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Standard Model corrections to Fermi decays in NCSM

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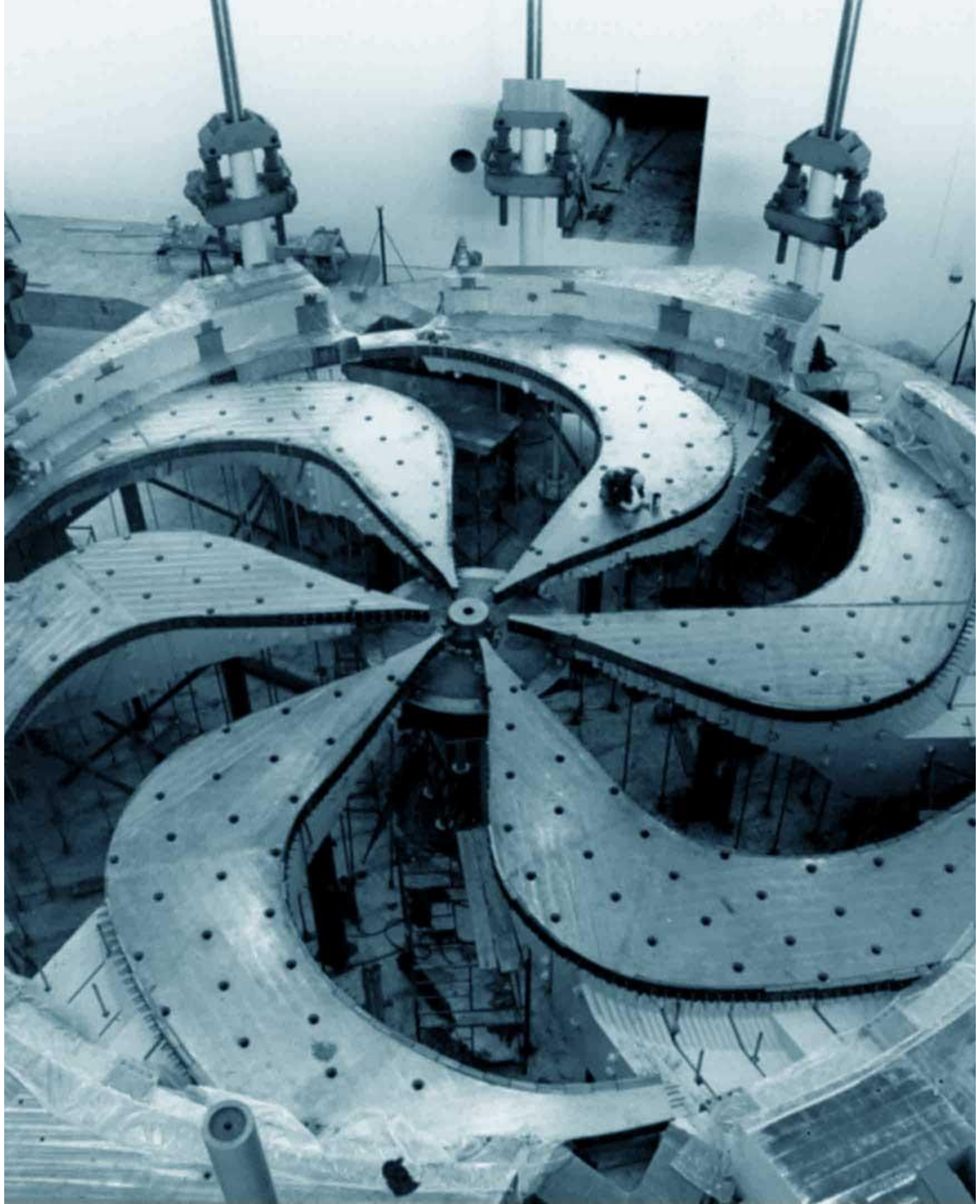
TRIUMF and University of Victoria



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Collaborators: Mehdi Drissi, Chien–Yeah Seng, Misha Gorchtein

2023-03-01

Discovery, accelerated

V_{ud} element of CKM matrix

	$\lvert V_{ud}\rvert$
superallowed	0.97373(31)[19]
n	0.97377(90)[20]
nuclear mirror	0.9739(10)[21]
π_{e3}	0.9740(28)[22]

	$\lvert V_{us}\rvert$
$K_{\ell 3}$	0.22309(56)[23]
τ	0.2221(13)[24]
Hyperon	0.2250(27)[25]

