What can we learn from precision positronium measurements?

John N Ng

Triumf

Triumf Experimental Group Meeting

April 4, 2023

Introduction

- Positronium (Ps) is the simplest atom made up of e^+e^-. Unlike hydrogen is not stable.
- Ps states are described by a number of quantum numbers,
  1. Principal quantum number n,
  2. Orbital angular momentum, L,
  3. Total spin of the e^- and e^+,S. $S = 0 \,\mathrm{or}\, 1$
  4. Total angular momentum is $J = L + S$.
  5. States are labeled by ${}^{S+1}L_J$ labels the states.
- The two lowest states have $n = 1$ and $L = 0$
  1. Para-positronium (pPs)

$$\mathbf{1\,{}^1S_0}\ \ (J^{PC} = 0^{-+})$$

  2. Ortho-positronium (oPs)

$$\mathbf{1\,{}^3S_1}\ \ (J^{PC} = 1^{--}).$$

General remarks

- Ps is a pure leptonic atom
- Its properties are governed by QED to very high accuracy.
- Hadronic uncertainties are several orders of magnitude lower than experimental accuracies.
- Electroweak physics enters at even higher order
- Ideal for new light degrees of freedom beyond the SM.

Ps lifetimes

- pPs

$$\Gamma(1\,^1S_0 \to 2\gamma) = \frac{\alpha^5 m}{2}\left[1 - \left(5 - \frac{\pi^2}{4}\right)\left(\frac{\alpha}{\pi}\right) + 2\alpha^2 \ln^2(\frac{1}{\alpha}) + 1.75(30)\left(\frac{\alpha}{\pi}\right)^2 + \mathcal{O}(\alpha^3)\right]$$

$$= 7989.50(20)\mu s^{-1}$$

$$\Gamma(\mathrm{pPs})_{\mathrm{exp}} = 7990.9(1.7)\mu s^{-1}$$

- oPs

$$\Gamma(1\,^3S_1 \to 3\gamma) = \frac{2(\pi^2 - 9)\alpha^6 m}{9\pi}\left[1 - 10.28861(1)\left(\frac{\alpha}{\pi}\right) + \frac{\alpha^2}{3}\ln\alpha + B_0\left(\left(\frac{\alpha}{\pi}\right)^2\right) + \mathcal{O}(\alpha^3)\right]$$

$$\simeq (7.0382 + 0.39 \times 10^{-4} B_0)\mu\mathrm{s}^{-1}.$$

$$\Gamma(\mathrm{oPs})_{\mathrm{exp}} = 7.0398(29)\mu\mathrm{s}^{-1}.$$

$\mathrm{Ps} \to \gamma + X$

$$e^- + e^+ \to \gamma + X \quad (X = S, a, \gamma_D),$$

$s = E_{\mathrm{cm}}^2$ and later set $s \to 4m^2$. We also note the energy of the outgoing photon is given by

$$E_\gamma = \frac{\sqrt{s}}{2}\left(1 - \frac{M_x^2}{s}\right).$$

The only SM background comes from the process $\mathrm{Ps} \to \gamma\bar{\nu}\nu$. The branching ratio is estimated as

$$\frac{\Gamma(\mathbf{1\,^3S_1} \to \gamma\bar{\nu}\nu)}{\Gamma(\mathbf{1\,^3S_1} \to 3\gamma)} \sim \left(\frac{G_F m^2}{\alpha}\right)^2 \simeq 10^{-19}.$$

Diagram for $\mathrm{Ps} \to \gamma + X$

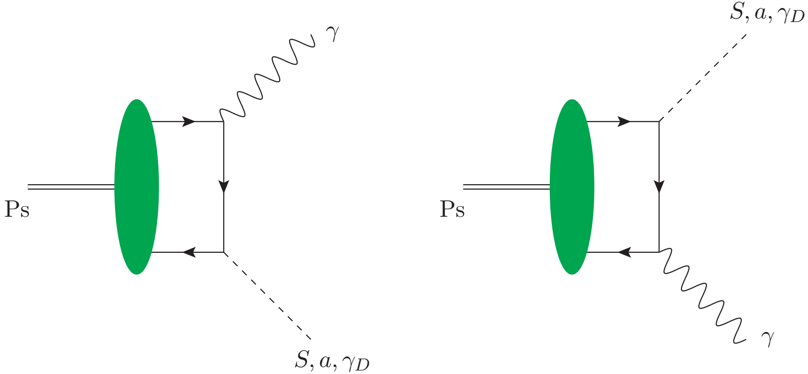

Figure : Positronium decay into a scalar (S), or a psuedoscalar (a) or a dark photon (γ_D)

