U.S. National Science Foundation

Toward extracting scattering phase shift from integrated correlation function in lattice QCD

Peng Guo

Dakota State University

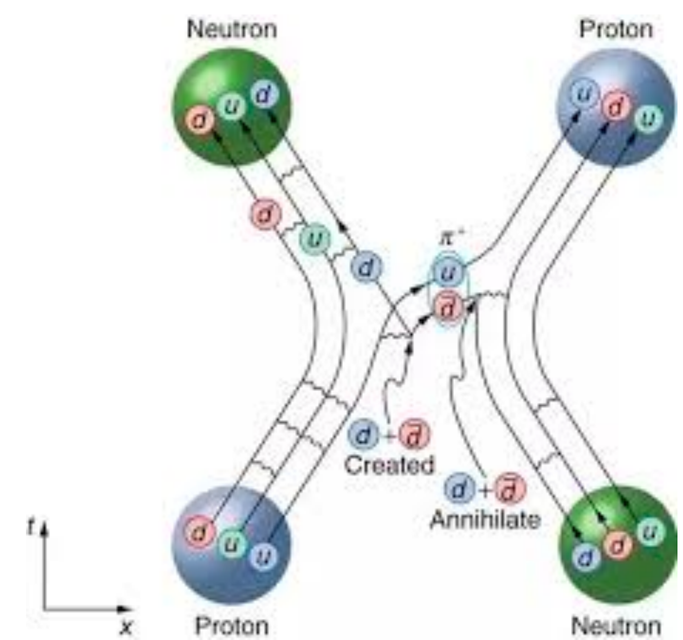
NN 2024

14th International Conference on NN Collisions 2024
Whistler, BC Canada

Major tasks in Nuclear/Hadron physics:

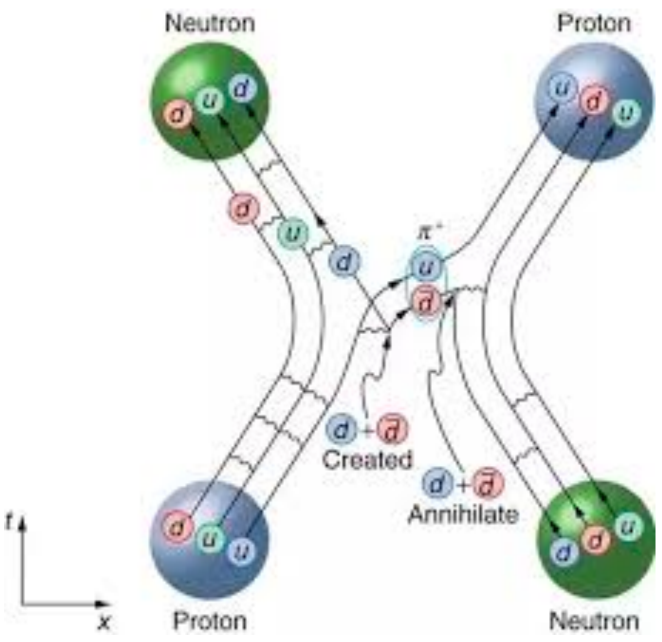
- Understand and extract few-body dynamics, such as scattering phase shift

- Understand resonances and bound states

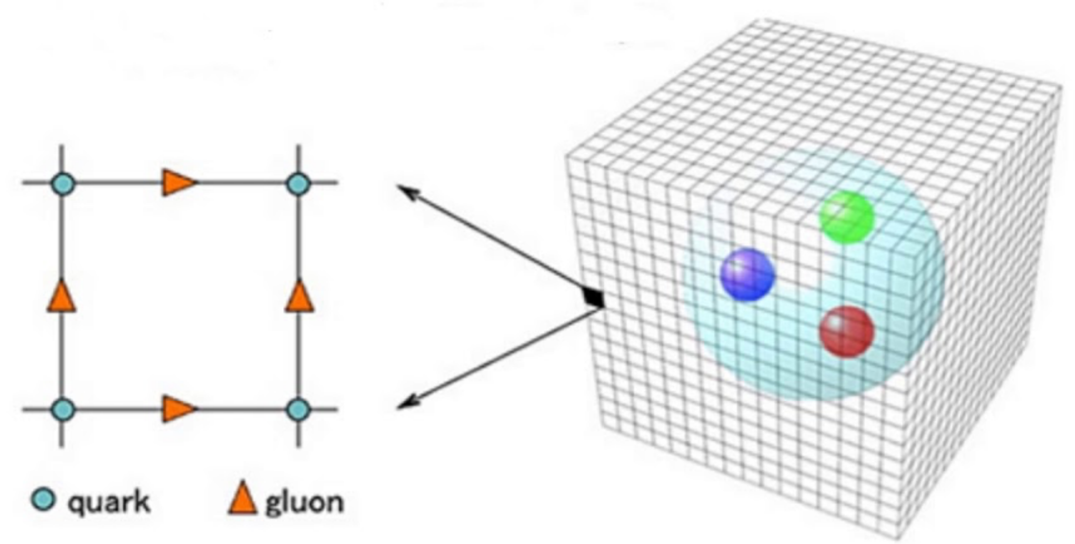


Major tasks in Nuclear/Hadron physics:

- Understand and extract few-body dynamics, such as scattering phase shift



- Understand resonances and bound states

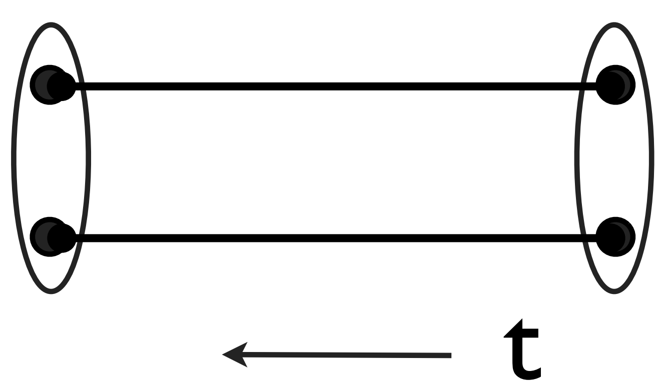


☑ **Periodic box in LQCD**

Lattice QCD: one route to non-perturbative dynamics

Lattice QCD: one route to non-perturbative dynamics

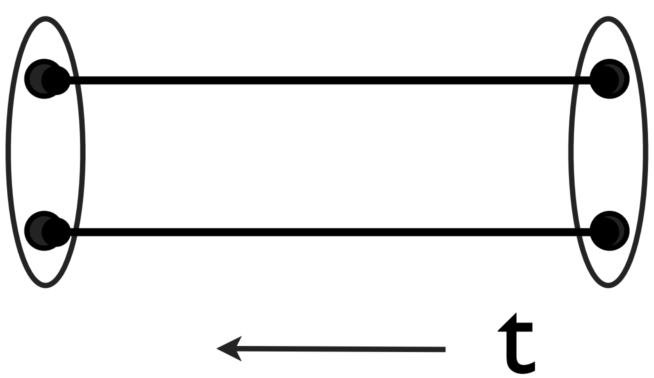
$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$



Lattice QCD: one route to non-perturbative dynamics

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \propto \int \mathcal{D}[U]\mathcal{D}[\psi,\bar{\psi}]e^{-S_G[U]-S_F[\psi,\bar{\psi},U]}\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

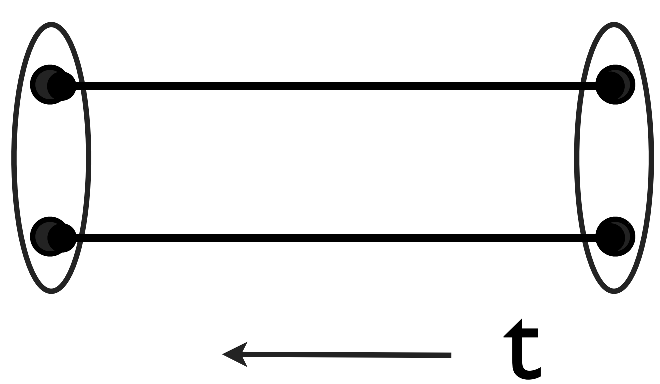


Lattice QCD: one route to non-perturbative dynamics

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \propto \int \mathcal{D}[U]\mathcal{D}[\psi,\bar{\psi}]e^{-S_G[U]-S_F[\psi,\bar{\psi},U]}\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

$$S_G[U]+S_F[\psi,\bar{\psi},U] \rightarrow \int d^4x\bar{\psi}(x)[\gamma_\mu(\partial_\mu+igA_\mu(x))+m]\psi(x)+\frac{1}{2}\int d^4x tr[F_{\mu\nu}(x)F_{\mu\nu}(x)]$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