	$\lvert V_{us}/V_{ud}\rvert$
$K_{\mu 2}/\pi_{\mu 2}$	0.23131(51)[23]
$K_{\ell 3}/\pi_{e3}$	0.22908(87)[23]

$$|0^+\rangle \to |0^+\rangle$$

$$n \to p e^- \bar{\nu}_e$$

$$\pi^+ \to \pi^0 e^+ \nu_e(\gamma)$$

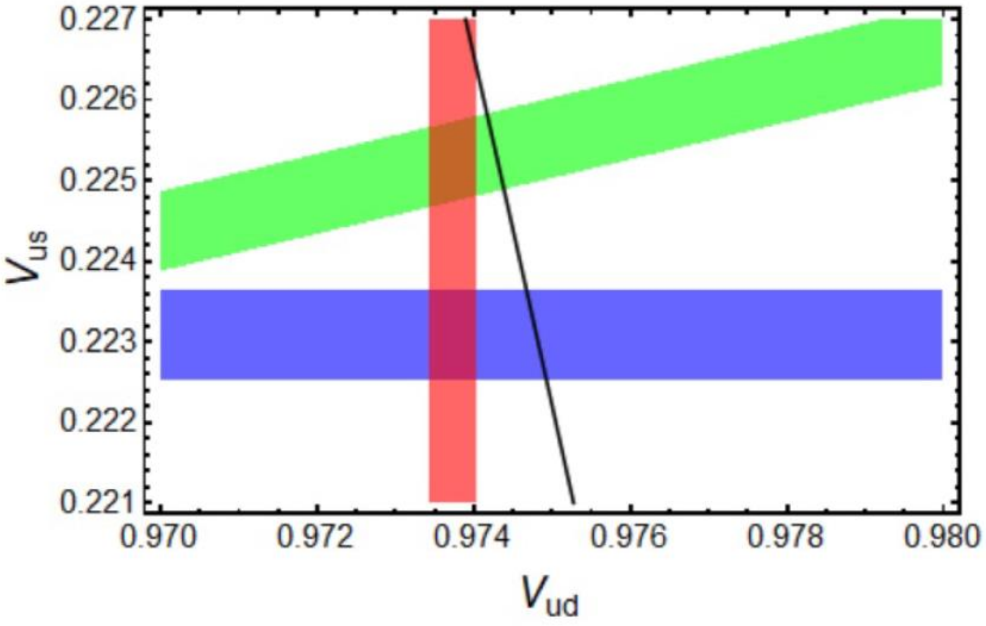


Fig. 1. A combined plot of $|V_{ud}|$ from superallowed decays (red band), $|V_{us}|$ from $K_{\ell 3}$ (blue band), $|V_{us}/V_{ud}|$ from $K_{\mu 2}/\pi_{\mu 2}$ (green band) and the SM unitarity requirement (black line).

V_{ud} element of CKM matrix

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}}(\bar{u}_L, \bar{c}_L, \bar{t}_L)\gamma^\mu W_\mu^+ V_{CKM}\begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} + h.c.$$

- Precise V_{ud} from super-allowed Fermi transitions **[1-2]**

$$|V_{ud}|^2 = \frac{\hbar^7}{G_F^2 m_e^5 c^4}\frac{\pi^3 \ln(2)}{\mathcal{F}t(1+\Delta_R^V)}$$

$G_F \equiv$ **Fermi coupling constant**
determined from muon β decay

  – hadronic matrix elements modified by nuclear environment
  – Fermi matrix element renormalized by isospin non–conserving forces

$$\mathcal{F}t = ft(1+\delta'_R)(1-\delta_C+\delta_{NS})$$

- Dispersion integral approach gives $(2-3)\sigma$ discrepancy **[3-4]**

[1] C. Y. Seng (2022)
[2] P.A. Zyla et al. (2020)
[3] C. Y. Seng et al. (2018)
[4] Gorchtein et al. (2019)

Corrections to Fermi transitions

$$\mathcal{F}t = ft(1+\delta'_R)(\underline{1-\delta_C+\delta_{NS}})$$

Historical treatment (Hardy and Towner)

- δ_{NS} from shell model and approximate single-nucleon currents
- δ_C from shell model with Woods-Saxon potential
- Dominant approach for three decades **[5]**

Evaluate SM corrections with *ab initio* NCSM

$^{10}C \rightarrow {}^{10}B$

[5] Hardy et al. (2020)

