

Charge-Exchange Reactions as Probes of Neutrinoless Double-Beta Decays

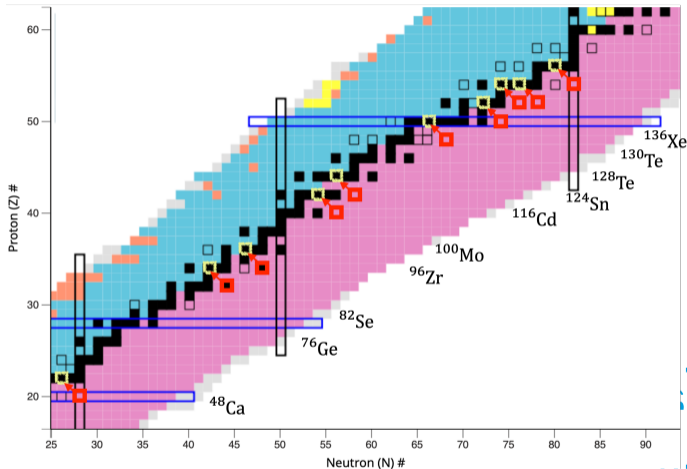
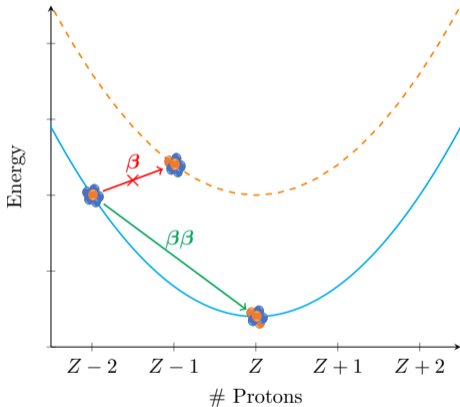
Lotta Jokiniemi
TRIUMF, Theory Department
NN2024 Conference, Whistler, BC Canada
22/08/2024



Arthur B. McDonald
Canadian Astroparticle Physics Research Institute



Double-Beta Decay



nndc.bnl.gov

Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- Violates lepton-number conservation

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- \text{ ~~+2}\nu_e~~$$

Maria Goeppert-Mayer



$2\nu\beta\beta$

1935

Ettore Majorana



Majorana particles

1937

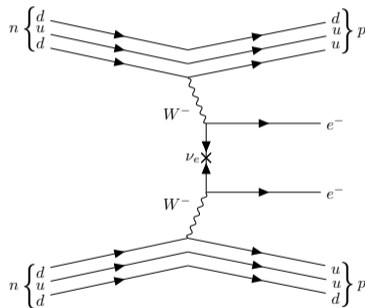
Wendell H. Furry



$0\nu\beta\beta$

1939

...



Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- Violates lepton-number conservation
- Requires that **neutrinos are Majorana particles**

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- \neq 2\nu_e$$

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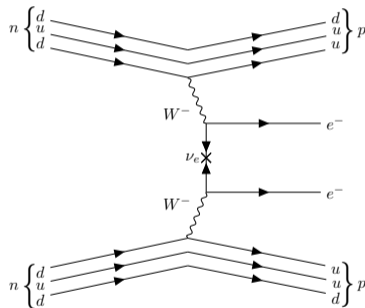
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Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- Violates lepton-number conservation
- Requires that **neutrinos are Majorana particles**
- If observed, $t_{1/2}^{0\nu} \gtrsim 10^{25}$ years

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- \quad \cancel{+2\nu_e}$$

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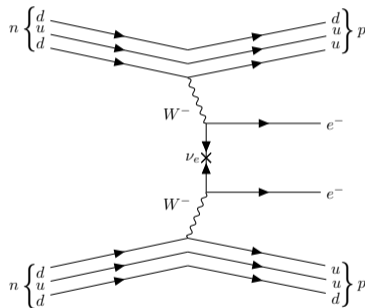
Wendell H. Furry



$0\nu\beta\beta$

1939

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Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- Violates lepton-number conservation
- Requires that **neutrinos are Majorana particles**
- If observed, $t_{1/2}^{0\nu} \gtrsim 10^{25}$ years
 ($t_{1/2}^{2\nu} \approx 10^{20}$ years,
 age of the Universe $\approx 10^{10}$ years)

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- \neq 2\nu_e$$

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Wendell H. Furry



$2\nu\beta\beta$

Majorana particles

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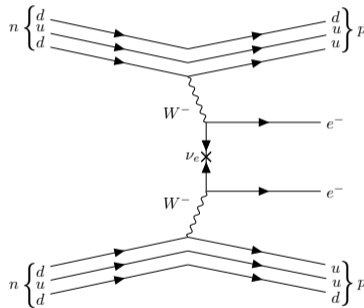
1937