$\mathrm{Ps} \to \gamma + S(a)$

The effective Lagrangians are

- For ALP

$$L_{\text{eff}} = -i\lambda_a \bar{e}\gamma^5 e a.$$

- For light scalar S

$$L_{\text{eff}} = -\lambda_s \bar{e} e S.$$

Both lead to the same branching ratio for $\mathbf{1\,{}^3S_1} \to \gamma + X$

$$\mathrm{Br} = \frac{6\pi^2}{\pi^2 - 9}\left(\frac{\lambda_X}{\alpha}\right)^2\left(1 - \frac{m_X^2}{4m^2}\right).$$

where m is the electron mass.

Constraint on λ_X vs m_X.

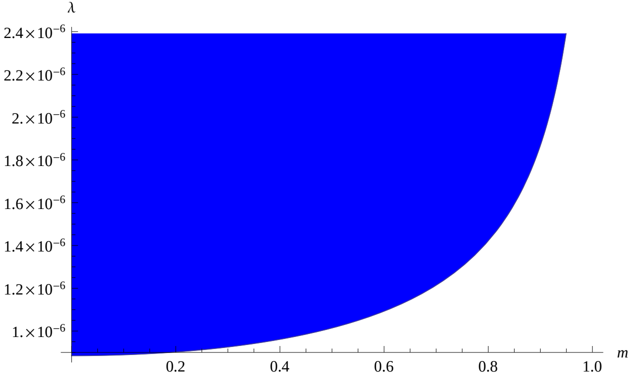

Figure : Constrain of $\mathbf{1\,^3S_1}$ decay into a photon plus scalar (S), or a psuedoscalar (a). λ is the coupling of S or a to the electrons and m denotes the mass of S or a in MeV.

The experimental limit is

$$\mathrm{Br}(\mathbf{1\,^3S_1} \to \gamma + \mathrm{inv}) < 1.1 \times 10^{-6}$$

$X = \gamma_D$ the dark photon

- The effective Lagrangian is

$$L_{\rm eff} = -e\epsilon A_D^{\mu}\bar{e}\gamma_{\mu}e$$

where ϵ is the small $\gamma - \gamma_D$ mixing parameter $\epsilon F_D^{\mu\nu}F_{\mu\nu}$.

- γ_D has universal couplings to all SM fermions.
- The branching ratio is

$$\mathrm{Br}(\mathbf{1\,^3S_1} \to \gamma + \gamma_D) = \frac{3\pi\epsilon^2}{2(\pi^2-9)\alpha}\left(1 - \frac{m_D^2}{4m^2}\right).$$

Constraint on ϵ vs m_D.

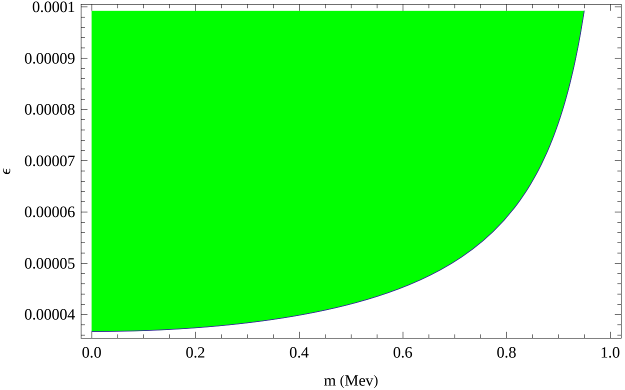

Figure : Constrain of $\mathbf{1\,^3S_1}$ decay into $\gamma + \gamma_D$. m is the mass of γ_D .

Remarks on $\gamma + X$

1. The constraints obtained have minimal model dependents.
2. They are kinematic constraints and hence not affected by whether X has couplings to other SM fields or BSM fields.
3. If seen measuring E_γ will give th mass m_X.
4. It cannot distinguish the nature of X.
5. It is limited to $m_X < 1\mathrm{MeV}$.x

Positronium invisible decays.