Δ_R^V and δ_{NS}

Lepton spinor

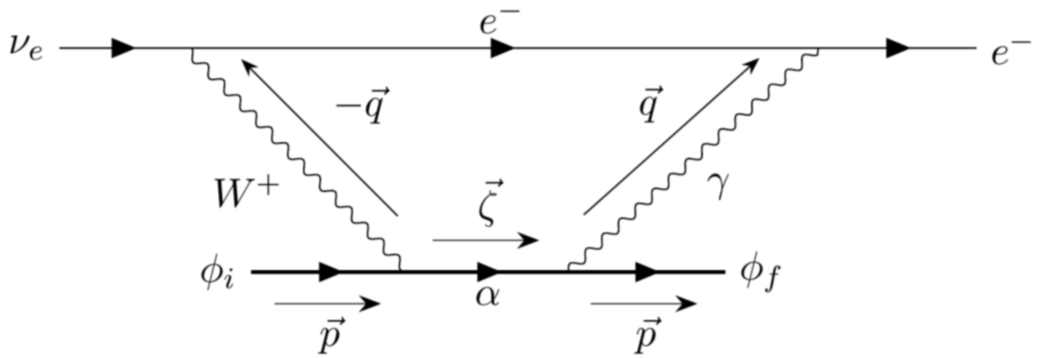
NME of charged weak current

- Tree level beta decay amplitude $M_{tree} = -\frac{G_F}{\sqrt{2}} L_\lambda F^\lambda(p', p)$

- Hadronic correction in forward scattering limit

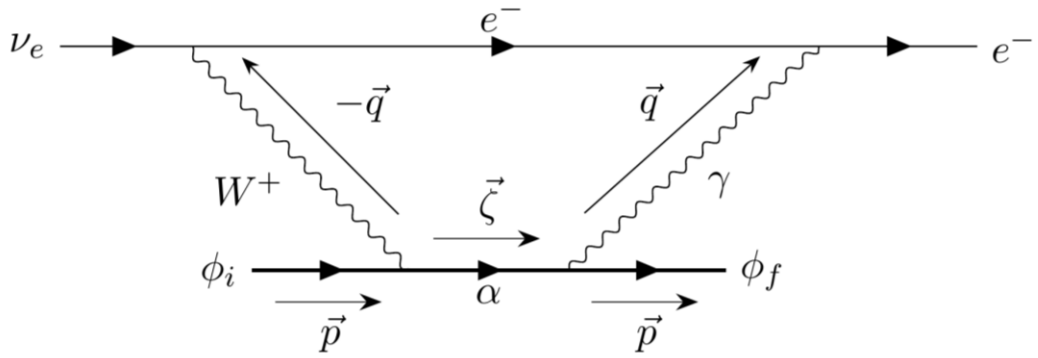
$$\delta M = -i\sqrt{2} G_F e^2 L_\lambda \int \frac{d^4 q}{(2\pi)^4} \frac{M_W^2}{M_W^2 - q^2} \frac{\epsilon^{\mu\nu\alpha\lambda} q_\alpha}{[(p_e - q)^2 - m_e^2] q^2} T_{\mu\nu}(p', p, q)$$

$$\delta M = \Box_{\gamma W}(E_e) M_{tree}$$



Calculating T_3 in the NCSM for ^{10}C → ^{10}B

1) FT currents into momentum space
2) Multipole expansion
3) General electroweak basis of operators **[6]**



$$T_3(q_0, Q^2) = -4\pi i \frac{q_0}{q} \sqrt{M_i M_f} \sum_{J=1}^{\infty} (2J+1)$$
$$\times \langle A\lambda_f J_f M_f \Big| \Big[T_{J0}^{mag}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,el}(q) + T_{J0}^{el}(q)\, G(M_f + q_0 + i\epsilon)\, T_{J0}^{5,mag}(q)$$
$$+ T_{J0}^{5,mag}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{el}(q) + T_{J0}^{5,el}(q)\, G(M_i - q_0 + i\epsilon)\, T_{J0}^{mag}(q) \Big] \Big| A\lambda_i J_i M_i \rangle$$

[6] Haxton et al. (2008)

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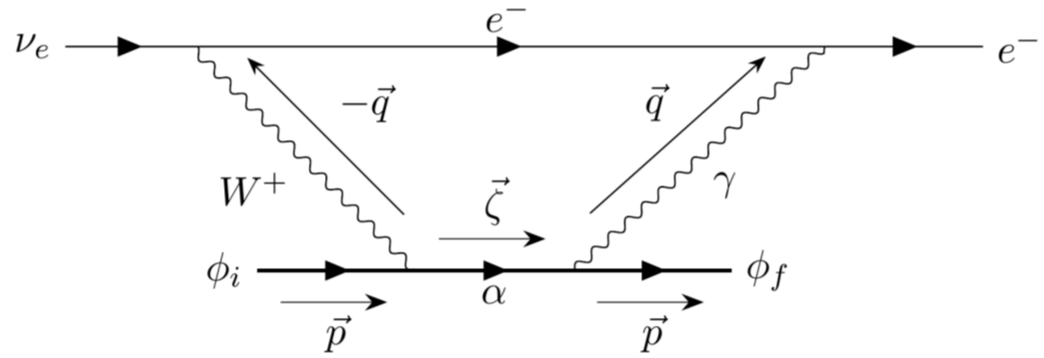

