TRIUMF

Nambu-Covariant Green's Functions

and its use for superfluid nuclear matter

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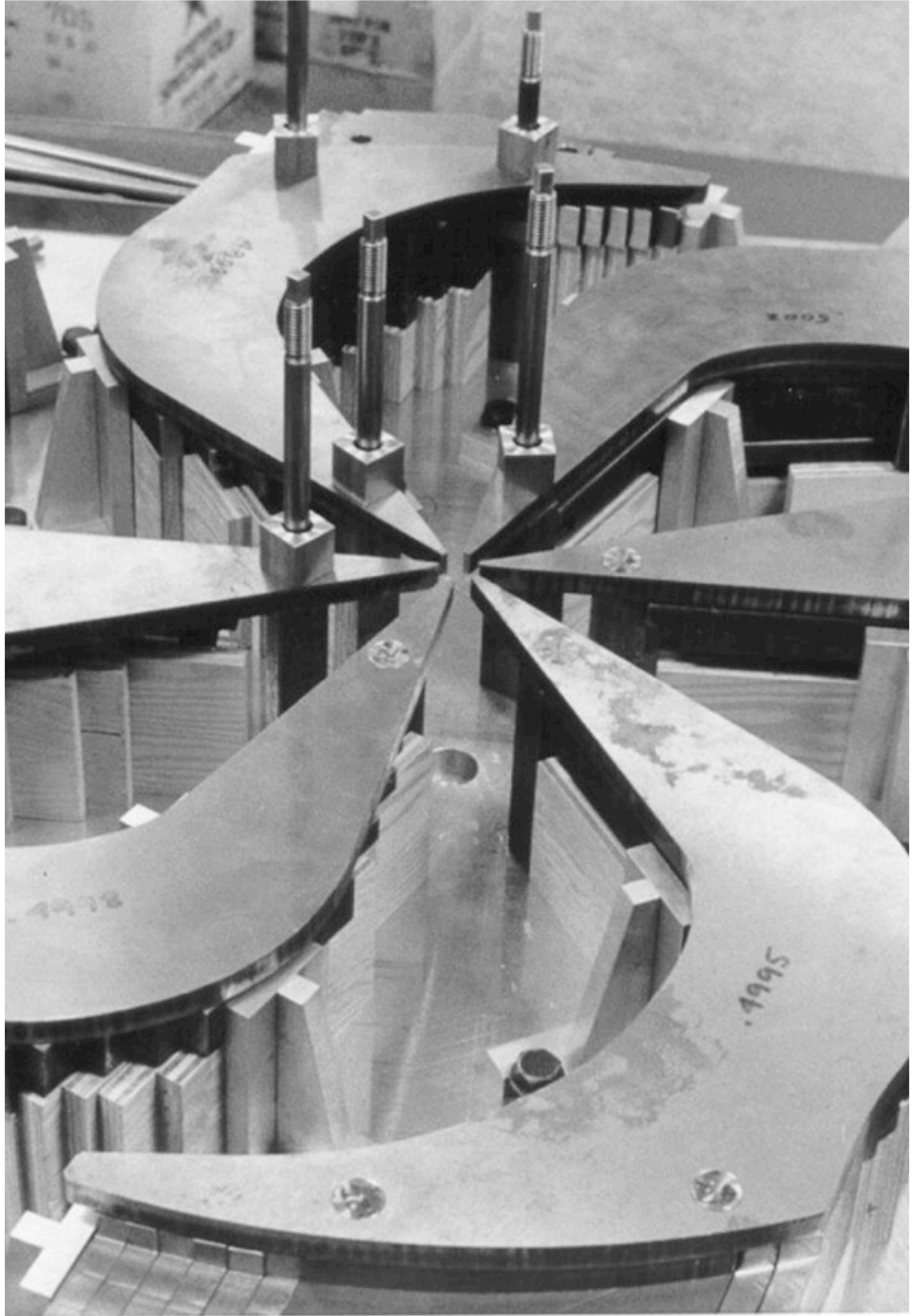
TRIUMF - Theory department

Progress in Ab Initio Nuclear Theory
3rd of March 2023

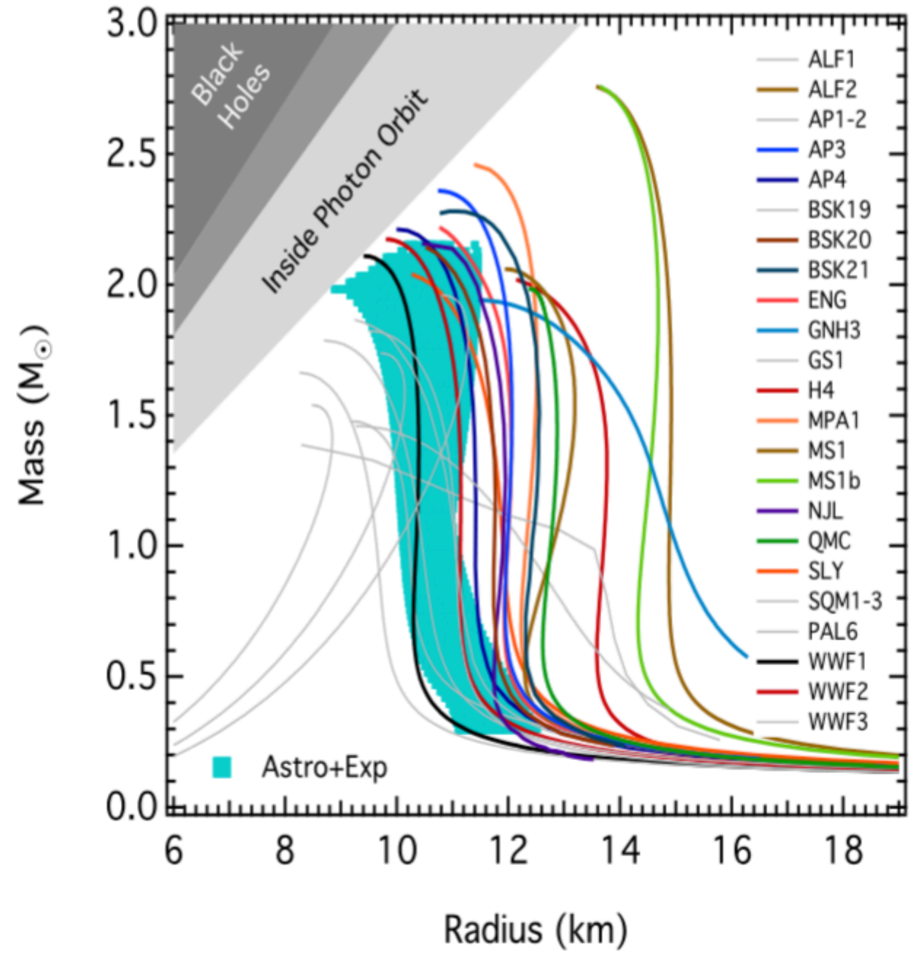
arXiv:2107.09763 [nucl-th]
arXiv:2107.09759 [nucl-th]



Discovery, accelerated

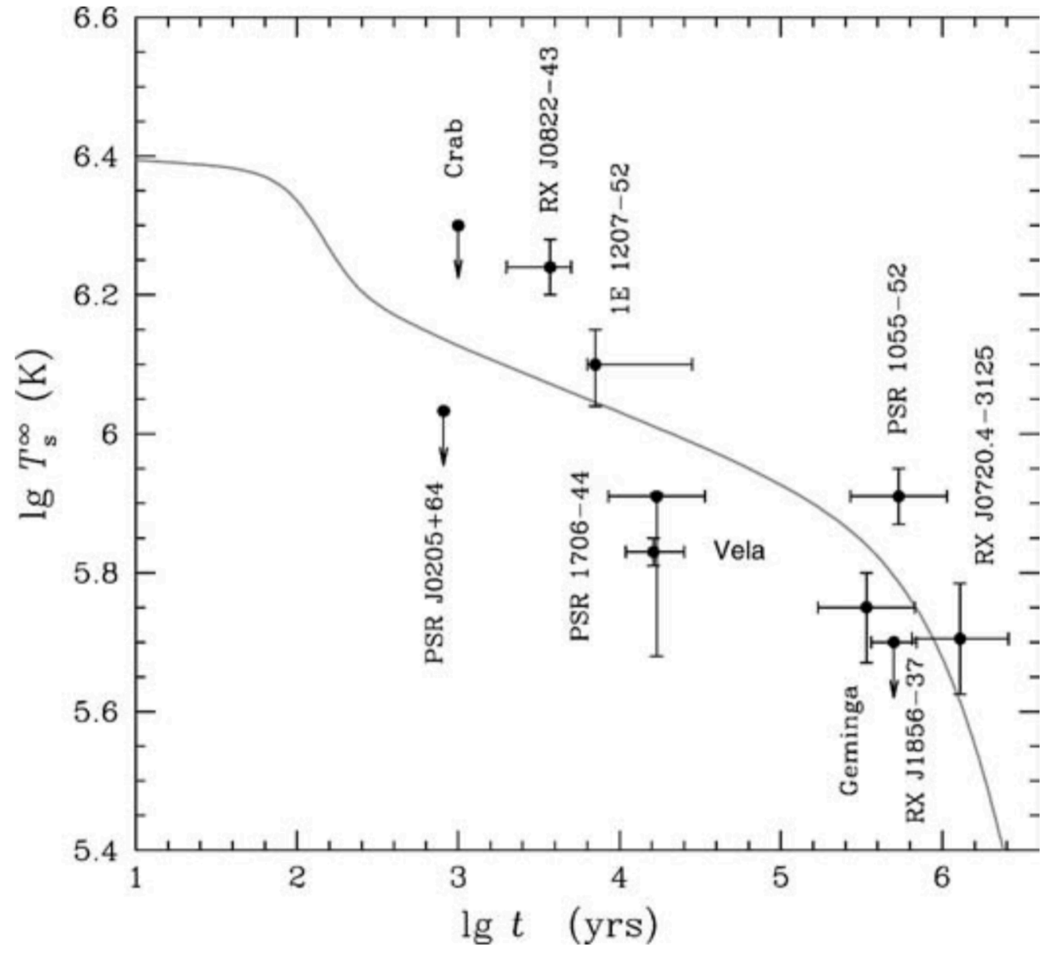
Goal: a consistent nuclear picture for neutron stars

Mass-Radius: $M(R)$



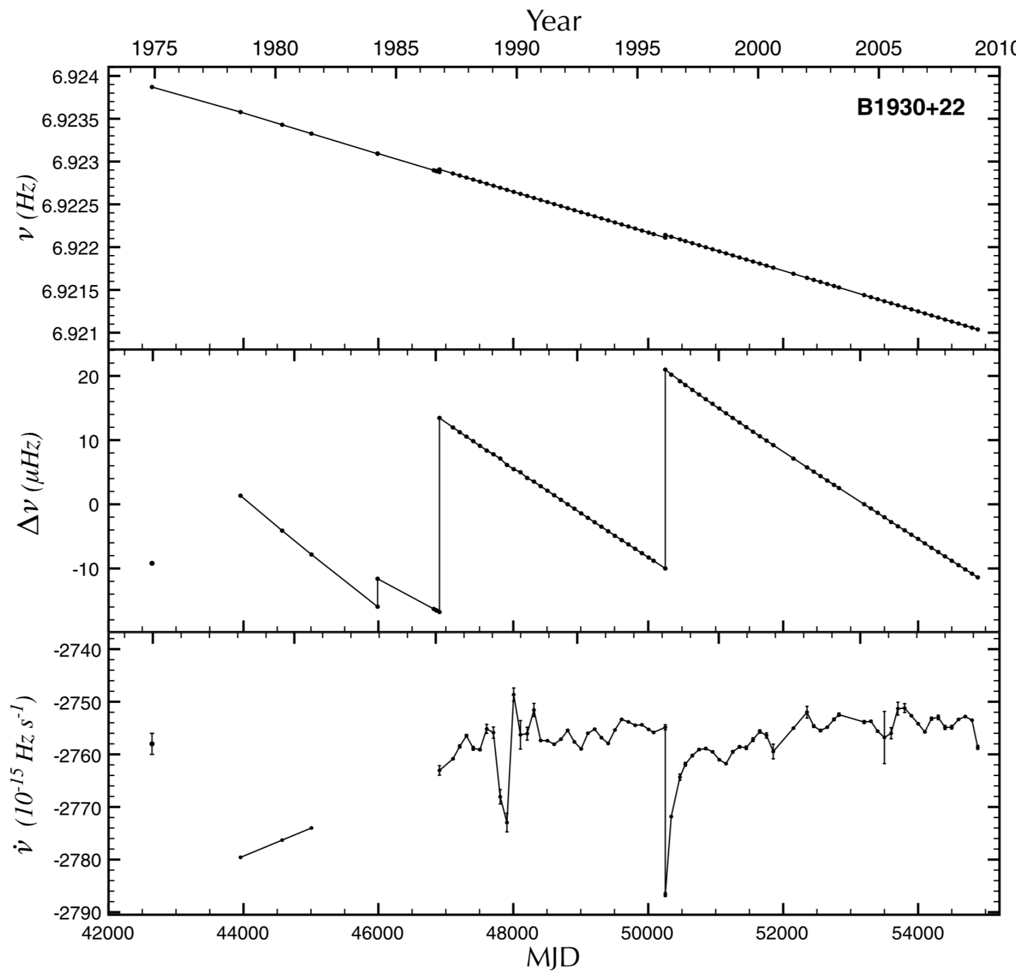
[Özel, Freire (2016)]

Cooling curve: $T(t)$



[Yakovlev, Pethick (2004)]

Frequency glitching: $\dot{\nu}(t)$



[Espinoza et al. (2011)]

Exp. observations

constrain ↓ ↑ predict

Nuclear model

Neutron star model

Equation of state

- Energy: $E(\rho)$
- Pressure: $P(\rho)$

Pairing gaps

- $\Delta\left({}^1S_0\right)(\rho, T)$
- $\Delta\left({}^3PF_2\right)(\rho, T)$
- In-medium reaction rates

Cluster-Superfluid

- Lattice: cluster inhomogeneities
- Rotational superfluid: vortices
- Pinning forces

Nuclear inputs

- Incoherent picture
- X Hinder constraint feedback

Goal: a consistent nuclear picture for neutron stars

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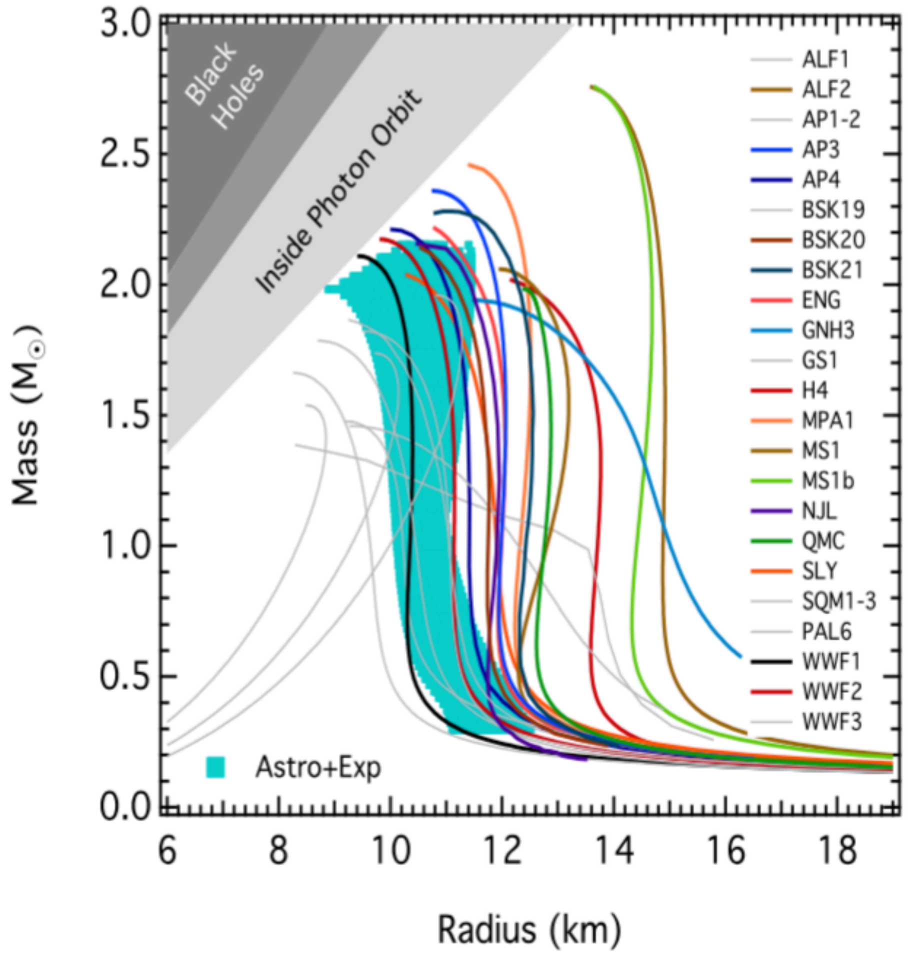

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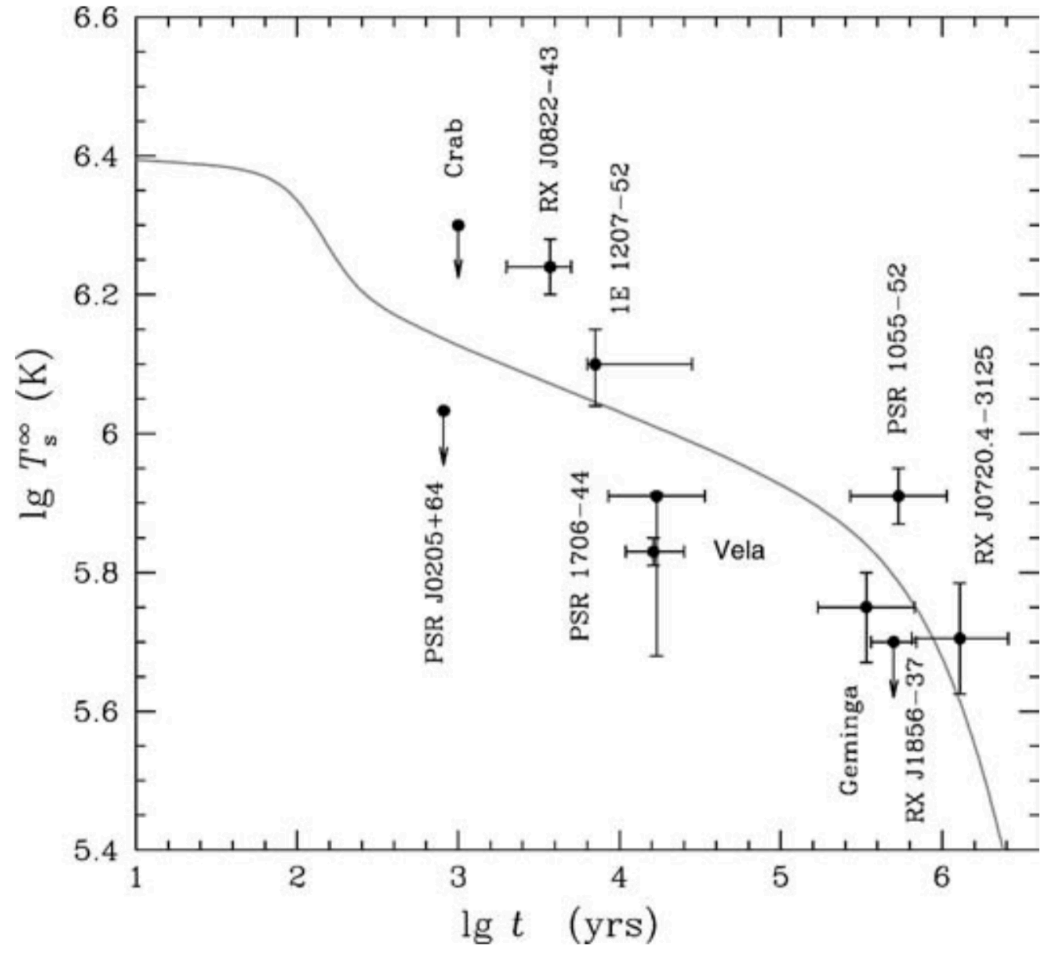

