



Machine Learning Transforms the Inference of the Nuclear Equation of State

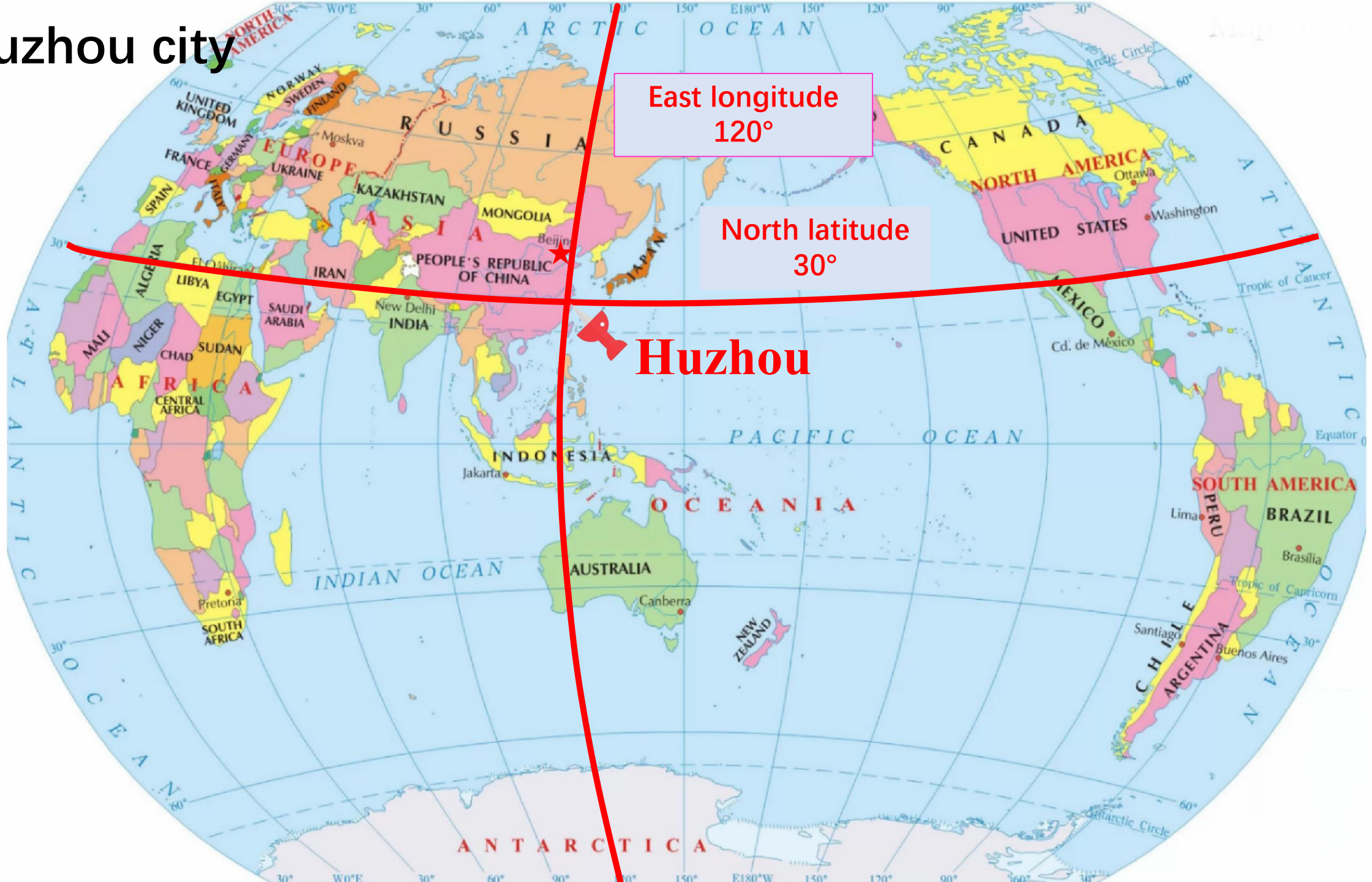
Yongjia Wang (Huzhou University)

Huzhou city

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North latitude
30°

Huzhou



Outline

Background

01

02

ML algorithm

- ✓ Bayesian inference
- ✓ Supervised learning

Contents

03

Results

- ✓ Bayesian inference on the in-medium nucleon-nucleon cross section
- ✓ Supervised learning on the nuclear symmetry energy