$$T_3(q_0, Q^2) = -4\pi i \frac{q_0}{q}\sqrt{M_i M_f}\sum_{J=1}^{\infty}(2J+1)$$

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$$+ T_{J0}^{5,mag}(q)\, \boxed{G(M_i - q_0 + i\epsilon)}\, T_{J0}^{el}(q) + T_{J0}^{5,el}(q)\, \boxed{G(M_i - q_0 + i\epsilon)}\, T_{J0}^{mag}(q) \Big] \big| A\lambda_i J_i M_i \rangle$$

[6] Haxton et al. (2008)

Lanczos continued fraction method

- Reformulate as inhomogeneous Schrödinger equation **[7]**

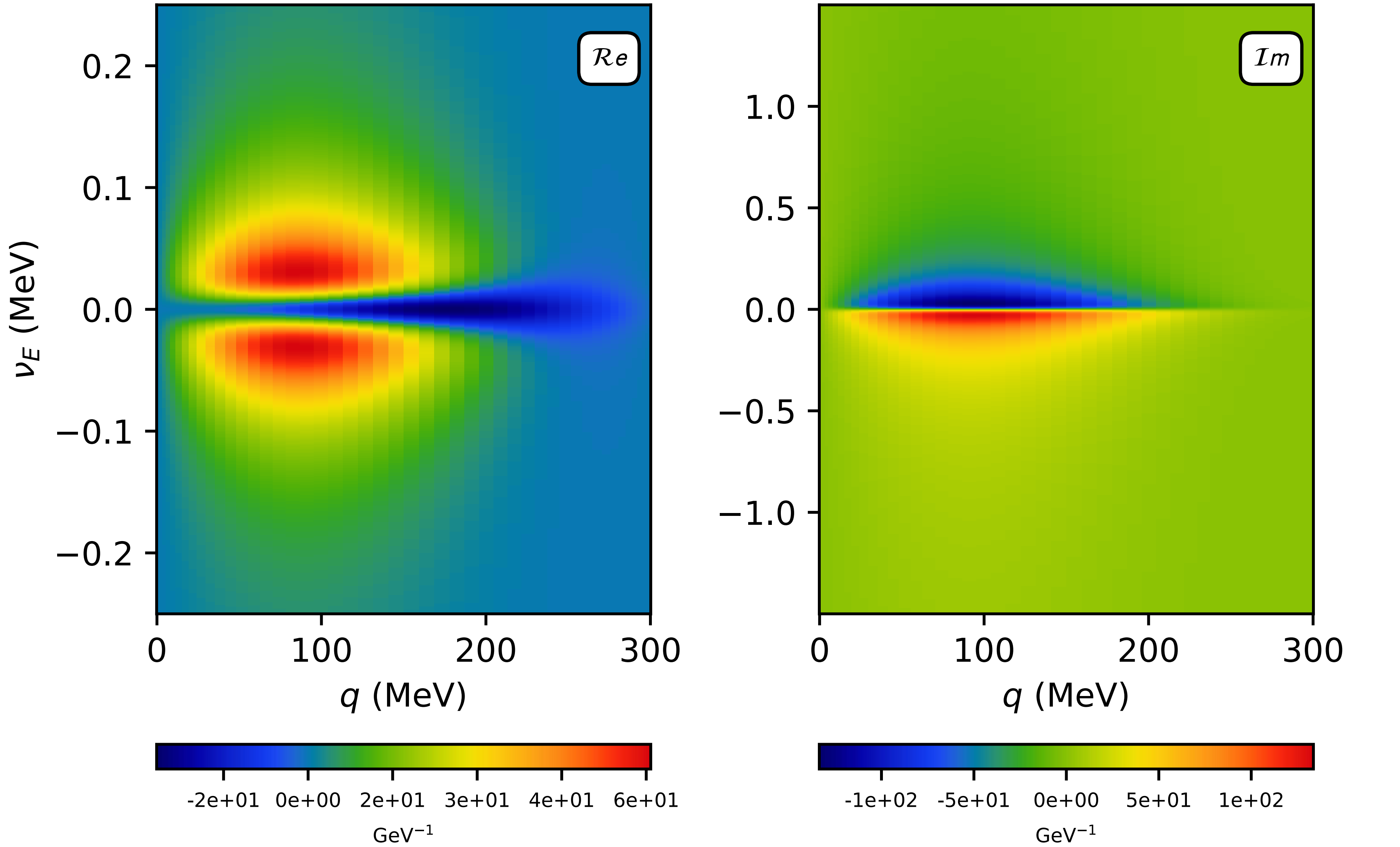
$$(H - E\mathbb{1})|\Phi\rangle = \hat{O}|\Psi\rangle$$

$$\begin{aligned}
H\mathbf{v}_1 &= \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2 \\
H\mathbf{v}_2 &= \beta_1\mathbf{v}_1 + \alpha_2\mathbf{v}_2 + \beta_2\mathbf{v}_3 \\
H\mathbf{v}_3 &= \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4 \\
H\mathbf{v}_4 &= \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5
\end{aligned}$$