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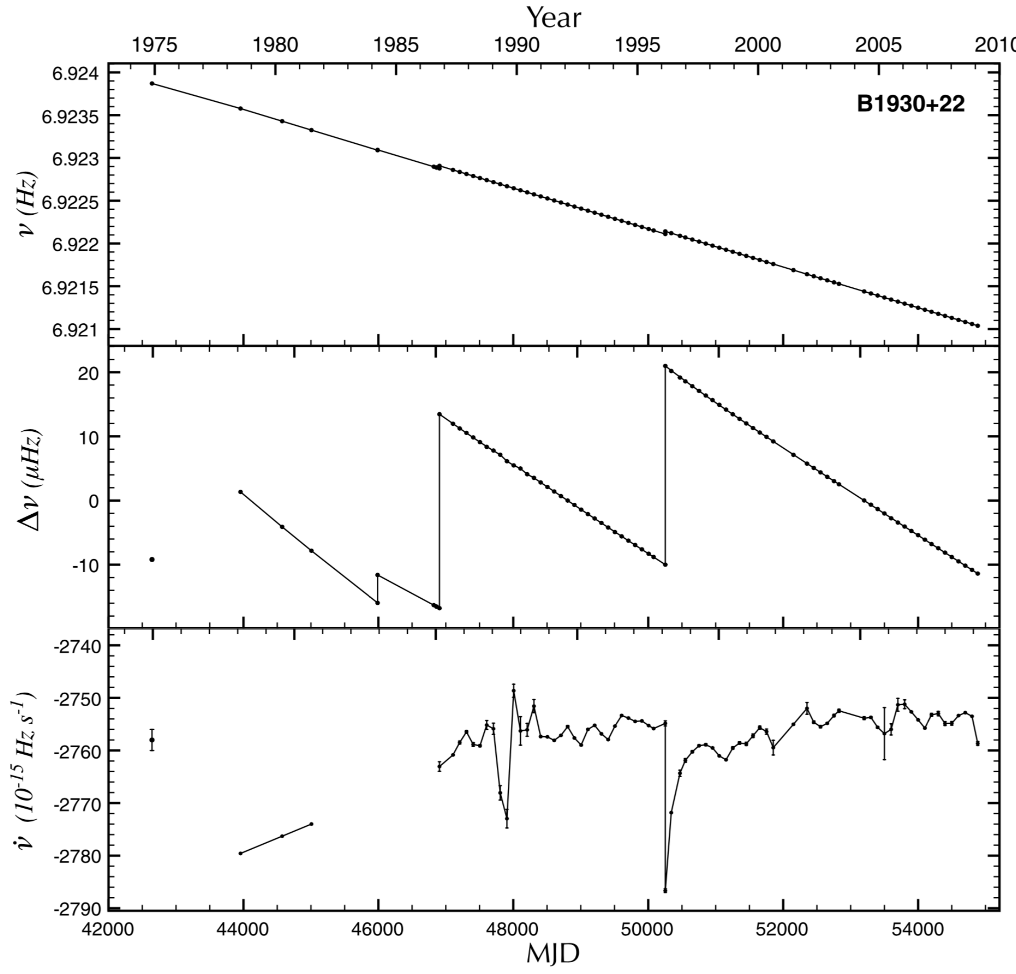


[Espinoza et al. (2011)]

Exp. observations

constrain

predict

Nuclear model

Neutron star model

Consistent nuclear model

Equation of state

Pairing gaps

Cluster-Superfluid

Nuclear inputs

- Coherent picture
- ✓ Relevant feedback

Goal: a consistent nuclear picture for neutron stars

Necessary requirements on PT-based MB approxs

- Ladder diagrams summation
  - High density $\Rightarrow$ $\rho \in [0, 4\rho_0]$
  - $k_F \sim 600$ MeV $\Rightarrow$ $\Lambda_b \gg 600$ MeV
  - Validity of soft χ-potentials unclear
- ➡ High cutoff/Hard-core potentials as a cross-check
- Temperature dependence
  - $T \in [0, 50$ MeV$]$
- Φ-derivability (dressed propagator)
  - Thermo consistency + continuity equations
- Symmetry-breaking partitioning
  - Superfluid regime + Thouless' criterion

[Thouless (1960)]

Well under control

- Ladder sum
- Finite temperature
- Dressed propagator

Theses: [T. Frick, 2004] [A. Rios, 2007] [V. Somà, 2009] [A. Carbone, 2014]

UNIVERSITY OF SURREY

Original goal:

Sum all ladder diagrams, at finite temperature, with symmetry-breaking, and in a self-consistent fashion

Complex interrelation between the features

UNIVERSITAT DE BARCELONA

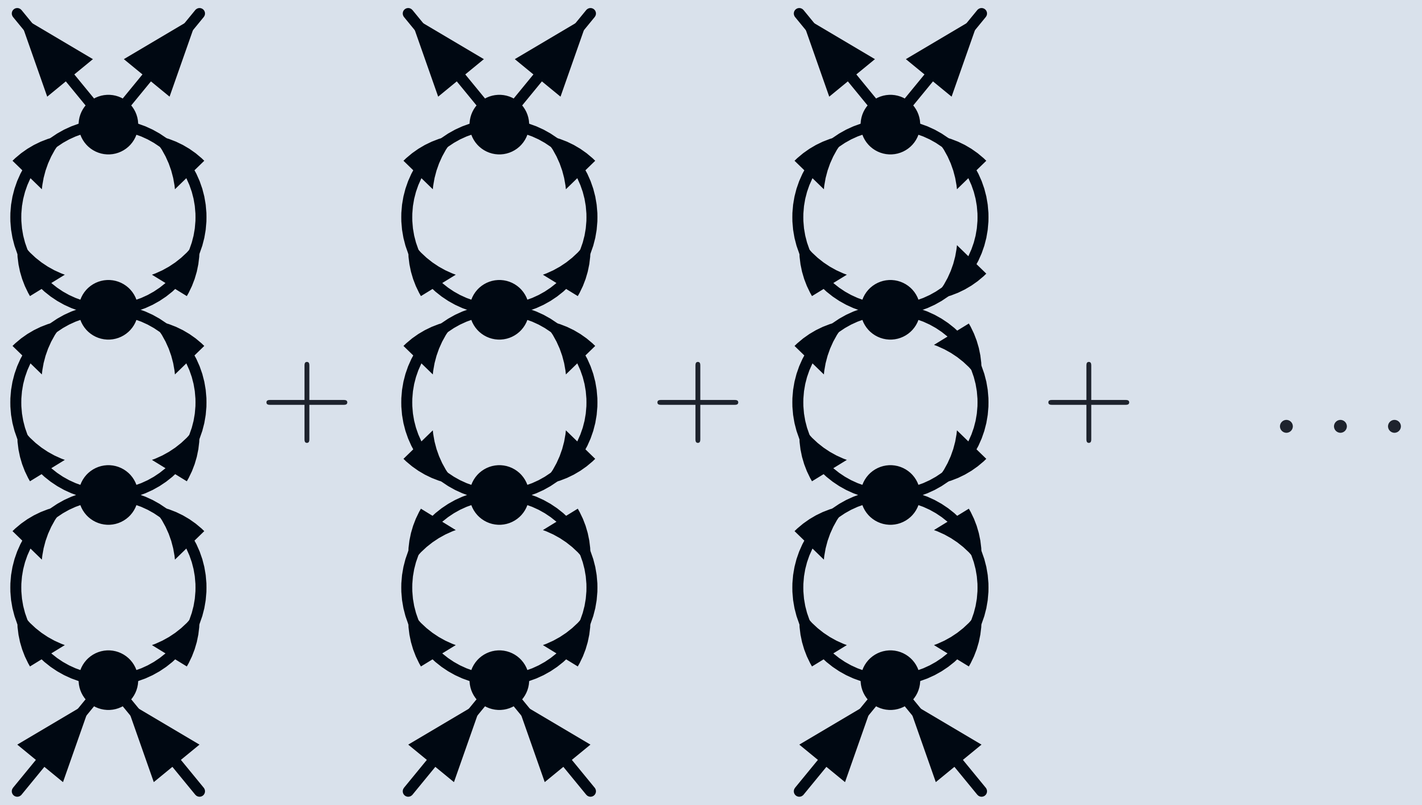
Arnau Rios

UNIVERSITÀ DEGLI STUDI DI MILANO

Carlo Barbieri

How to sum all ladder diagrams?

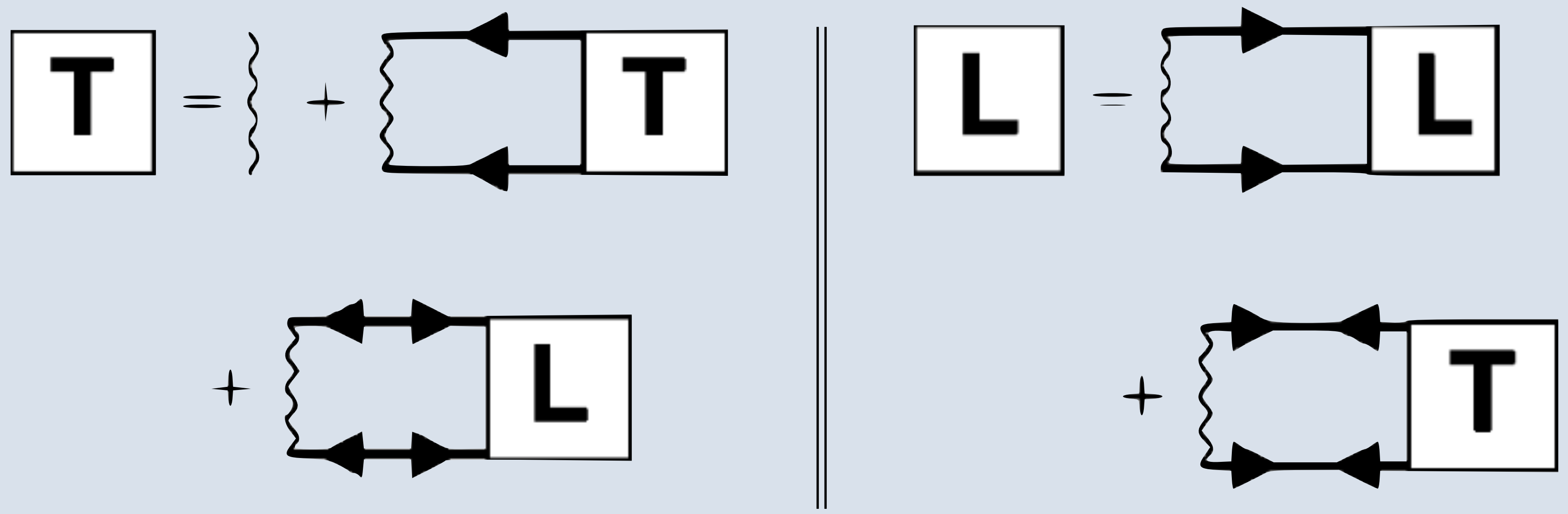
Summing *all* ladder diagrams



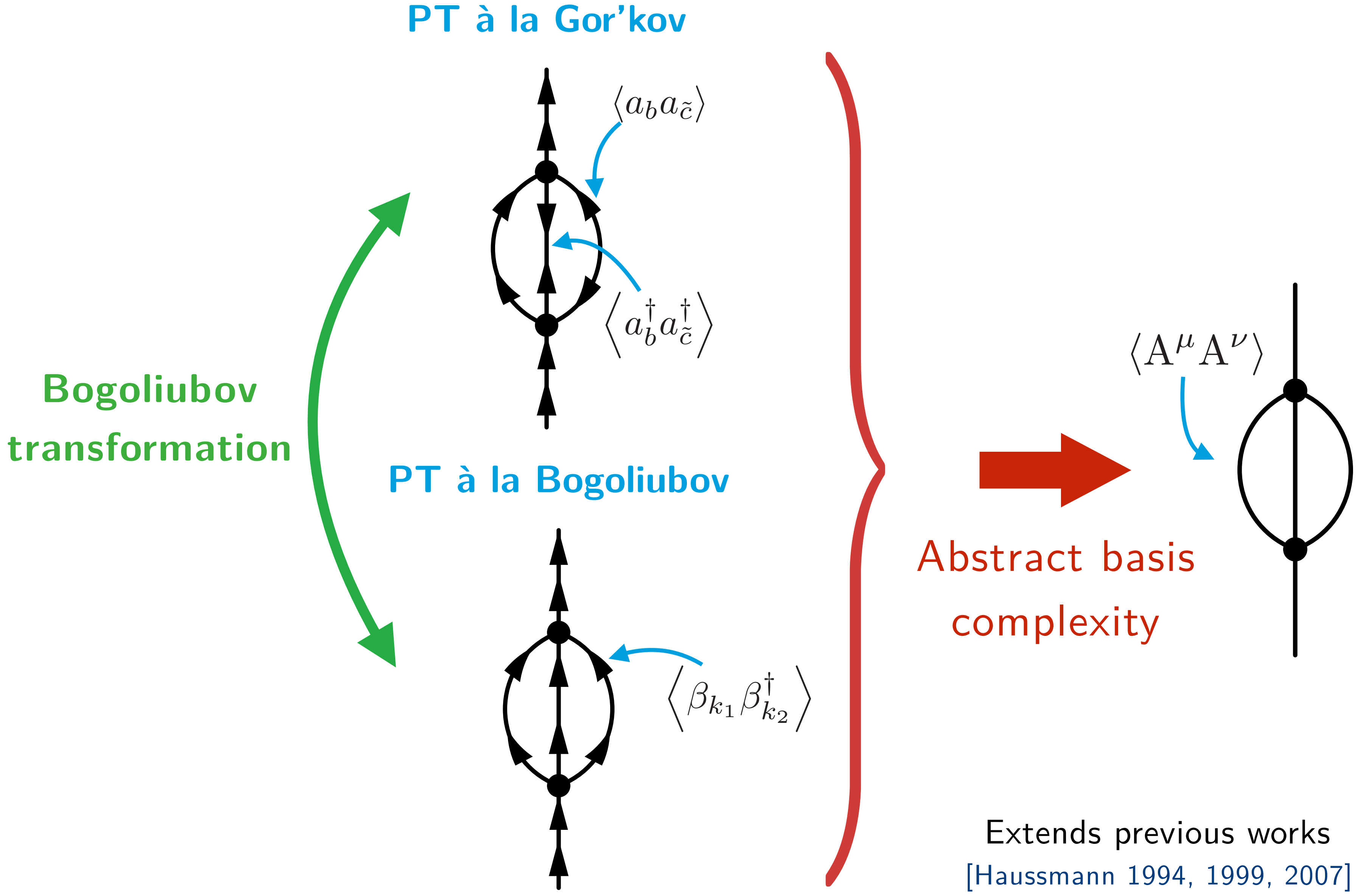
- Mixed pp/hh/ph/anomalous/hybrid
- Tedious combinatorics
  - Track conservation laws
  - Avoid double-counting
- Dressed prop. ⇒ No basis simplification

Previous attempts: partial sums

- In nuclear physics [Bożek, 1999, 2002]



Alternative path: unifying perturbative frameworks



Extends previous works [Haussmann 1994, 1999, 2007]

Advantage of reformulation

- Practical aspects
  - Un-oriented diagrammatic
  - Dramatic formal simplification
  - Decouples: Basis vs MB approx
  - Economy of thoughts [Mach, Poincaré, etc]
- Theoretical aspects
  - Contravariant propagators
  - Covariant vertices
  - ➡ Bogoliubov invariant equations

Outline

- **Nambu-covariant formalism**
  - Nambu-covariant perturbation theory
  - Self-consistent ladder approximation
- **Selected applications**
  - First approximation: general complex HFB
  - Conditions for the convergence of the series of ladders

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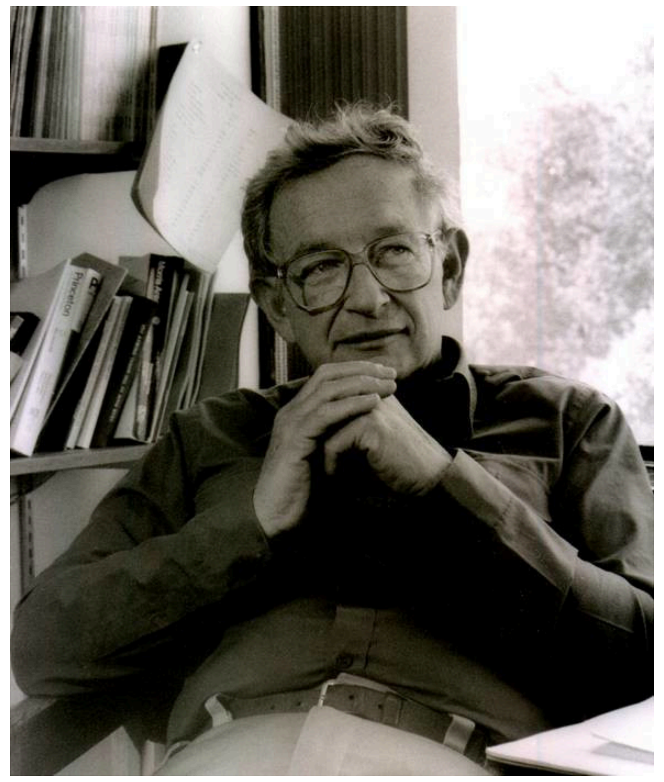
Nambu-tensors and where to find them