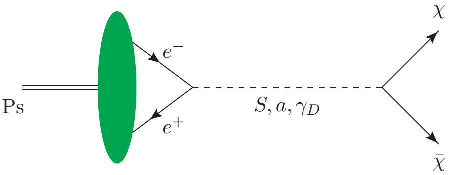

Figure : Positronium annihilation into invisible dark matter or neutrinos

$X = S, a, \gamma_D$ and χ denotes DM or neutrinos.

Background processes for invisible decays

the SM background comes from $\mathbf{1\,^3S_1} \to \bar{\nu}_i \nu_i,\ i = e, \mu, \tau$. The width is

$$\Gamma(\mathbf{1\,^3S_1} \to \bar{\nu}_\mu \nu_\mu) = \frac{G_F^2 \alpha^3 m^5}{24\pi^2} \left(1 - 4\sin^2\theta_w\right)^2 \quad (1)$$

$$\Gamma(\mathbf{1\,^3S_1} \to \bar{\nu}_e \nu_e) = \frac{G_F^2 \alpha^3 m^5}{24\pi^2} \left(1 + 4\sin^2\theta_w\right)^2$$

The respective branching ratios are 9.6×10^{-21} and 6.1×10^{-18} whereas the experimental limit is

$$\mathrm{Br}(\mathbf{1\,^3S_1} \to \mathrm{inv}) < 2.8 \times 10^{-6}$$

.

Invisible decays : $X = S$

- For S since it has spin 0 the exchange picks $\mathbf{1\,{}^1S_0}$
- oPs is CP odd but S is CP even.

Invisible decays via s-channel scalar exchange is forbidden.

Invs decays : $X = a$

- a is CP odd so $\mathbf{1\,^1S_0} \to a^* \to \text{invisibles}$ is allowed.
- Additional coupling is assumed

$$L_{\text{eff}} = -i\lambda_\chi a\bar{\chi}\gamma^5\chi,$$

  for fermionic χ.
- The branching ratio is

$$\text{BR}(\mathbf{1\,^1S_0} \to \bar{\chi}\chi) = \frac{\lambda_e^2\lambda_\chi^2}{16\pi^2\alpha^2}\frac{1-\frac{m_\chi^2}{m^2}}{\left(1-\frac{m_a^2}{4m^2}\right)^2}$$

Constraint on $\lambda\lambda_\chi$

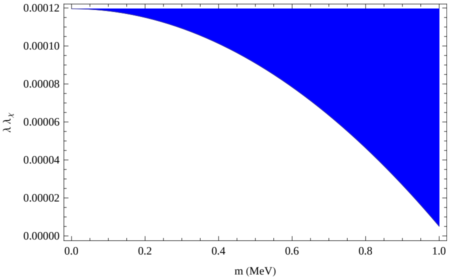

Figure : Constraint on $\lambda_a\lambda_\chi$ vs m_a. m_χ is set to 0.

- $\mathbf{1\,^3S_1} \to \gamma_D^* \to \chi\chi$ is allowed.
- χ can be dark bosons or dark fermions.
- Additional coupling for dark fermions is

$$L_{\text{eff}} = -g_D A_D^{\mu}\ \bar{\chi}\gamma_\mu\chi$$

- The branching ratio is

$$Br(\mathbf{1\,^3S_1} \to \text{invisibles})_{\text{DP}} = \frac{3\epsilon^2 g_D^2}{16(\pi^2-9)\alpha^2}\frac{\left(1-\frac{m_\chi^2}{m^2}\right)^{\frac{1}{2}}}{\left(1-\frac{m_D^2}{4m^2}\right)^2}\left(1+\frac{m_\chi^2}{2m^2}\right)$$

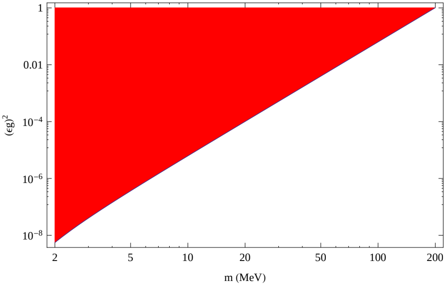

Figure : Constraint on ϵg_D vs m_D. m_χ is set to 0.

Comments on Ps invisible decays.