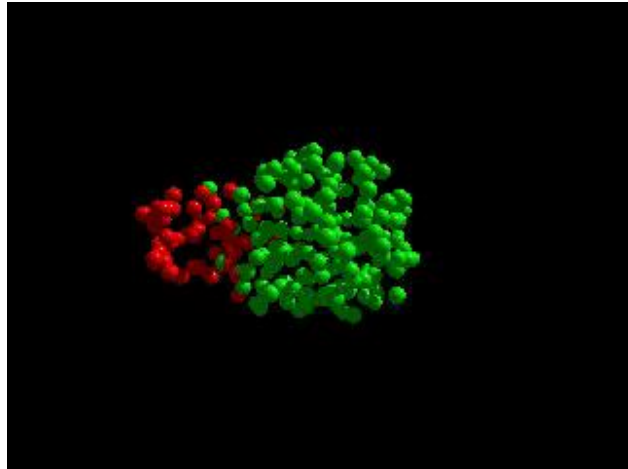
04

Summary and Outlook

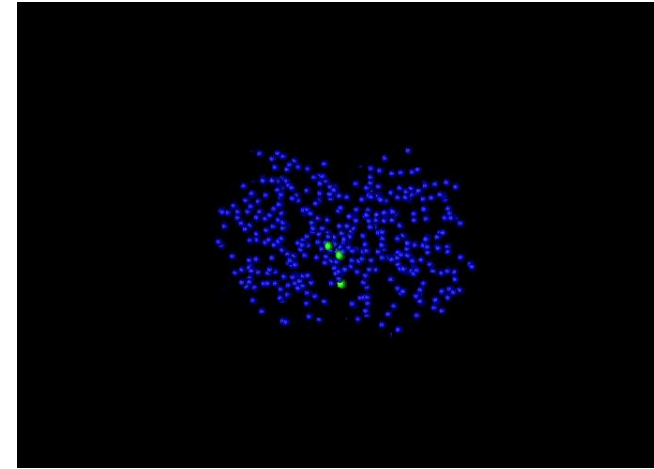
01

Overview of HIC

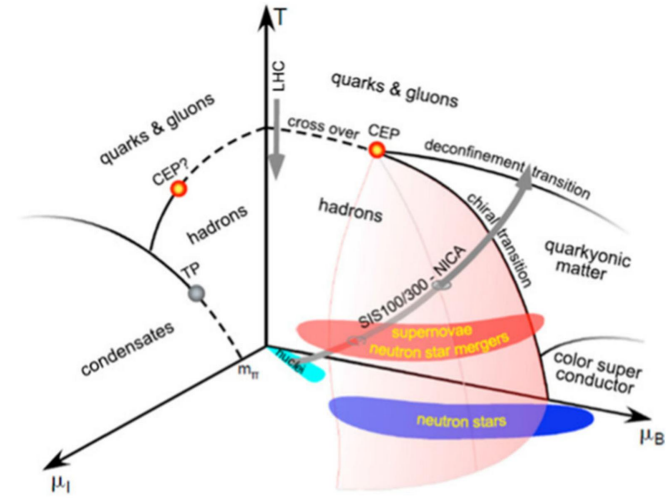
◆ intermediate energies HIC



Nuclear equation of state



~100 MeV- a few GeV/nucleon

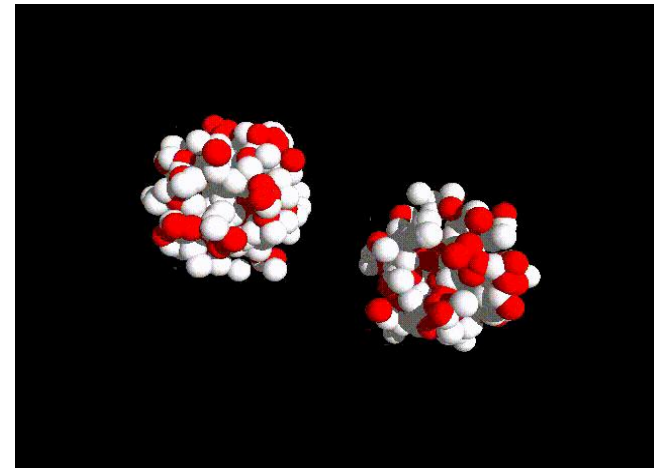
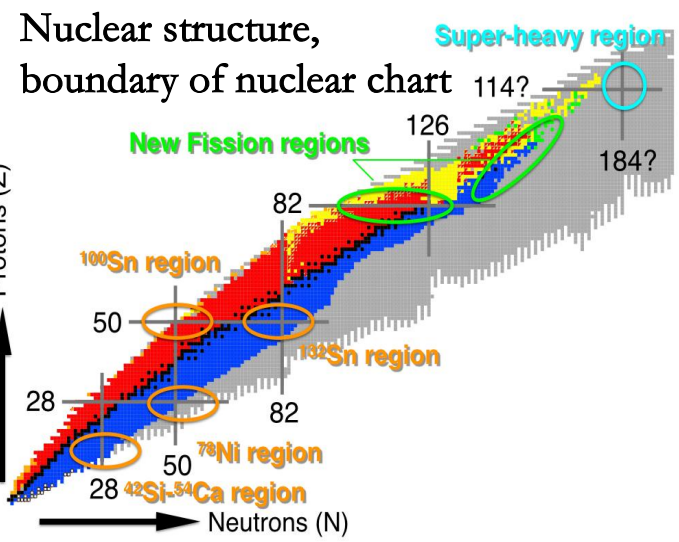


QCD phase diagram, QGP properties

◆ Low energy fusion reaction

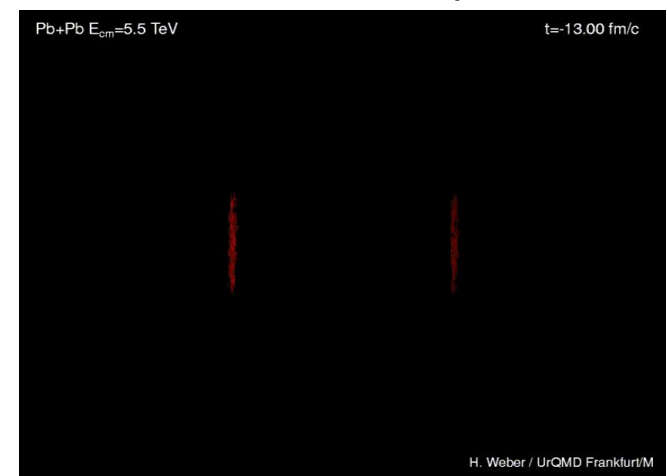


~100 MeV/nucleon



> hundreds of GeV/nucleon

beam energy



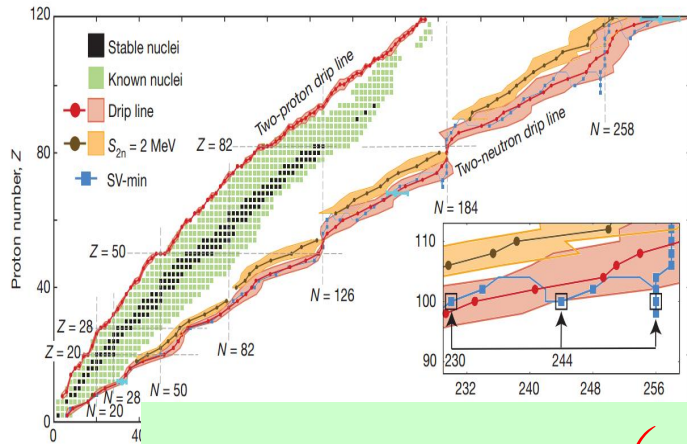
◆ Relativistic HIC

Nuclear equation of state (EOS)

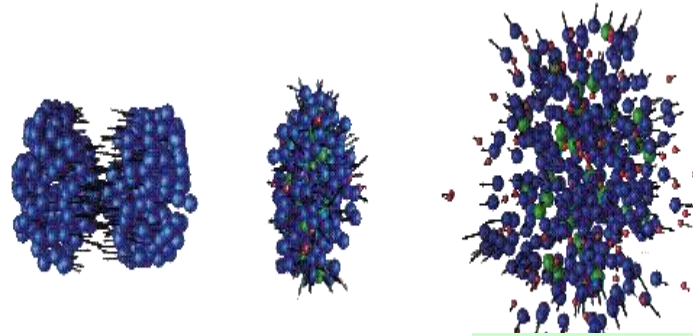
The thermodynamic relationship between the binding energy E (or pressure P) and density ρ , as well as the isospin asymmetry δ .

$$E(\rho, \delta) = E(\rho, 0) + E_{sym}(\rho)\delta^2 + \dots, \quad \delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

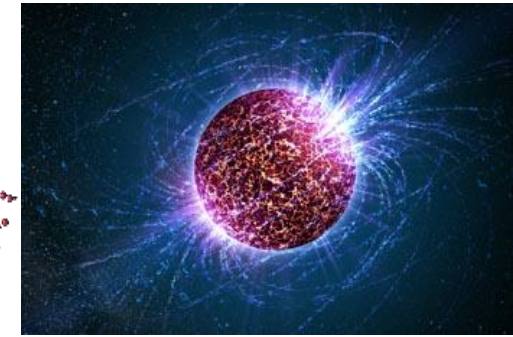
Nuclear landscape



Heavy ion collision



Neutron star



$$E(\rho, 0) = E_0 + \frac{K_0}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \dots,$$