- Resolvent becomes sum over Lanczos vectors with continued fraction coefficients **[8]**
- Avoids brute force calculation of intermediate states

[7] Marchisio et al. (2003)
[8] Haydock (1974)

$$\left\langle {}^{10}\mathrm{B}\right|\, T^{\mathrm{mag}}_{J=1}(q)\, G(M_f + i\nu_E)\, T^{5,\mathrm{el}}_{J=1}(q)\, \left|{}^{10}\mathrm{C}\right\rangle$$

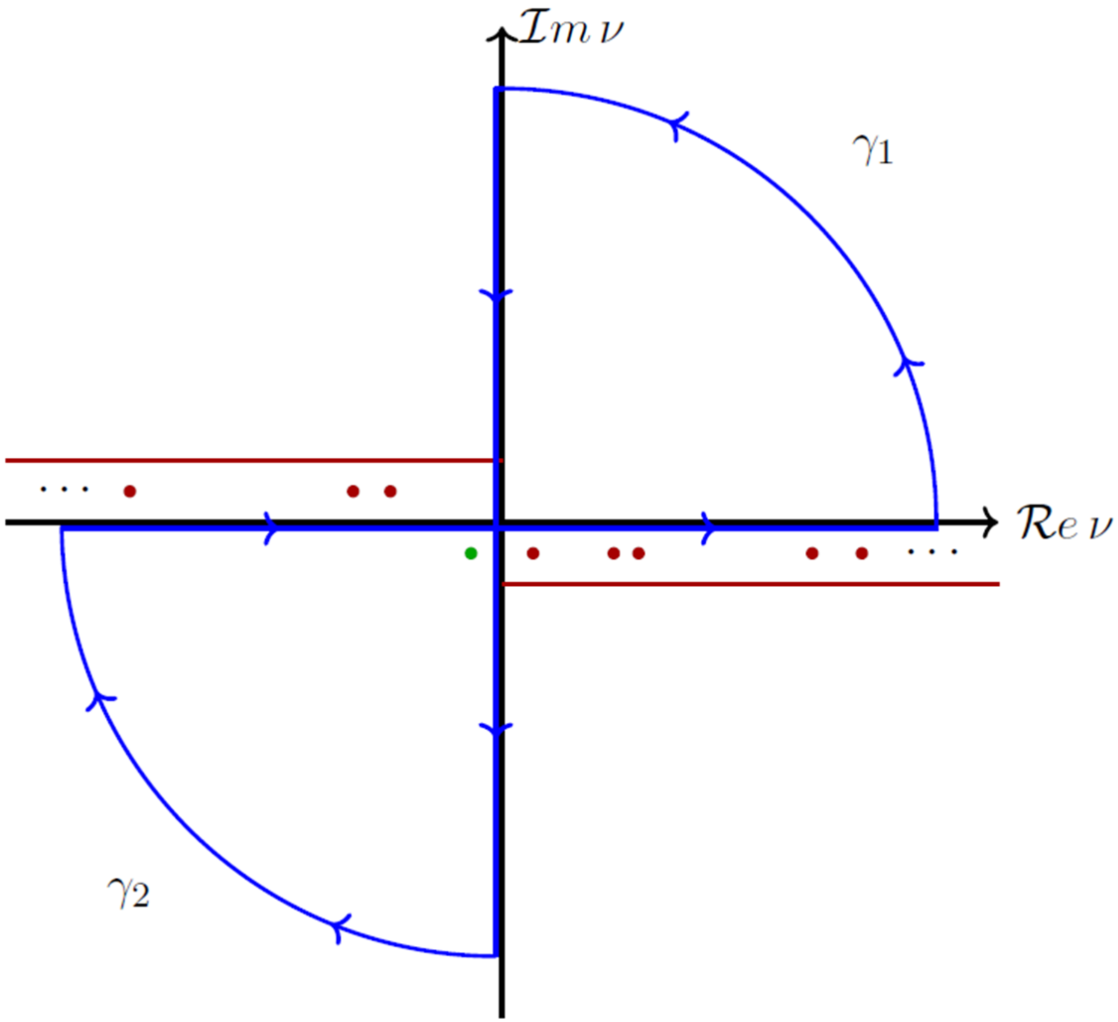


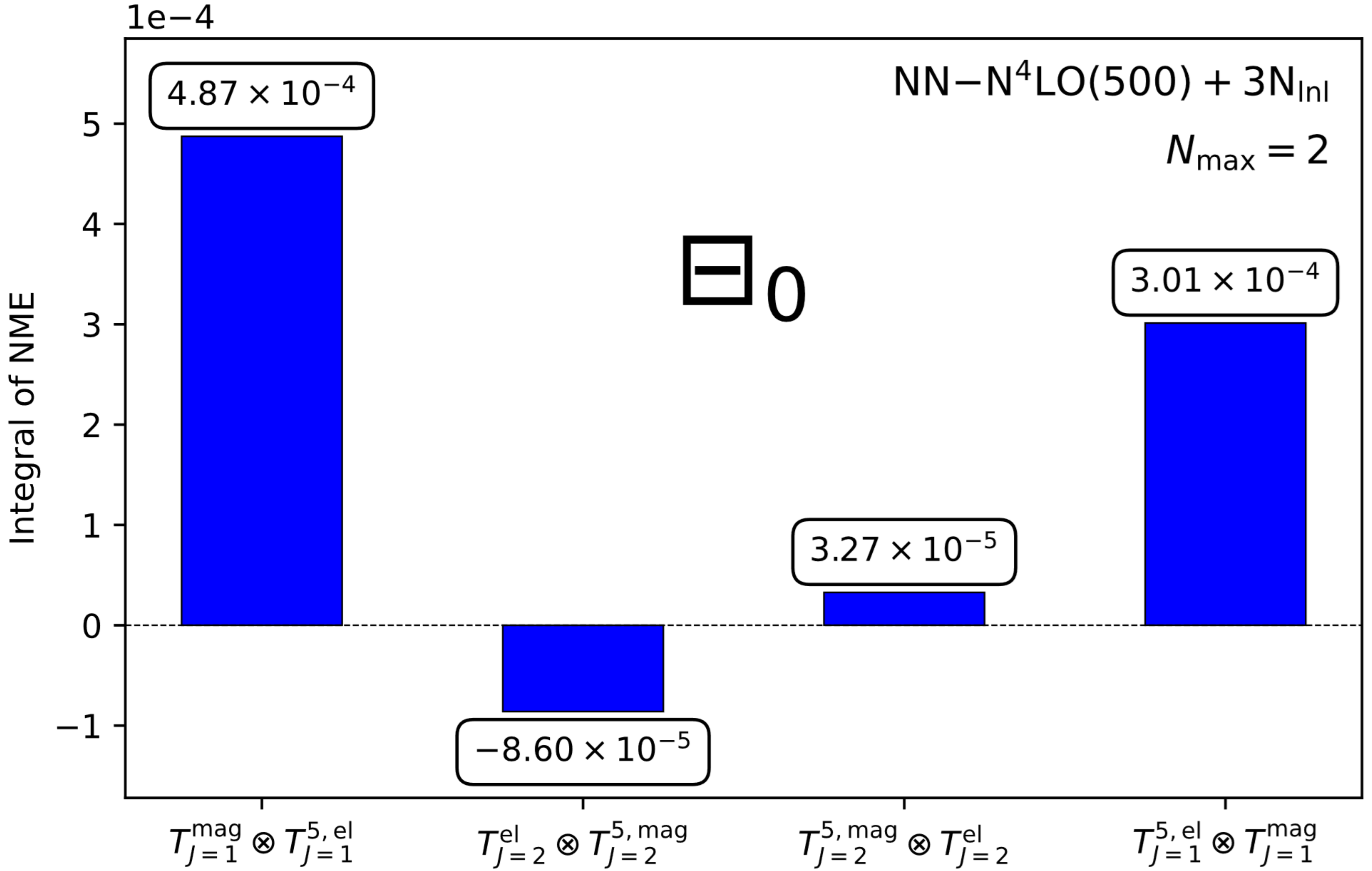
Integrating T_3 in the NCSM for $^{10}C \to {}^{10}B$

- T_3 contains poles in deformed contour!
- Ground state 3^+ and low-lying 1^+ have residues after Wick rotation
- Remaining poles in residue terms must also be treated

Poles	$n = 1$	$n = 2$	$n = 3$	$n = 4$
P_- [MeV]	-0.048	0.0187	9.148	10.712
P_+ [MeV]	-8.9346	-10.975	-18.965	-22.354

Table 1: Pole locations along ν axis corresponding to the n–th excited 1^+ state in T_3 amplitude for $^{10}C \to {}^{10}B$ Fermi transition at $N_{max} = 2$.