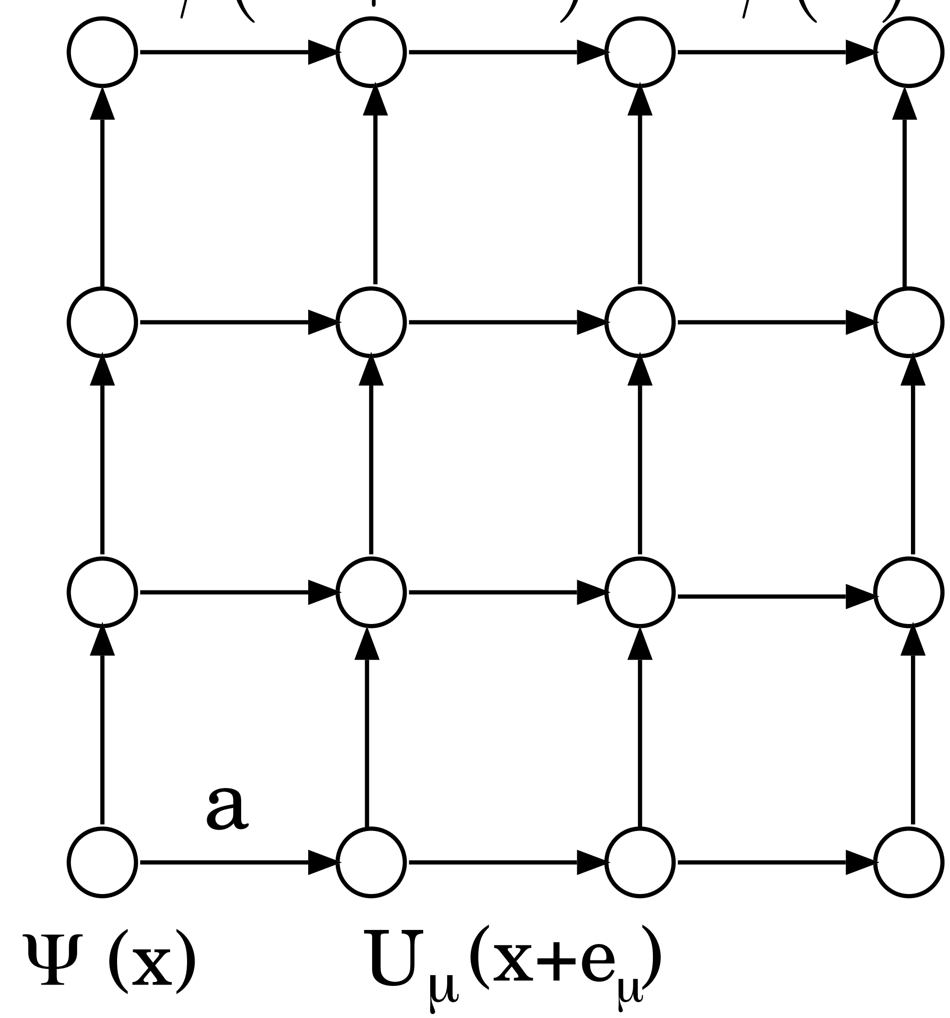


Lattice QCD: one route to non-perturbative dynamics

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \propto \int \mathcal{D}[U]\mathcal{D}[\psi,\bar{\psi}]e^{-S_G[U]-S_F[\psi,\bar{\psi},U]}\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

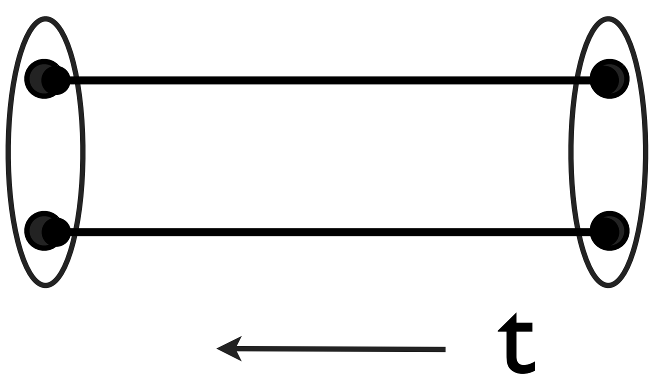
$$S_G[U]+S_F[\psi,\bar{\psi},U] \rightarrow \int d^4x\bar{\psi}(x)[\gamma_\mu(\partial_\mu+igA_\mu(x))+m]\psi(x)+\frac{1}{2}\int d^4x tr[F_{\mu\nu}(x)F_{\mu\nu}(x)]$$

$$\psi(x+nL)=\psi(x)$$



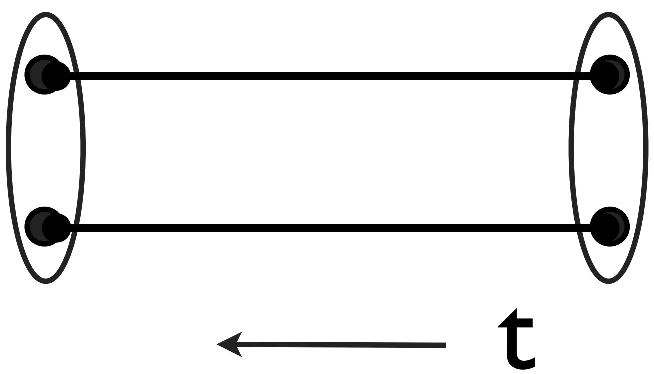
$$U_\mu(n)=e^{iaA_\mu(n)}$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$



Spectroscopy from Lattice QCD

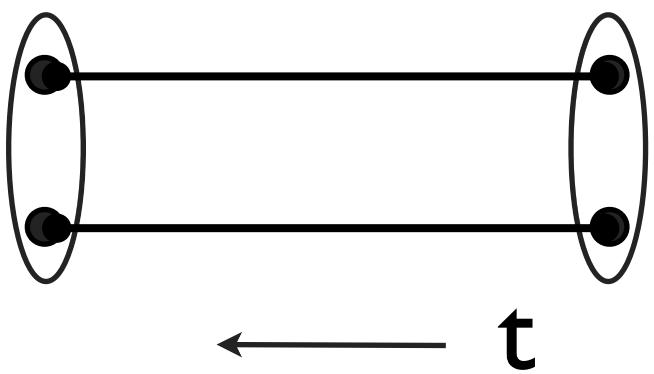
$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$



Spectroscopy from Lattice QCD

$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

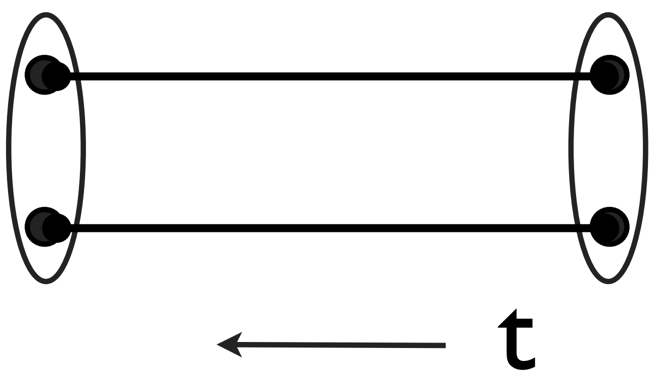


Spectroscopy from Lattice QCD

$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

$$\sum_n |n\rangle\langle n| = 1$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

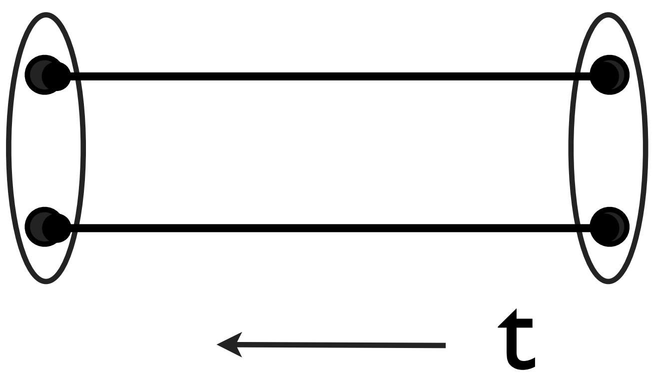


Spectroscopy from Lattice QCD

$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

$$\sum_n |n\rangle\langle n| = 1$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \sim \sum_n z_n^* z_n e^{-E_n t}, \; z_n = \langle n|\mathcal{O}^\dagger(0)|0\rangle$$