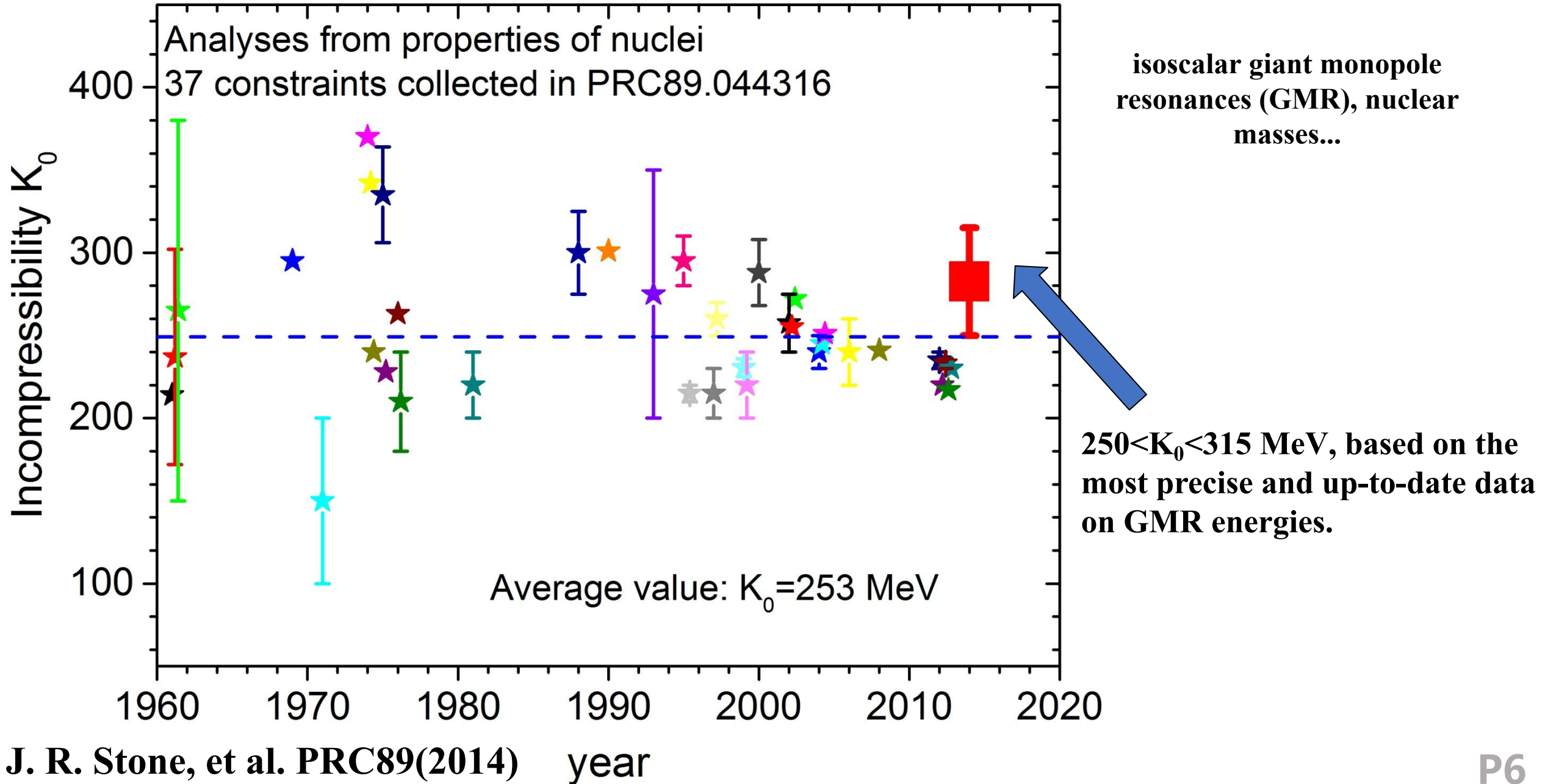
$$K_0 = 9\rho^2 \left(\frac{\partial^2 E}{\partial \rho^2} \right) \Big|_{\rho=\rho_0}$$

$$E_{sym}(\rho) = S_0 + L \left(\frac{\rho - \rho_0}{3\rho_0} \right) + \frac{K_{sym}}{2} \left(\frac{\rho - \rho_0}{3\rho_0} \right)^2 + \dots$$

$$L = 3\rho \frac{dE_{sym}(\rho)}{d\rho} \Big|_{\rho=\rho_0}$$

K_0 and L determine the EOS in the vicinity of the saturation density.

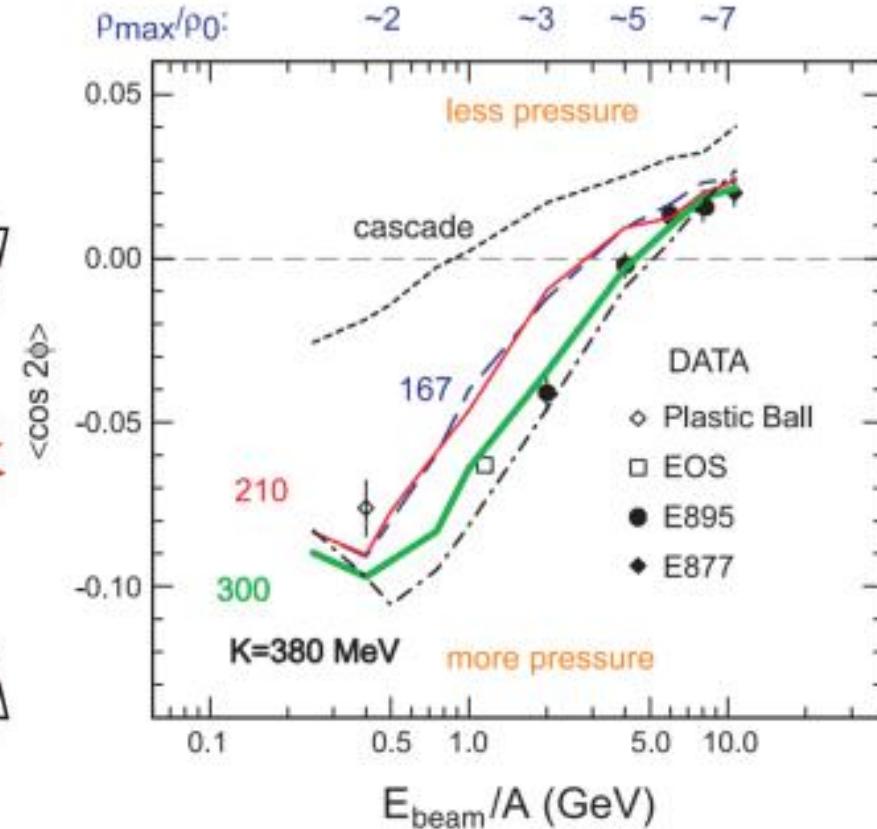
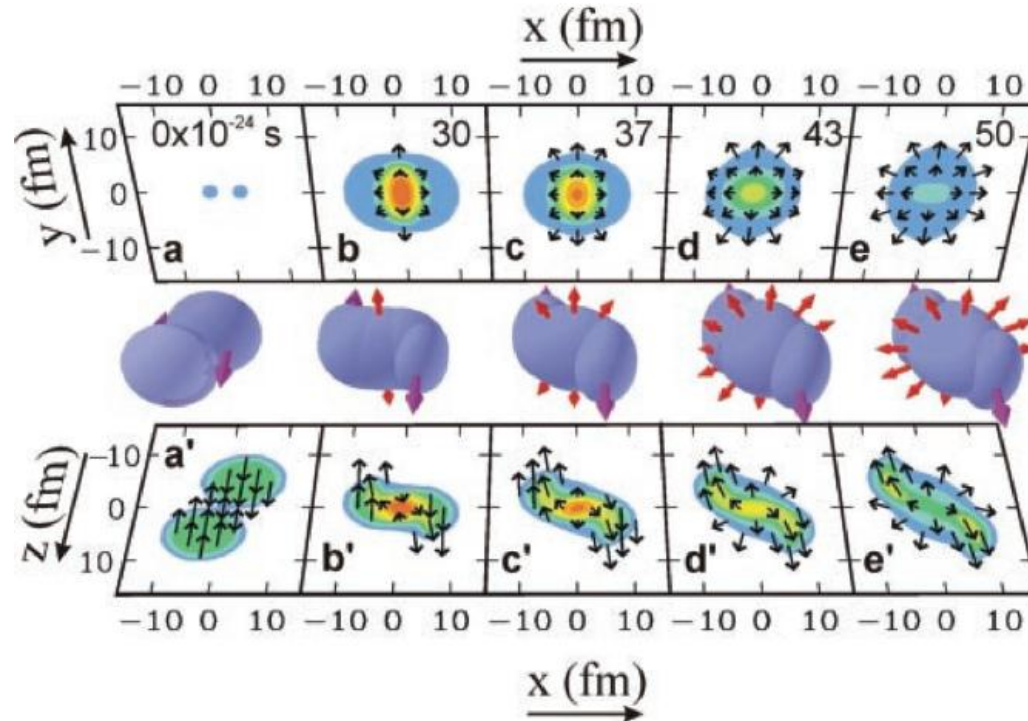
The incompressibility K_0 from properties of nuclei



HIC offers a unique way to create nuclear matter with high density and isospin asymmetry in laboratory.



