rates in ^{12}B and ^{16}N

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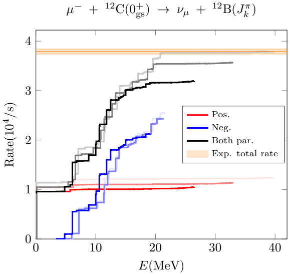
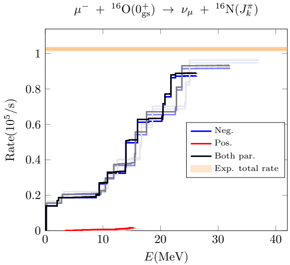
- Rates obtained summing over ~ 50 final states of each parity





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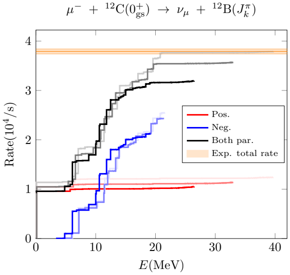
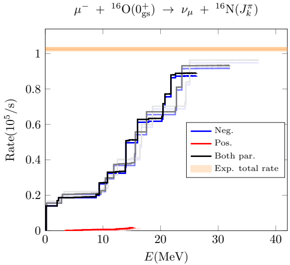
- Rates obtained summing over ~ 50 final states of each parity
- Summing up **the rates up to ~ 20 MeV**, we capture **$\sim 85\%$ of the total rate** in both ^{12}B and ^{16}N





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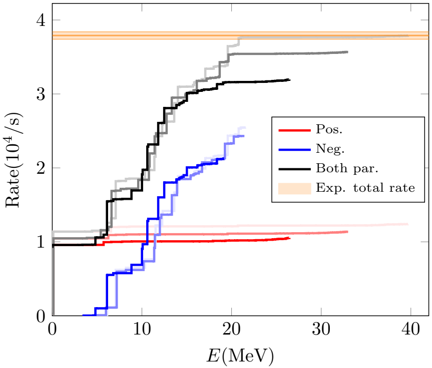
Total Muon-Capture Rates

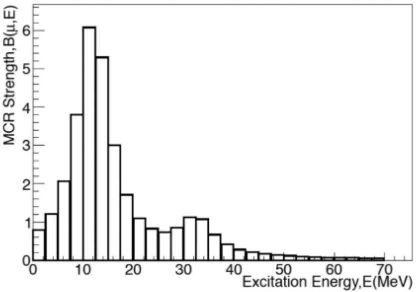
Calculation:

$$\mu^- + {}^{12}\mathrm{C}(0^+_{\mathrm{gs}}) \rightarrow \nu_\mu + {}^{12}\mathrm{B}(J^\pi_k)$$



Experiment:

$$\mu^- + {}^{100}\mathrm{Mo} \rightarrow \nu_\mu + {}^{100}\mathrm{Nb}$$



Hashim et al., Phys. Rev. C ***97****, 014617 (2018)*

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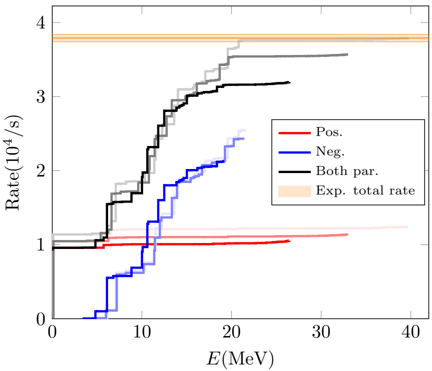

Missing potentially important contribution from high energies

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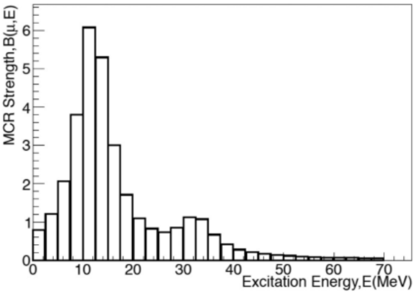