Spectroscopy from Lattice QCD

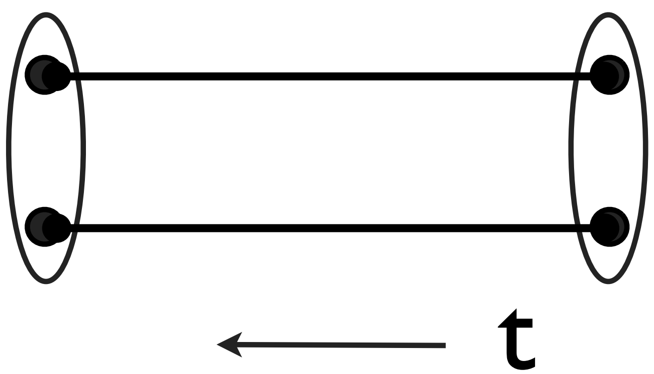
$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

$$\sum_n |n\rangle\langle n| = 1$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle$$

$$\sim \sum_n z_n^* z_n e^{-E_n t}, \; z_n = \langle n|\mathcal{O}^\dagger(0)|0\rangle$$

$$\to |z_0|^2 e^{-E_0 t}$$



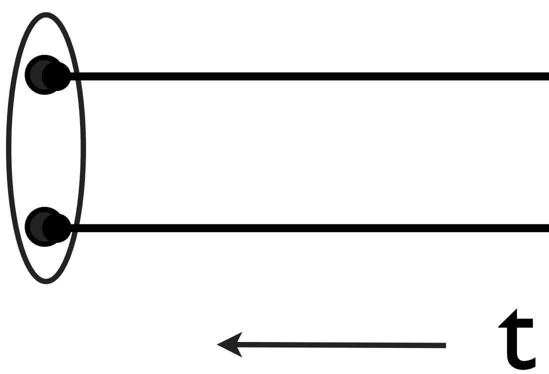
Spectroscopy from Lattice QCD

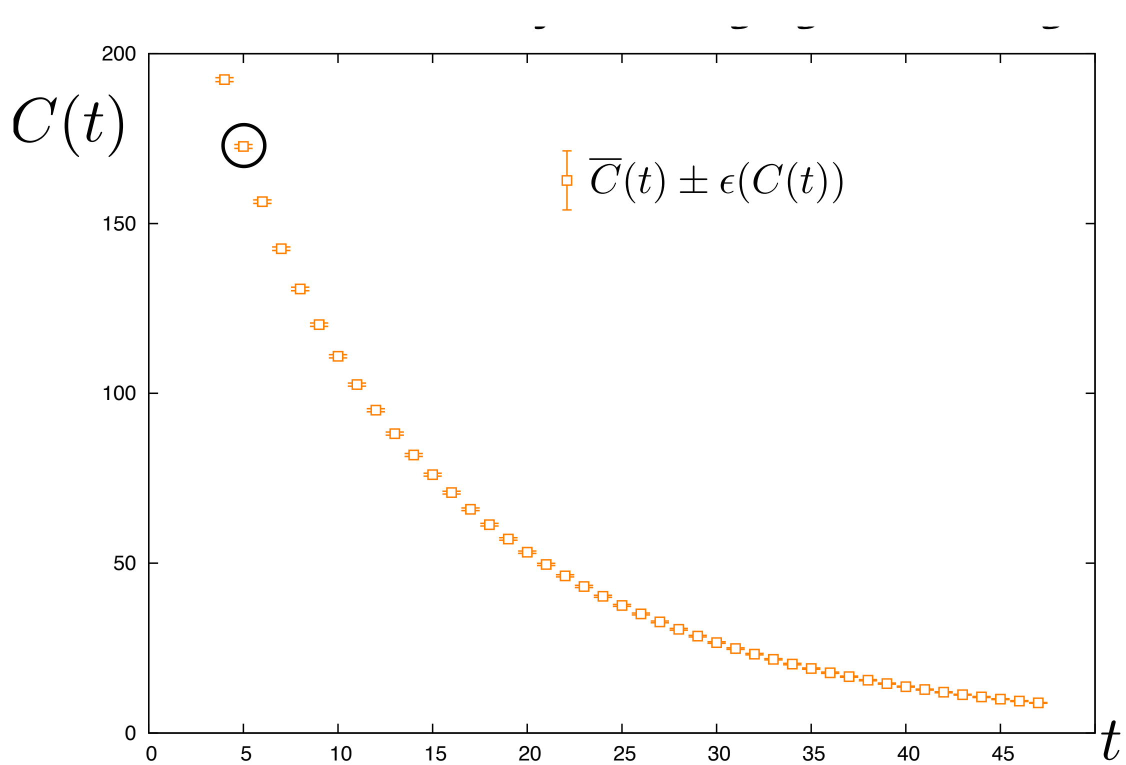
$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

$$\sum_n |n\rangle\langle n| = 1$$

$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \sim \sum_n z_n^* z_n e^{-E_n t}, \; z_n = \langle n|\mathcal{O}^\dagger(0)|0\rangle$$

$$\rightarrow |z_0|^2 e^{-E_0 t}$$



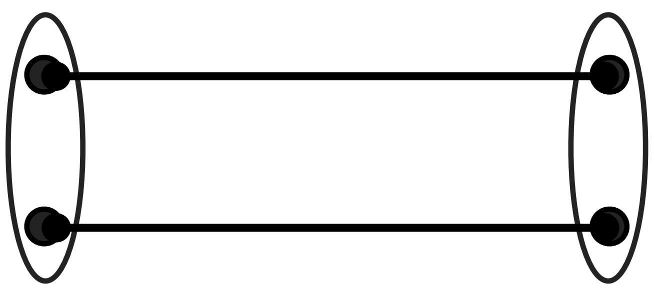


Spectroscopy from Lattice QCD

$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

$$\sum_n |n\rangle\langle n| = 1$$

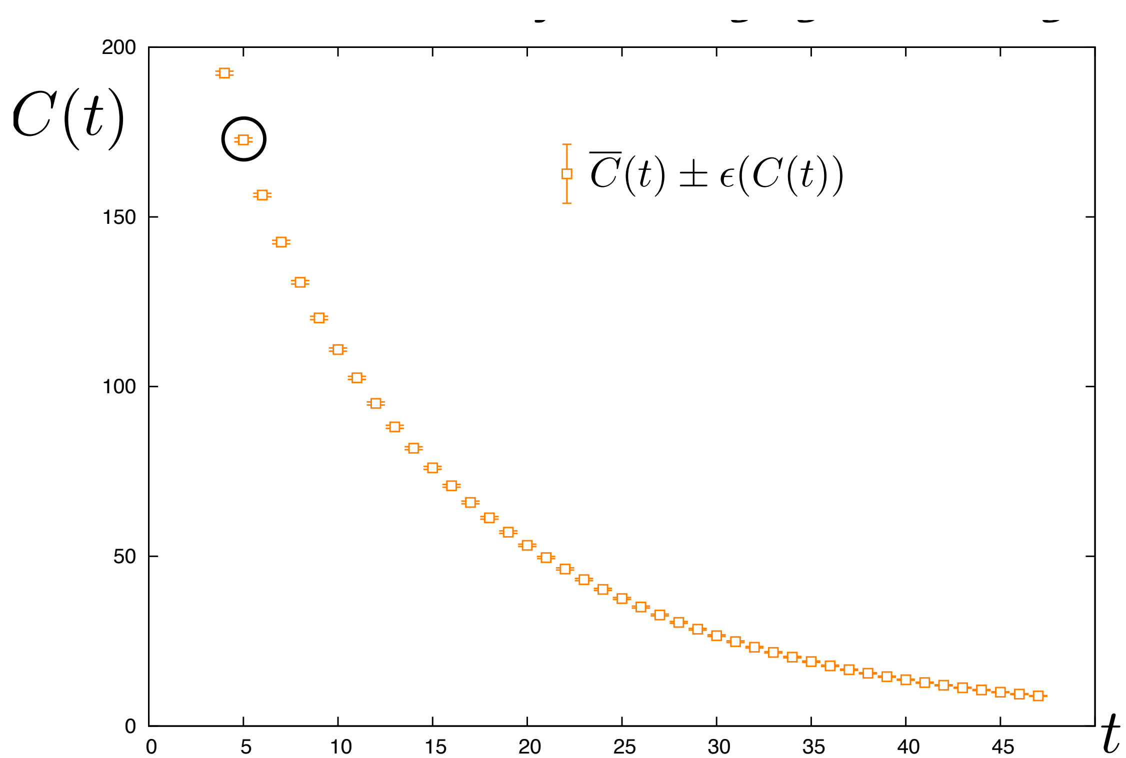
$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \sim \sum_n z_n^* z_n e^{-E_n t},\ z_n = \langle n|\mathcal{O}^\dagger(0)|0\rangle$$