1. They are s-channel annihilations so they can probe values of $m_X > 1\mathrm{MeV}$.
2. They are now more model dependent.
3. Scalar exchange is forbidden.
4. ALP exchange is allowed only for pPs
5. Dark photon exchange is allowed for both pPs and oPs

Precision Ps spectroscopy

Ps is very well described by QED. The most accurate experimental measure for $\mathbf{1\,^3S_1} \rightarrow 2\,^3S_1$ is

$$E^{\mathrm{exp}_{\mathbf{1\,^3S_1}\rightarrow 2\,^3S_1}} = 1233607216.4 \pm 3.2\,\mathrm{MHz}$$

and the most precise theory calculation is

$$E^{\mathrm{th}_{\mathbf{1\,^3S_1}\rightarrow 2\,^3S_1}} = 1233607222.12 \pm 0.58\,\mathrm{MHz}.$$

State of art of precision Ps measurements

Transitions	Exp(MHz)	Year	U(exp)	U(th)
$\mathbf{1\,^3S_1} \rightarrow 2^3S_1$	1233607216.4 ± 3.2	1993	2.6 ppb	0.8 ppb
$\mathbf{1\,^3S_1} \rightarrow \mathbf{1\,^1S_0}$	$203394.2 \pm 1.6 \pm 1.3$	2014	10.1 ppm	2.5 ppm
$2^3S_1 \rightarrow 2^3P_0$	$18499.65 \pm 1.2 \pm 4.0$	1993	266 ppm	7.0 ppm
$2^3S_1 \rightarrow 2^3P_1$	$13012.42 \pm 0.67 \pm 1.54$	1993	129 ppm	10.0 ppm
$2^3S_1 \rightarrow 2^3p_2$	$8624.38 \pm 0.54 \pm 1.40$	1993	174 ppm	15.1 ppm

U is the fractional uncertainty.

Uses of precision spect.

Ways to make use of such beautiful results

1. A a probe of fifth force due to exchange of light bosons such as S, A, γ_D.
2. Use with even more impressive spectroscopy of hydrogen which has hadronic uncertainties such as the proton radius and improve our understanding of QCD at low energies. This will require improving Ps spectroscopy at least by an order of magnitude.
3. Use it as a measurement of fundamental parameter such as α. This will require improvement of Ps Rydberg measurement.
4. If the above can be achieved a comparison with electron $g-2$ can be a powerful probe of new light degrees of freedom

Ps spect. as fifth force probe

The physics is depicted below

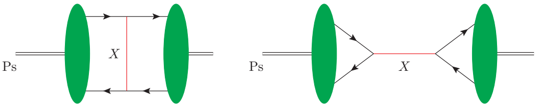

Figure : The one quantum exchange contribution to positronium potential. $X = S, a, \gamma_D$.

- The QED part is just $X \to \gamma$ and is very accurately calculated for several transitions.
- The annihilation term is not present for hydrogen atom and muonium.
- Any modification of Coulomb force by one particle exchange of X will give a contribution to electron $g-2$ but not vice versa.

scalar exchange force

The potential from scalar exchange is

$$\delta V_s(r) = -\lambda_s^2 \left\{ \left(1 + \frac{m_s^2}{4m^2}\right) \frac{1}{4\pi r}\, e^{-m_s r} - \frac{1}{4m^2}\delta^3(\mathbf{r}) \right\}$$

The two transitions used are

1. $\mathbf{1\,{}^3S_1} \to 2\,{}^3S_1$ which agrees well with theory.
2. $2\,{}^3S_1 \to 2\,{}^3P_0$ which has a discrepancy with theory equal to a shift of $2.77\mathrm{MHz}$.

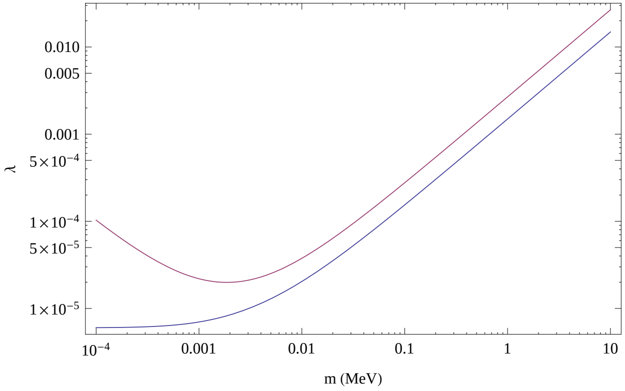

Figure : Coupling λ_S vs m_S Blue line is for $1S \to 2S$ and magenta line is for $2S \to 2P$.