Philip W. Anderson

Yoichiro Nambu

Extended space and bases

- Extended one-body space: $\mathscr{H}_1^e \equiv \mathscr{H}_1 \times \mathscr{H}_1^\dagger$
- Extended one-body basis: $\mathscr{B}^e \equiv \mathscr{B} \cup \mathscr{B}^\dagger$
  - where: $\mathscr{B} \equiv \{|b\rangle\}$ and $\mathscr{B}^\dagger \equiv \{\langle b|\}$
  - such that: $\langle b|c\rangle = \delta_{bc}$

2nd quantization view

$\mathscr{H}_1^e \cong \text{Span}\{a_b^\dagger\} \oplus \text{Span}\{a_b\}$

Nambu fields

[Anderson, 1958] [Nambu, 1960]

- Define $\mu \equiv (b, g)$, where $g \in \{1,2\}$ is a Nambu index
- Then Nambu fields A^μ and A_μ are then defined as

$$\left.\begin{aligned} \mathrm{A}^{(b,1)} &\equiv a_b \\ \mathrm{A}^{(b,2)} &\equiv a_b^\dagger \\ \mathrm{A}_{(b,1)} &\equiv a_b^\dagger \\ \mathrm{A}_{(b,2)} &\equiv a_b \end{aligned}\right\} \mathscr{B}^e \longleftrightarrow \mathscr{B}^{e'} \left\{\begin{aligned} \mathrm{A}'^\mu &\equiv \sum_\nu (\mathcal{W}^{-1})^\mu{}_\nu \, \mathrm{A}^\nu \\ \mathrm{A}'_\mu &\equiv \sum_\nu \mathcal{W}^\nu{}_\mu \, \mathrm{A}_\nu \end{aligned}\right.$$

Change of extended basis $\mathscr{W}^\mu{}_\nu$

Tensor definition

- Def: (p,q)-tensor $t \equiv$ multi-dim array of elts s.t.

$$t'^{\mu_1\ldots\mu_p}{}_{\nu_1\ldots\nu_q} \equiv \sum_{\kappa_1\ldots\kappa_p} \sum_{\lambda_1\ldots\lambda_q} \left(\mathscr{W}^{-1}\right)^{\mu_1}{}_{\kappa_1} \ldots \left(\mathscr{W}^{-1}\right)^{\mu_p}{}_{\kappa_p} \times t^{\kappa_1\ldots\kappa_p}{}_{\lambda_1\ldots\lambda_q} \left(\mathscr{W}\right)^{\lambda_1}{}_{\nu_1} \ldots \left(\mathscr{W}\right)^{\lambda_q}{}_{\nu_q}$$

- p *contravariant* and q *covariant* indices

Operators' expression

- Operators as polynomial of Nambu fields

$$O \equiv \sum_{\mu_1\ldots\mu_{2k}} o^{\mu_1\ldots\mu_k}{}_{\mu_{k+1}\ldots\mu_{2k}} \mathrm{A}_{\mu_1}\ldots\mathrm{A}_{\mu_k}\mathrm{A}^{\mu_{k+1}}\ldots\mathrm{A}^{\mu_{2k}}$$

$$O \equiv \sum_{\mu_1\ldots\mu_{2k}} o_{\mu_1\ldots\mu_{2k}} \mathrm{A}^{\mu_1}\ldots\mathrm{A}^{\mu_{2k}}$$

$$O \equiv \sum_{\mu_1\ldots\mu_{2k}} o^{\mu_1\ldots\mu_{2k}} \mathrm{A}_{\mu_1}\ldots\mathrm{A}_{\mu_{2k}}$$

Metric tensor

$g_{\mu\nu} \equiv \{\mathrm{A}_\mu, \mathrm{A}_\nu\}$

Perturbation expansion of Green's functions

Partitioning of the Hamiltonian

$$H \equiv H_0 + H_1$$

$$H_0 \equiv \frac{1}{2} \sum_{\mu\nu} U_{\mu\nu} \; \mathrm{A}^{\mu}\mathrm{A}^{\nu}$$

$$H_1 \equiv \sum_{k=1}^{n} \frac{1}{(2k)!} \sum_{\mu_1\dots\mu_{2k}} v^{(k)}_{\mu_1\dots\mu_{2k}} \; \mathrm{A}^{\mu_1}\dots\mathrm{A}^{\mu_{2k}}$$

Covariant vertices

Contravariant Green's functions

- Contravariant k-body Green's function

$$(-1)^k \; \mathcal{G}^{\mu_1\dots\mu_{2k}}(\tau_1, \dots, \tau_{2k}) \; \equiv \; \left\langle \mathrm{T}\left[\mathrm{A}^{\mu_1}(\tau_1) \; \dots \; \mathrm{A}^{\mu_{2k}}(\tau_{2k})\right] \right\rangle$$

with $\langle \, . \, \rangle = \mathrm{Tr}\left(\, . \, \rho\right)$ and $\rho \equiv \dfrac{e^{-\beta H}}{\mathrm{Tr}\left(e^{-\beta H}\right)}$

- Unperturbed case: $H \longleftrightarrow H_0$

Green's functions expansion

- Interaction picture expression

$$(-1)^k \; \mathcal{G}^{\mu_1\dots\mu_{2k}}(\tau_1, \dots, \tau_{2k}) \; = \; \frac{\left\langle \mathrm{T}\left[e^{-\int_0^\beta \mathrm{d}s \; H_1(s)} \; \mathrm{A}^{\mu_1}(\tau_1) \; \dots \; \mathrm{A}^{\mu_{2k}}(\tau_{2k}) \right] \right\rangle_0}{\left\langle \mathrm{T} e^{-\int_0^\tau \mathrm{d}s \; H_1(s)} \right\rangle_0}$$

- Perturbation expansion

$$\left\langle \mathrm{T}\left[e^{-\int_0^\beta \mathrm{d}s \; H_1(s)} \; \mathrm{A}^{\mu_1}(\tau_1) \; \dots \; \mathrm{A}^{\mu_{2k}}(\tau_{2k}) \right] \right\rangle_0 =$$

$$\sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \int_0^\beta \mathrm{d}\tau_1' \dots \int_0^\beta \mathrm{d}\tau_n' \left\langle \mathrm{T}\left[H_1(\tau_1') \dots H_1(\tau_n') \; \mathrm{A}^{\mu_1}(\tau_1) \; \dots \; \mathrm{A}^{\mu_{2k}}(\tau_{2k}) \right] \right\rangle_0$$

- Statistical time-dependent Wick theorem + Linked-cluster theorem

⇒ Feynman diagrammatic *almost* as usual

Building block of Feynman's diagrams

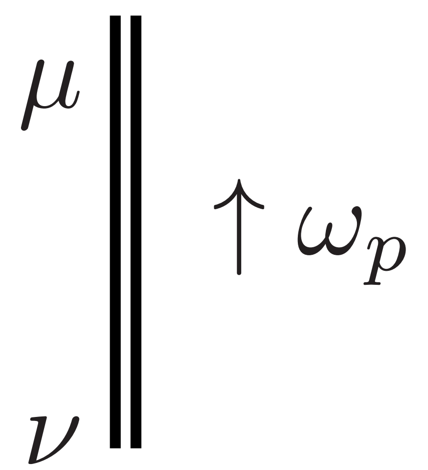
Several formulations

- Time-dependent partitioning
  - Out of the scope of this presentation
- Time-independent partitioning
  - Time rep
  - **Energy rep**

Fourier Transformation

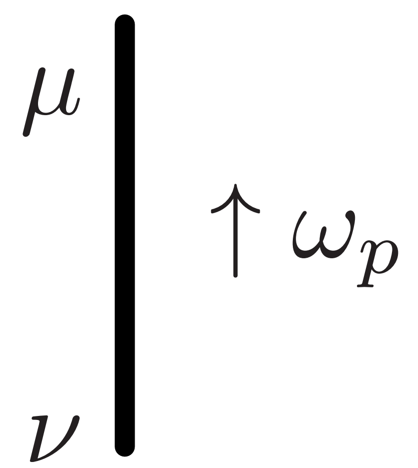
Particle propagators

$$-\mathscr{G}^{\mu\nu}(\omega_p) =$$



;

$$-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_p) =$$



Fully antisymmetric vertex

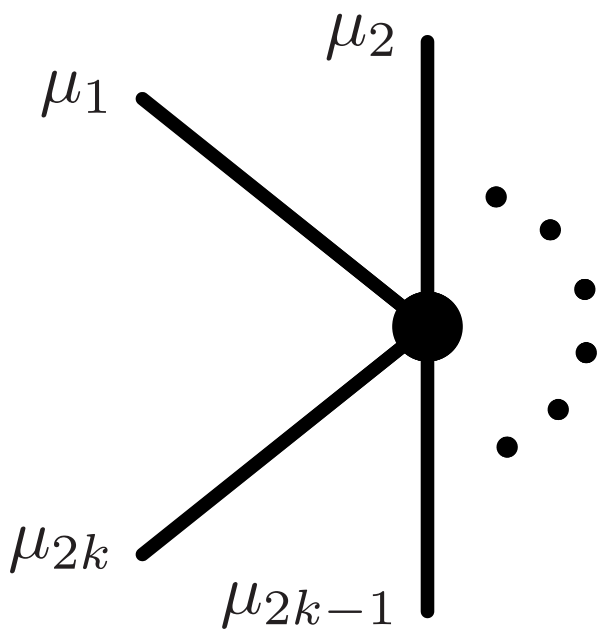
- Definition

$$v^{(k)}_{[\mu_1\ \mu_2\ \dots\ \mu_{2k-1}\ \mu_{2k}]} \equiv \frac{1}{(2k)!} \sum_{\sigma \in S_{2k}} \epsilon(\sigma)\ v^{(k)}_{\mu_{\sigma(1)}\ \mu_{\sigma(2)}\ \dots\ \mu_{\sigma(2k-1)}\ \mu_{\sigma(2k)}}$$

- Antisymmetrization defines a new (0,2k)-tensor
- Would *not* be the case in a mixed representation

k-body vertex

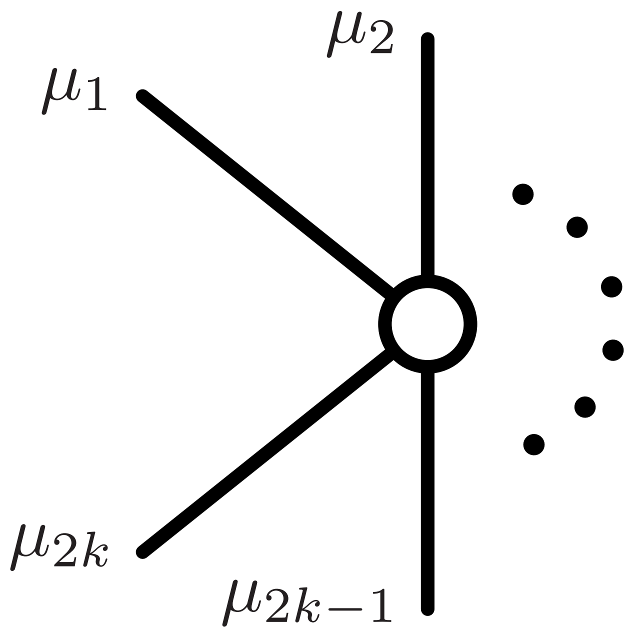
$$v^{(k)}_{[\mu_1\ \mu_2\ \dots\ \mu_{2k-1}\ \mu_{2k}]} =$$



Why fully antisymmetric vertices?

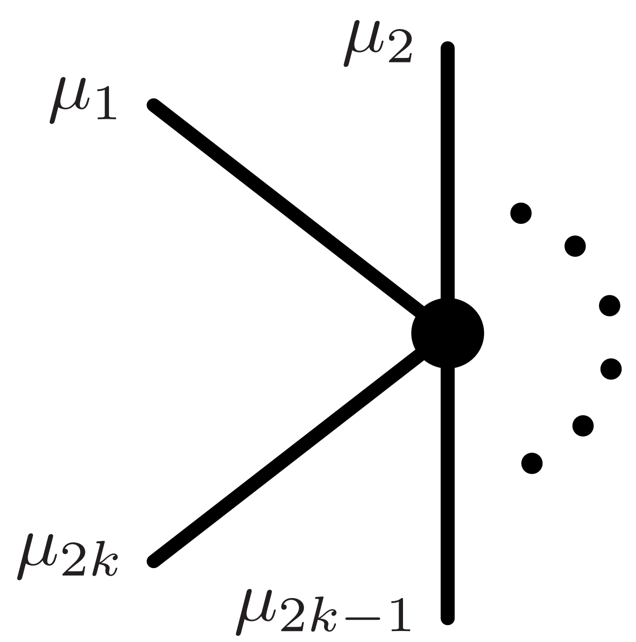
un-symmetrized k-body vertex

$$v^{(k)}_{\mu_1\,\mu_2\,\dots\,\mu_{2k-1}\,\mu_{2k}} =$$



Fully antisymmetrized k-body vertex

$$v^{(k)}_{[\mu_1\,\mu_2\,\dots\,\mu_{2k-1}\,\mu_{2k}]} =$$



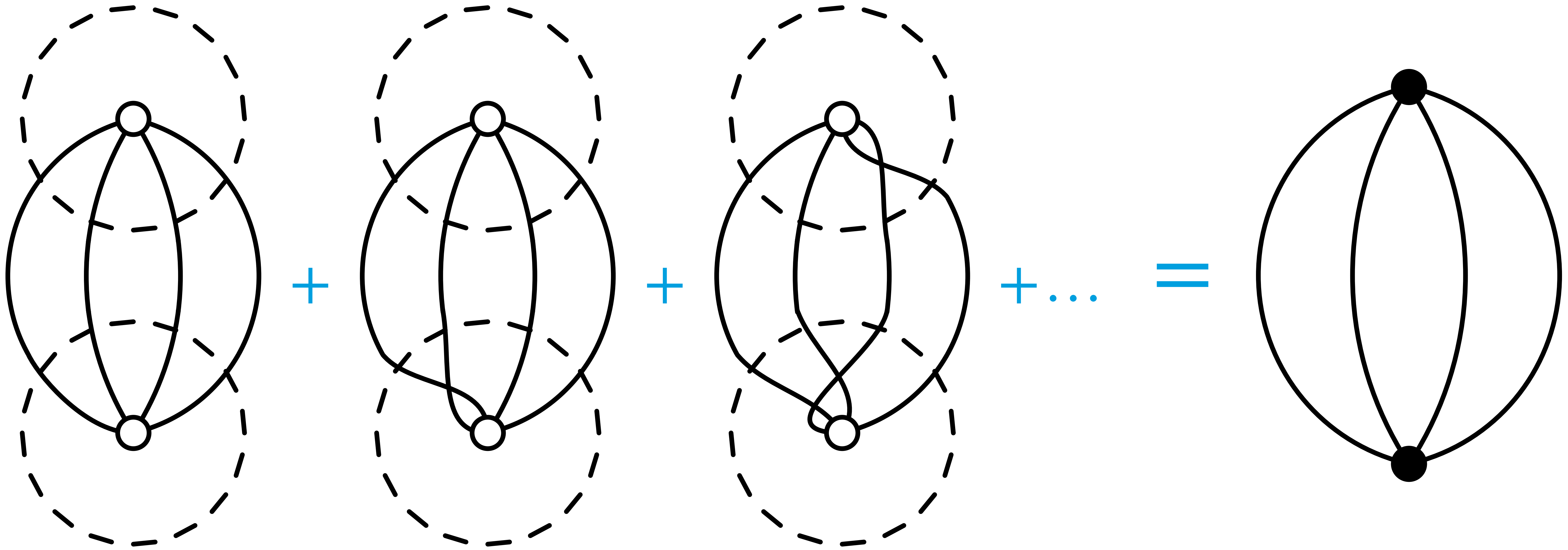


Diagram factorisation

- Derivation rely on
  - Wick theorem ⇒ sum over pairing
  - Sum over single-particle and Nambu indices

➡ **Extends Hugenholtz antisymmetrization**

Simpler diagrammatic !