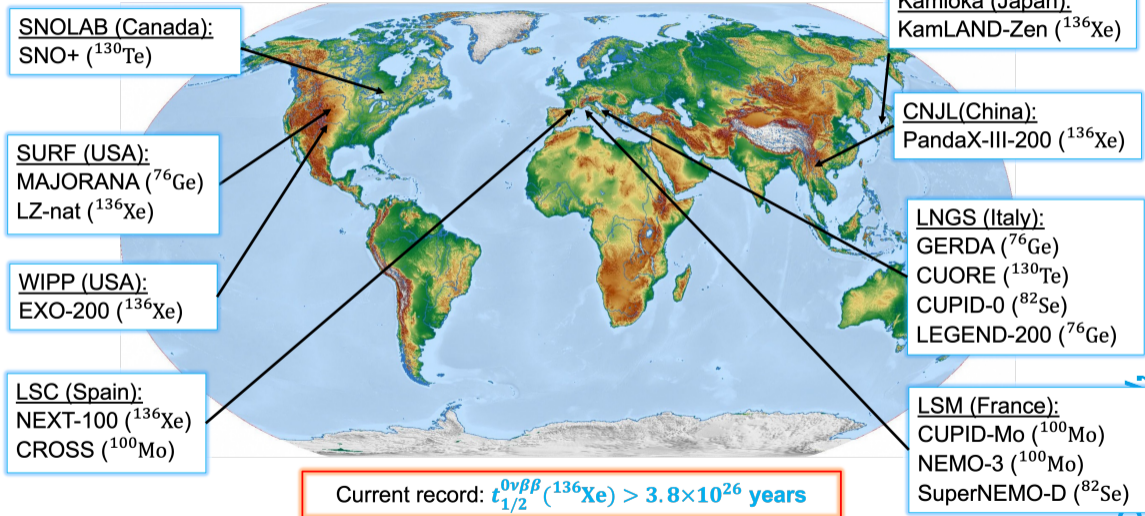
1939



...



$0\nu\beta\beta$ -Decay Experiments



KamLAND-Zen, arXiv:2407:11438

accelerated

Next-Generation $0\nu\beta\beta$ -Decay Experiments

SNOLAB (Canada):
SNO+II (^{130}Te)

Kamioka (Japan):
KamLAND2-Zen (^{136}Xe)

Yemilab (Korea):
PandaX-III-200 (^{136}Xe)

LNGS (Italy):
CUPID (^{100}Mo)

LSM (France):
SuperNEMO (^{82}Se)

+nEXO (^{136}Xe), LEGEND-1000 (^{76}Ge), NEXT-HD (^{136}Xe), Darwin (^{136}Xe), ...

M. Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

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Aim: $t_{1/2}^{0\nu} \approx 10^{28}$ years

LSM (France):
SuperNEMO (^{82}Se)

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*What would be
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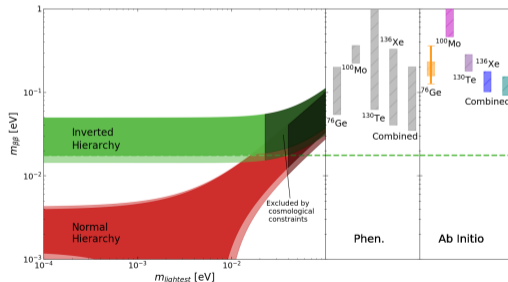
$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

$0\nu\beta\beta$ -Decay Half-Life

What would be measured

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

Majorana mass
 $m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$



T. Shickele, L.J. A. Belley, J. D. Holt, in preparation

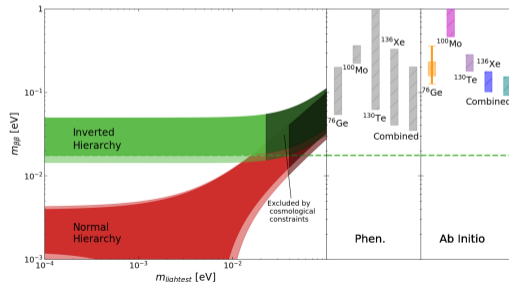
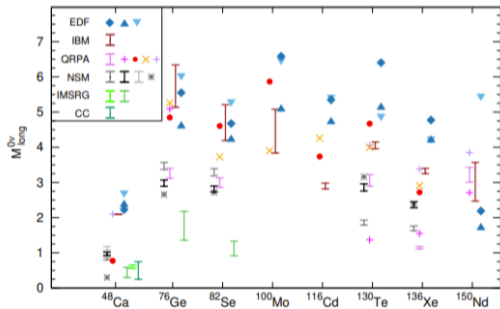
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Majorana mass
 $m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$

Nuclear matrix element



What Can We Learn from Double-Charge-Exchange Reactions?

$0\nu\beta\beta$ Decay vs Double-Charge-Exchange Reactions

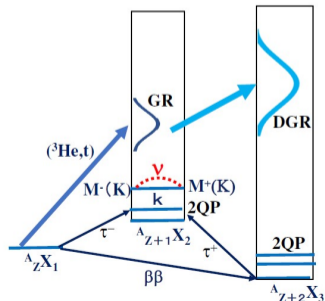
$$M^{0\nu} = M_{\text{GT}}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{\text{F}}^{0\nu} + M_{\text{T}}^{0\nu} + M_{\text{S}}^{0\nu}$$

Leading contribution

$$M_{\text{GT}}^{0\nu} = \langle f || \sum_{jk} \tau_j^- \tau_k^- \sigma_j^- \sigma_k^- V_{\text{GT}}(r_{jk}) || i \rangle$$

- Double-Gamow-Teller (DGT) strength function

$$B(\text{DGT}; \lambda) = \frac{1}{2J_i + 1} |\langle f || [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(\lambda)} || i \rangle|^2$$



$0\nu\beta\beta$ Decay vs Double-Charge-Exchange Reactions

$$M^{0\nu} = M_{\text{GT}}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{\text{F}}^{0\nu} + M_{\text{T}}^{0\nu} + M_{\text{S}}^{0\nu}$$