Determination of the Equation of State of Dense Matter
 Pawel Danielewicz, *et al.*
Science **298**, 1592 (2002);
 DOI: 10.1126/science.1078070



EOS can be deduced from the comparison between experimental observables and transport model calculations.

01 Nuclear equation of state

Some of highly cited papers

Recent progress and new challenges in isospin physics with heavy-ion reactions

[BA Li](#), [LW Chen](#), [CM Ko](#) - *Physics Reports*, 2008 - Elsevier

... on the reaction aspect of isospin physics, especially heavy-... of isospin physics is to determine the isospin dependence of ...) of isospin asymmetric nuclear matter, particularly its isospin-...

☆ 保存 羽 引用 被引用次数: 1597 相关文章 所有 11 个版本 免费在线GPT 》》

Equations of state for supernovae and compact stars

[M Oertel](#), [M Hempel](#), [T Klähn](#), [S Typel](#) - *Reviews of Modern Physics*, 2017 - APS

A review is given of various theoretical approaches for the equation of state (EoS) of dense matter, relevant for the description of core-collapse supernovae, compact stars, and compact ...

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Constraints on the density dependence of the symmetry energy

[MB Tsang](#), [Y Zhang](#), [P Danielewicz](#), [M Famiano](#), [Z Li...](#) - *Physical review ...*, 2009 - APS

... over a range of symmetry energies at saturation density and different representations of the density dependence of the symmetry energy, constraints on the density dependence of the ...

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Progress in Particle and Nuclear Physics

Available online 19 September 2023, 104080

In Press, Journal Pre-proof [?](#) [What's this?](#)



Review

Dense nuclear matter equation of state from heavy-ion collisions

[Agnieszka Sorensen](#)¹ , [Kshitij Agarwal](#)², [Kyle W. Brown](#)^{3,4}, [Zbigniew Chajecski](#)⁵, [Paweł Danielewicz](#)^{3,6}, [Christian Drischler](#)⁷, [Stefano Gandolfi](#)⁸, [Jeremy W. Holt](#)^{9,10}, [Matthias Kaminski](#)¹¹, [Che-Ming Ko](#)^{9,10}, [Rohit Kumar](#)³, [Bao-An Li](#)¹², [William G. Lynch](#)^{3,6}, [Alan B. McIntosh](#)¹⁰, [William G. Newton](#)¹², [Scott Pratt](#)^{3,6}, [Oleh Savchuk](#)^{3,13}, [Maria Stefaniak](#)^{14,15}, [Ingo Tews](#)⁸, [ManYee Betty Tsang](#)^{3,6}...[Yi Yin](#)^{9,4}

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<https://doi.org/10.1016/j.pnpnp.2023.104080>

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Abstract

This [White Paper](#) highlights the essential role of hadronic transport simulations of heavy-ion collisions in studies involving the equation of state of nuclear matter. It also elucidates many connections between inferences of the equation of state from heavy-ion collision data and other efforts aiming to understand the properties of nuclear matter.

arXiv > nucl-th > arXiv:2211.02224

Nuclear Theory

[Submitted on 4 Nov 2022 (v1), last revised 8 Nov 2022 (this version, v2)]

Long Range Plan: Dense matter theory for heavy-ion collisions and neutron stars

[Alessandro Lovato](#), [Travis Dore](#), [Robert D. Pisarski](#), [Bjoern Schenke](#), [Katerina Chatziioannou](#), [Jocelyn S. Read](#), [Philippe Landry](#), [Pav Hannah Elfner](#), [Veronica Dexheimer](#), [Rajesh Kumar](#), [Michael Strickland](#), [Johannes Jahan](#), [Claudia Ratti](#), [Volodymyr Vovchenko](#), [Mikh Hippert](#), [Jacquelyn Noronha-Hostler](#), [Jorge Noronha](#), [Enrico Speranza](#), [Nicolas Yunes](#), [Chuck J. Horowitz](#), [Steven P. Harris](#), [Larry McStefano Gandolfi](#), [Ingo Tews](#), [M. Coleman Miller](#), [Cecilia Chirenti](#), [Zohreh Davoudi](#), [Jamie M. Karthein](#), [Krishna Rajagopal](#), [Salvatore Vladimirov](#), [Vladimir Skokov](#), [Ulrich Heinz](#), [Christian Drischler](#), [Daniel R. Phillips](#), [Madappa Prakash](#), [Zoltan Fodor](#), [David Radice](#), [Christopher Plu Fraga](#), [Aleksi Kurkela](#), [James M. Lattimer](#), [Andrew W. Steiner](#), [Jeremy W. Holt](#), [Bao-An Li](#), [Chun Shen](#), [Mark Alford](#), [Alexander Haber](#),

Since the release of the 2015 Long Range Plan in Nuclear Physics, major events have occurred that reshaped our understanding of quantum chromodynamics of equilibrium. The US nuclear community has an opportunity to capitalize on advances in astrophysical observations and nuclear experiments and engage matter that connects low- and high-energy nuclear physics, astrophysics, gravitational waves physics, and data science

Comments: 70 pages, 3 figures, [White Paper for the Long Range Plan for Nuclear Science](#)

Subjects: **Nuclear Theory (nucl-th)**; High Energy Astrophysical Phenomena (astro-ph.HE); High Energy Physics - Phenomenology (hep-ph)

Report number: LA-UR-22-31648

Cite as: arXiv:2211.02224 [nucl-th]

(or arXiv:2211.02224v2 [nucl-th] for this version)

<https://doi.org/10.48550/arXiv.2211.02224>

Submission history

From: [Jacquelyn Noronha-Hostler](#) [[view email](#)]

[v1] Fri, 4 Nov 2022 02:15:29 UTC (2,372 KB)

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Nuclear equation of state

nature

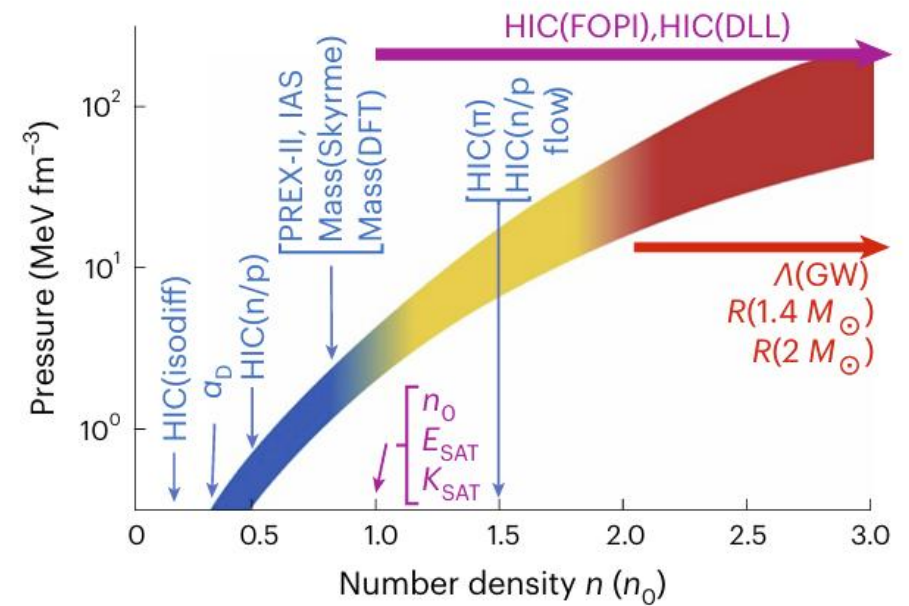
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Constraining neutron-star matter with microscopic and macroscopic collisions