Diagrammatic rules for Green's functions

Graphical rules for connected k-body Green's function

- Draw all topologically distinct unlabelled diagrams:
  - with 2k external legs
  - with n vertices (for order n contributions)
  - which is connected

Algebraic rules for connected k-body Green's function

- Label vertices from 1 to n
  - S ≡ number of vertex labels permutations leaving invariant the diagram
- For each line multiply by $-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_e)$
- For each k-body vertex multiply by $v^{(k)}_{[\mu_1\,\mu_2\,\ldots\,\mu_{2k-1}\,\mu_{2k}]}$
- Sum over each internal μ index and each independent ω_e frequency
- Multiply by $\dfrac{(-1)^{n+L}}{S \times 2^T\ \prod_{l=2}^{l_{\max}} (l!)^{m_l}}$

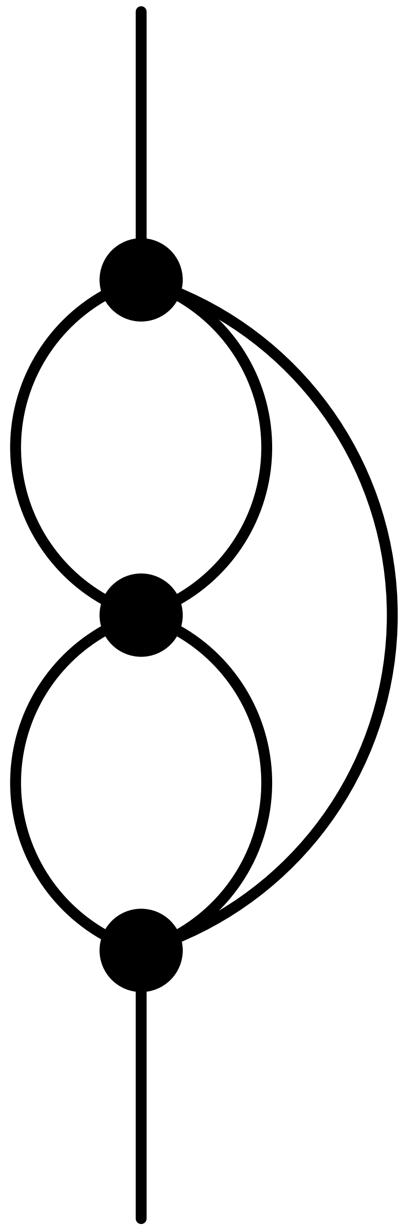
Tadpole case

- Propagator on a tadpole
  - Divergent Matsubara sum ⇒ ambiguity to be lifted
  - Rule: $-\left(\mathscr{G}^{(0)}\right)^{\mu\nu}(\omega_e)\ e^{-\omega_e \eta}$
- Vertex with tadpole
  - Antisymmetrization must be partial
  - Rule for p tadpoles:

$$v^{(k)}_{[\mu_1\ldots\dot{\mu}_x\ldots\dot{\mu}_y\ldots\mu_{2k}]} \equiv \frac{2^p p!}{(2k)!} \sum_{\sigma\in S_{2k}/S_2^p\times S_p} \epsilon(\sigma)\ v^{(k)}_{\mu_{\sigma(1)}\ldots\dot{\mu}_x\ldots\dot{\mu}_y\ldots\mu_{\sigma(2k)}}$$

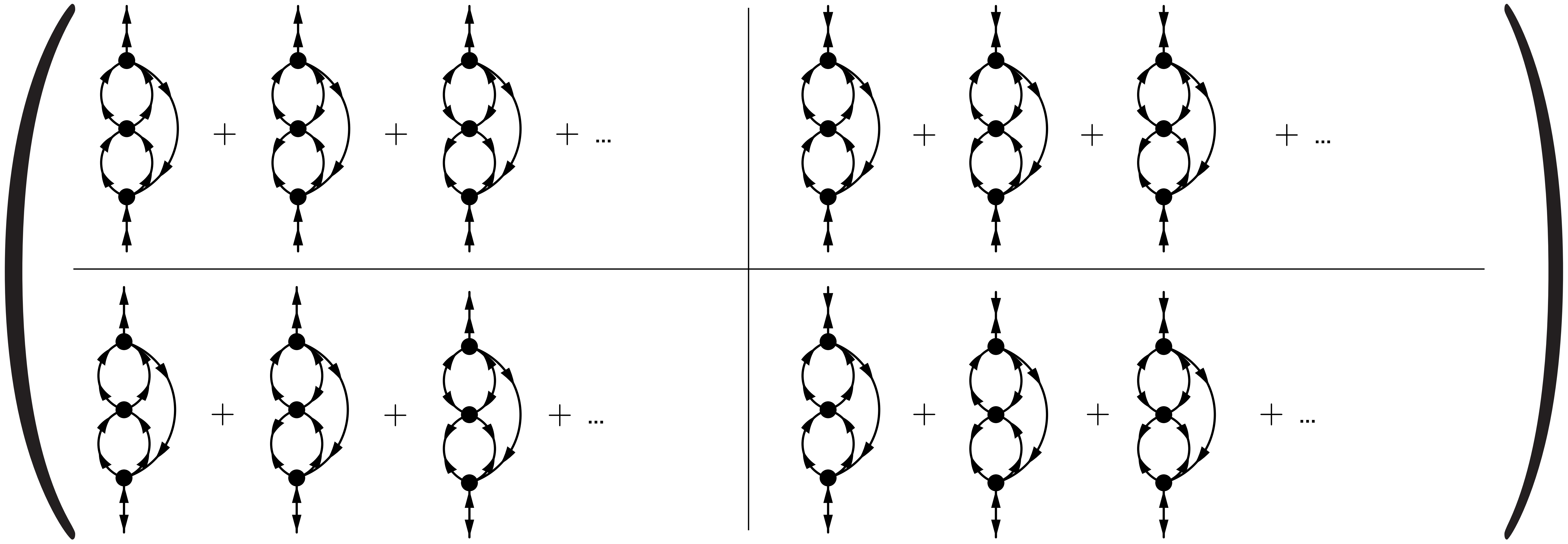
  - Tadpoles permutation or inside one not taken into account

Example and connection with Gorkov diagrams



Using
$\mathscr{B}^e = \mathscr{B} \cup \mathscr{B}^\dagger$

Fixing external
Nambu indices



Unperturbed propagator

- $H_0 \equiv \frac{1}{2} \sum_{\mu\nu} U_{\mu\nu} \, \mathrm{A}^\mu \mathrm{A}^\nu$
- $\mathscr{G}^{(0)}(\omega_p) = \left(i\omega_p - U \right)^{-1}$

A diagram contributing to the propagator at 3rd order

- $\mathscr{A}^{\mu\nu}_{(3)}(\omega_m) = \frac{1}{(2!)^2} \sum_{\lambda_1 \lambda_4''} \mathscr{G}^{(0)\mu\lambda_1}(\omega_m) \times \sum_{\substack{\lambda_2\lambda_3\lambda_4 \\ \lambda_1'\lambda_2'\lambda_3'\lambda_4' \\ \lambda_1''\lambda_2''\lambda_3''}} v^{(2)}_{[\lambda_1\lambda_2\lambda_3\lambda_4]} v^{(2)}_{[\lambda_4'\lambda_3'\lambda_2'\lambda_1']} v^{(2)}_{[\lambda_1''\lambda_2''\lambda_3''\lambda_4'']} \, I^{\lambda_2\lambda_3\lambda_4\lambda_1'\lambda_2'\lambda_3'\lambda_4'\lambda_1''\lambda_2''\lambda_3''}_{3,\mathrm{Matsubara}} \times \mathscr{G}^{(0)\lambda_4''\nu}(\omega_m)$
- where $I_{3,\mathrm{Matsubara}}$ is the sum over Matsubara frequencies of a product of $\mathscr{G}^{(0)}(\omega_p)$

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  - Nambu-covariant perturbation theory
  - Self-consistent ladder approximation
- **Selected applications**
  - First approximation: general complex HFB
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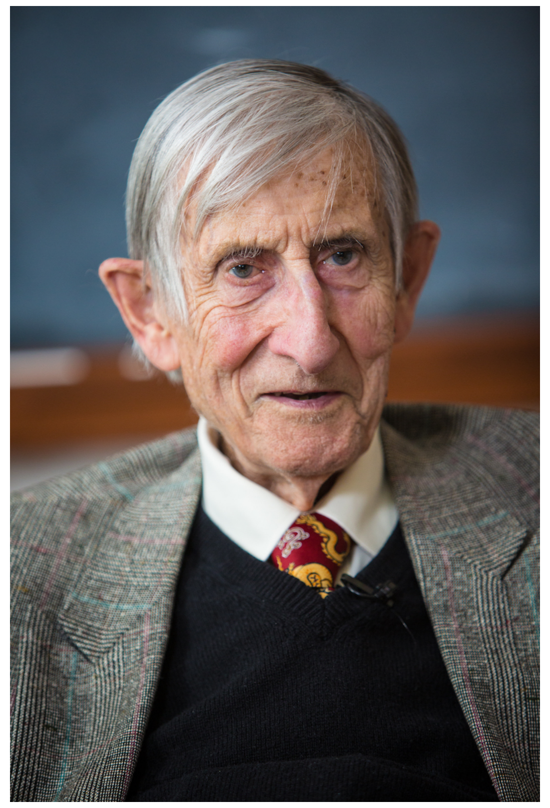
Self-consistent Green's functions



Freeman Dyson

Dyson-Schwinger equation

- Partitioning considered

$$H = \underbrace{\frac{1}{2!}\sum_{\mu\nu} U_{\mu\nu}\, \mathrm{A}^{\mu}\mathrm{A}^{\nu}}_{H_0} + \underbrace{\frac{1}{4!}\sum_{\alpha\beta\gamma\delta} v^{(2)}_{\alpha\beta\gamma\delta}\, \mathrm{A}^{\alpha}\mathrm{A}^{\beta}\mathrm{A}^{\gamma}\mathrm{A}^{\delta}}_{H_1}$$

- Dyson-Schwinger equation [Dyson, 1949] [Schwinger, 1951]

$$\mathscr{G}^{\mu\nu}(\omega_n) = \mathscr{G}^{(0)\mu\nu}(\omega_n) + \sum_{\lambda_1\lambda_2} \mathscr{G}^{(0)\mu\lambda_1}(\omega_n)\, \Sigma_{\lambda_1\lambda_2}(\omega_n)\, \mathscr{G}^{\lambda_2\nu}(\omega_n)$$

Diagrammatic expansion of $\Sigma_{\mu\nu}(\omega_n)$

- with unperturbed propagators

$$-\mathscr{G}^{\mu\nu}(\omega_p) = \sum_{\mathscr{D}\in\mathscr{S}} \mathscr{A}^{\mu\nu}[\mathscr{G}^{(0)}](\omega_p)$$

$$-\Sigma_{\mu\nu}(\omega_p) = \sum_{\mathscr{D}\in\mathscr{S}'_{1PI}} \mathscr{A}_{\mu\nu}[\mathscr{G}^{(0)}](\omega_p)$$

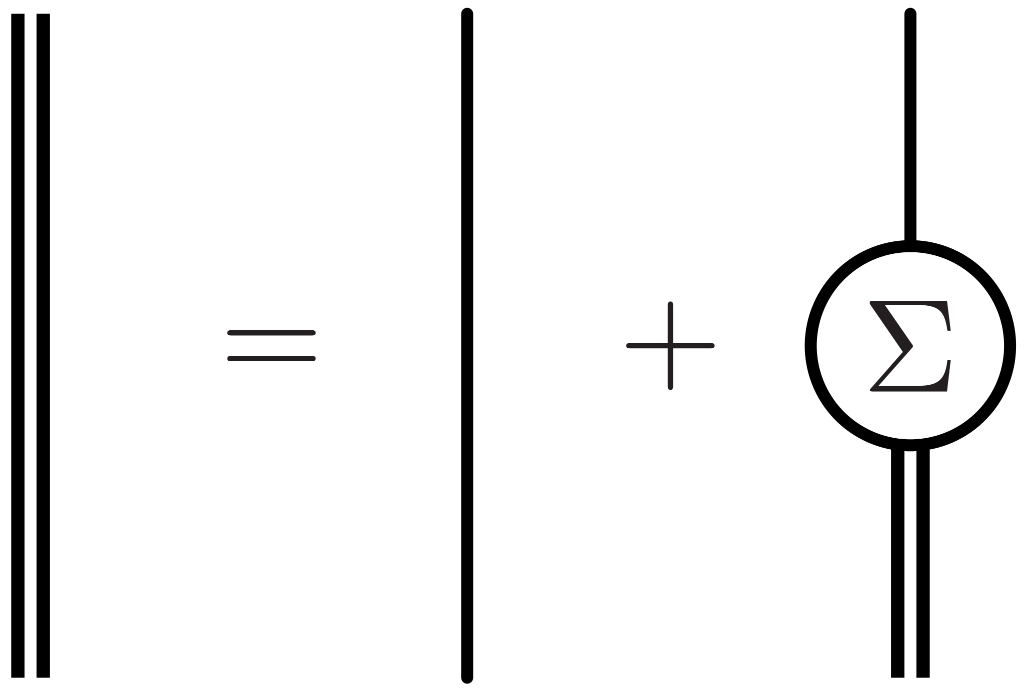
- with self-consistent propagators

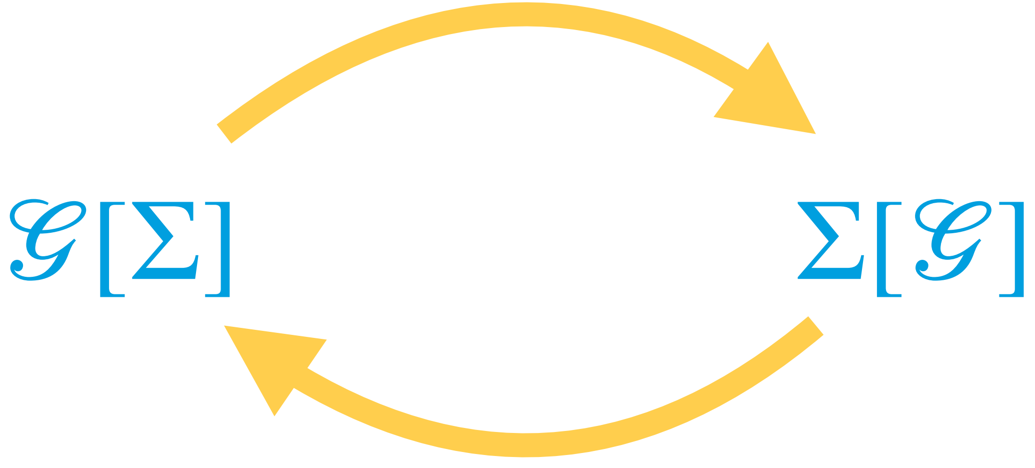
$$-\Sigma_{\mu\nu}(\omega_p) = \sum_{\mathscr{D}\in\mathscr{S}'_{SK}} \mathscr{A}_{\mu\nu}[\mathscr{G}](\omega_p)$$