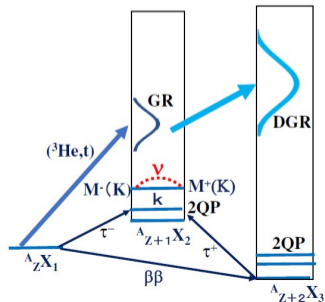
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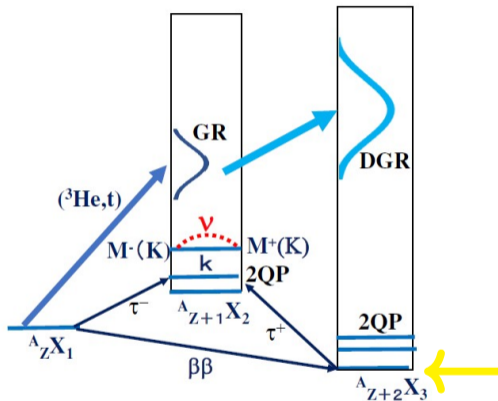
$$B(\text{DGT}; \lambda) = \frac{1}{2J_i + 1} \left| \langle f || \left[\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^- \right]^{(\lambda)} || i \rangle \right|^2$$

- ▶ Could we probe $0\nu\beta\beta$ decay by DGT reactions?



Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ || [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} || 0_{\text{gs},i}^+ \rangle$$

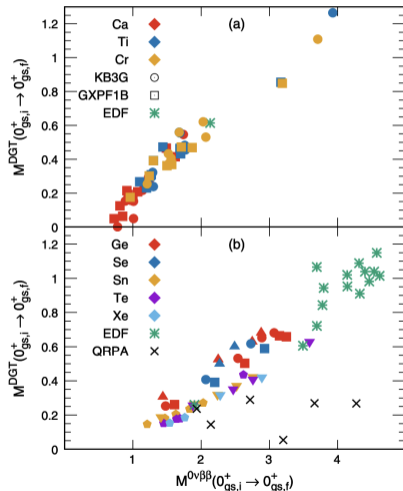


H. Ejiri, LJ, J. Suhonen, Phys. Rev. C 105, L022501 (2022)

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- Correlation between $M^{0\nu}$ and M_{DGT} found in **nuclear shell model** and **EFT**

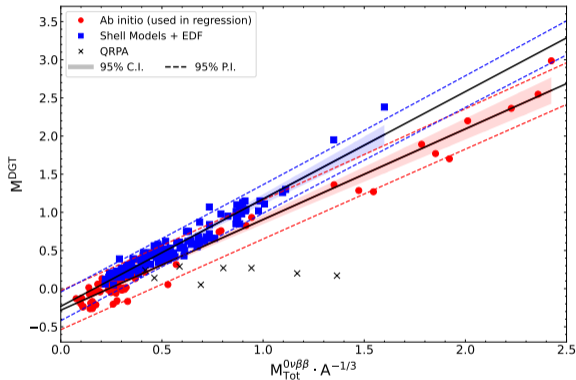


N. Shimizu, J. Menéndez, K. Yako, *Phys. Rev. Lett.* 120, 142502 (2018)

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- Correlation also holds in *ab initio* **VS-IMSRG**

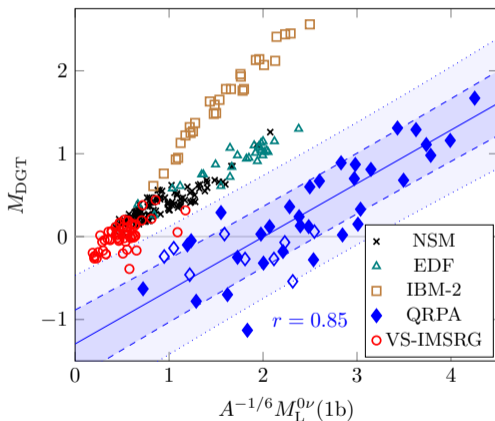


J. M. Yao, I. Ginnett, A. Belley et al., *Phys. Rev. C* 106, 014315 (2022)

Correlations Between DGT and $0\nu\beta\beta$ Decay

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- Correlation between $M^{0\nu}$ and M_{DGT} found in **nuclear shell model** and **EFT**
- Correlation also holds in *ab initio* **VS-IMSRG**
- ...and **QRPA**, when proton-neutron pairing varied
 - ▶ **Observation of M_{DGT} → constraints for $M^{0\nu}$**



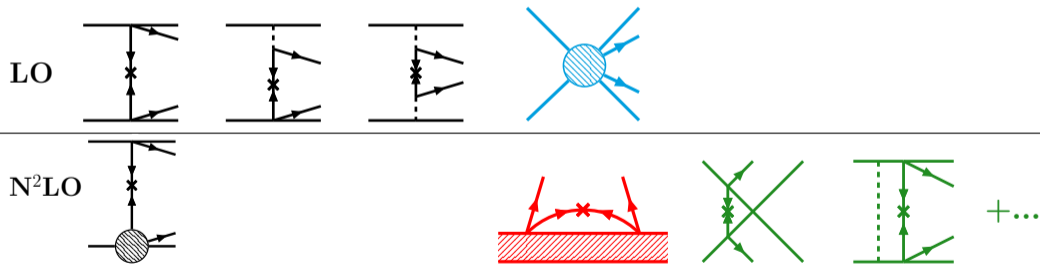
LJ, J. Menéndez, *Phys. Rev. C* 107, 044316 (2023)

Could We Learn Something from Single-Charge-Exchange Reactions?