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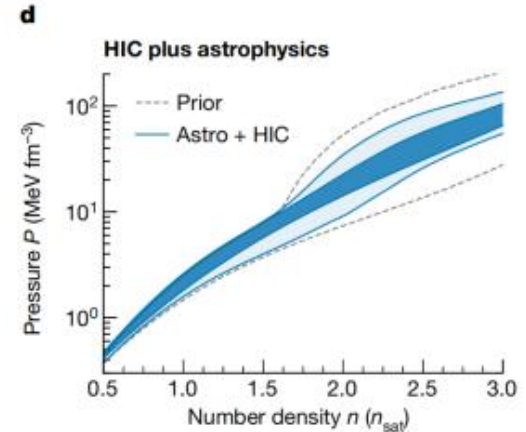
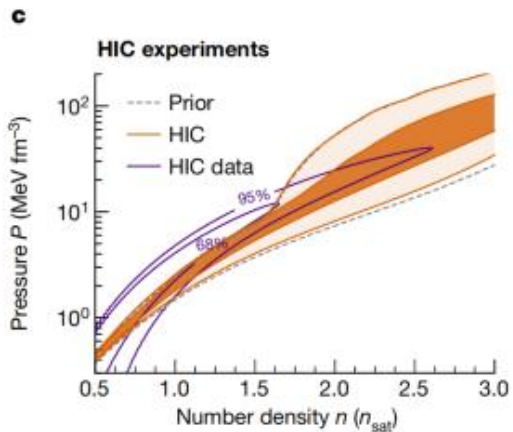
nature > nature astronomy > articles > article

Article | Published: 05 January 2024

Determination of the equation of state from nuclear experiments and neutron star observations

Chun Yuen Tsang, ManYee Betty Tsang, William G. Lynch, Rohit Kumar & Charles J. Horowitz

Nature Astronomy 8, 328–336 (2024) | Cite this article

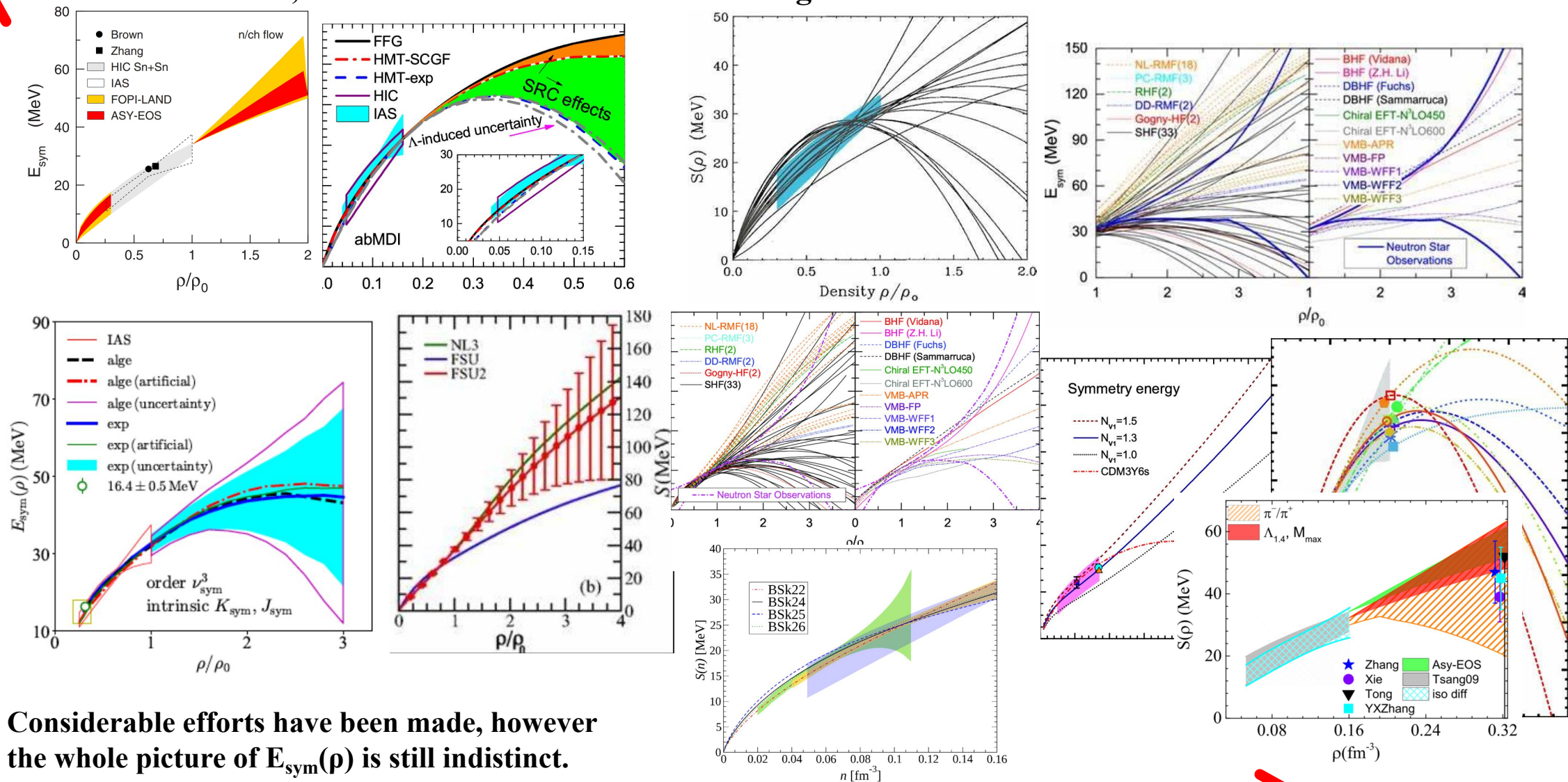


The density-dependent nuclear symmetry energy $E_{\text{sym}}(\rho)$

$$E(\rho, \delta) = E(\rho, 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4),$$

$E_{\text{sym}}(\rho)$ is crucial for our understanding of diverse phenomena observed in rare isotopes, nuclear reactions with exotic nuclei, as well as neutron star and its merger.