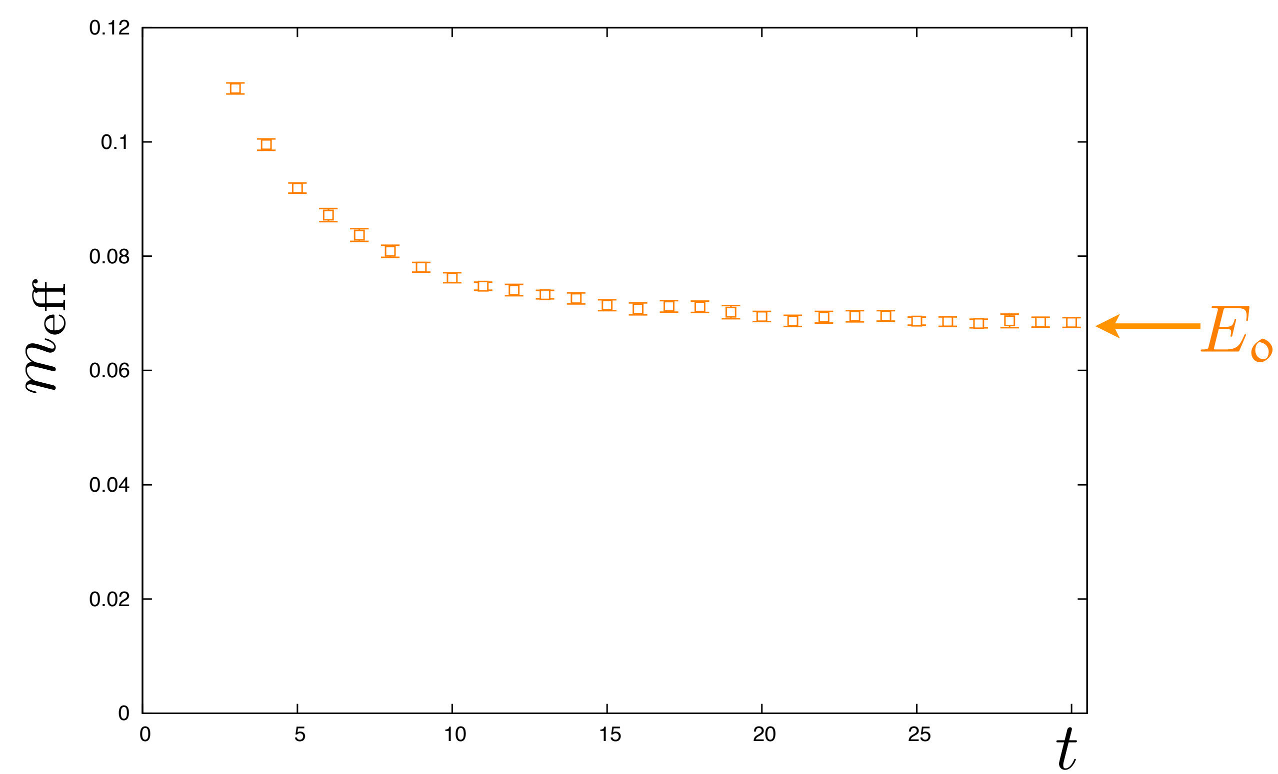


$\longleftarrow$ **t**

$$\rightarrow |z_0|^2 e^{-E_0 t}$$

$$m_{\text{eff}}(t) = -\frac{1}{\delta t}\ln\frac{C(t+\delta t)}{C(t)}$$





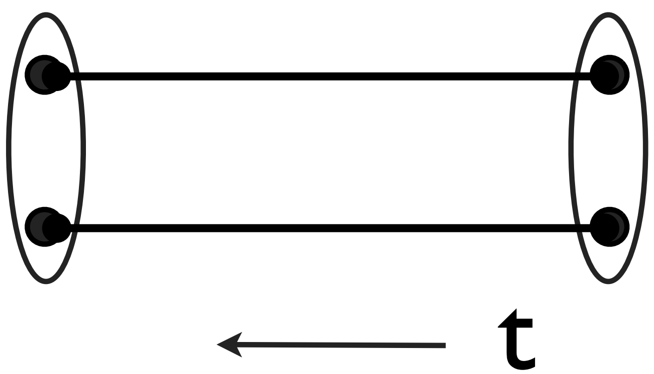
Spectroscopy from Lattice QCD

$$\mathcal{O}(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

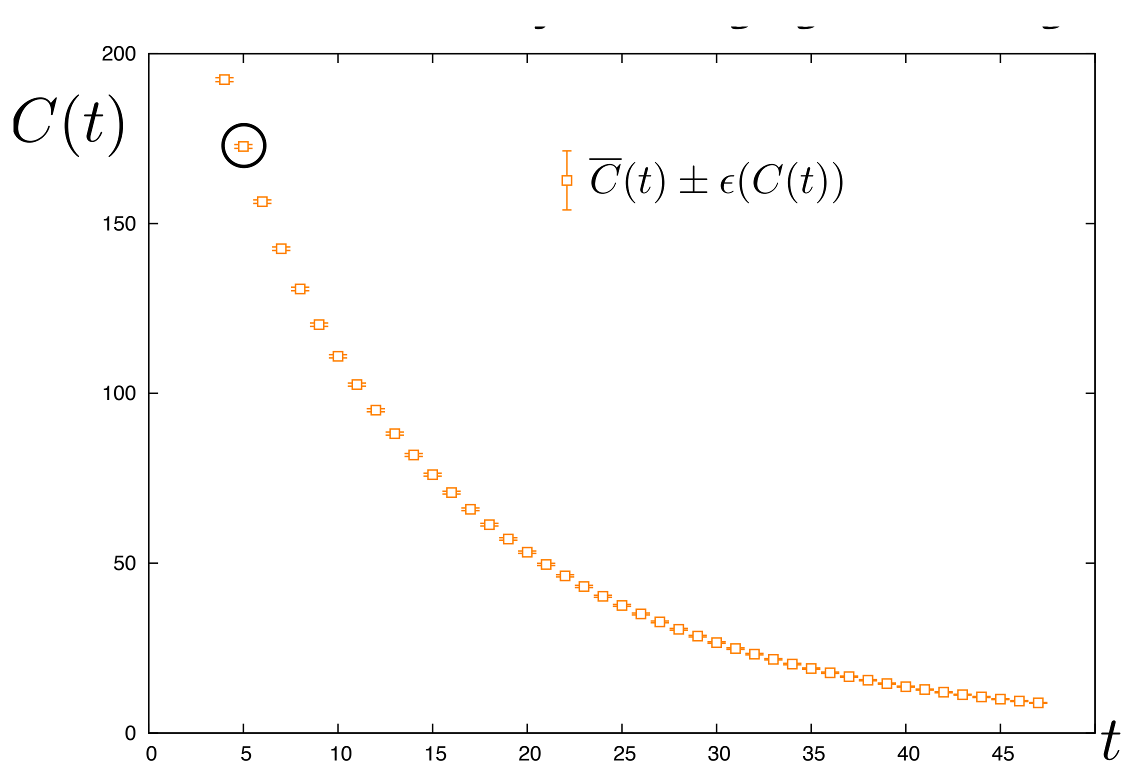
$$\sum_n |n\rangle\langle n| = 1$$

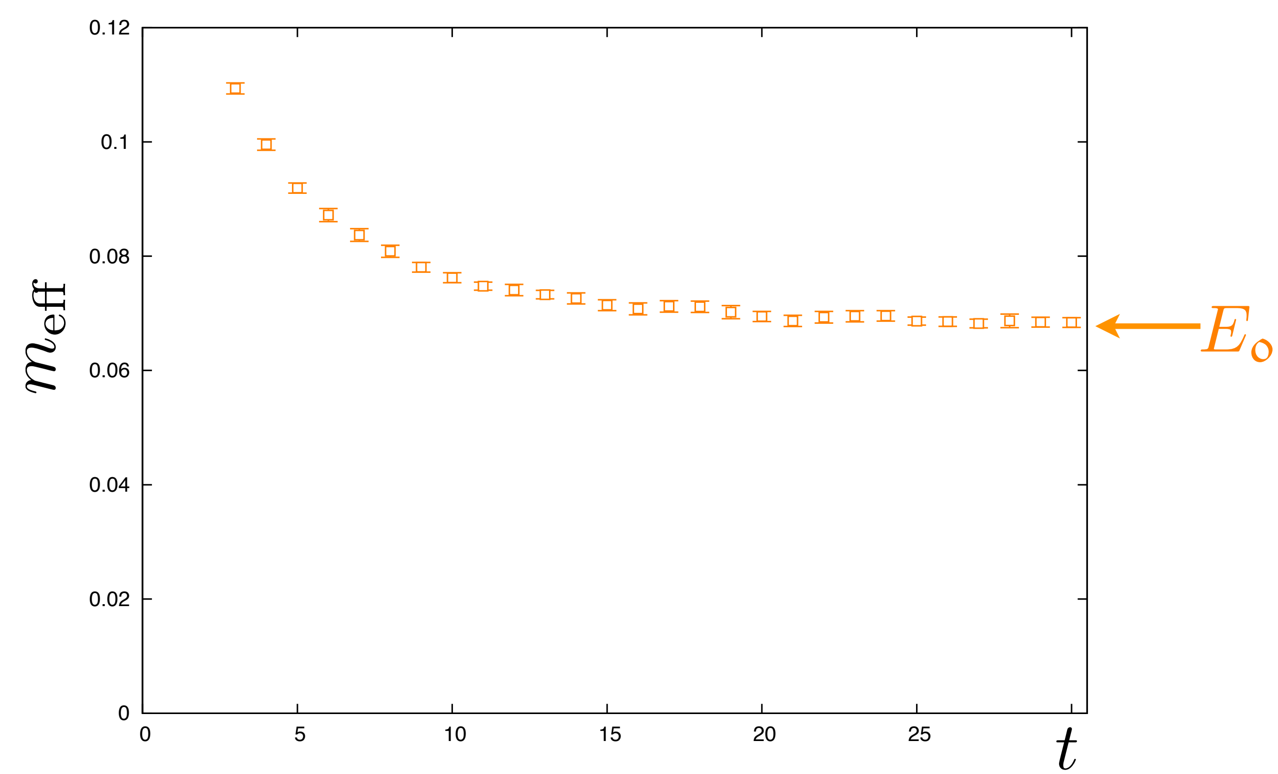
$$\langle 0|\mathcal{O}(t)\mathcal{O}^\dagger(0)|0\rangle \sim \sum_n z_n^* z_n e^{-E_n t}, \; z_n = \langle n|\mathcal{O}^\dagger(0)|0\rangle$$