χ EFT Analysis of $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{\text{loops}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

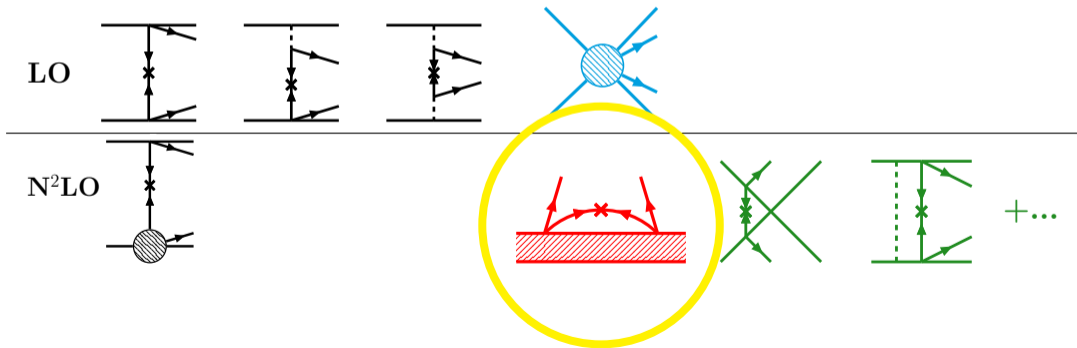
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



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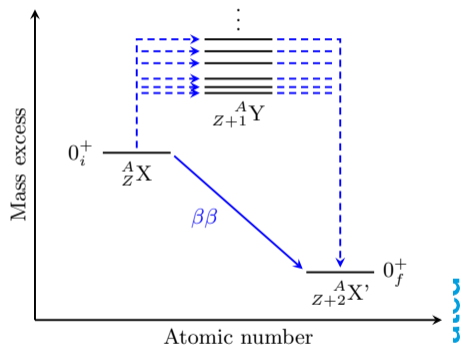
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- A $N^2\text{LO}$ correction from “ultrasoft” ($|\mathbf{k}| \ll k_F \approx 100 \text{ MeV}$) neutrinos:

$$M_{\text{usoft}}^{0\nu} = -\frac{2R}{\pi} \sum_n \langle f | \sum_a \sigma_a \tau_a^+ | n \rangle \langle n | \sum_b \sigma_b \tau_b^+ | i \rangle \times (E_e + E_n - E_i) \left(\ln \frac{\mu_{\text{us}}}{2(E_e + E_n - E_i)} + 1 \right)$$

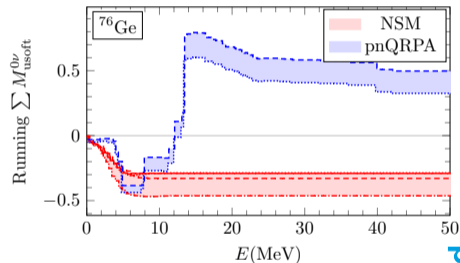


$M_{\text{usoft}}^{0\nu}$ Evaluated in pnQRPA and NSM

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- A $\approx 10\%$ increase in pnQRPA, and $\approx 10\%$ decrease in nuclear shell model



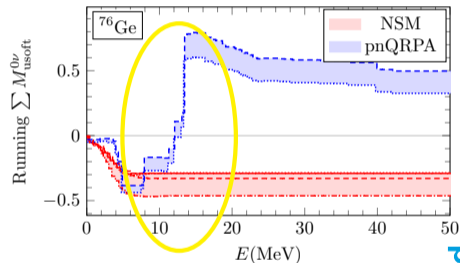
D. Castillo, L.J. P. Soriano, J. Menéndez, arXiv:2408:03373

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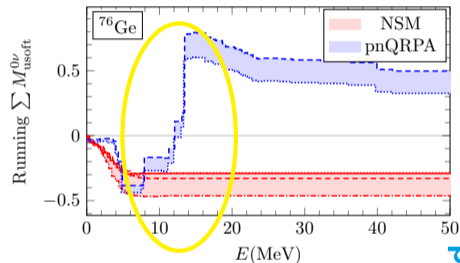
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- Many potential explanations:



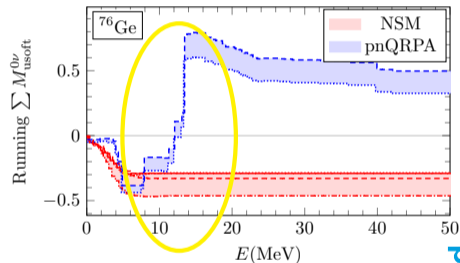
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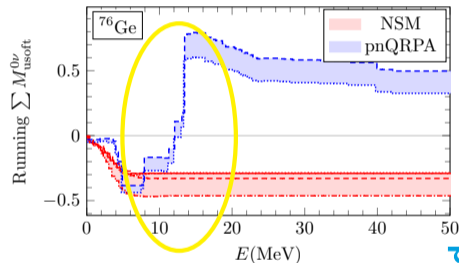
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 - ▶ Missing correlations in pnQRPA?