Nuclear symmetry energy



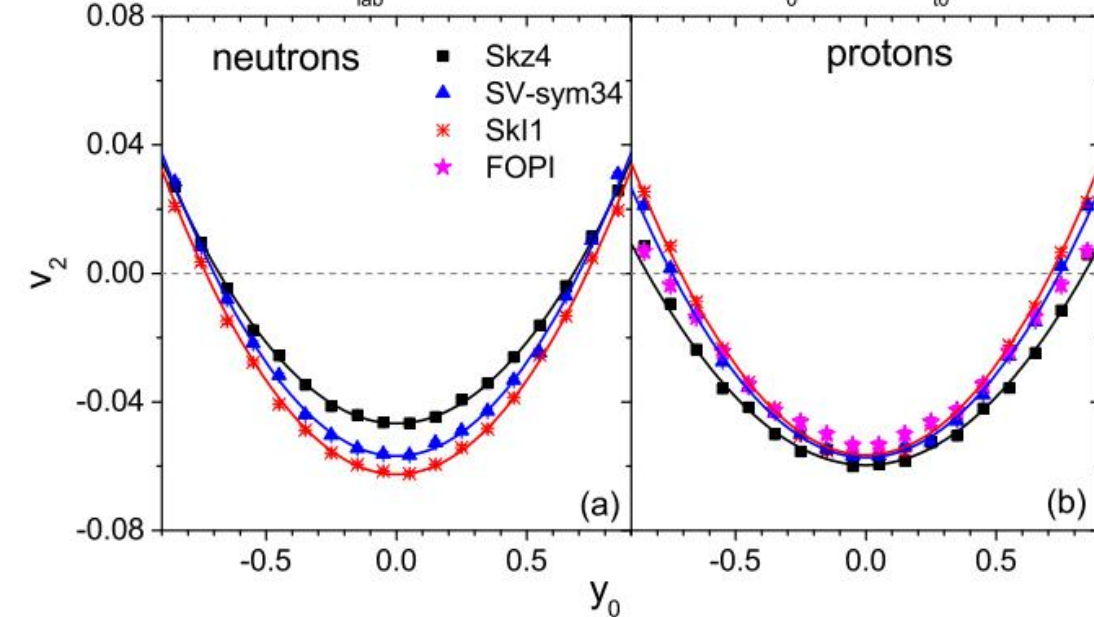
Considerable efforts have been made, however the whole picture of $E_{\text{sym}}(\rho)$ is still indistinct.

Nuclear density

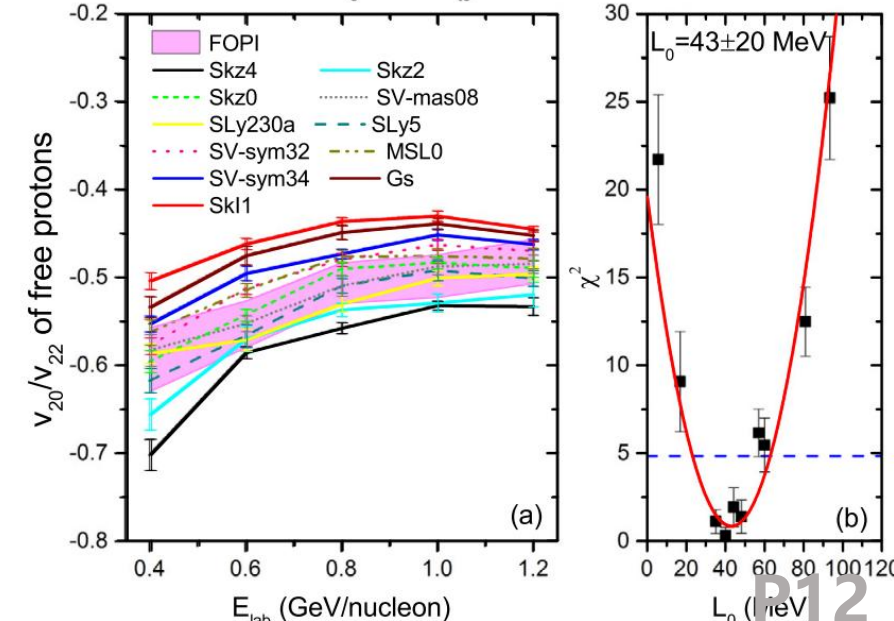
The density-dependent nuclear symmetry energy $E_{\text{sym}}(\rho)$

$$v_2 = v_{20} + v_{22} \cdot y_0^2.$$

Au+Au $E_{\text{lab}} = 0.4$ GeV/nucleon $0.25 < b_0 < 0.45$ $u_{t0} > 0.4$



Au+Au $0.25 < b_0 < 0.45$ $u_{t0} > 0.4$



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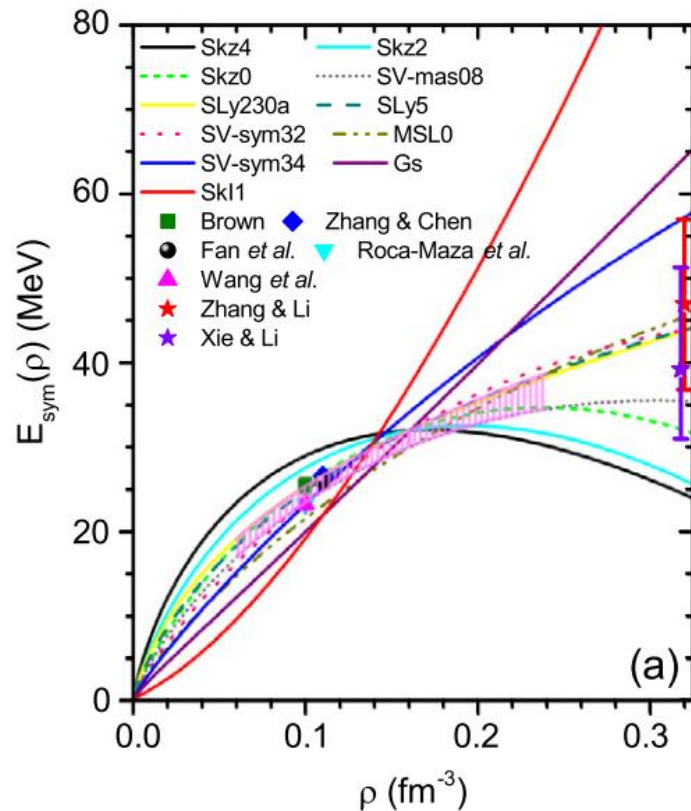
Physics Letters B

www.elsevier.com/locate/physletb

Physics Letters B 802 (2020) 135249

Study of the nuclear symmetry energy from the rapidity-dependent elliptic flow in heavy-ion collisions around 1 GeV/nucleon regime

Yongjia Wang^a, Qingfeng Li^{a,b,*}, Yvonne Leifels^c, Arnaud Le Fèvre^c



Model calculations: considering different interactions that exhibit different types of $E_{\text{sym}}(\rho)$.

Experimental data: the rapidity-dependent elliptic flow.

By using UrQMD model, together with the FOPI data on elliptic flow, the slope parameter of $E_{\text{sym}}(\rho)$ can be constrained.

Background

Because of the update and iteration of computer techniques, the paradigm of scientific research has changed.