1e−4
Integral of NME
4.87×10^{-4}
-8.60×10^{-5}
3.27×10^{-5}
3.01×10^{-4}
NN−N^{4}LO(500) + 3N$_{\rm lnl}$
$N_{\rm max} = 2$
Θ_0
$T^{\rm mag}_{J=1} \otimes T^{5,\rm el}_{J=1}$
$T^{\rm el}_{J=2} \otimes T^{5,\rm mag}_{J=2}$
$T^{5,\rm mag}_{J=2} \otimes T^{\rm el}_{J=2}$
$T^{5,\rm el}_{J=1} \otimes T^{\rm mag}_{J=1}$

Conclusions

- **Goal:** consistent nuclear theory corrections to Fermi transitions
- NCSM calculations of δ_{NS} underway
- NCSMC calculations for δ_{C} ongoing (with Mack Atkinson)

Outlook

- Residue contributions to γW–box
- Tackle large number of many-body calculations with realistic N_{max}
- Improve limited uncertainty quantification
- $^{14}O \rightarrow {}^{14}N$ transition

[17] Atkinson et al. (2022)

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Thank you
Merci

Discovery,
accelerated

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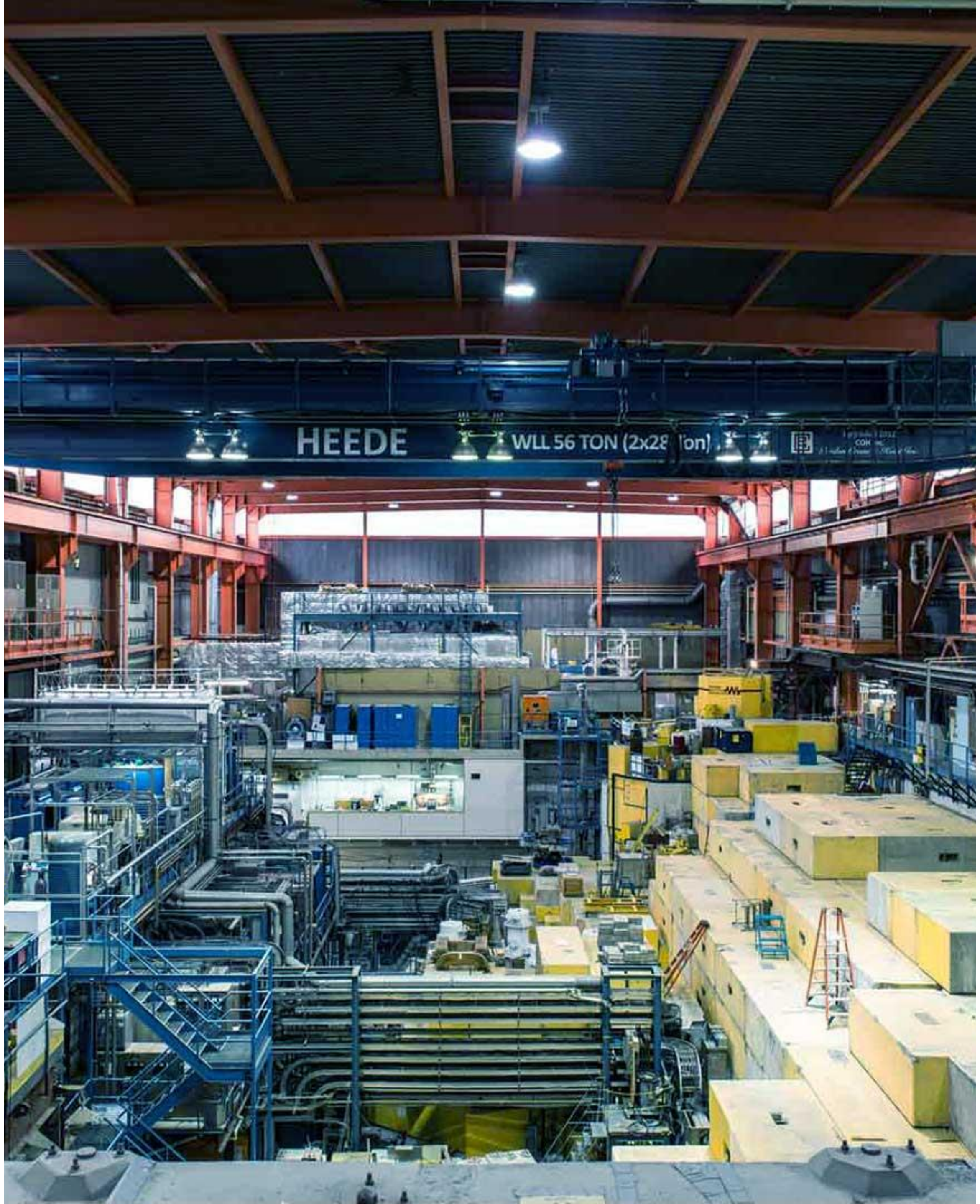
Backup