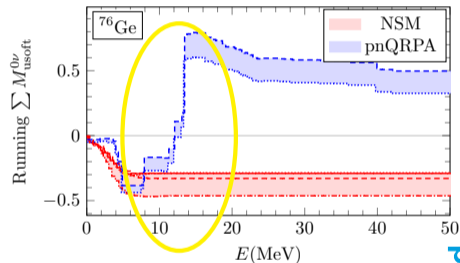
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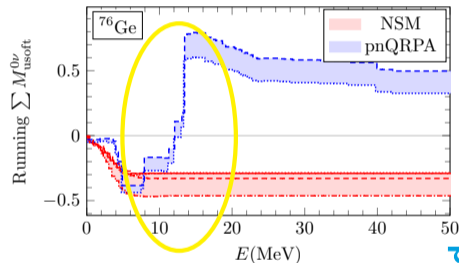
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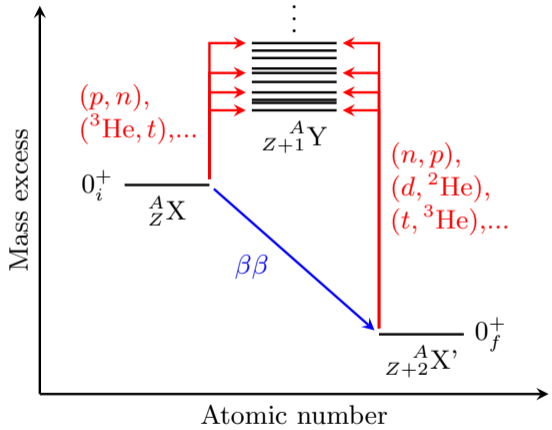
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 - ▶ ...



D. Castillo, L.J. P. Soriano, J. Menéndez, arXiv:2408:03373

- Charge-exchange reactions can probe the virtual transitions

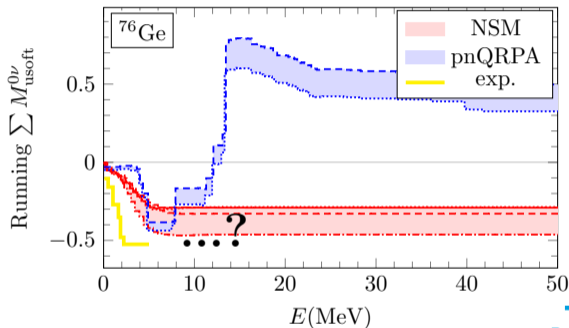


- Charge-exchange reactions can probe the virtual transitions

- **Available data ends at $E \approx 5$ MeV**

$^{76}\text{Ge}(^3\text{He}, t)^{76}\text{As}$: J. H. Hies et al., *Phys. Rev. C* 86, 014304 (2012)

$^{76}\text{Se}(d, ^2\text{He})^{76}\text{As}$: E.-W. Grewe et al., *Phys. Rev. C* 78, 044301 (2008)



Modified from: D. Castillo, L.J. P. Soriano, J. Menéndez, arXiv:2408:03373

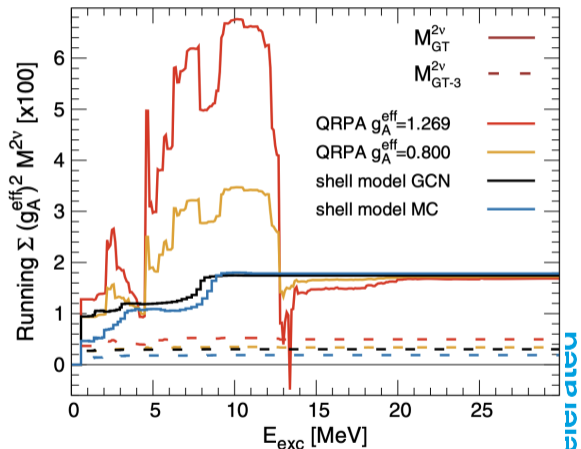
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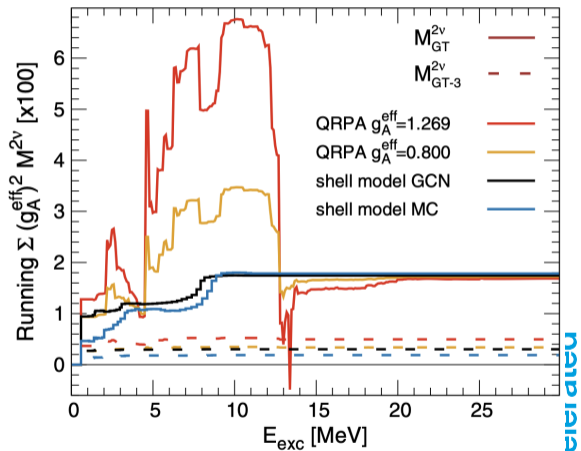
$^{76}\text{Se}(d, ^2\text{He})^{76}\text{As}$: E.-W. Grewe et al., *Phys. Rev. C* 78, 044301 (2008)

- Charge-exchange reactions can also probe $2\nu\beta\beta$ decays



KamLAND-Zen, Phys. Rev. Lett. 122, 192501 (2019)