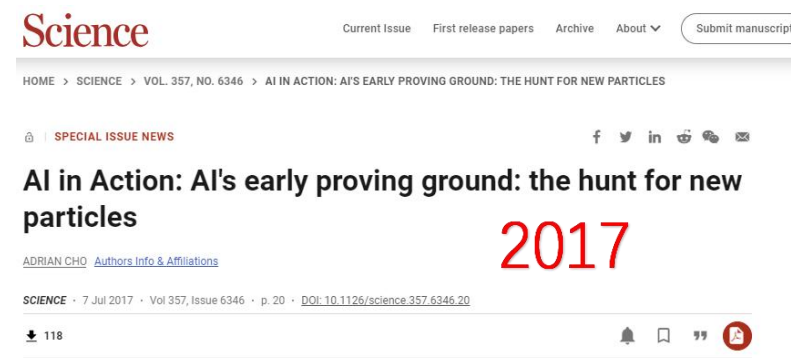
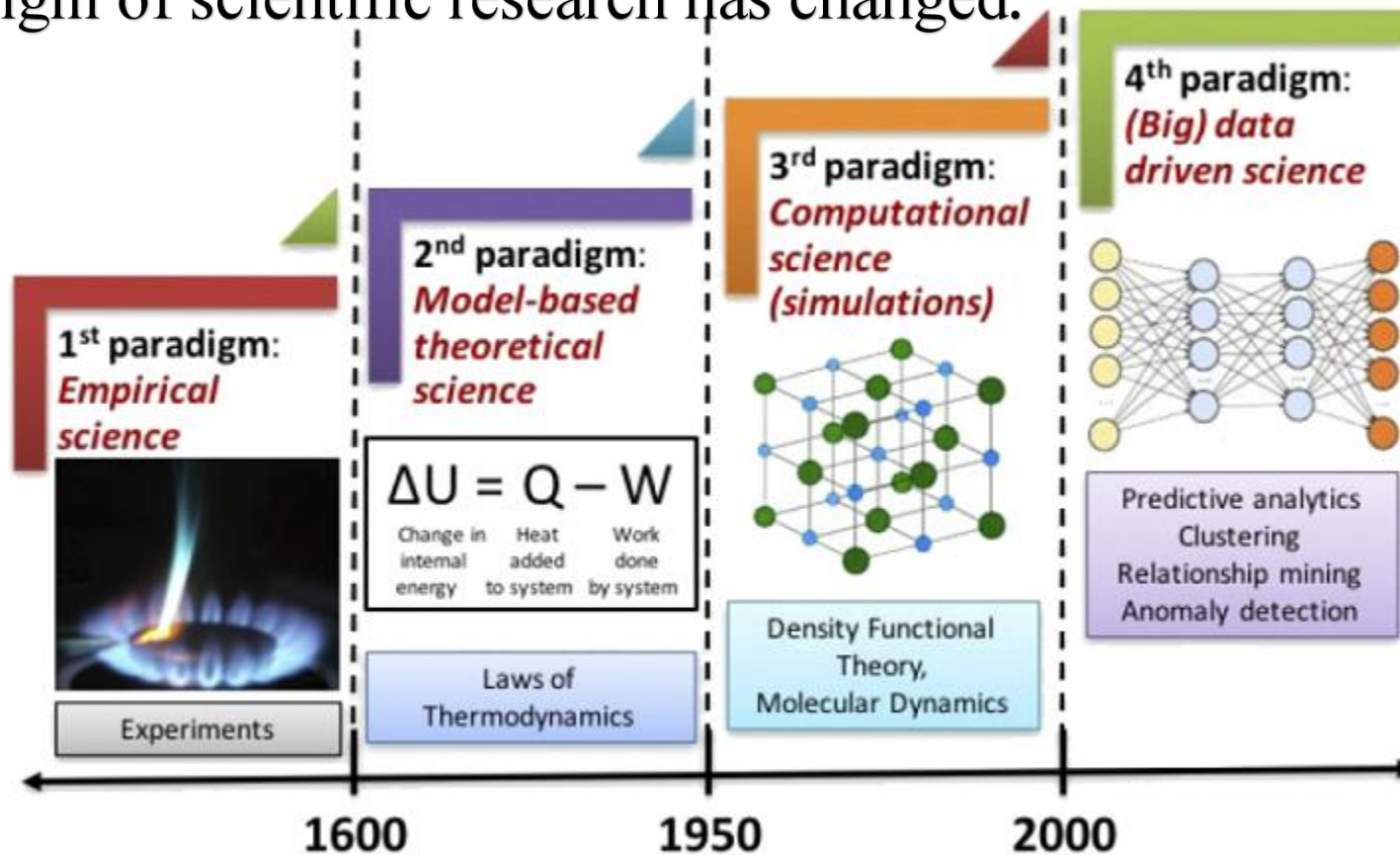


FIG. 1. The four paradigms of science: empirical, theoretical, computational, and data-driven.