$$\to |z_0|^2 e^{-E_0 t}$$

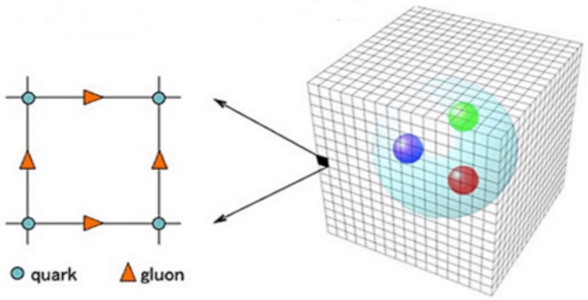


$$m_{\text{eff}}(t) = -\frac{1}{\delta t}\ln\frac{C(t+\delta t)}{C(t)}$$





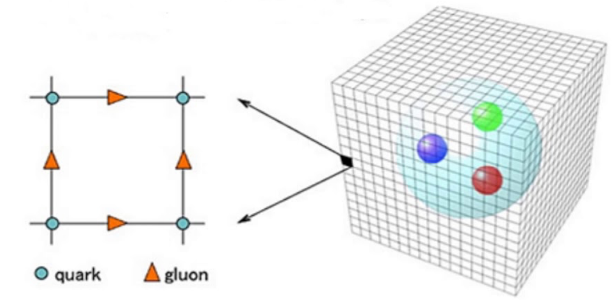
Few-body interaction in a box:



Challenges:

- ☑ No asymptotic states
- ☑ Stationary solutions instead of scattering solutions
- ☑ Discrete energies

Few-body interaction in a box:

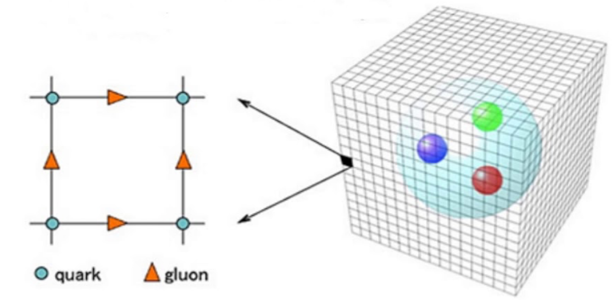


Challenges:

- ☑ No asymptotic states
- ☑ Stationary solutions instead of scattering solutions
- ☑ Discrete energies

- Extracting Two-body dynamics from discrete energy levels

Few-body interaction in a box:



Challenges:

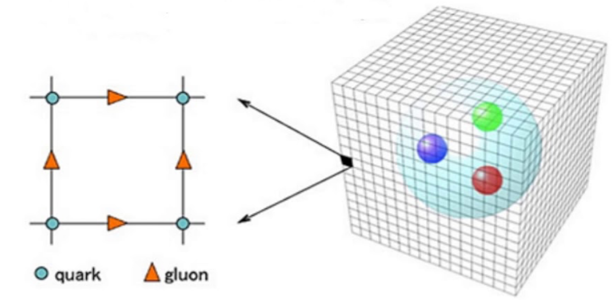
- No asymptotic states
- Stationary solutions instead of scattering solutions
- Discrete energies

Extracting Two-body dynamics from discrete energy levels

- Lusecher formula-like QC as result of factorization of long-range effect and short-range dynamics

$$\det\left[\cot\delta(E) - \mathcal{M}(E)\right] = 0$$

Few-body interaction in a box:



Challenges:

- No asymptotic states
- Stationary solutions instead of scattering solutions
- Discrete energies

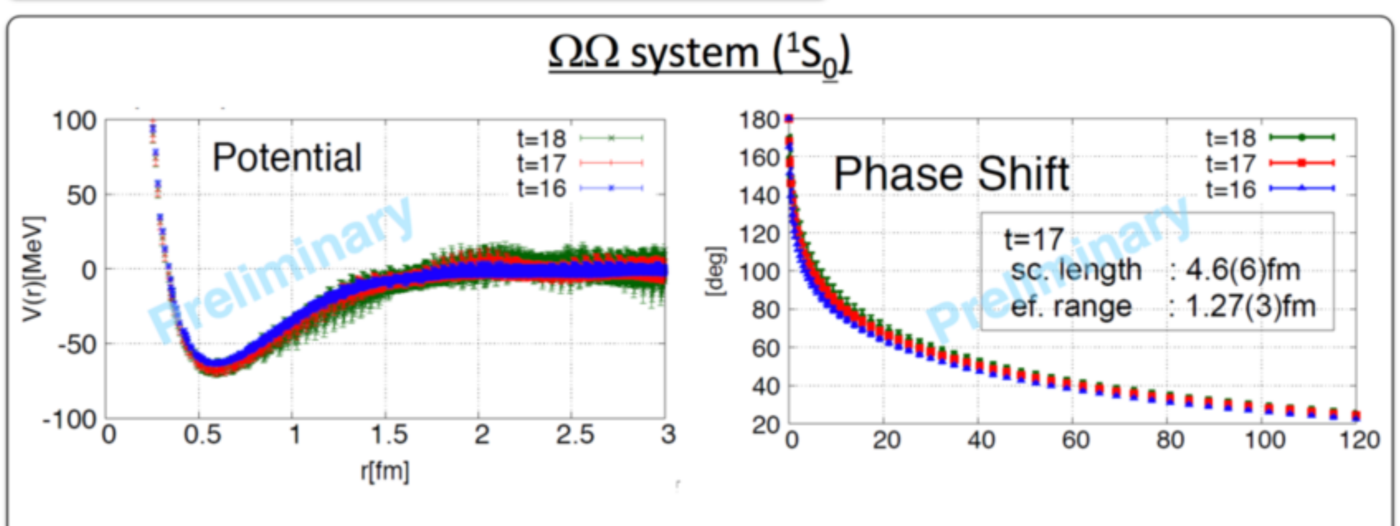
- Extracting Two-body dynamics from discrete energy levels

  - Lusecher formula-like QC as result of factorization of long-range effect and short-range dynamics

$$\det\left[\cot\delta(E) - \mathcal{M}(E)\right] = 0$$

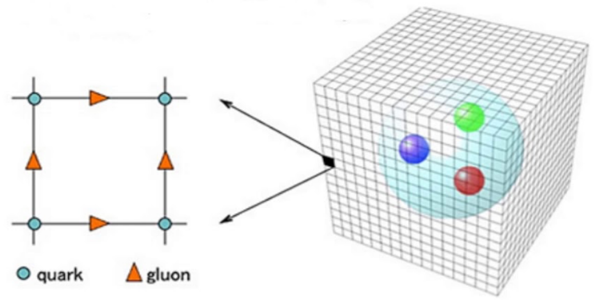
  - HAL QCD collaboration potential method

$N_f = 2+1,\ m_\pi = 146$ MeV, $a \approx 0.085$ fm, $L \approx 8$ fm



EPJ Web of Conferences **175**, 01022 (2018)
Lattice 2017

NN controversy:



- Two-Nucleon correlation function suffer Signal-to-noise ratio issue:

$$\mathcal{R}(t) \overset{t\to\infty}{\sim} e^{-(m_N - \frac{3}{2}m_\pi)t}$$

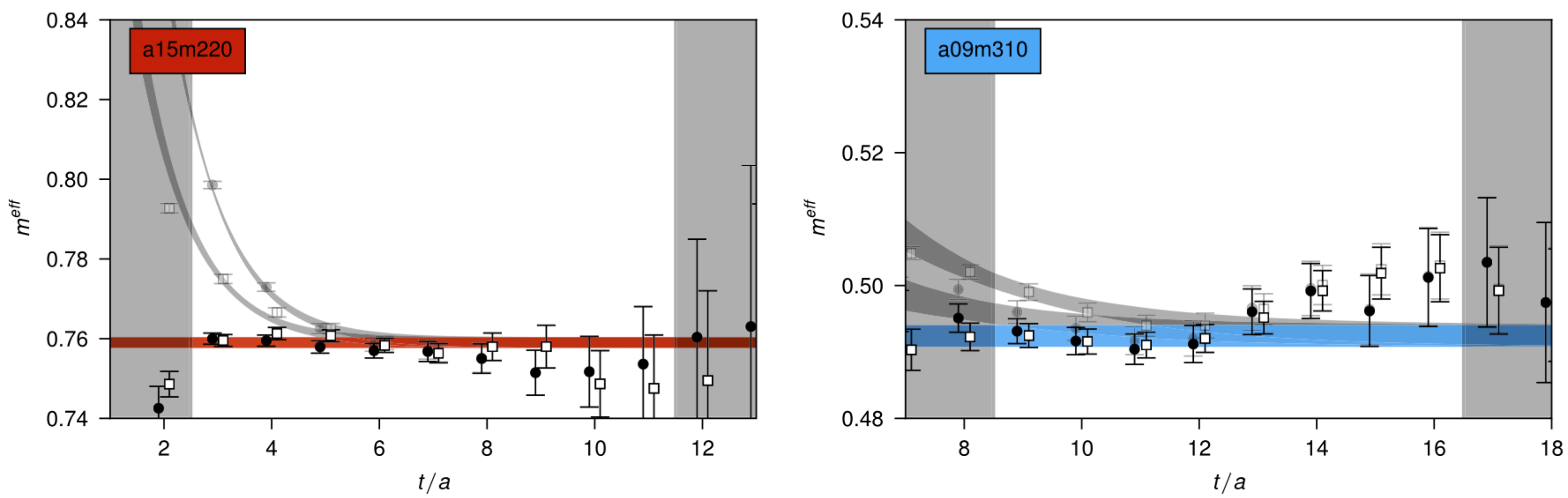


Fig. 1. Effective mass plots of the nucleon from Ref. [64] which suffer from correlated, late time fluctuations, making it more challenging to identify the ground state. The top plot is from a calculation with $a \sim 0.15$ fm and $m_\pi \sim 220$ MeV while the bottom is for $a \sim 0.09$ fm and $m_\pi \sim 310$ MeV.

Progress in Particle and Nuclear Physics 121 (2021) 103888

- Lusecher formula and HAL QCD collaboration potential method disagree on whether or not two-nucleon form a bound state with pion masses as heavy as 800 MeV

Progress in Particle and Nuclear Physics 121 (2021) 103888

- Lusecher formula face difficulties at large volume limit due to increasing density of states

Alternative to Lusecher formula method:

- The difference of integrated correlation functions between interacting and non-interacting systems approaches rapidly to its infinite volume limit which is related to scattering phase shift

$$C(t) - C_0(t) = \sum_n [e^{-\epsilon_n t} - e^{-\epsilon_n^{(0)} t}] \stackrel{L\to\infty}{\to} \frac{t}{\pi}\int_0^\infty d\epsilon\delta(\epsilon)e^{-\epsilon t}$$

where $$C(t) = \int_0^L drC(rt;r0)$$

$$C(rt;r'0) = \langle 0|T\left[\widehat{\mathcal{O}}_H(r,t)\widehat{\mathcal{O}}_H^\dagger(r',0)\right]|0\rangle$$

Two-particle creation operator

PHYSICAL REVIEW D **108,** 074504 (2023)

- e-Print: 2402.15628 [hep-lat]

Alternative to Lusecher formula method:

- Exactly solvable model with contact interaction: $$\left[-\frac{1}{2\mu}\frac{d^2}{dr^2}+V_0\sum_{n\in\mathbb{Z}}\delta(r+nL)\right]\psi_\epsilon^{(\mathrm{rel})}(r)=\epsilon\psi_\epsilon^{(\mathrm{rel})}(r)$$

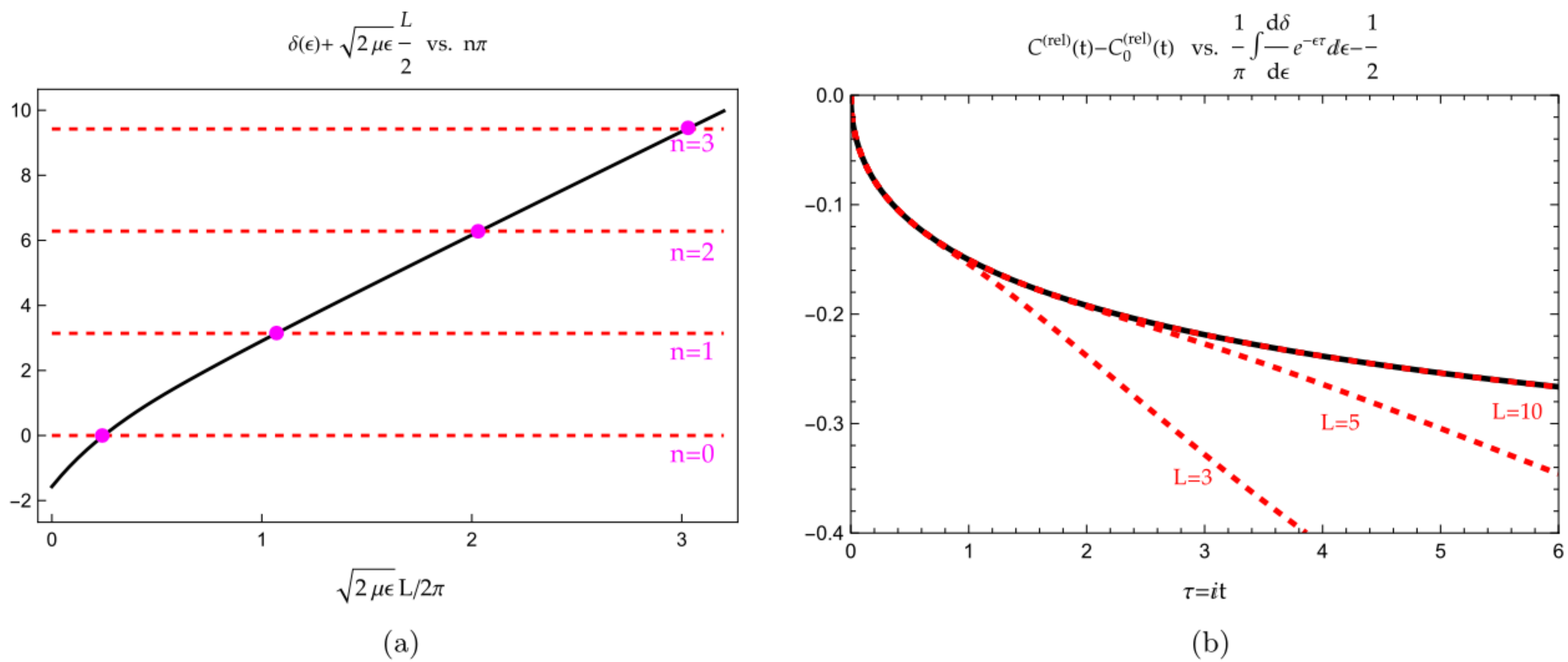


FIG. 1. The energy spectra and difference of integrated correlation function plots for particles interaction in a periodic box: (a) $\delta(\epsilon_n)+\sqrt{2\mu\epsilon_n}\frac{L}{2}$ (solid black) vs $n\pi$ (dashed red) with $L=3$, energy spectra are located at intersection points of black and red curves; (b) $\frac{1}{\pi}\int_0^\infty d\epsilon\frac{d\delta(\epsilon)}{d\epsilon}e^{-\epsilon\tau}-\frac{1}{2}$ (solid black) vs $C^{(\mathrm{rel})}(t)-C_0^{(\mathrm{rel})}(t)$ (dashed red) with $L=3,5,10$. The rest of parameters are taken as $V_0=0.5$ and $\mu=1$.

PHYSICAL REVIEW D **108,** 074504 (2023)

- e-Print: 2402.15628 [hep-lat]

Alternative to Lusecher formula method:

- Exactly solvable model with contact interaction: $\left[-\frac{1}{2\mu}\frac{d^2}{dr^2}+V_0\sum_{n\in\mathbb{Z}}\delta(r+nL)\right]\psi_\epsilon^{(\mathrm{rel})}(r)=\epsilon\psi_\epsilon^{(\mathrm{rel})}(r)$

$$\longrightarrow \quad C(t)-C_0(t)\overset{L\to\infty}{\longrightarrow}\frac{1}{2}\mathrm{erfc}(\mu V_0\sqrt{\frac{t}{2\mu}})e^{(\mu V_0)^2\frac{t}{2\mu}}-\frac{1}{2}$$