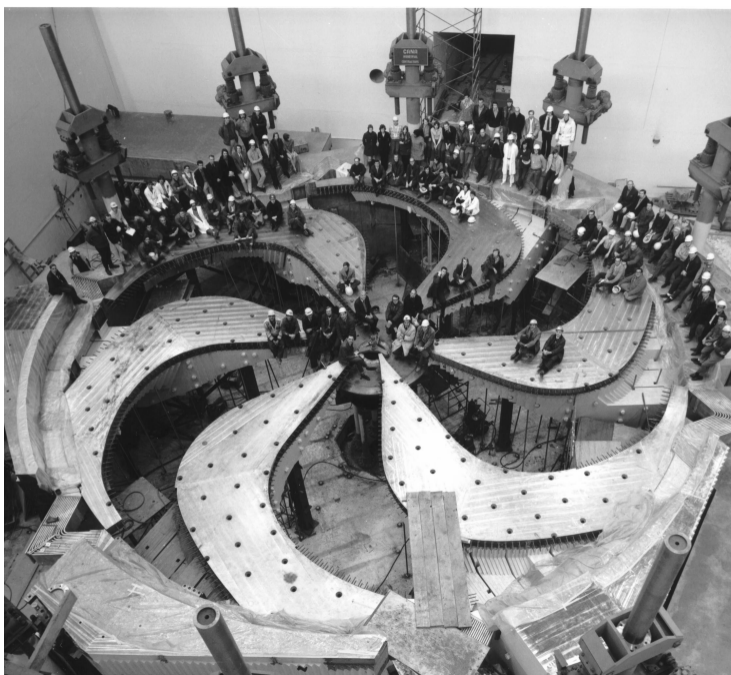
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- Charge-exchange reactions **can also probe $2\nu\beta\beta$ decays**
 - ▶ **Good benchmark for future ab initio studies**



KamLAND-Zen, Phys. Rev. Lett. 122, 192501 (2019)

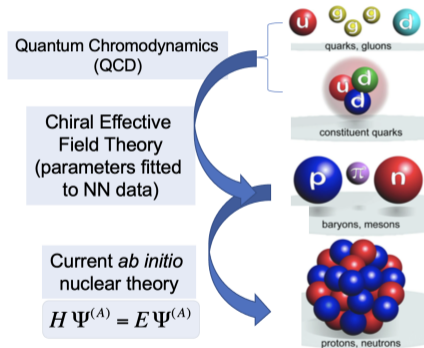
- Theoretical efforts needed in the hunt for $0\nu\beta\beta$ decay
- **Correlations between $0\nu\beta\beta$ decay and double charge-exchange reactions** may help constrain the $0\nu\beta\beta$ -decay nuclear matrix elements
- Measuring **single-charge-exchange reactions up to high excitation energies** would help probe
 - N²LO corrections to $0\nu\beta\beta$ decays
 - $2\nu\beta\beta$ -decay calculations

Thank you
Merci



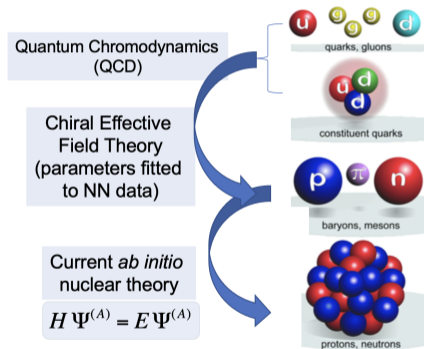
Nuclear Many-body Methods

- *Ab initio methods* (IMSRG, NCSM,...)