Hashim et al., Phys. Rev. C ***97****, 014617 (2018)*

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Outline

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Summary

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Summary

- Ab initio muon-capture studies could shed light on g_A quenching at finite momentum exchange regime
- Discrepancies between calculated and measured muon capture rates to ^{24}Na yet to be understood
- No-core shell-model describes well partial muon-capture rates in light nuclei ^{6}He, ^{12}B and ^{16}N

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Outlook

- Study potential OMC candidates ^{48}Ti, ^{40}Ca, ^{40}Ti in VS-IMSRG

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  - ^{16}N potential candidate for forbidden β-decay studies
  - ^{12}C and ^{16}O are both of interest in neutrino-scattering experiments

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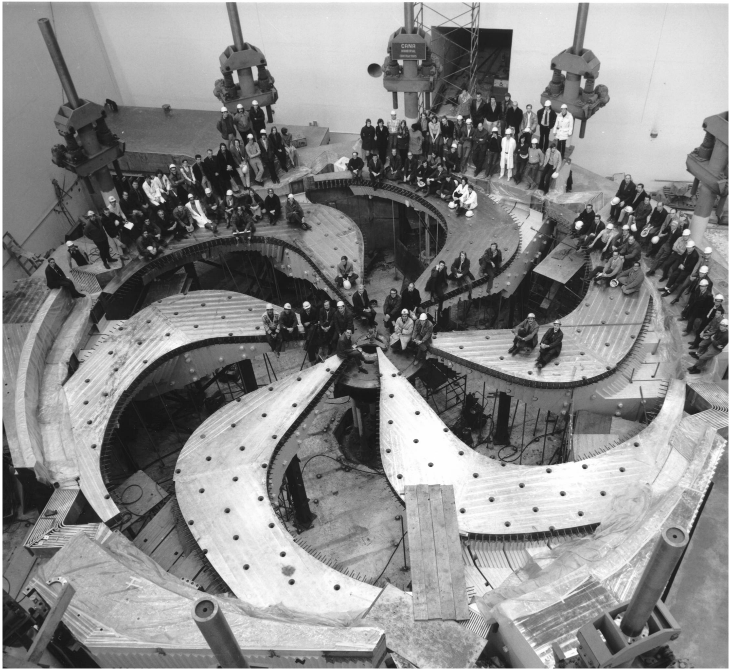
Thank you

Merci



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OMC operators

$$(\Psi_f||\sum_{s=1}^{A}\hat{O}_{kwux}(\mathbf{r}_s,\mathbf{p}_s)||\Psi_i) = \frac{1}{\sqrt{2J_f+1}}\sum_{pn}(n||\hat{O}_{kwux}(\mathbf{r}_s,\mathbf{p}_s)||p)\frac{1}{\sqrt{2u+1}}(\Psi_f||[a_n^\dagger\tilde{a}_p]_u||\Psi_i)$$

NME	$\mathcal{O}_s$
$\mathcal{M}[0\,w\,u]$	$j_w(qr_s)G_{-1}(r_s)\mathcal{Y}^{M_f-M_i}_{0wu}(\hat{\mathbf{r}}_s)\delta_{wu}$
$\mathcal{M}[1\,w\,u]$	$j_w(qr_s)G_{-1}(r_s)\mathcal{Y}^{M_f-M_i}_{1wu}(\hat{\mathbf{r}}_s,\boldsymbol{\sigma}_s)$
$\mathcal{M}[0\,w\,u\,\pm]$	$[j_w(qr_s)G_{-1}(r_s)\mp\frac{1}{q}j_{w\mp1}(qr_s)\frac{d}{dr_s}G_{-1}(r_s)]\mathcal{Y}^{M_f-M_i}_{0wu}(\hat{\mathbf{r}}_s)\delta_{wu}$
$\mathcal{M}[1\,w\,u\,\pm]$	$[j_w(qr_s)G_{-1}(r_s)\mp\frac{1}{q}j_{w\mp1}(qr_s)\frac{d}{dr_s}G_{-1}(r_s)]\mathcal{Y}^{M_f-M_i}_{1wu}(\hat{\mathbf{r}}_s,\boldsymbol{\sigma}_s)$
$\mathcal{M}[0\,w\,u\,p]$	$ij_w(qr_s)G_{-1}(r_s)\mathcal{Y}^{M_f-M_i}_{0wu}(\hat{\mathbf{r}}_s)\boldsymbol{\sigma}_s\cdot\mathbf{p}_s\delta_{wu}$
$\mathcal{M}[1\,w\,u\,p]$	$ij_w(qr_s)G_{-1}(r_s)\mathcal{Y}^{M_f-M_i}_{1wu}(\hat{\mathbf{r}}_s,\mathbf{p}_s)$