ALP exchange

This affects the hyperfine splittings (HFS) due to spin-spin correlations. The induced potential is

$$\delta V_a = -\frac{\lambda_a^2}{16\pi m^2}\left\{(2\mathbf{S}^2-3)\left[\frac{4\pi}{3}\delta^3(\mathbf{r}) - \frac{m_a^2}{3r}\right]e^{-m_a r} + 4\pi\frac{\mathbf{S}^2-2}{1-\frac{m_a^2}{4m^2}}\,\delta^3(\mathbf{r})\right\}$$

where $\mathbf{S} = \frac{1}{2}(\boldsymbol{\sigma}_+ + \boldsymbol{\sigma}_-)$ and $\mathbf{S}^2 = S(S+1)$.

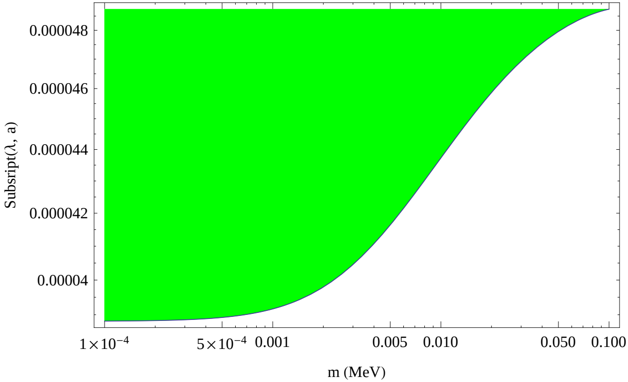

Figure : Coupling λ_a vs m_a using $\mathbf{1\,^1S_0} \rightarrow \mathbf{1\,^3S_1}$ transition.

The Rydberg

Currently the Rydberg from Ps is

$$R_{\mathrm{Ps}} = 3289841945(15)\,\mathrm{MHz}\ \ (4.6\mathrm{ppb})$$

The CODATA value for R is 1.9 (ppt). Consider energy of a hydrogen state with q.n. $n, \ell, \ldots$ can be written as

$$E(n, \ell, \ldots) = -\frac{R}{n^2} + \frac{2m^3\alpha^4}{2n^3} r_p^2 \delta_{\ell,0} + \tilde{E}$$

where δE due to uncertainty in r_p (δr_p) is given by

$$\delta E = \frac{8}{3} R m^2 \alpha^2 r_p \delta r_p$$

Energy shift due to uncertainly in R is δR. For $\delta r_9 = .034$ fm and $r_p = 0.84037$ fm this corresponds to a shift of $\frac{\delta R}{R} \sim 27$ (ppt). If we can get R from Ps to (ppt) level then one can have a separate determination of δr_p. It appears that 80 (ppt) is possible.

Looking forward

1. $Ps \rightarrow \gamma + \text{inv}$ is a good probe of light X feeble coupled to electrons. It probes parameter space not accessible by colliders. However, the measurements are done in 1990's . Improvements are necessary.
2. $Ps \rightarrow \text{inv}$ can the ranged of X masses to tens of MeV. Beyond which no meaningful constraints can be made. Again the measurements are dated. Improvements are necessary.
3. Ps spectroscopy can be viewed as fifth force probes. Recent (after 2014) have pushed sensitivity to ppm and ppd level depending on transitions.
4. The real challenge is extract the Rydgerg from Ps at 10 ppt level. This can then be an independent clean measurement of α much like the recent measurement of a_e.
5. This also requires next order α^7 calculation.
6. This discussion has been focussed on one particle exchange forces. Other possibilties such as quantum forces or Yukawa-like forces have yet to be studied.

References

1. G.S. Adkins et al ,"Precision spectroscopy of Positronium", Phys. Rept. **C 975** 1 (2022).
2. D.B. Cassidy," Experimental progress in positronium laser physics", Eur. Phys. J **D 72** 53 (2018)
3. S.D. Bass et al, "Positronium physics and biomedical applications", arXiv 2302.09246 (2023) [physics.med − ph]