

Bayesian Analysis of χ EFT at Leading Order in a Modified Weinberg Power Counting

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Introduction

- Make predictions of nuclear properties in a well founded EFT framework
 - A model: χ EFT — from low energy QCD
 - Power counting (PC): orders contributions to observables; LO, NLO, ...
 - Data: to constrain low energy constants (LECs) \implies construct interaction models
 - Weinberg PC have problems already at LO caused by the singular attraction of the one-pion exchange potential

A. Nogga, R. G. E. Timmermans, and U. van Kolck, Phys. Rev. C **72**, 054006 (2005)
- \implies One solution is to promote additional contact terms to LO to counteract the singular behavior

Modified LO potential in χ EFT

$$\langle \mathbf{p}' | V | \mathbf{p} \rangle = \frac{1}{(2\pi)^3} \left[\underbrace{-\frac{g_A^2}{4f_\pi^2} \frac{(\boldsymbol{\sigma}_1 \cdot \mathbf{q})(\boldsymbol{\sigma}_2 \cdot \mathbf{q})}{\mathbf{q}^2 + m_\pi^2} (\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}_{\substack{\text{One pion exchange} \\ \text{(OPE)}}} + \underbrace{\tilde{C}_{1S_0} + \tilde{C}_{3S_1} + (C_{3P_0} + C_{3P_2}) p'p}_{\text{Contact interactions}} \right]$$

B. Long, C. J. Yang, Phys. Rev. C **85**, 034002 (2012)

B. Long, C. J. Yang, Phys. Rev. C **86**, 024001 (2012)

LO contains only partial waves with $l \leq 1$ as well as 3D_1 and 3F_2

- This potential studied by Yang *et al.* C. J. Yang, A. Ekström, C. Forssén, and G. Hagen, Phys. Rev. C **103**, 054304 (2021)
 - Found that at LO the nuclear-binding mechanism fails for some nuclei with $A > 4$
 - LECs overfitted to phase shifts is a potential cause

Systematic Bayesian Inference

- Bayesian inference of LECs across a large range of momentum cutoffs

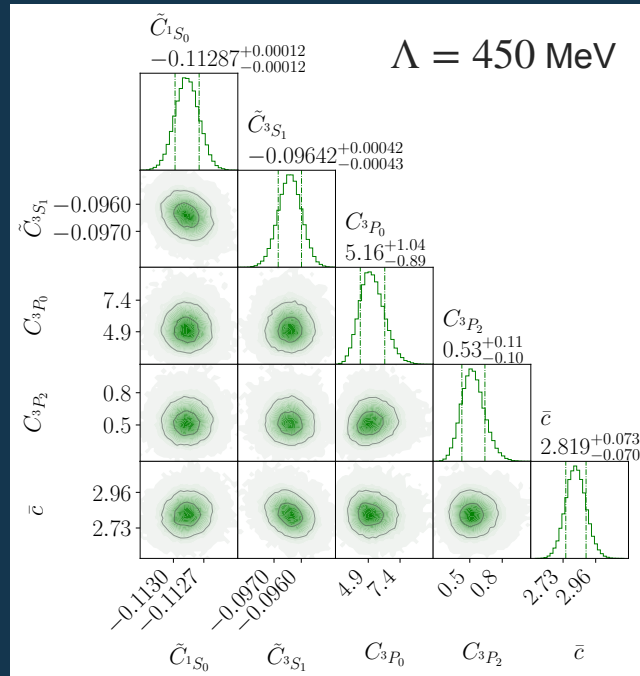
O. Thim, E. May, A. Ekström, C. Forssén, arXiv:2302.12624 (2023)

- Theory of renormalization of singular potentials and study appearing limit cycles
- History matching
- Sample posteriors for LECs across momentum cutoffs $\Lambda = 400$ to 4000 MeV
- Infer the magnitude of the EFT truncation error (\bar{c}) at each cutoff

R. J. Furnstahl *et al.*, Phys. Rev. C **92**, 024005 (2015)

- Demonstrate RG-invariant posterior predictive distributions (ppds) for np - observables

Bayesian Posterior



Posterior $p(\theta|D, I)$

$$\theta = \left(\underbrace{\tilde{C}_{1S_0}, \tilde{C}_{3S_1}, C_{3P_0}, C_{3P_2}}_{\text{LECs}}, \bar{c} \right)$$