Morita, Fujii, *Phys. Rev.* **118**, 606 (1960)

Two-Body Currents

$$\mathbf{J}^{\mathrm{eff}}_{i,2\mathrm{b}}(\rho,\mathbf{p}) = g_A\tau_i^-\left[\delta_a(p^2)\boldsymbol{\sigma}_i + \frac{\delta_a^P(p^2)}{p^2}(\mathbf{p}\cdot\boldsymbol{\sigma}_i)\mathbf{p}\right]$$

with two-body functions $\delta_a(p^2)$, $\delta_a^P(p^2)$ dependent on the Fermi-gas density ρ:

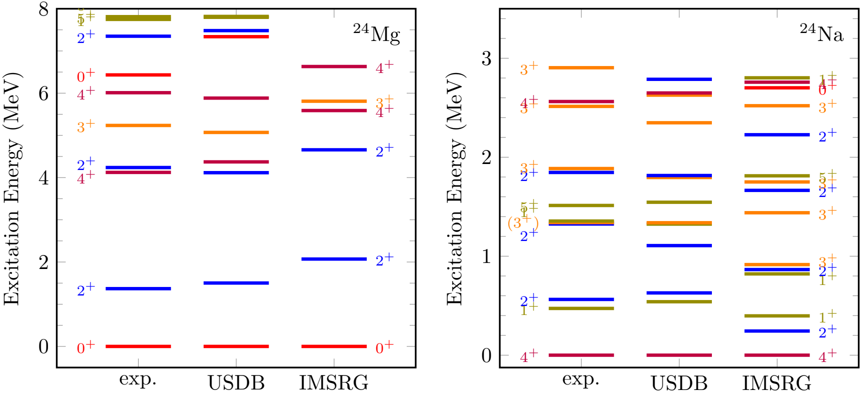
$$\delta_a(p^2) = -\frac{\rho}{F_\pi^2}\left[\frac{c_4}{3}[3I_2^\sigma(\rho,p) - I_1^\sigma(\rho,p)] - \frac{1}{3}\left(c_3 - \frac{1}{4m_\mathrm{N}}\right)I_1^\sigma(\rho,p) - \frac{c_6}{12}I_{c6}(\rho,p) - \frac{c_D}{4g_A\Lambda_\chi}\right]$$

and

$$\delta_a^P(p^2) = \frac{\rho}{F_\pi^2}\left[-2(c_3-2c_1)\frac{m_\pi^2p^2}{(m_\pi^2+p^2)^2} + \frac{1}{3}\left(c_3+c_4-\frac{1}{4m_\mathrm{N}}\right)I^P(\rho,p) - \left(\frac{c_6}{12} - \frac{2}{3}\frac{c_1m_\pi^2}{m_\pi^2+p^2}\right)I_{c6}(\rho,p)\right.$$
$$\left. - \frac{p^2}{m_\pi^2+p^2}\left(\frac{c_3}{3}[I_1^\sigma(\rho,p)+I^P(\rho,p)] + \frac{c_4}{3}[I_1^\sigma(\rho,p)+I^P(\rho,p)-3I_2^\sigma(\rho,p)]\right) - \frac{c_D}{4g_A\Lambda_\chi}\frac{p^2}{m_\pi^2+p^2}\right]$$