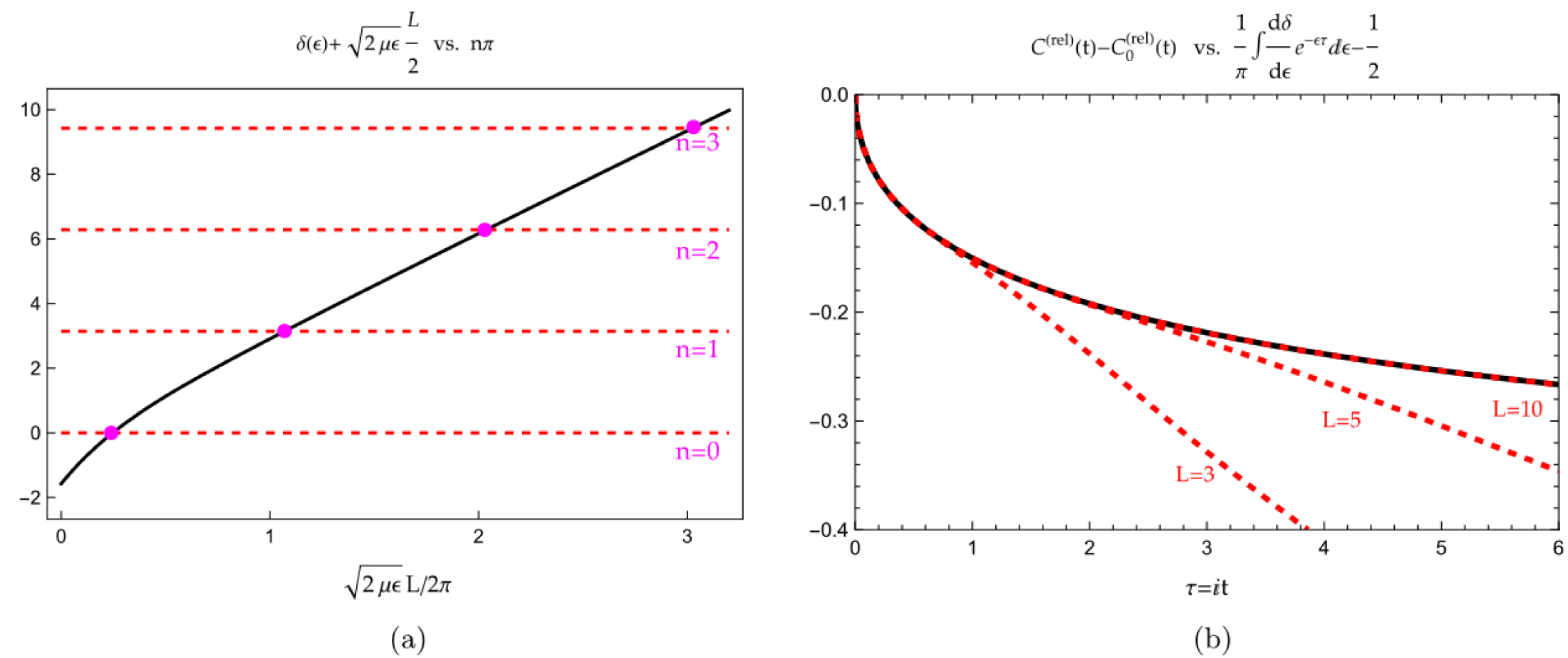


FIG. 1. The energy spectra and difference of integrated correlation function plots for particles interaction in a periodic box: (a) $\delta(\epsilon_n)+\sqrt{2\mu\epsilon_n}\frac{L}{2}$ (solid black) vs $n\pi$ (dashed red) with $L=3$, energy spectra are located at intersection points of black and red curves; (b) $\frac{1}{\pi}\int_0^\infty d\epsilon\frac{d\delta(\epsilon)}{d\epsilon}e^{-\epsilon\tau}-\frac{1}{2}$ (solid black) vs $C^{(\mathrm{rel})}(t)-C_0^{(\mathrm{rel})}(t)$ (dashed red) with $L=3,5,10$. The rest of parameters are taken as $V_0=0.5$ and $\mu=1$.

PHYSICAL REVIEW D **108,** 074504 (2023)

- e-Print: 2402.15628 [hep-lat]

Alternative to Lusecher formula method:

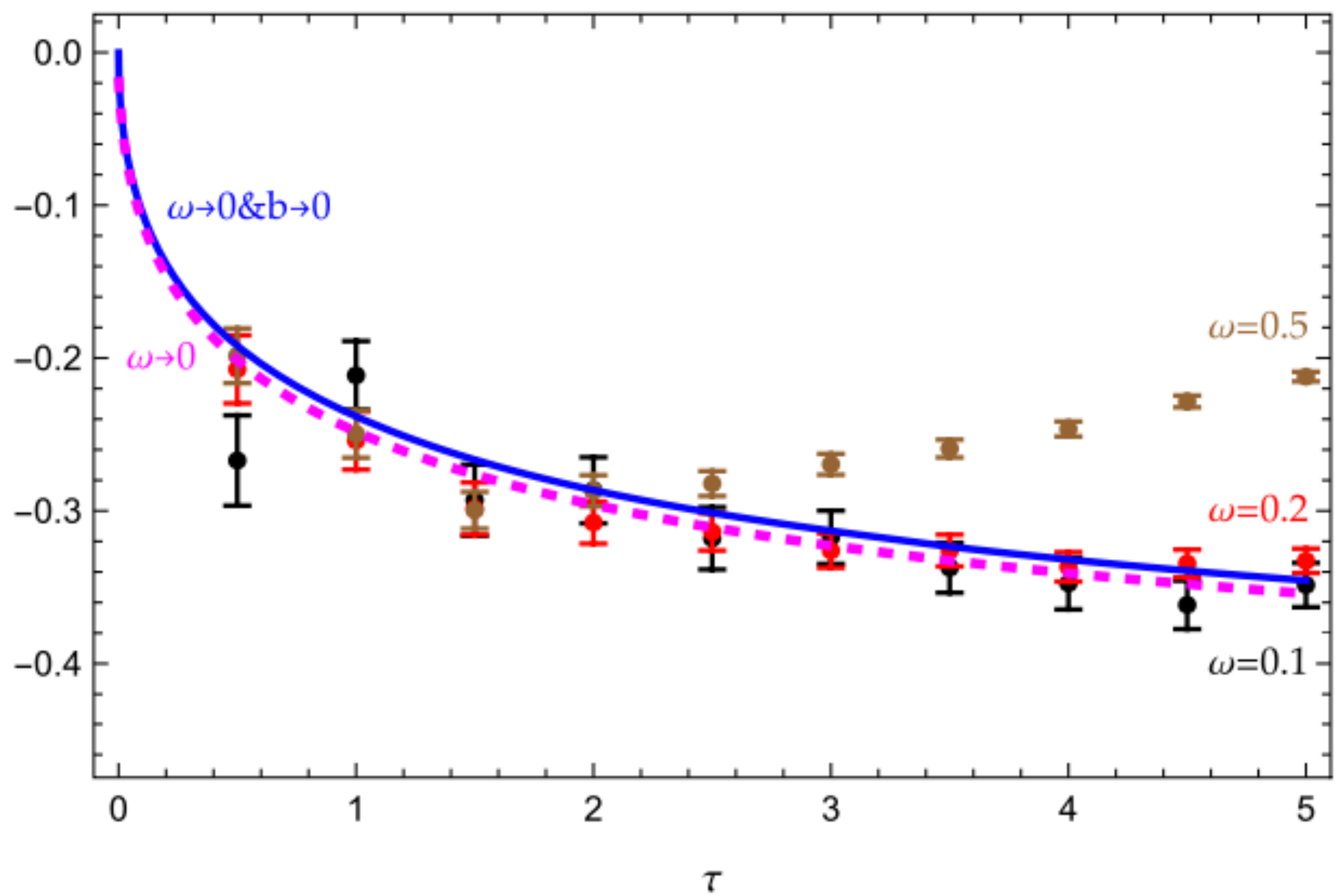
$$\left[-\frac{1}{2\mu}\frac{d^2}{dr^2}+\frac{1}{2}\mu\omega^2 r^2+V(r)\right]\psi_n(r)=\epsilon_n\psi_n(r),$$

where

$$V(r)=\begin{cases}\frac{V_0}{b}, & r\in[-\frac{b}{2},\frac{b}{2}]\\ 0, & \text{otherwise}\end{cases}, \quad \overset{b\to 0}{\to} V_0\delta(r).$$

- Monte Carlo simulation test with square well potential in harmonic trap:

$$c(\tau)-c_0(\tau) \text{ vs. } \frac{1}{\pi}\int d\epsilon \frac{d(\delta_+ + \delta_-)}{d\epsilon} e^{-\epsilon\tau}-\frac{1}{2}$$



PHYSICAL REVIEW D **108,** 074504 (2023)

- e-Print: 2402.15628 [hep-lat]

Alternative to Lusecher formula method:

- Relativistic extension for complex scalar lattice field theory model

$$S_E = -\kappa \sum_{x,t,\hat{n}_x,\hat{n}_t} \hat{\phi}^*(x,t)\hat{\phi}(x+\hat{n}_x, t+\hat{n}_t) + c.c. + (1-2\lambda)\sum_{x,t}|\hat{\phi}(x,t)|^2 + \lambda \sum_{x,t}|\hat{\phi}(x,t)|^4$$