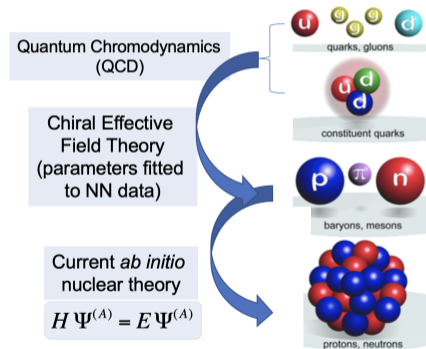
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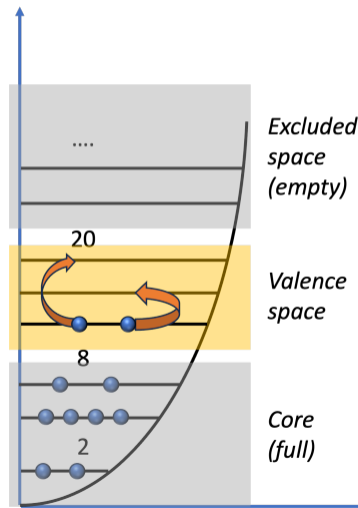
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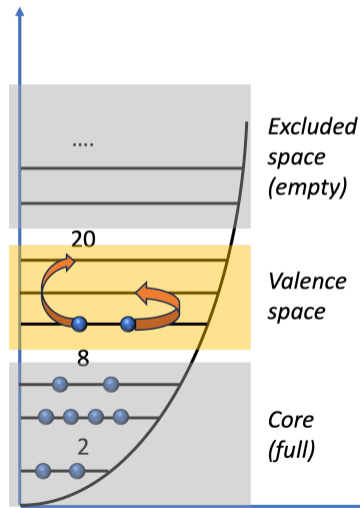
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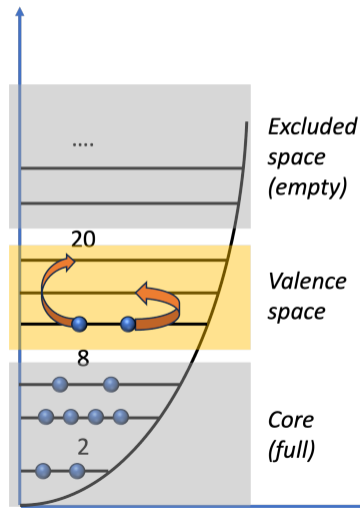
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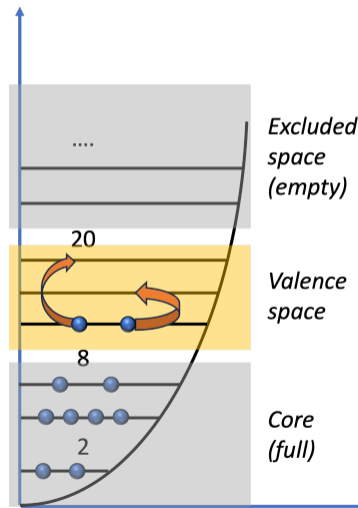
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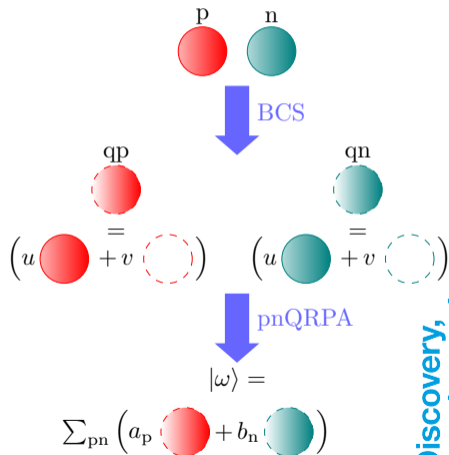
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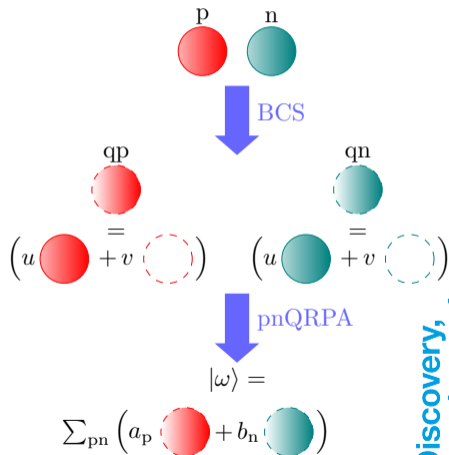
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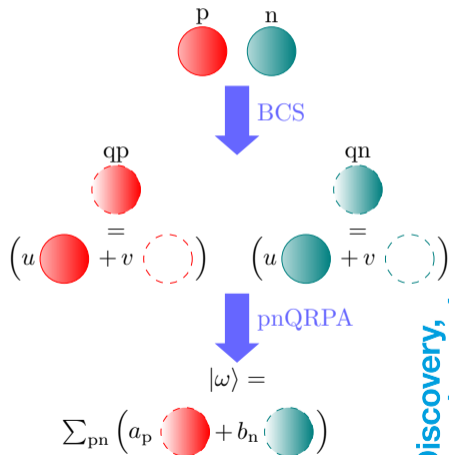
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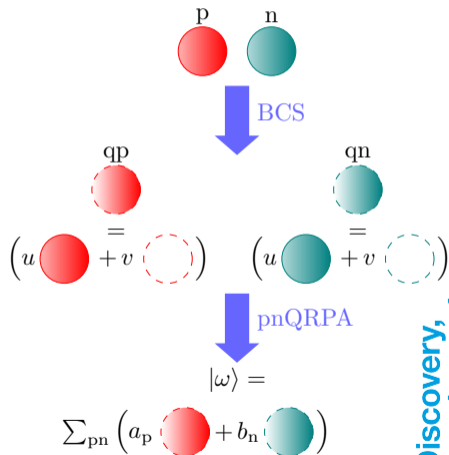
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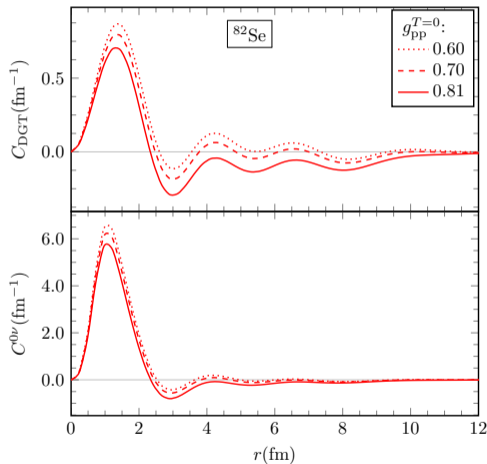
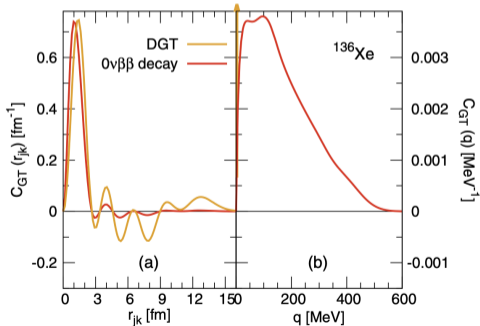
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● ...