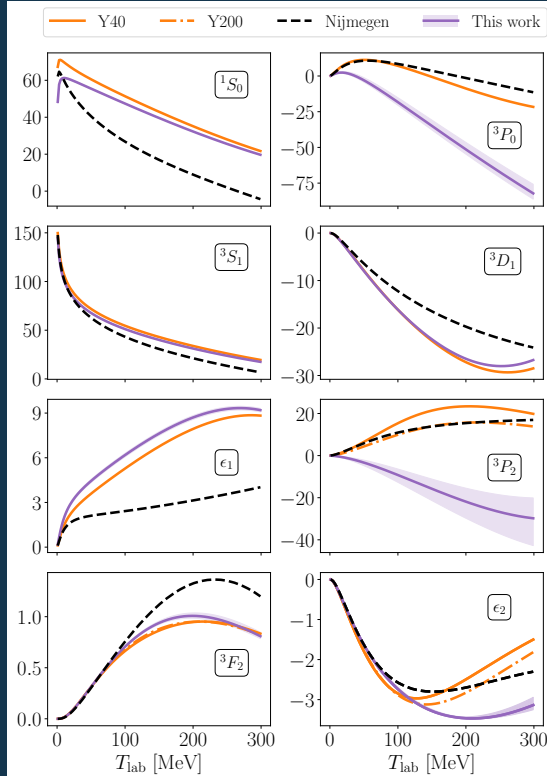
LECs

magnitude of EFT error

$$S: 10^4 \text{ GeV}^{-2}, P: 10^4 \text{ GeV}^{-4}$$

Bayesian ppds

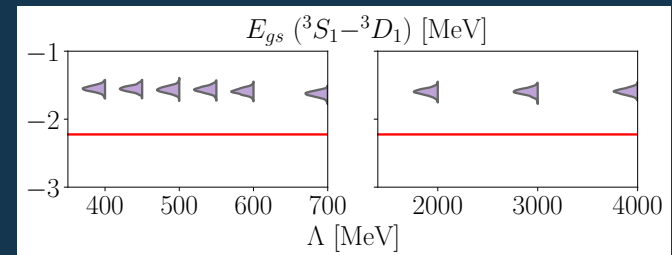
δ [deg], $\Lambda = 450$ MeV



$$p(y|D, I) = \int d\theta p(y|\theta, D, I) p(\theta|D, I)$$

- Y40 and Y200 from C. J. Yang *et al.* Phys. Rev. C **103**, (2021)

Posterior predictive distribution for the
deuteron ground state energy



Outlook

- Include subleading orders in distorted wave perturbation theory
- Investigate EFT convergence, e.g. Lepage plots
- Compute predictions in heavier nuclei
- Develop and use more sophisticated error models
- Compare different power counting proposals in a quantitative Bayesian framework

Bayesian Analysis of χ EFT at Leading Order a Modified Weinberg Power Counting

Oliver Thim, Eleanor May, Andreas Ekström, Christian Forssén

Introduction

A well-defined power counting in χ EFT is crucial for:
• sound a priori estimates of the EFT truncation error.
In several studies, starting with Nogga et al., the canonical power counting due to Weinberg has been up for debate. Numerous works have investigated the issue and Yang et al. investigated the following renormalization group (RG) invariant modified leading order (LO) nuclear-nucleon potential and found that the nuclear-binding mechanism fails for nuclei with mass number $A > 4$. Low energy phase shifts identified by constants (LECs) is a potential cause. A more detailed and robust inference is highly warranted.

$$V(\vec{r}, p) = \frac{1}{(2\pi)^3} \int d^3q \left[C_0 + C_1 \frac{q^2}{\Lambda^2} + C_2 \frac{q^4}{\Lambda^4} + \dots \right] e^{i\vec{q}\cdot\vec{r}}$$

Include only terms with $|\vec{q}| \leq \Lambda$, as we are working in the Λ -regime.

- We perform a robust Bayesian inference for the momentum cutoffs, Λ .
- We assess the quality of the inference by comparing the predictive distributions (ppds) for different priors.

Limit Cycles and High-Exchange Potentials

• LECs display limit cycle behavior due to the non-renormalizability of the $1/r$ potential.
• The nature of the limit cycle depends on the exchange potentials used.



Bayesian Posteriors

- Posterior ppd for LECs and Λ for momentum scale cutoff $\Lambda = 400$ MeV.
- Conservative naturalness prior used for the LECs and inverse gamma prior for Λ with mode around one.
- Inferred $\lambda \approx 2.3 \Rightarrow 15\%$ EFT error for $p < m_\pi$.



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Thank you!

- Goal: Bayesian inference for the LECs (α) and Λ using scattering data across the range of cutoffs.
- We use History Matching (Vernon et al.) to reduce the number of evaluations of the model evidence to analyze multimodalities.
- We perform Markov Chain Monte Carlo sampling where we also include the model evidence to analyze multimodalities.
- We use History Matching (Vernon et al.) to reduce the number of evaluations of the model evidence to analyze multimodalities.

Outlook

- Include higher-order χ EFT potentials in distorted-wave perturbation theory.
- Develop a more sophisticated error model with correlations taken into account.
- Sample ppds for observables in heavier mass nuclear systems.

References

C. J. Vale, A. Ekström, C. Forssén, and O. Hansen, Phys. Rev. C **100**, 054004 (2019)
A. Nogga, H. G. C. Timmermans, and U. van Kolck, Phys. Rev. C **72**, 054008 (2005)
I. Vernon, M. Golubov, and R. Bowser, Bayesian Anal. **9**(1), 93 (2018)

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