Hoferichter, Menéndez, Schwenk, *Phys. Rev. D* **102**, 074018 (2020)

Excitation Energies in the $A = 24$ Systems



Electromagnetic Moments in the $A = 24$ Systems

Nucleus	J_i^π	$E(\mathrm{MeV})$			$\mu(\mu_\mathrm{N})$			$Q(e^2\mathrm{fm}^2)$		
		exp.	NSM	IMSRG	exp.	NSM	IMSRG	exp.	NSM	IMSRG
^{24}Mg	2^+	1.369	1.502	1.981	1.08(3)	1.008	1.033	$-29(3)$	-19.346	-12.9
^{24}Mg	4^+	4.123	4.372	5.327	1.7(12)	2.021	2.096	-		
^{24}Mg	2^+	4.238	4.116	4.327	1.3(4)	1.011	1.085	-		
^{24}Mg	4^+	6.010	5.882	6.347	2.1(16)	2.015	2.089	-		
^{24}Na	4^+	0.0	0.0	0.0	1.6903(8)	1.533	1.485	-		
^{24}Na	1^+	0.472	0.540	0.397	$-1.931(3)$	-1.385	-0.344	-		

β Decays of the $A = 24$ Systems

Nucleus	$J_i \rightarrow J_f$	log ft		
		exp.	NSM	IMSRG
^{24}Na	$1_1^+ \rightarrow 0_1^+$	5.80	5.188–5.223	4.448–4.545
^{24}Na	$4_{gs}^+ \rightarrow 4_1^+$	6.11	5.416–5.461	5.795–5.866
^{24}Na	$4_{gs}^+ \rightarrow 3_1^+$	6.60	5.727–5.773	6.342–6.422

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Excitation Energies of ^{12}B

J_i^π	Interaction	$E_{\mathrm{exc.}}$ (MeV) $N_{\mathrm{max}} = 4$	$N_{\mathrm{max}} = 6$	$N_{\mathrm{max}} = 8$	Exp.
1_1^+	NN(N^{4}LO)-3NlnI	0.0	0.0	0.0	0.0
	NN(N^{4}LO)-3NlnlE7	0.135	0.000	0.000	
2_1^+	NN(N^{4}LO)-3NlnI	0.251	0.465	0.538	0.953
	NN(N^{4}LO)-3NlnlE7	0.000	0.027	0.097	
0_1^+	NN(N^{4}LO)-3NlnI	2.073	1.831	1.713	2.723
	NN(N^{4}LO)-3NlnlE7	3.306	2.909	2.761	
2_2^+	NN(N^{4}LO)-3NlnI	3.816	3.490	3.344	3.760
	NN(N^{4}LO)-3NlnlE7	4.919	4.463	4.281	

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Excitation Energies of ^{16}N

J_i^π	Interaction	$E_{\mathrm{exc.}}$ (MeV) $N_{\max}=4$	$N_{\max}=6$	$N_{\max}=8$	Exp.
2_1^-	NN(N^4LO)-3NInl	0.154	0.087	0.064	0.0
	NN(N^4LO)-3NInlE7	0.214	0.146	0.133	
0_1^-	NN(N^4LO)-3NInl	2.245	1.487	1.010	0.120
	NN(N^4LO)-3NInlE7	2.807	2.065	1.606	
3_1^-	NN(N^4LO)-3NInl	0.000	0.000	0.000	0.298
	NN(N^4LO)-3NInlE7	0.000	0.000	0.000	
1_1^-	NN(N^4LO)-3NInl	2.561	1.833	1.363	0.397
	NN(N^4LO)-3NInlE7	2.985	2.310	1.869	