Reduced set of diagrams

Julian Schwinger

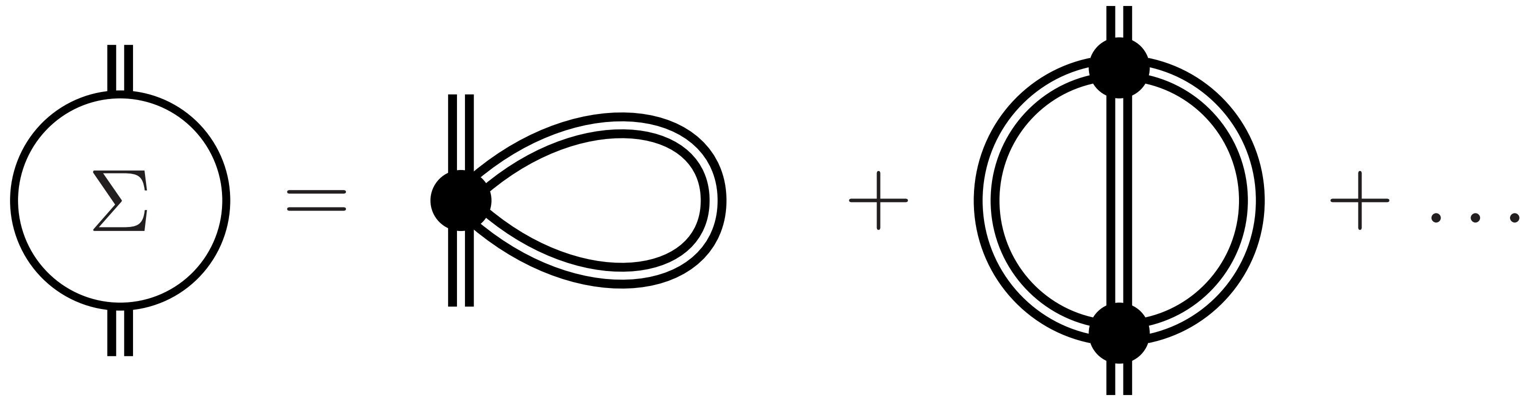
Diagrammatic representation





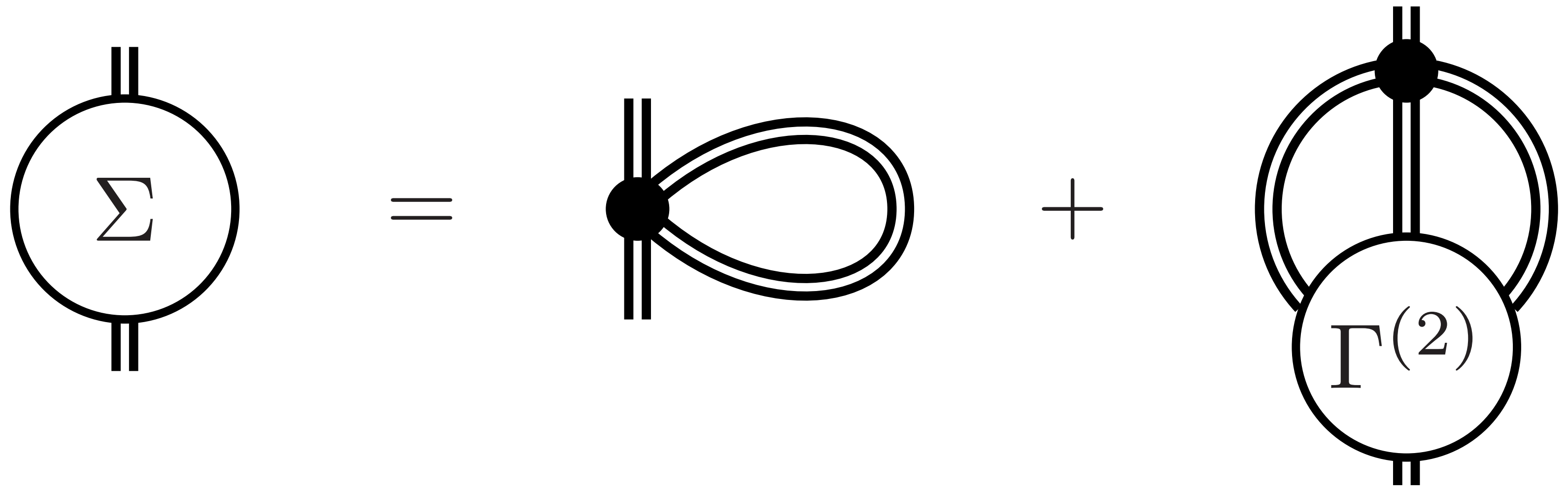
SCGF cycle

Self-energy expression

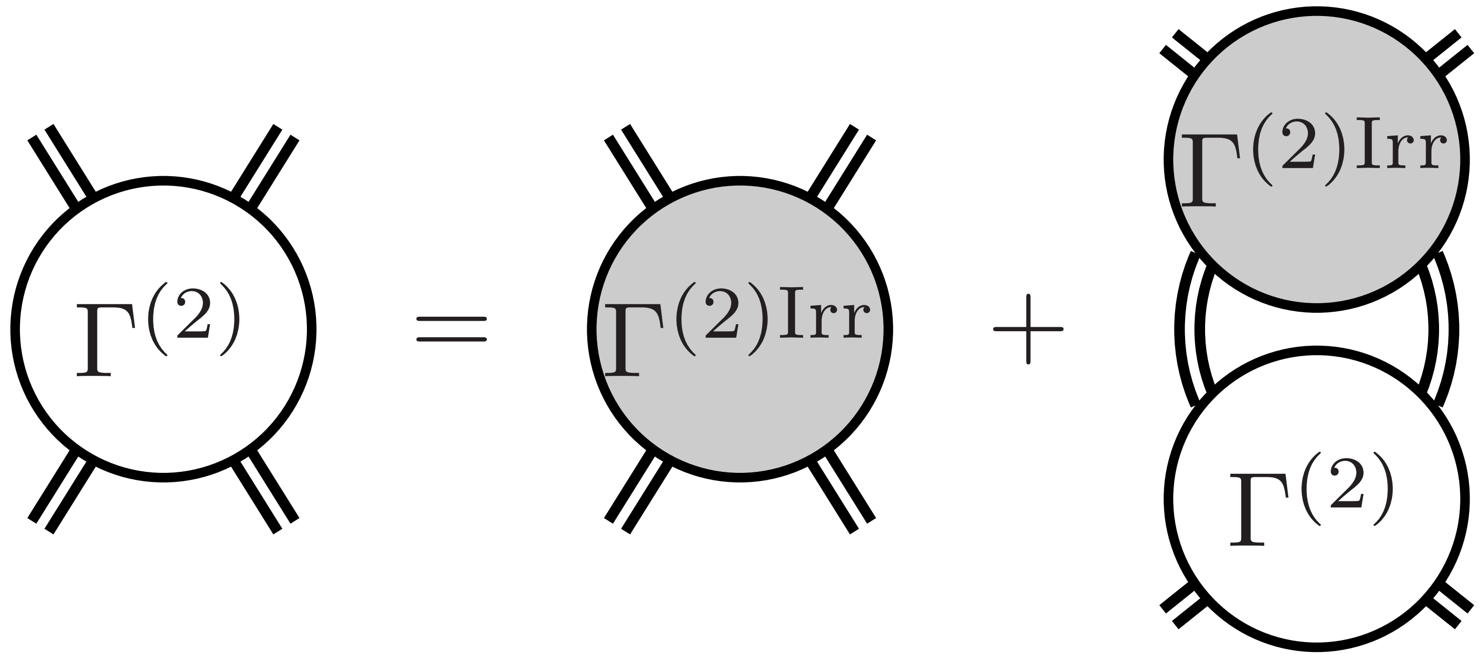


Self-consistent ladder approximation

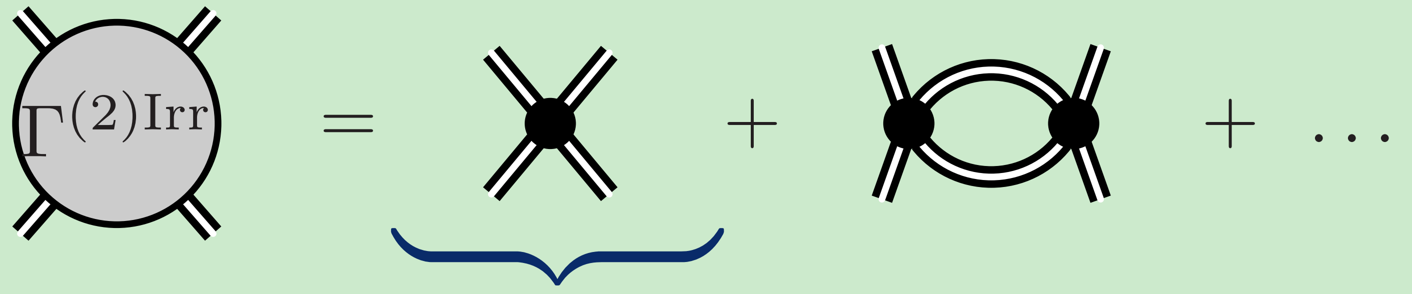
Equation of Motion: $\Sigma\left[\mathscr{G},\Gamma^{(2)}\right]$



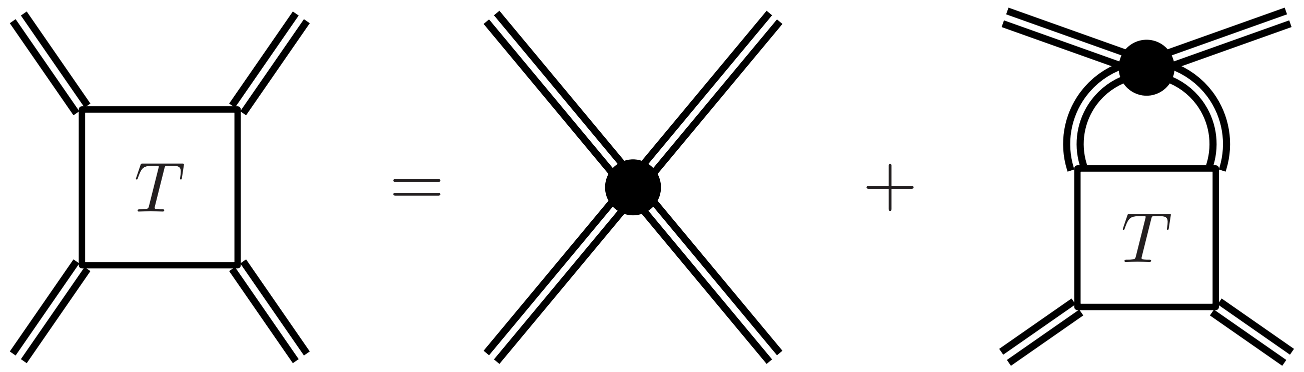
Bethe-Salpeter equation: $\Gamma^{(2)}[\mathscr{G},\Gamma^{(2)\mathrm{Irr}}]$

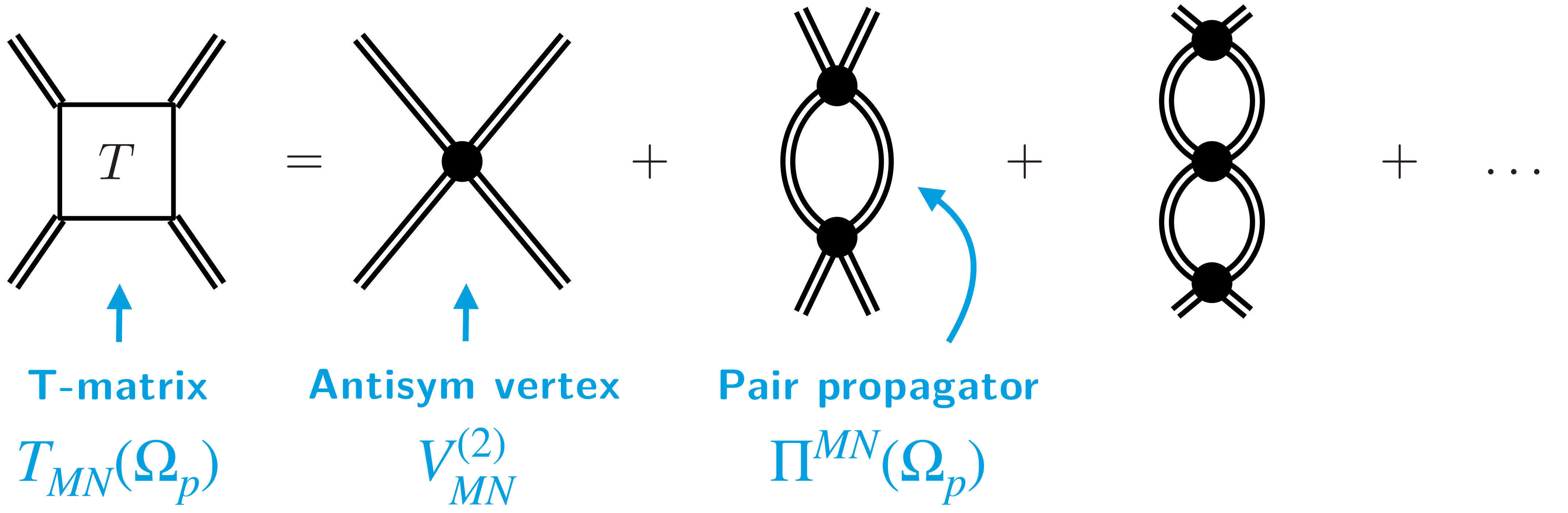


Approximations on $\Gamma^{(2)\mathrm{Irr}}$: ladder's rung



T-matrix $\equiv \Gamma^{(2)}$ in ladder approximation





Ladder approximation

- T-matrix equation
  - $T_{MN}(\Omega_p) \;=\; V^{(2)}_{MN} + \frac{1}{2}\sum_{LL'} V^{(2)}_{ML}\,\Pi^{LL'}(\Omega_p)\,T_{L'N}(\Omega_p)$

    where $V^{(2)}_{MN} \equiv v^{(2)}_{[\mu_1\mu_2\nu_1\nu_2]}$, $M \equiv (\mu_1,\mu_2)$ and $N \equiv (\nu_1,\nu_2)$
- Explicit solution
  - $T(\Omega_p) = V^{(2)}\left(1 - \frac{1}{2}\Pi(\Omega_p)V^{(2)}\right)^{-1}$

Self-consistent ladder approximation

Self-consistent ladder approximation

Initial guess

Unperturbed $S^{(0)}(\omega)$

or

Refined initial $S(\omega)$

Pair propagator Π

$$P^{MN}(\Omega) = \frac{1}{b(\Omega)}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{2\pi}\, S^{\mu_1\nu_1}(\omega)\, f(\omega) \times S^{\mu_2\nu_2}(\Omega-\omega)\, f(\Omega-\omega)$$

$$\Pi^{MN}(\Omega_p) = \int_{-\infty}^{+\infty}\frac{\mathrm{d}\Omega'}{2\pi}\frac{P^{MN}(\Omega')}{i\Omega_p-\Omega'}$$

T-matrix

$$\mathcal{T}(\Omega) = iV^{(2)}\left\{\left(1-\frac{1}{2}\Pi^R(\Omega)V^{(2)}\right)^{-1} - \left(1-\frac{1}{2}\Pi^A(\Omega)V^{(2)}\right)^{-1}\right\}$$

Self-energy Σ

$$\Gamma_{\mu\nu}(\omega) = -\frac{1}{3}\sum_{\lambda_1\lambda_2}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega'}{2\pi}\left[f(\omega')+b(\omega'-\omega)\right] \times \mathcal{T}_{\mu\lambda_1\lambda_2\nu}(\omega-\omega')\, S^{\lambda_1\lambda_2}(\omega')$$

$$\Sigma^{\infty}_{\mu\nu} = \frac{1}{2}\sum_{\mu_2\mu_3} v^{(2)}_{[\mu\dot\mu_2\dot\mu_3\nu]}\int_{-\infty}^{+\infty}\frac{\mathrm{d}\epsilon}{2\pi}\, f(-\epsilon)\, S^{\mu_2\mu_3}(\epsilon)$$

$$\Sigma_{\mu\nu}(\omega_p) = \Sigma^{\infty}_{\mu\nu} + \int_{-\infty}^{+\infty}\frac{\mathrm{d}\omega}{2\pi}\frac{\Gamma_{\mu\nu}(\omega)}{i\omega_p-\omega}$$

Propagator $\mathcal{G}$

$$S(\omega) = i\left(\omega-\left(U+\Sigma^R(\omega)\right)\right)^{-1} - i\left(\omega-\left(U+\Sigma^A(\omega)\right)\right)^{-1}$$

Example: T-matrix equation in a plane-wave basis

Plane-wave basis

- Single-particle plane-wave basis

$$\mathscr{B}_{\text{pw}} \equiv \{\, |\, \vec{k}, s, \sigma, t, \tau > \,\}$$

- Time-reversed basis

$$\tilde{\mathscr{B}}_{\text{pw}} \equiv \{\, |\, (-\vec{k}), s, (-\sigma), t, \tau > \,\}$$

- Extended one-body basis

$$\mathscr{B}^{e}_{\text{pw}} \equiv \mathscr{B}_{\text{pw}} \cup \tilde{\mathscr{B}}^{\dagger}_{\text{pw}}$$

- Two-body potential

$$\bar{V}_{(\vec{k}_1\sigma_1\tau_1)(\vec{k}_2\sigma_2\tau_2)(\vec{k}'_1\sigma'_1\tau'_1)(\vec{k}'_2\sigma'_2\tau'_2)} \equiv \left\langle \vec{k}_1\sigma_1\tau_1, \vec{k}_2\sigma_2\tau_2 \,\middle|\, V \,\middle|\, \vec{k}'_1\sigma'_1\tau'_1, \vec{k}'_2\sigma'_2\tau'_2 \right\rangle$$

- Assuming time-reversal invariant potential

$$\begin{aligned} v^{(2)}{}_{[(\vec{k}_1\sigma_1\tau_1,l_1)(\vec{k}_2\sigma_2\tau_2,l_2)(\vec{k}_3\sigma_3\tau_3,l_3)(\vec{k}_4\sigma_4\tau_4,l_4)]} &= \bar{V}_{(-\vec{k}_1-\sigma_1\tau_1)(-\vec{k}_2-\sigma_2\tau_2)(\vec{k}_4\sigma_4\tau_4)(\vec{k}_3\sigma_3\tau_3)}(E^{21}_{l_1l_4}E^{21}_{l_2l_3} + E^{21}_{l_3l_2}E^{21}_{l_4l_1}) \\ &- \bar{V}_{(-\vec{k}_1-\sigma_1\tau_1)(-\vec{k}_3-\sigma_3\tau_3)(\vec{k}_4\sigma_4\tau_4)(\vec{k}_2\sigma_2\tau_2)}(E^{21}_{l_1l_4}E^{21}_{l_3l_2} + E^{21}_{l_2l_3}E^{21}_{l_4l_1}) \\ &+ \bar{V}_{(-\vec{k}_1-\sigma_1\tau_1)(-\vec{k}_4-\sigma_4\tau_4)(\vec{k}_3\sigma_3\tau_3)(\vec{k}_2\sigma_2\tau_2)}(E^{21}_{l_1l_3}E^{21}_{l_4l_2} + E^{21}_{l_2l_4}E^{21}_{l_3l_1}) \end{aligned}$$

T-matrix equation

$$T^R_{MN}(\Omega) = V^{(2)}_{MN} + \frac{1}{2}\sum_{LL'} V^{(2)}_{ML}\, \Pi^{RLL'}(\Omega)\, T^R_{L'N}(\Omega)$$

⬇

$$\begin{aligned} &(T^R)^{g_1g_2,g'_1g'_2}_{\lambda_1\lambda_2,\lambda'_1\lambda'_2}\left(\vec{K},\vec{k},\vec{k}',\Omega\right) \\ &= \Big[\bar{V}_{(\vec{k}\lambda_1)(-\vec{k}\lambda_2)(-\vec{k}'\lambda'_2)(\vec{k}'\lambda'_1)}\left(E^{11}_{g_1g_2}E^{11}_{g'_1g'_2} + E^{22}_{g_1g_2}E^{22}_{g'_1g'_2}\right) \\ &\quad -\bar{V}_{(\vec{K}+\vec{k}\lambda_1)(-\vec{K}-\vec{k}'\tilde{\lambda}'_1)(\vec{K}-\vec{k}'\lambda'_2)(-\vec{K}+\vec{k}\tilde{\lambda}_2)}\left(E^{12}_{g_1g_2}E^{21}_{g'_1g'_2} + E^{21}_{g_1g_2}E^{12}_{g'_1g'_2}\right) \\ &\quad +\bar{V}_{(\vec{K}+\vec{k}\lambda_1)(-\vec{K}+\vec{k}'\tilde{\lambda}'_2)(\vec{K}+\vec{k}'\lambda'_1)(-\vec{K}+\vec{k}\tilde{\lambda}_2)}\left(E^{12}_{g_1g_2}E^{12}_{g'_1g'_2} + E^{21}_{g_1g_2}E^{21}_{g'_1g'_2}\right)\Big] \\ &+\frac{1}{2}\int\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\sum_{\substack{\kappa_1\kappa_2\\ \kappa'_1\kappa'_2}}\sum_{h'_1h'_2} \\ &\quad \Big[\bar{V}_{(\vec{k}\lambda_1)(-\vec{k}\lambda_2)(-\vec{q}\kappa_2)(\vec{q}\kappa_1)}\Big((\Pi^R)^{11,h'_1h'_2}_{\kappa_1\kappa_2,\kappa'_1\kappa'_2}\left(\vec{K}+\vec{q},\vec{K}-\vec{q},\Omega\right)E^{11}_{g_1g_2} \\ &\qquad +(\Pi^R)^{22,h'_1h'_2}_{\kappa_1\kappa_2,\kappa'_1\kappa'_2}\left(\vec{K}+\vec{q},\vec{K}-\vec{q},\Omega\right)E^{22}_{g_1g_2}\Big) \\ &\quad -\bar{V}_{(\vec{K}+\vec{k}\lambda_1)(-\vec{K}-\vec{q}\tilde{\kappa}_1)(\vec{K}-\vec{q}\kappa_2)(-\vec{K}+\vec{k}\tilde{\lambda}_2)}\Big((\Pi^R)^{21,h'_1h'_2}_{\kappa_1\kappa_2,\kappa'_1\kappa'_2}\left(\vec{K}+\vec{q},\vec{K}-\vec{q},\Omega\right)E^{12}_{g_1g_2} \\ &\qquad +(\Pi^R)^{12,h'_1h'_2}_{\kappa_1\kappa_2,\kappa'_1\kappa'_2}\left(\vec{K}+\vec{q},\vec{K}-\vec{q},\Omega\right)E^{21}_{g_1g_2}\Big) \\ &\quad +\bar{V}_{(\vec{K}+\vec{k}\lambda_1)(-\vec{K}+\vec{q}\tilde{\kappa}_2)(\vec{K}+\vec{q}\kappa_1)(-\vec{K}+\vec{k}\tilde{\lambda}_2)}\Big((\Pi^R)^{12,h'_1h'_2}_{\kappa_1\kappa_2,\kappa'_1\kappa'_2}\left(\vec{K}+\vec{q},\vec{K}-\vec{q},\Omega\right)E^{12}_{g_1g_2} \\ &\qquad +(\Pi^R)^{21,h'_1h'_2}_{\kappa_1\kappa_2,\kappa'_1\kappa'_2}\left(\vec{K}+\vec{q},\vec{K}-\vec{q},\Omega\right)E^{21}_{g_1g_2}\Big)\Big] \\ &\times(T^R)^{h'_1h'_2,g'_1g'_2}_{\kappa'_1\kappa'_2,\lambda'_1\lambda'_2}\left(\vec{K},\vec{q},\vec{k}',\Omega\right) \end{aligned}$$