Radial Densities of $M^{0\nu}$ and M_{DGT}

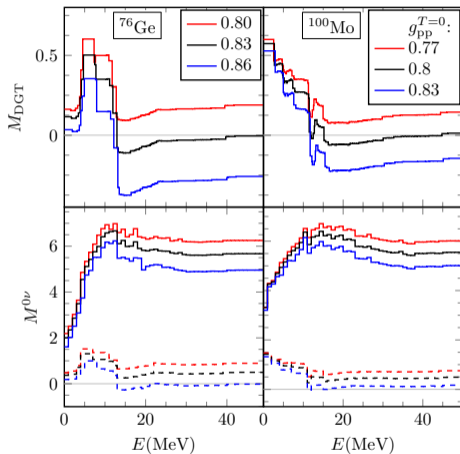
$$M_L^{0\nu} = \int_0^\infty C^{0\nu}(r) dr,$$

$$M_{\text{DGT}} = \int_0^\infty C_{\text{DGT}}(r) dr$$



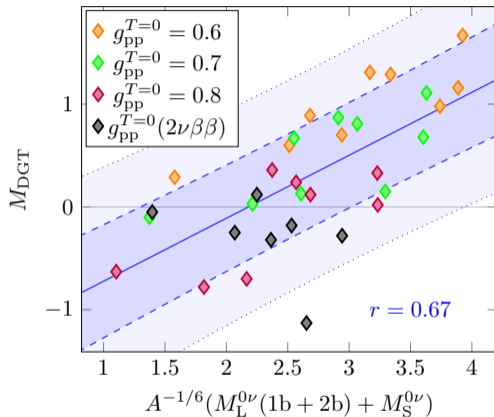
LJ, J. Menéndez, *Phys. Rev. C* **107**, 044316 (2023)

Running Sums of $M^{0\nu}$ and M_{DGT}



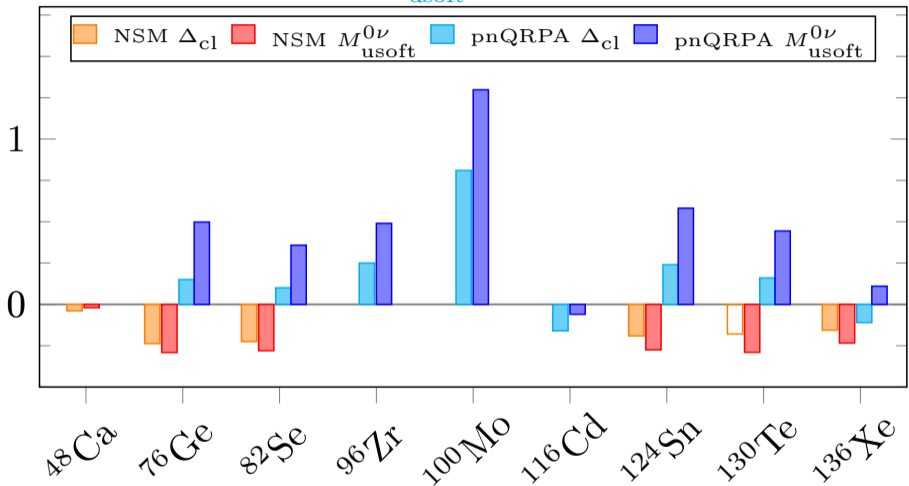
LJ, J. Menéndez, *Phys. Rev. C* **107**, 044316 (2023)

Correlation Survives 2BCs and Short-Range



LJ, J. Menéndez, *Phys. Rev. C* **107**, 044316 (2023)

$M_{\text{usoft}}^{0\nu}$ as a Closure Correction



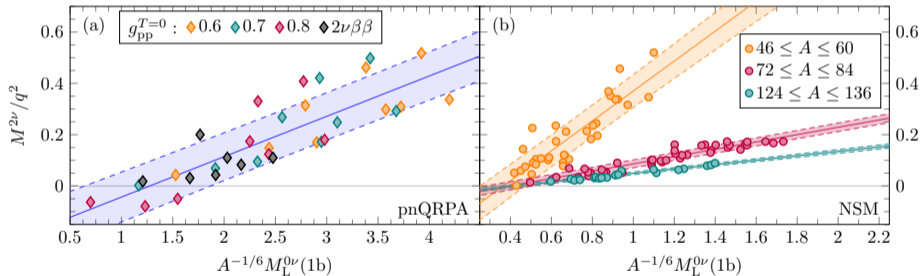
D. Castillo, L.J. P. Soriano, J. Menéndez, arXiv:2408:03373

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- *How about $2\nu\beta\beta$ decay?*

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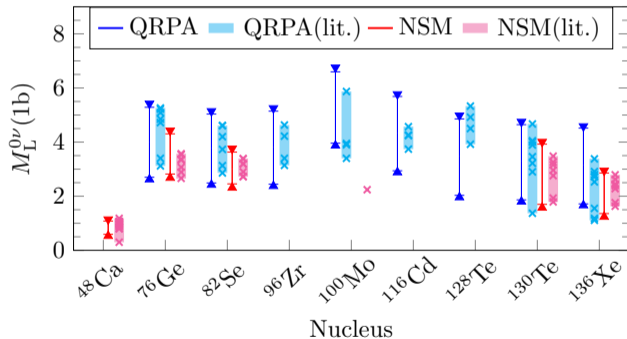
- *How about $2\nu\beta\beta$ decay?*
- $2\nu\beta\beta$ -decay also correlated with $0\nu\beta\beta$ -decay!



LJ, B. Romeo, P. Soriano and J. Menéndez, *Phys. Rev. C* **107**, 044305 (2023)

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- **How about $2\nu\beta\beta$ decay?**
- $2\nu\beta\beta$ -decay also correlated with $0\nu\beta\beta$ -decay!
- We can use the existing data to estimate $0\nu\beta\beta$ -decay NMEs!



LJ, B. Romeo, P. Soriano and J. Menéndez, *Phys. Rev. C* **107**, 044305 (2023)