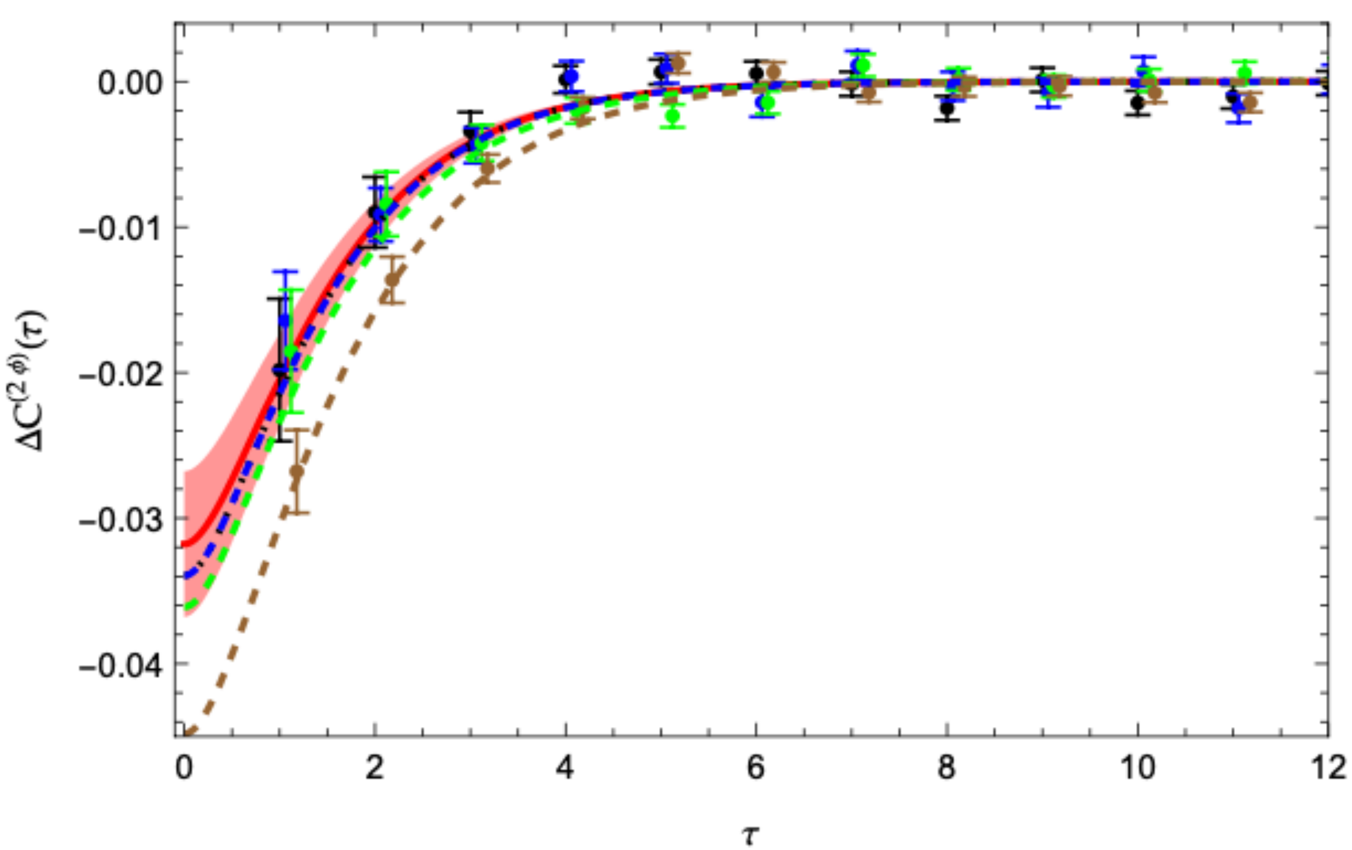
PHYSICAL REVIEW D **110,** 014504 (2024)

- e-Print: 2402.15628 [hep-lat]

Alternative to Lusecher formula method:

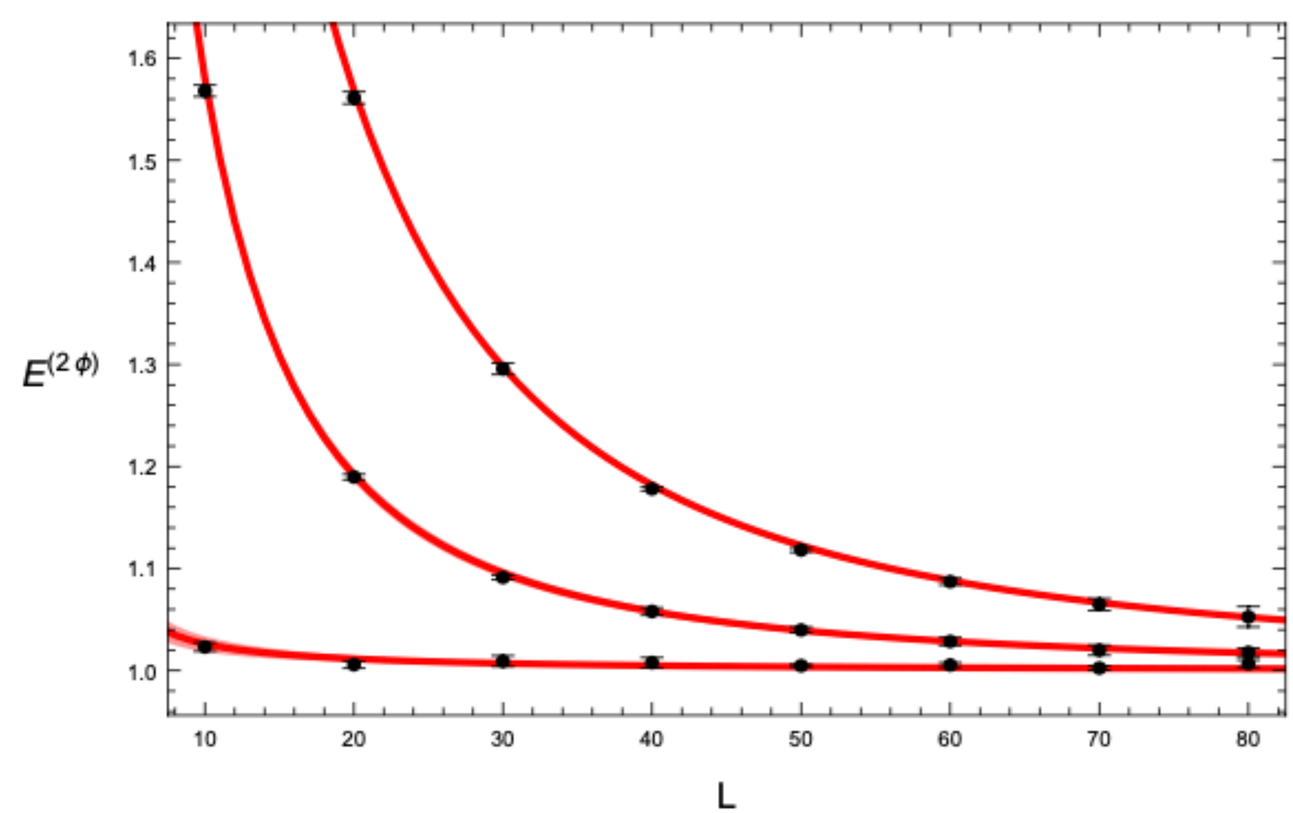
- Relativistic extension for complex scalar lattice field theory model

$$S_E = -\kappa \sum_{x,t,\hat{n}_x,\hat{n}_t} \hat{\phi}^*(x,t)\hat{\phi}(x+\hat{n}_x, t+\hat{n}_t) + c.c.$$
$$+ (1-2\lambda)\sum_{x,t} |\hat{\phi}(x,t)|^2 + \lambda \sum_{x,t} |\hat{\phi}(x,t)|^4$$



(a) Heavy ϕ field with $m \sim 0.500$ vs. lattice data.

PHYSICAL REVIEW D **110**, 014504 (2024)



(b) $E^{(2\phi)}$ with $m \sim 0.500$ and $V_0 \sim 0.271$

- e-Print: 2402.15628 [hep-lat]

Summary

Alternative approach to Lusecher formula method

- The difference of integrated correlation functions between interacting and non-interacting systems converge rapidly to its infinite volume limit that is related to scattering phase shift through an integral;

U.S. National Science Foundation

Summary

Alternative approach to Lusecher formula method

- The difference of integrated correlation functions between interacting and non-interacting systems converge rapidly to its infinite volume limit that is related to scattering phase shift through an integral;

- Good candidate to overcome Lusecher formula method at large volume limit;

U.S. National Science Foundation

Summary

Alternative approach to Lusecher formula method

- ☑ **The difference of integrated correlation functions between interacting and non-interacting systems converge rapidly to its infinite volume limit that is related to scattering phase shift through an integral;**

- ☑ **Good candidate to overcome Lusecher formula method at large volume limit;**

- ☑ **Inelastic effect may be important in some cases, and need to be build in;**

U.S. National Science Foundation

Summary

Alternative approach to Lusecher formula method

- ☑ The difference of integrated correlation functions between interacting and non-interacting systems converge rapidly to its infinite volume limit that is related to scattering phase shift through an integral;

- ☑ Good candidate to overcome Lusecher formula method at large volume limit;

- ☑ Inelastic effect may be important in some cases, and need to be build in;

- ☑ May have potential to overcome S/N problem.

U.S. National Science Foundation