Many-body system

- Homogeneous nuclear matter
- Conserved symmetry
  - Only translation invariance
  - ➡ Polarized asymmetric nuclear matter
- Simplifications from assumed homogeneity

$$\begin{aligned} (T^R)^{(\vec{p}_1\sigma_1\tau_1g_1,\vec{p}_2\sigma_2\tau_2g_2)}{}_{(\vec{p}'_1\sigma'_1\tau'_1g'_1,\vec{p}'_2\sigma'_2\tau'_2g'_2)}(\Omega) &\equiv (T^R)^{g_1g_2,g'_1g'_2}_{(\sigma_1\tau_1)(\sigma_2\tau_2),(\sigma'_1\tau'_1)(\sigma'_2\tau'_2)}\left(\vec{K},\vec{k},\vec{k}',\Omega\right) \\ &\times\frac{(2\pi)^3}{2^3}\,\delta^{(3)}\left(\vec{K}-\vec{K}'\right), \\ (\Pi^R)^{(\vec{p}_1\sigma_1\tau_1g_1,\vec{p}_2\sigma_2\tau_2g_2)}{}_{(\vec{p}'_1\sigma'_1\tau'_1g'_1,\vec{p}'_2\sigma'_2\tau'_2g'_2)}(\Omega) &\equiv (\Pi^R)^{g_1g_2,g'_1g'_2}_{(\sigma_1\tau_1)(\sigma_2\tau_2),(\sigma'_1\tau'_1)(\sigma'_2\tau'_2)}(\vec{p}_1,\vec{p}_2,\Omega) \\ &\times(2\pi)^6\,\delta^{(3)}\left(\vec{p}_1-\vec{p}'_1\right)\delta^{(3)}\left(\vec{p}_2-\vec{p}'_2\right) \end{aligned}$$

Advantages

- Simpler
- Faster

} During formal developments

- Obvious Nambu-covariance
- Re-usable for other $\mathscr{B}^{e'}$
  - Harmonic oscillators
  - Quasiparticles
  - Berggren basis
  - …

Example: T-matrix equation in a plane-wave basis

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  $\mathscr{B}^{e}_{\text{pw}} \equiv \mathscr{B}_{\text{pw}} \cup \tilde{\mathscr{B}}^{\dagger}_{\text{pw}}$

- Two-body potential

  $$\bar{V}_{(\vec{k}_1\sigma_1\tau_1)(\vec{k}_2\sigma_2\tau_2)(\vec{k}'_1\sigma'_1\tau'_1)(\vec{k}'_2\sigma'_2\tau'_2)} \equiv \left\langle \vec{k}_1\sigma_1\tau_1, \vec{k}_2\sigma_2\tau_2 \middle| V \middle| \vec{k}'_1\sigma'_1\tau'_1, \vec{k}'_2\sigma'_2\tau'_2 \right\rangle$$

- Assuming time-reversal invariant potential

$$\begin{aligned} v^{(2)}&{}_{[(\vec{k}_1\sigma_1\tau_1,l_1)(\vec{k}_2\sigma_2\tau_2,l_2)(\vec{k}_3\sigma_3\tau_3,l_3)(\vec{k}_4\sigma_4\tau_4,l_4)]} \\ &= \bar{V}_{(-\vec{k}_1-\sigma_1\tau_1)(-\vec{k}_2-\sigma_2\tau_2)(\vec{k}_4\sigma_4\tau_4)(\vec{k}_3\sigma_3\tau_3)}(E^{21}_{l_1l_4}E^{21}_{l_2l_3} + E^{21}_{l_3l_2}E^{21}_{l_4l_1}) \\ &- \bar{V}_{(-\vec{k}_1-\sigma_1\tau_1)(-\vec{k}_3-\sigma_3\tau_3)(\vec{k}_4\sigma_4\tau_4)(\vec{k}_2\sigma_2\tau_2)}(E^{21}_{l_1l_4}E^{21}_{l_3l_2} + E^{21}_{l_2l_3}E^{21}_{l_4l_1}) \\ &+ \bar{V}_{(-\vec{k}_1-\sigma_1\tau_1)(-\vec{k}_4-\sigma_4\tau_4)(\vec{k}_3\sigma_3\tau_3)(\vec{k}_2\sigma_2\tau_2)}(E^{21}_{l_1l_3}E^{21}_{l_4l_2} + E^{21}_{l_2l_4}E^{21}_{l_3l_1}) \end{aligned}$$

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$$T^R_{MN}(\Omega) = V^{(2)}_{MN} + \frac{1}{2}\sum_{LL'} V^{(2)}_{ML}\, \Pi^{RLL'}(\Omega)\, T^R_{L'N}(\Omega)$$

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Extends previous "partial sums" of ladders [Bożek, 1999, 2002]

Many-body system

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- Conserved symmetry
  - Only translation invariance
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$$\begin{aligned} (T^R)^{(\vec{p}_1\sigma_1\tau_1g_1,\vec{p}_2\sigma_2\tau_2g_2)}{}_{(\vec{p}'_1\sigma'_1\tau'_1g'_1,\vec{p}'_2\sigma'_2\tau'_2g'_2)}(\Omega) &\equiv (T^R)^{g_1g_2,g'_1g'_2}_{(\sigma_1\tau_1)(\sigma_2\tau_2),(\sigma'_1\tau'_1)(\sigma'_2\tau'_2)}\left(\vec{K},\vec{k},\vec{k}',\Omega\right) \\ &\times\frac{(2\pi)^3}{2^3}\,\delta^{(3)}\left(\vec{K}-\vec{K}'\right)\ , \\ (\Pi^R)^{(\vec{p}_1\sigma_1\tau_1g_1,\vec{p}_2\sigma_2\tau_2g_2)}{}_{(\vec{p}'_1\sigma'_1\tau'_1g'_1,\vec{p}'_2\sigma'_2\tau'_2g'_2)}(\Omega) &\equiv (\Pi^R)^{g_1g_2,g'_1g'_2}_{(\sigma_1\tau_1)(\sigma_2\tau_2),(\sigma'_1\tau'_1)(\sigma'_2\tau'_2)}\left(\vec{p}_1,\vec{p}_2,\Omega\right) \\ &\times(2\pi)^6\,\delta^{(3)}\left(\vec{p}_1-\vec{p'}_1\right)\delta^{(3)}\left(\vec{p}_2-\vec{p'}_2\right) \end{aligned}$$

Advantages

- Simpler } During formal developments
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- Obvious Nambu-covariance
- Re-usable for other $\mathscr{B}^{e'}$
  - Harmonic oscillators
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Outline

- **Nambu-covariant formalism**
  - Nambu-covariant perturbation theory
  - Self-consistent ladder approximation
- **Selected applications**
  - First approximation: general complex HFB
  - Conditions for the convergence of the series of ladders

Hartree-Fock-Bogoliubov approximation

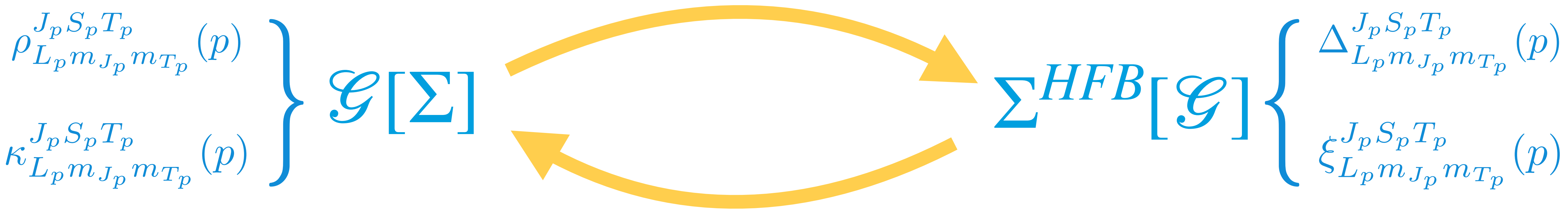
Hartree-Fock-Bogoliubov (HFB) propagator

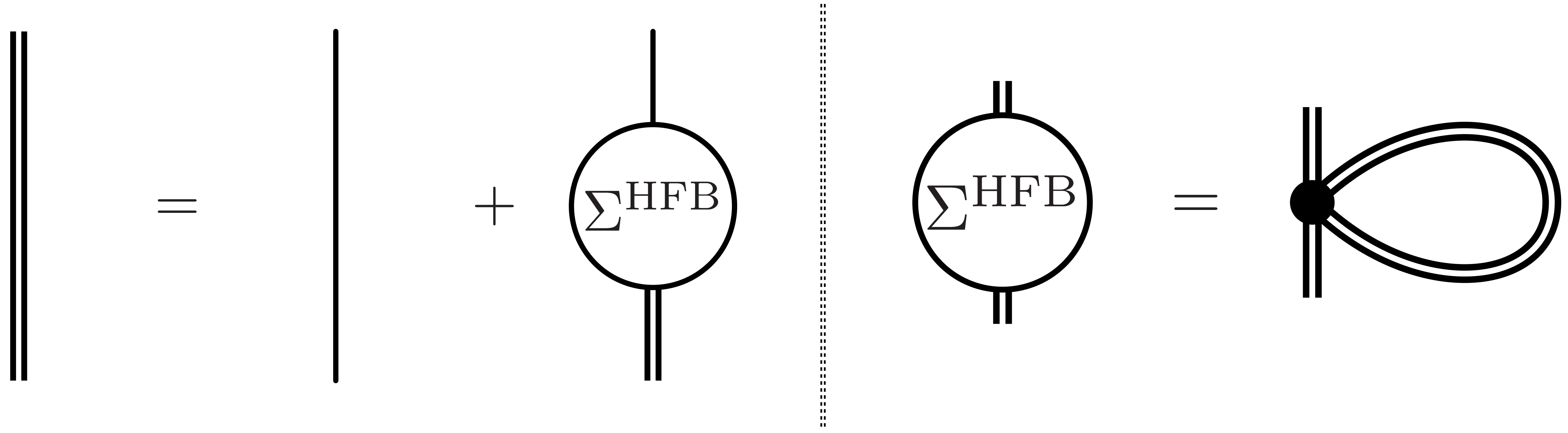
- Unperturbed propagator

$$\mathcal{G}^{HFB}(\omega_p) = \Big(i\omega_p - (U + \Sigma^{HFB})\Big)^{-1}$$

- HFB self-energy

Σ^{HFB} solution of SCGF with $\Gamma^{(2)\,\mathrm{Irr}}_{\mu_1\mu_2\mu_3\mu_4}(\tau_1,\tau_2,\tau_3,\tau_4) \equiv 0$





BCS + fixed single-particle spectrum

- Standard calculation for superfluid nuclear matter

$$\Delta^{J_pS_pT_p}_{L_pm_{J_p}m_{T_p}}(p) = \{T_p\ \tfrac12\ \tfrac12\} \times \{S_p\ \tfrac12\ \tfrac12\} \times \{J_p\ L_p\ S_p\} \times \int_0^{+\infty} \frac{(p')^2\mathrm{d}p'}{(2\pi)^3} \sum_{L_{p'}} \Bigg\{ \frac{[1-(-1)^{L_p+S_p+T_p}]}{2} \frac{[1-(-1)^{L_{p'}+S_p+T_p}]}{2} \{J_p\ L_{p'}\ S_p\} \times \left\langle p \left| V^{J_pS_pT_p}_{L_pL_{p'}} \right| p' \right\rangle \times \kappa^{J_pS_pT_p}_{L_{p'}m_{J_p}m_{T_p}}(p') \Bigg\}$$

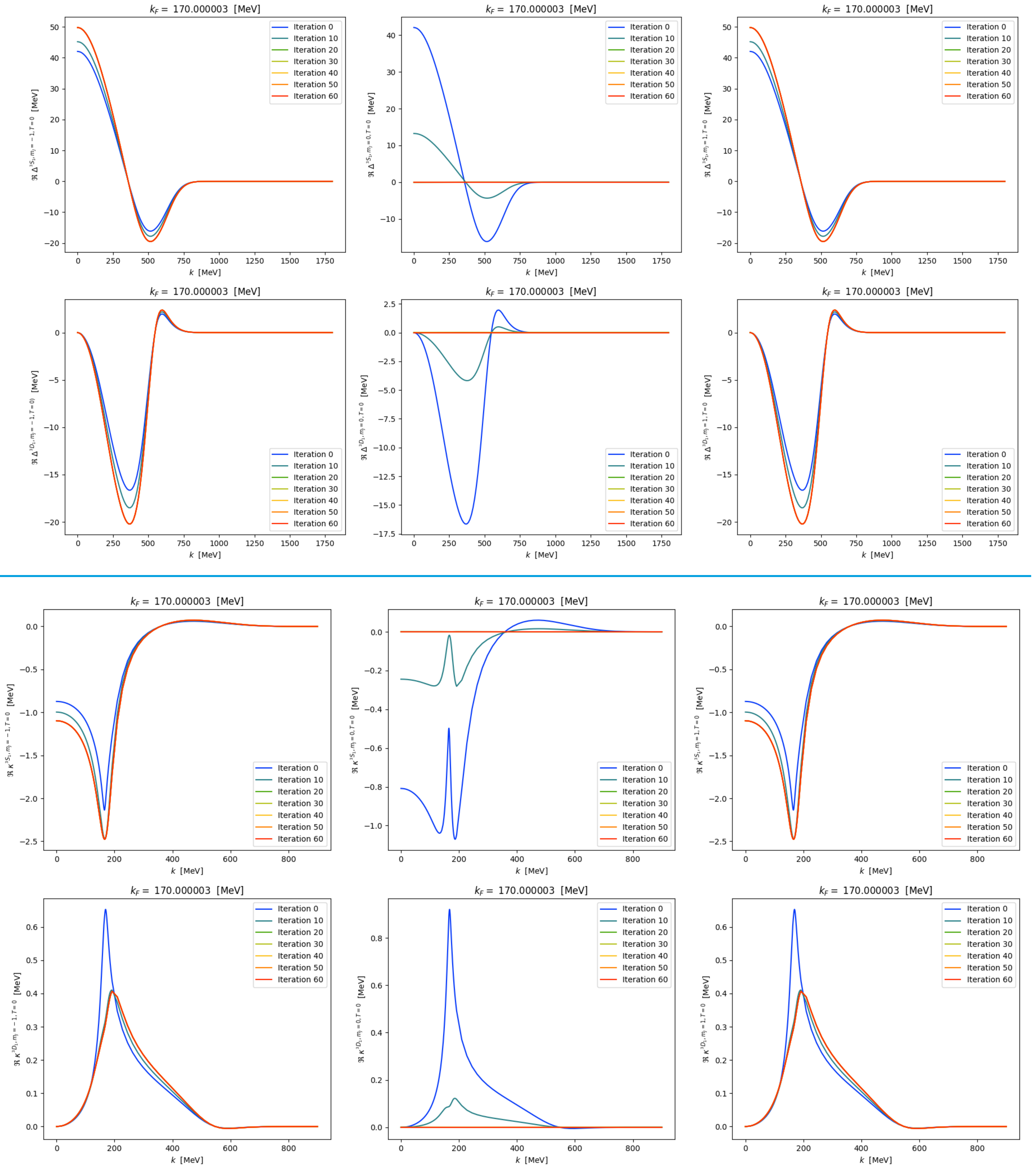
- Unitary BCS-like: ξ fixed + $\kappa[\Delta]$ ⇒ closed gap equation