Discovery, accelerated

Symmetry tests of nuclear T_3

Nuclei	Nucleons	Pions
$T_3^{(0)}(-\nu,Q^2) = -T_3^{(0)}(\nu,Q^2)$	$T_3^{(0)}(-\nu,Q^2) = -T_3^{(0)}(\nu,Q^2)$	$T_3^{(0)}(-\nu,Q^2) = -T_3^{(0)}(\nu,Q^2)$
$T_3^{(1)}(-\nu,Q^2) = \cdots$	$T_3^{(1)}(-\nu,Q^2) = T_3^{(1)}(\nu,Q^2)$	$T_3^{(1)}(\nu,Q^2) = 0$

- Symmetries of T_3 different in nuclei
- Currents can couple to
  - $T = 1 \rightarrow$ even with respect to ν
  - $T = 2 \rightarrow$ odd with respect to ν
- **Important** since previously assumed nuclear T_3 had same symmetries as nucleonic system

Lanczos continued fractions method

- Reformulate as inhomogeneous Schrödinger equation **[13]**

$$(H - E\mathbb{1})|\Phi\rangle = \hat{O}|\Psi\rangle$$

$$\begin{aligned}
H\mathbf{v}_1 &= \alpha_1\mathbf{v}_1 + \beta_1\mathbf{v}_2 \\
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H\mathbf{v}_3 &= \beta_2\mathbf{v}_2 + \alpha_3\mathbf{v}_3 + \beta_3\mathbf{v}_4 \\
H\mathbf{v}_4 &= \beta_3\mathbf{v}_3 + \alpha_4\mathbf{v}_4 + \beta_4\mathbf{v}_5
\end{aligned}$$

Choose specific starting vector

$$|v_1\rangle = \frac{\hat{O}|\Psi\rangle}{\langle\Psi|\hat{O}^\dagger\hat{O}|\Psi\rangle}$$

[13] Marchisio et al. (2003)
[14] Haydock (1974)

[8] Barrett et al. (2013)
[9] Weinberg (1991)
[10] Epelbaum (2009)
[11] Entem et al. (2017)
[12] Somà et al. (2020)

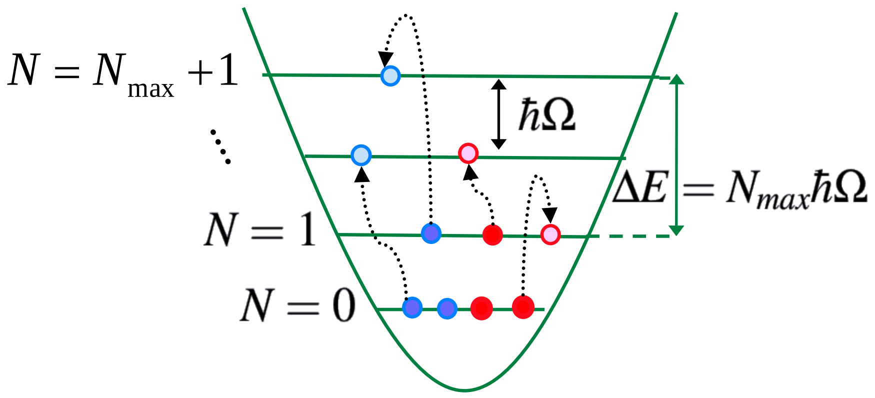
No–core shell model (NCSM)

- *Ab initio* approach to many–body Schrödinger equation for bound states and narrow resonances **[8]**
- Nuclear interactions sole input **[9-10]**

$$H\left|\Psi_A^{J^\pi T}\right\rangle = E^{J^\pi T}\left|\Psi_A^{J^\pi T}\right\rangle$$

$$\left|\Psi_A^{J^\pi T}\right\rangle = \sum_{N=0}^{N_{max}} \sum_{\alpha} c_{N\alpha}^{J^\pi T}\left|\Phi_{N\alpha}^{J^\pi T}\right\rangle$$

Anti-symmetrized products of many-body HO states



  – Two body: NN-N^4LO(500) **[11]**
  – Three body: $3N_{lnl}$ **[12]**

Accessible transitions

$^{10}C \rightarrow {}^{10}B$ and $^{14}O \rightarrow {}^{14}N$