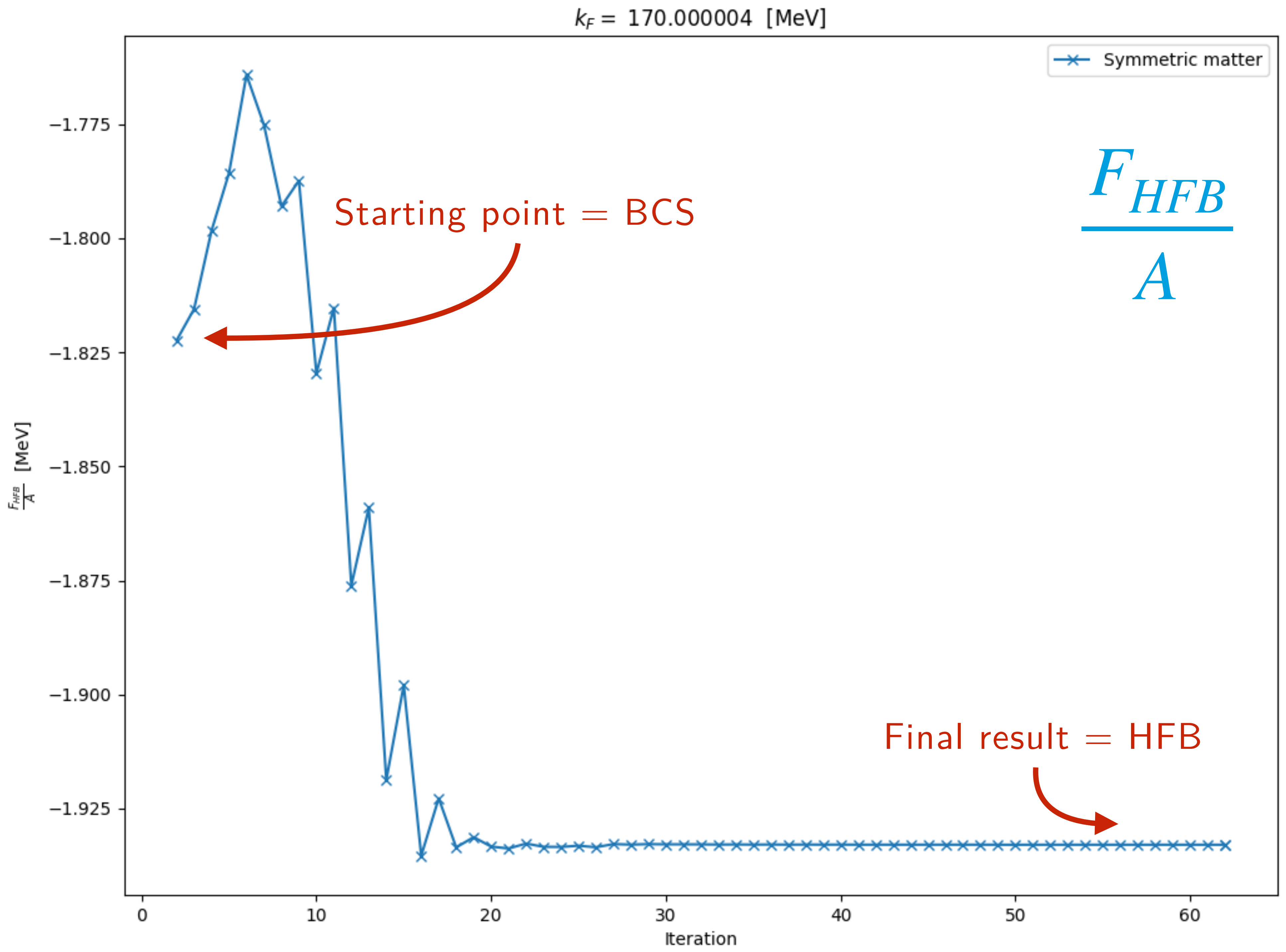
General HFB equation: the ugly truth

$$\xi^{J_pS_pT_p}_{L_pm_{J_p}m_{T_p}}(p) = (U^{11})^{J_pS_pT_p}_{L_pm_{J_p}m_{T_p}}(p) + (-1)^{L_p+S_p}(-1)^{m_{J_p}} \frac{2}{\sqrt{4\pi}} \int_0^{+\infty} \frac{(p')^2\mathrm{d}p'}{(2\pi)^3}$$
$$\times \sum_{\substack{J_{p'}S_{p'}T_{p'} \\ L_{p'}m_{J_{p'}}m_{T_{p'}}}} \sum_{\substack{JST \\ LL'm_T}} \sum_{L_V} \frac{[1-(-1)^{L+S+T}]}{2} \frac{[1-(-1)^{L'+S+T}]}{2}\, i^{L_p+L_{p'}} R^{JST,LL',m_T}_{L_VL_pL_{p'}}\left(\frac{p}{2},\frac{p'}{2}\right)$$
$$\times (\hat{L}\hat{L}'\hat{L}_p\hat{L}_{p'}) \times (\hat{L}_V)^3 \times (\hat{J}\hat{S}\hat{T})^2 \times (\hat{J}_p\hat{S}_p\hat{T}_p) \times (\hat{J}_{p'}\hat{S}_{p'}\hat{T}_{p'})$$
$$\times (-1)^{J+S+S_{p'}+T_{p'}} \begin{pmatrix} L & L' & L_V \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} L_p & L_{p'} & L_V \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} S & S & L_V \\ L & L' & J \end{Bmatrix}$$
$$\times \rho^{J_{p'}S_{p'}T_{p'}}_{L_{p'}m_{J_{p'}}m_{T_{p'}}}(p')$$
$$\times \Bigg(\sum_{T_xm_{T_x}} (-1)^{T_x-m_{T_x}} \hat{T}_x^2 \begin{pmatrix} T & T_x & T_{p'} \\ m_T & -m_{T_x} & m_{T_{p'}} \end{pmatrix} \begin{pmatrix} T_p & T_x & T \\ m_{T_p} & m_{T_x} & m_T \end{pmatrix} \times \begin{Bmatrix} T & T_x & T_{p'} \\ \frac12 & \frac12 & \frac12 \end{Bmatrix} \begin{Bmatrix} T_p & T_x & T \\ \frac12 & \frac12 & \frac12 \end{Bmatrix} \Bigg)$$
$$\times \Bigg(\sum_{\substack{S_xL_x \\ m_{S_x}m_{L_x}}} (-1)^{L_x-m_{L_x}} \hat{S}_x^2\hat{L}_x^2 \begin{pmatrix} S_x & L_x & J_{p'} \\ -m_{S_x} & m_{L_x} & -m_{J_{p'}} \end{pmatrix} \begin{pmatrix} J_p & L_x & S_x \\ m_{J_p} & -m_{L_x} & -m_{S_x} \end{pmatrix}$$
$$\times \begin{Bmatrix} S & S_x & S_p \\ \frac12 & \frac12 & \frac12 \end{Bmatrix} \begin{Bmatrix} S_{p'} & S_x & S \\ \frac12 & \frac12 & \frac12 \end{Bmatrix} \begin{Bmatrix} S_x & L_x & J_{p'} \\ L_{p'} & S_{p'} & S \end{Bmatrix} \begin{Bmatrix} J_p & L_x & S_x \\ S & S_p & L_p \end{Bmatrix} \begin{Bmatrix} S & L_x & L_p \\ L_{p'} & L_V & S \end{Bmatrix} \Bigg)$$

Pairing in symmetric matter with HFB

	$m_J = -1$	$m_J = 0$	$m_J = +1$
Re $\Delta^{{}^3S_1,\,T=0}_{m_J=-1,0,1}(p)$			
Re $\Delta^{{}^3D_1,\,T=0}_{m_J=-1,0,1}(p)$			
Re $\kappa^{{}^3S_1,\,T=0}_{m_J=-1,0,1}(p)$			
Re $\kappa^{{}^3D_1,\,T=0}_{m_J=-1,0,1}(p)$			





$$\frac{F_{HFB}}{A}$$

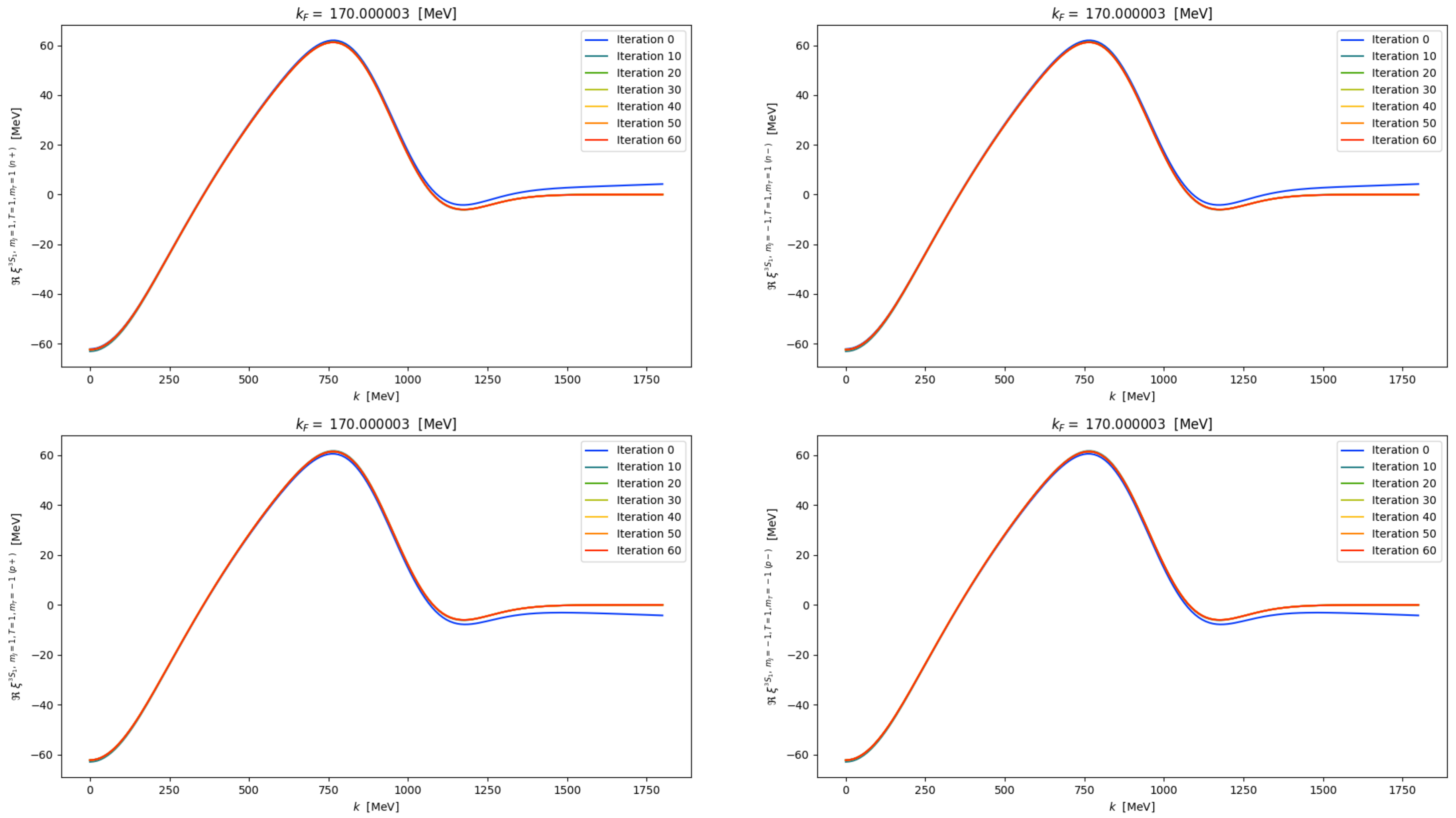
General features of HFB calculations

- Calculations here: EM500 @ $k_F = 170$ MeV @ $T = 0.2$ MeV
- Minor reduction of $\frac{F}{A}$ (~ 0.1 MeV) → **small effect on EoS**
- But important impact on gaps → **NS cooling curves impact?**
  - Partial-waves can see an increase of ~ 10 MeV
  - m_J -dependence → some partial-waves completely vanish
- And more: quadrupole deformation of the Fermi surface !

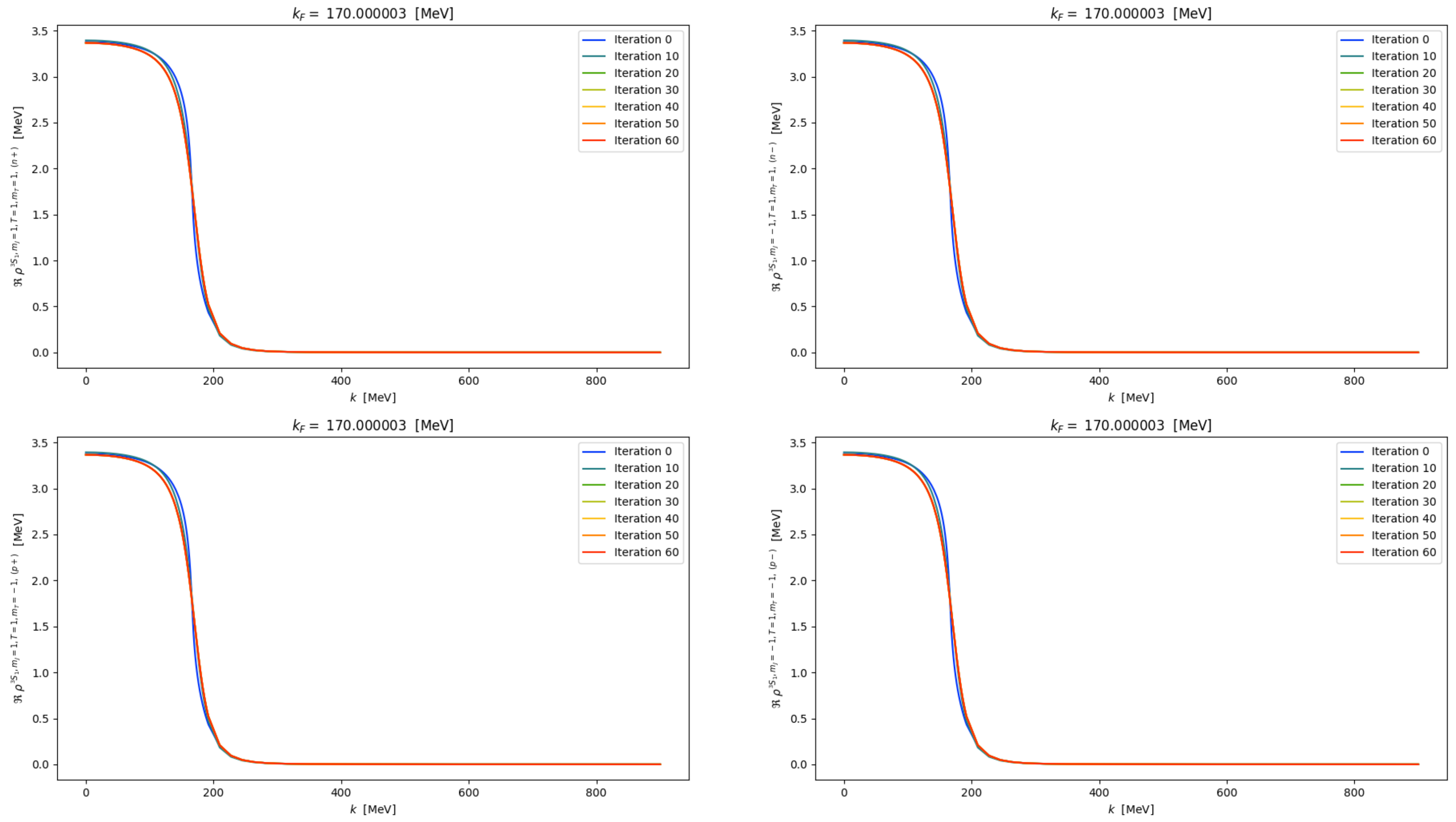
Deformation in symmetric matter with HFB

Main spherical contribution to ξ and ρ

Re $\xi^{^3S_1}_{nucleon}(p)$

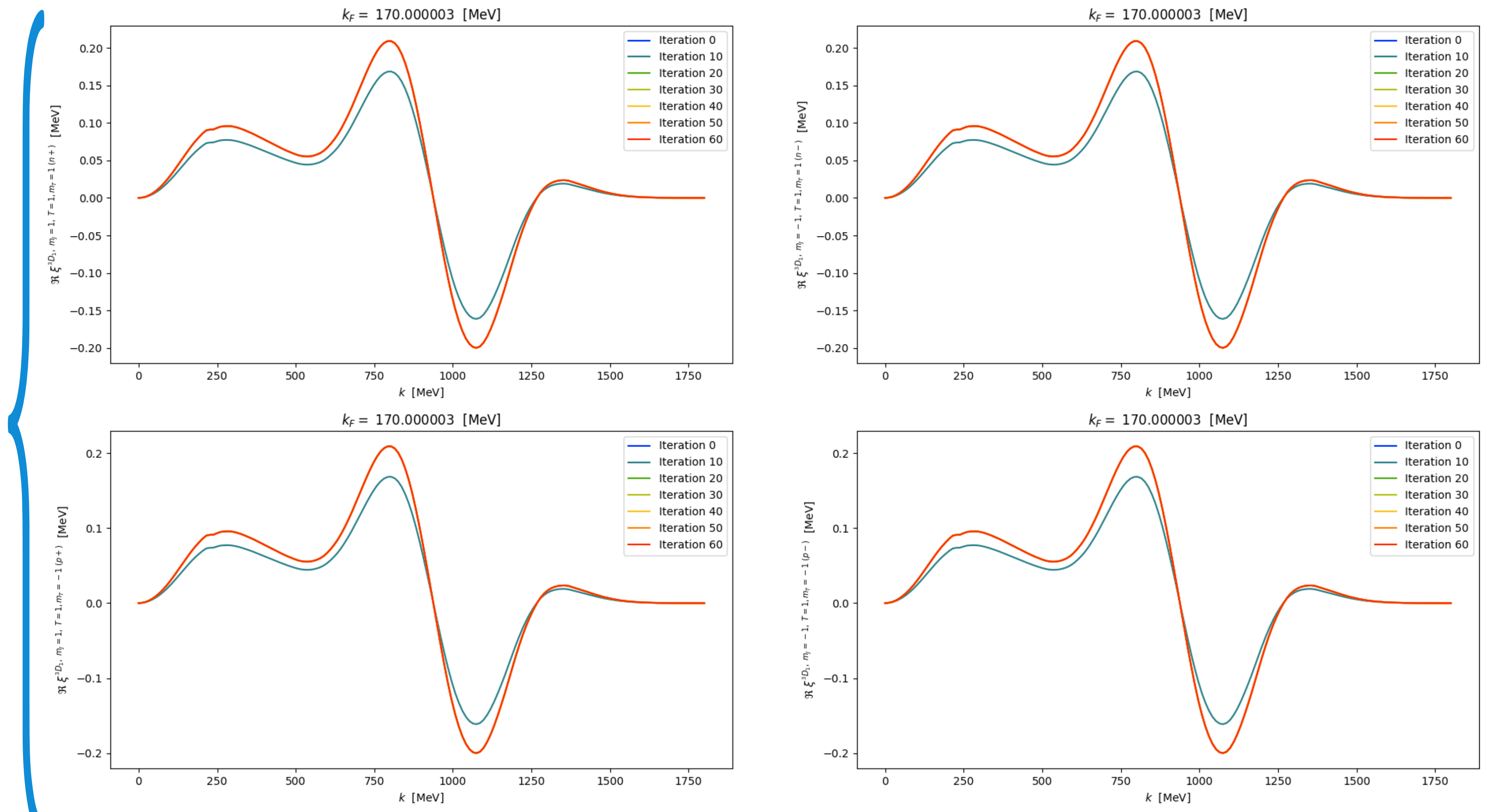


Re $\rho^{^3S_1}_{nucleon}(p)$

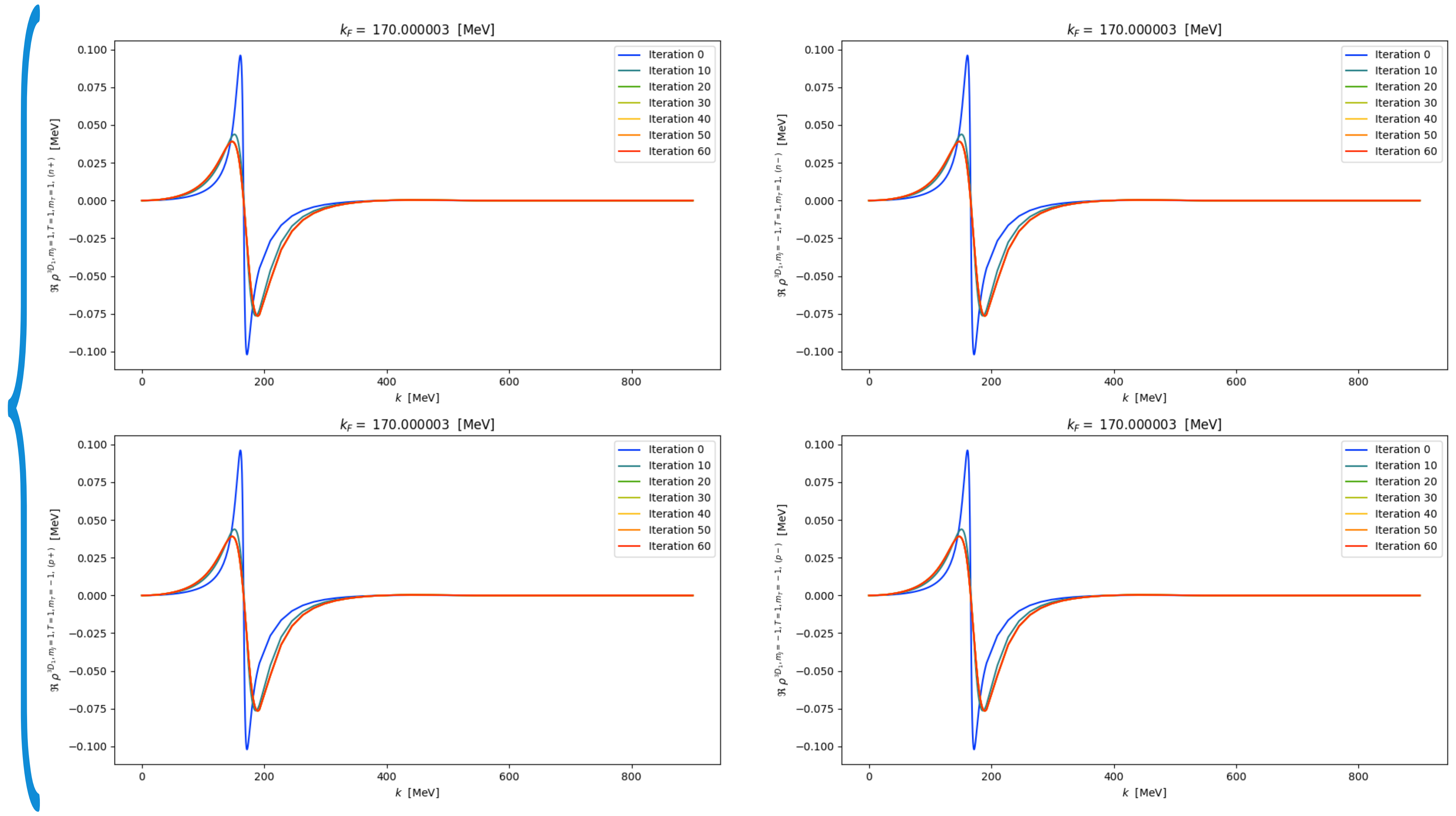


Main deformed contribution to ξ and ρ

Re $\xi^{^3D_1}_{nucleon}(p)$



Re $\rho^{^3D_1}_{nucleon}(p)$



$\sim 1\,\%$ correction around the Fermi-surface

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Revisiting Thouless' criterion

David J. Thouless

Thouless' criterion

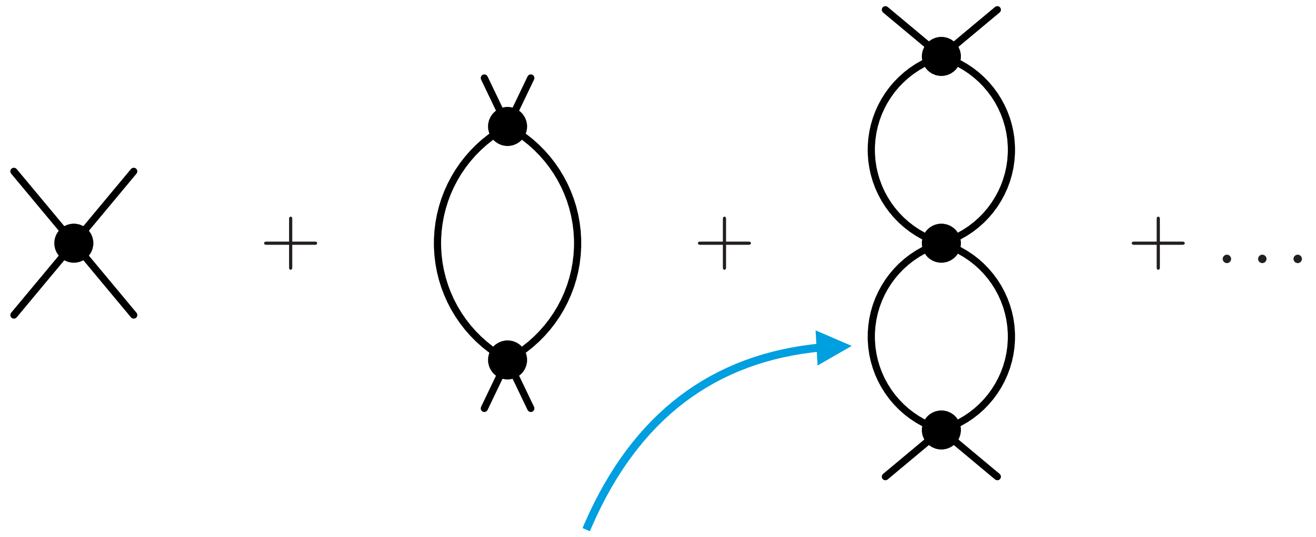
[Thouless, 1960]

- Homogeneous system of fermions + two-body interaction $\bar{V}_{bcde}$
- Finite-temperature: $T > 0$
- Thouless' claim: (in the abstract)

Several assumptions on the potential

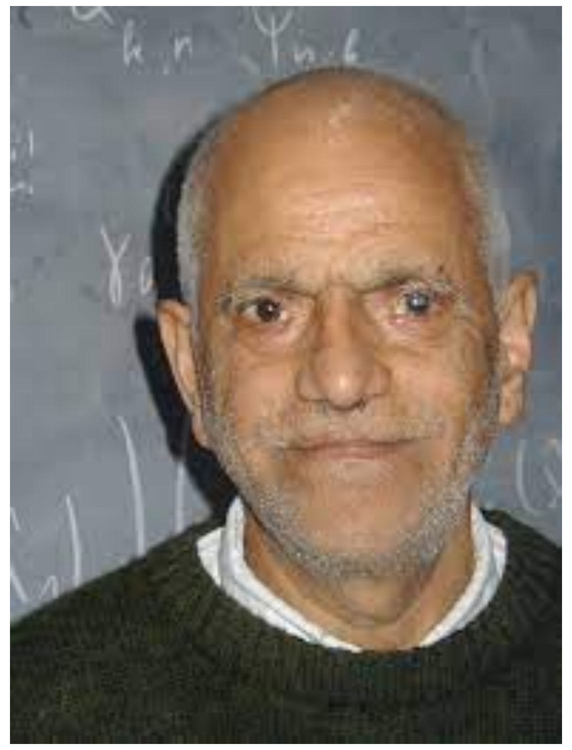
The convergence of the ladder diagrams is suggested as a criterion which the BCS solution must satisfy, and it is shown that this is equivalent to requiring the BCS solution to give a local minimum of the thermodynamic potential.

Sum of ladder diagrams



Mean-field propagator: $\mathscr{G}^{(0)BCS/HFB}(\omega_p)$

Roger Balian



Madan Lal Mehta

Balian and Mehta's work on the convergence of ladders

[Balian and Mehta, 1961, 1962]

- General many-body system of fermions + pair interaction
- Zero-temperature calculations
- Found counter-examples to their proof (eg: D-wave interaction)

➡ **For which systems Thouless' criterion is valid?**

➡ **What about nuclear matter?**

Nambu-covariant formulation

- Proof of necessary condition
  - ✓ General case straightforward
- Exploring sufficient conditions?
  - ✓ Becomes tractable

Conditions for the convergence of HFB-ladders

HFB self-energy as a SCGF fixed point

- HFB self-energy

$$\Sigma^{HFB}_{\mu\nu} = -\frac{1}{2}\sum_{\lambda_2\lambda_3} v^{(2)}_{[\mu\bar{\lambda}_2\bar{\lambda}_3\nu]}\frac{1}{\beta}\sum_{\omega_l}\left(i\omega_l - (U+\Sigma^{HFB})\right)^{-1\,\lambda_2\lambda_3} e^{-i\omega_l\eta}$$

- Functional such that $\mathscr{F}[\Sigma^{HFB}] = \Sigma^{HFB}$

$$\mathscr{F}[\Sigma]_{\mu\nu} = -\frac{1}{2}\sum_{\lambda_2\lambda_3} v^{(2)}_{[\mu\bar{\lambda}_2\bar{\lambda}_3\nu]}\frac{1}{\beta}\sum_{\omega_l}\left(i\omega_l - (U+\Sigma)\right)^{-1\,\lambda_2\lambda_3} e^{-i\omega_l\eta}$$

Fixed point stability

- Linear stability of Σ^{HFB} $\Leftrightarrow$ $r\left(\frac{\delta\mathscr{F}}{\delta\Sigma}[\Sigma^{HFB}]\right) < 1$
- After some algebra: $\frac{\delta\mathscr{F}}{\delta\Sigma}[\Sigma^{HFB}] = \frac{1}{2}V^{(2)}\Pi(0)$ ← **Kernel of the ladders !**
- Stability of HFB ⇔ Convergence of HFB-ladders at $\Omega_p = 0$

➡ Only a necessary condition for the convergence $\forall\Omega_p$!

How to extend to all energies?

- Original case considered by Thouless
  - Separable interaction in singlet channel:

$$\bar{V}_{(\vec{k}'_1\uparrow)(\vec{k}'_2\downarrow)(\vec{k}_1\downarrow)(\vec{k}_2\uparrow)} = g\; v(\vec{q}\,')^* \times v(\vec{q}) \times \delta^{(3)}(\vec{P}\,' - \vec{P})$$

  - Additional assumption: $\bar{V} = cst \neq 0$ **only** for $||\vec{P}||$ small and $||\vec{q}|| \sim ||\vec{q}\,'|| \sim k_F$

A new sufficient criterion

- Unsuccessful attempts to prove it in the general case
- At $T = 0$: counter-examples to a tentative general proof [Balian, Mehta, 1962]
- Investigations guided by the dictionary
  - Symmetry-conserving: z ; $|z|^2$; Re ; Im ; > 0
  - Symmetry-breaking: M ; $MM^\dagger$; $\bar{\text{Re}}$; $\bar{\text{Im}}$; $\succ 0$

  Nambu-covariant reformulation
- A new criterion proposed

$$\left\|\frac{1}{2}\Pi(0)V^{(2)}\right\|_{\mathcal{S}_\infty} < 1$$

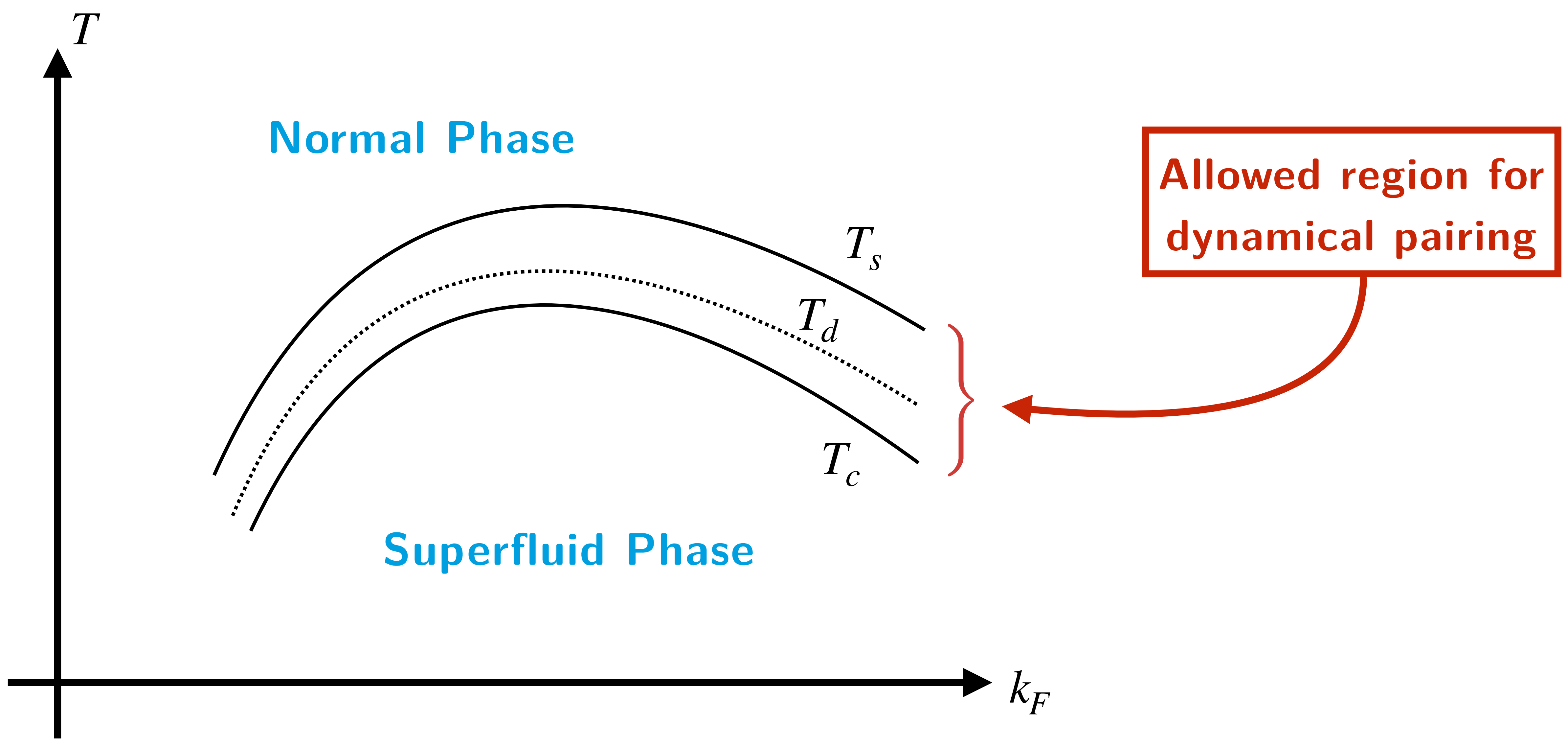
Largest singular value (norm $\|\cdot\|_{\mathcal{S}_\infty}$)

Strong stability condition on HFB

Physical interpretation: unfolding Thouless' criterion

Pairing temperatures

- Critical temperature
  - T_c such that $r\left(\frac{1}{2}\Pi(0)V^{(2)}\right) = 1$
- Dynamical pairing temperature
  - T_d such that $\exists\Omega_p$, $r\left(\frac{1}{2}\Pi(\Omega_p)V^{(2)}\right) = 1$
- Upper-bound on dynamical pairing temperature
  - T_s such that $\left\|\frac{1}{2}\Pi(0)V^{(2)}\right\|_{\mathcal{S}_\infty} = 1$
- Opening of possible regions of interest !
  - In general: $T_c \leq T_d \leq T_s$
  - Recover Thouless' criterion when $T_c = T_s$



Open questions to be investigated

- Are T_s and T_d close for relevant physical systems ?
- What are the characteristic properties when $T_c < T < T_s$?
- Pre-pairing effects such as pseudo-gap in $S(\omega)$?

Conclusions

Conclusions

Nambu-covariant many-body theory

- Based on Nambu tensors
- **Perturbation theory**
  - Covariance with Bogoliubov transformations
  - Un-oriented lines
  - Fully antisymmetric vertices

  } **Simpler diagrammatic**

- **Self-consistent ladder approximation**
  - Finite-temperature
  - Self-consistency
  - Symmetry-breaking

  } **Covariant formalism** ↓ **As simple as symmetry-conserving**

- **Towards the calculation of HFB-ladders**
  - ✓ Numerical implementation of HFB
  - ✓ Sufficient condition for convergence of HFB-ladders

Other developments not mentioned here

- Simplifies formal development for other many-body approximations
- **Several exact results revisited**
  - Gaudin's diagrammatic rule for evaluation of Matsubara sums
  - Spectral function positivity bounds
  - Matrix Fano shape of quasiparticle peaks
  - New tensor $\Theta(\omega)$ characterizing qp-background interferences
- **Efficient numerical implementation**
  - ✓ Partial-wave equations for polarized asymmetric nuclear matter
  - On-going numerical implementation of ladders

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Thank you
Merci

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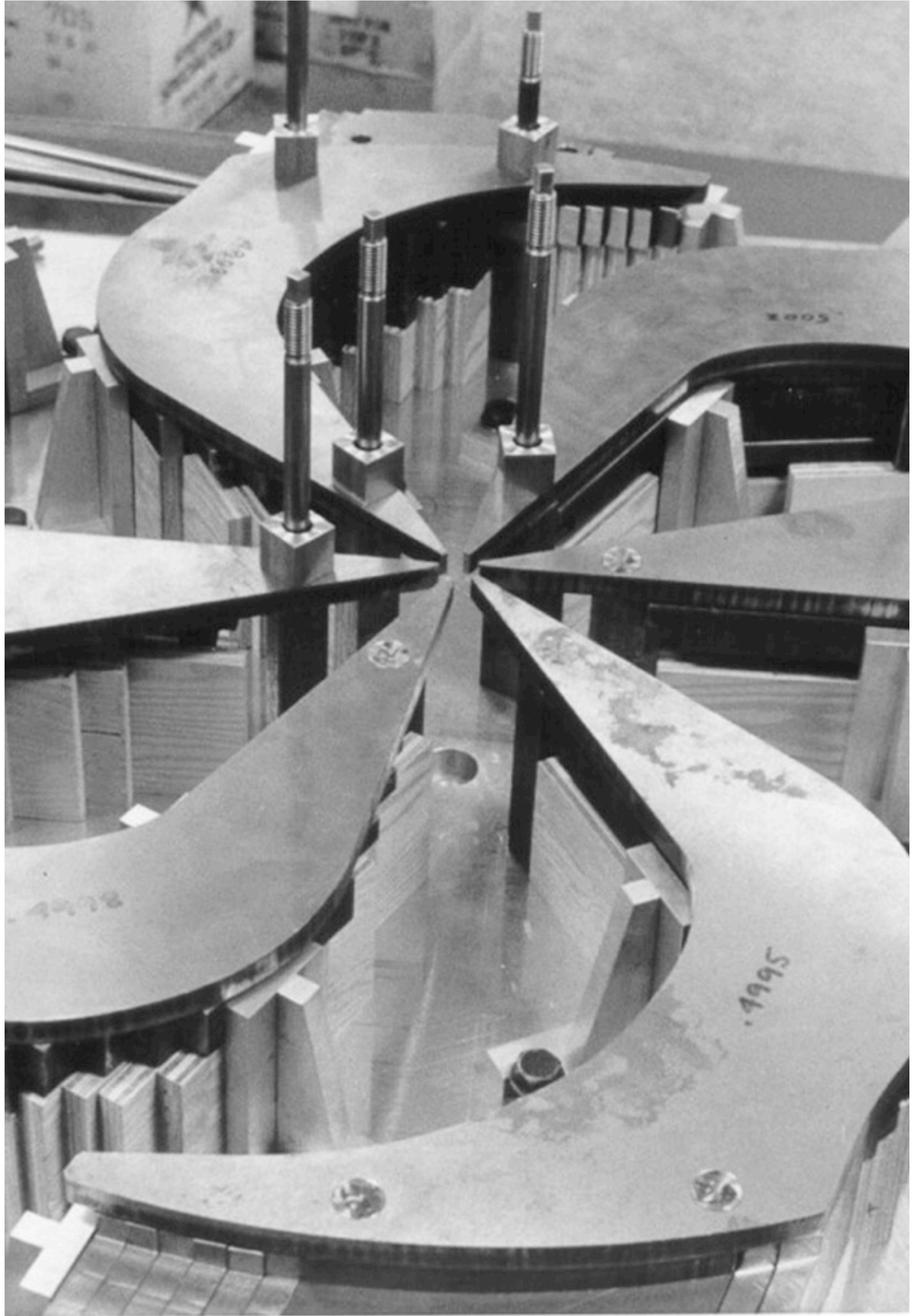

Discovery, accelerated