APL Mater. 4, 053208 (2016); <https://doi.org/10.1063/1.4946894>

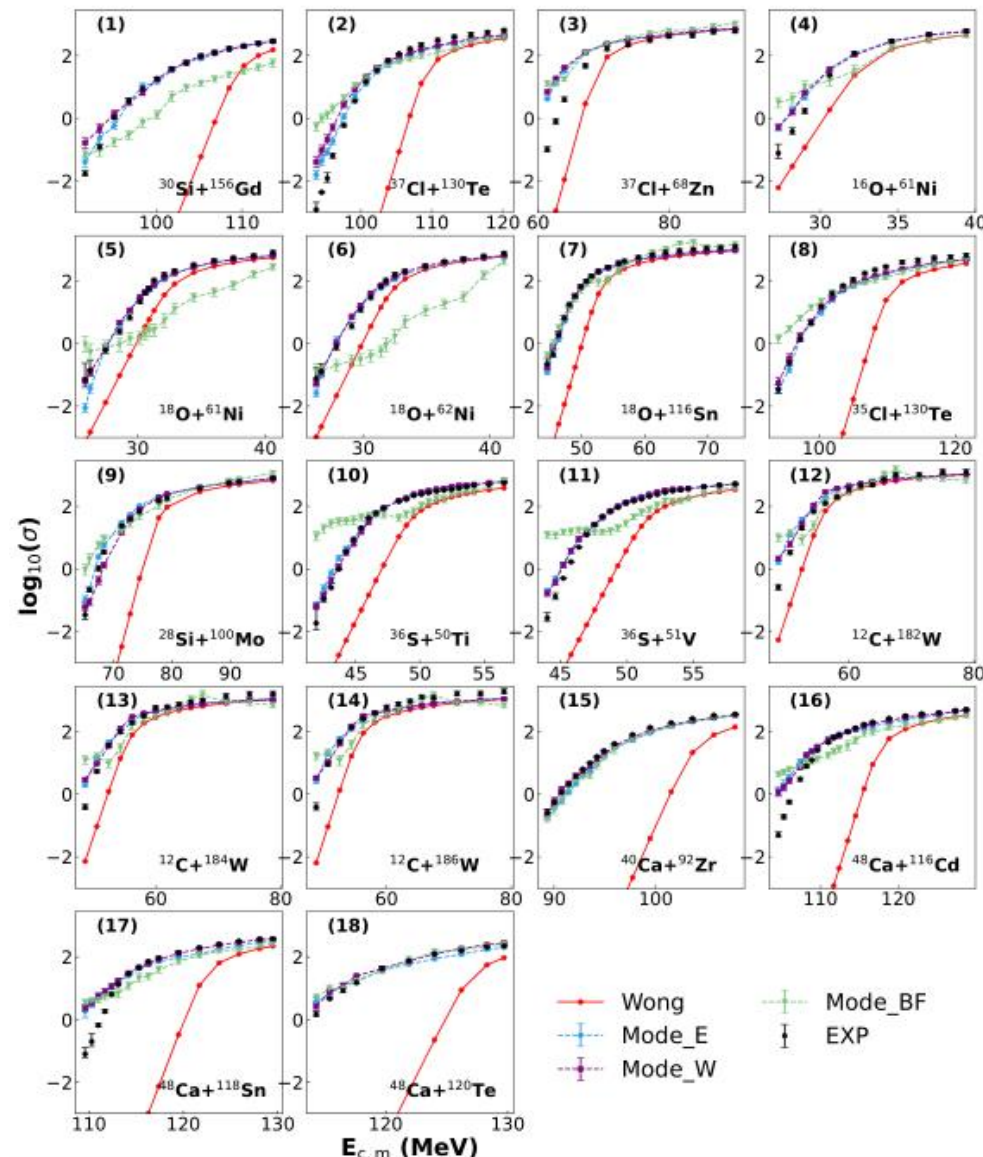
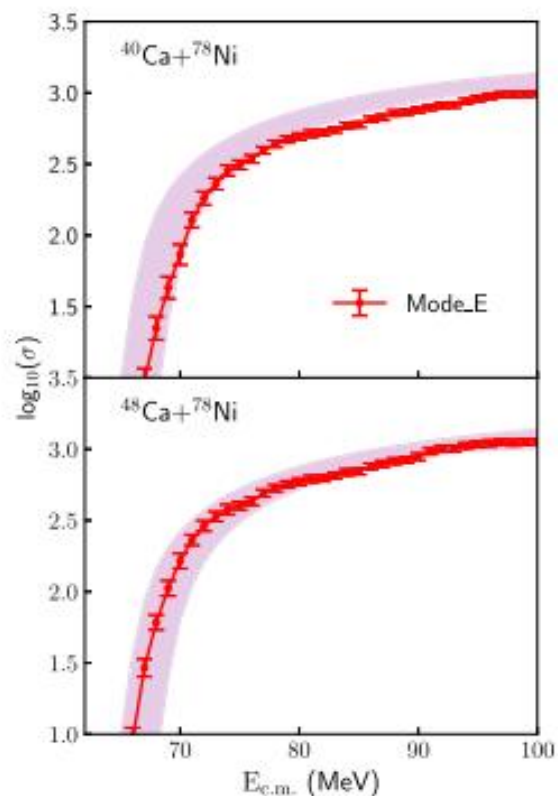
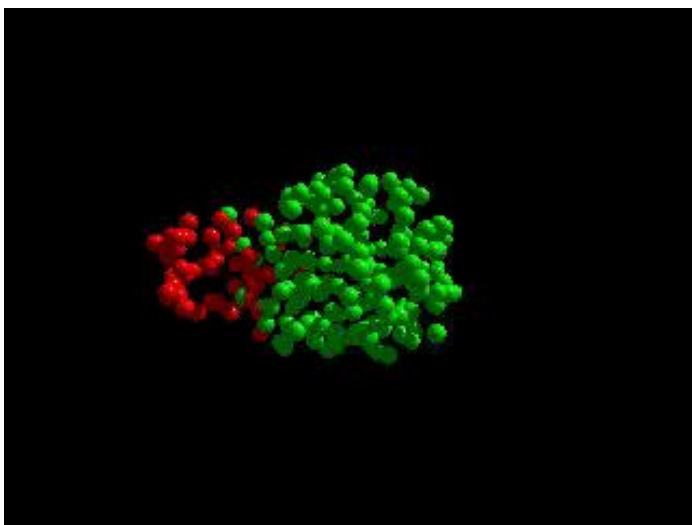
The cross section of heavy-ion fusion

PHYSICAL REVIEW C 109, 024604 (2024)

Importance of physical information on the prediction of heavy-ion fusion cross sections with machine learning

Zhilong Li^{1,2}, Zepeng Gao³, Ling Liu^{1,*}, Yongjia Wang^{2,†}, Long Zhu³, and Qingfeng Li^{2,4}

More than 1000 excitation functions for different reaction system have been measured.



Shaded band: calculations from TDHF.

The strong ability of ML algorithm is verified by both predictions from theoretical model and newly measured (unseen) experimental data.

01

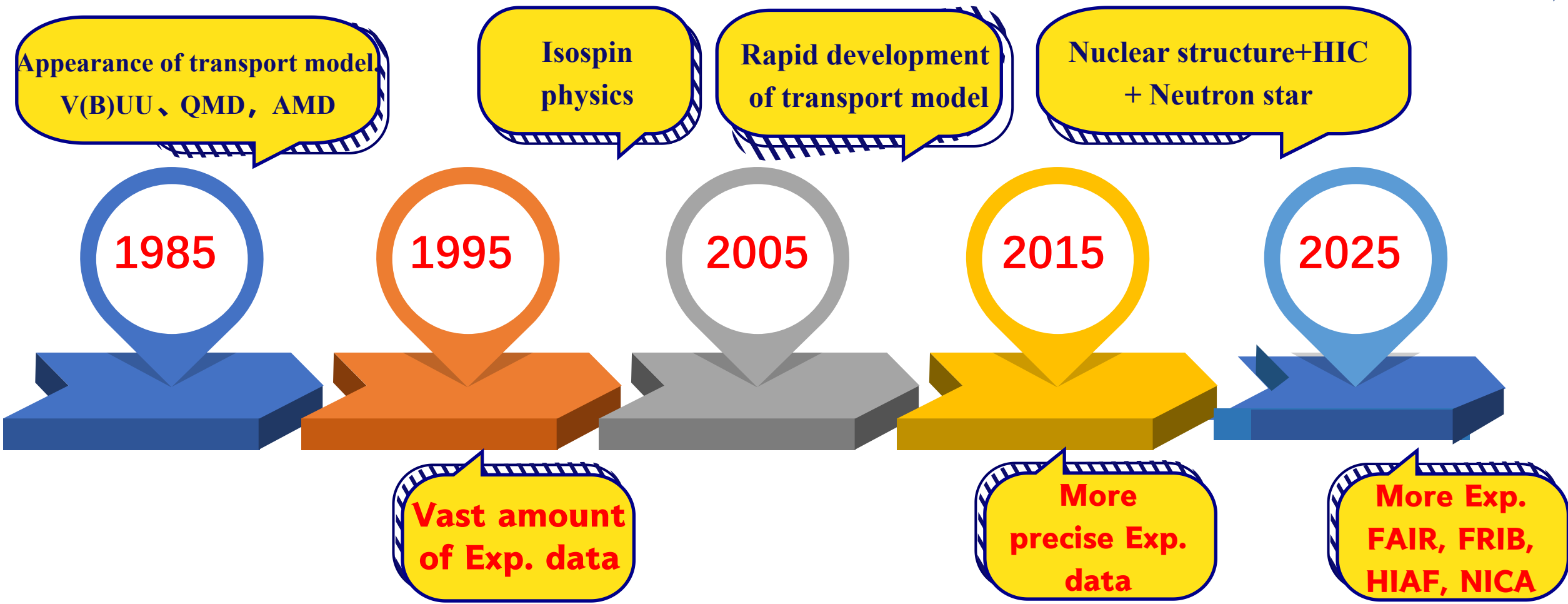
Background

Use of computer

Personal compute

computer cluster

GPU



Higher density, higher orders, higher accuracy, higher dimension

Machine learning

ARTIFICIAL INTELLIGENCE

IS NOT NEW

ARTIFICIAL INTELLIGENCE

Any technique which enables computers to mimic human behavior



1950's

1960's

1970's

1980's

1990's

2000's

2010s

Big data driven

MACHINE LEARNING

AI techniques that give computers the ability to learn without being explicitly programmed to do so

Statistical
methods



DEEP LEARNING

A subset of ML which make the computation of multi-layer neural networks feasible



Convolutional Neural Network (CNN)

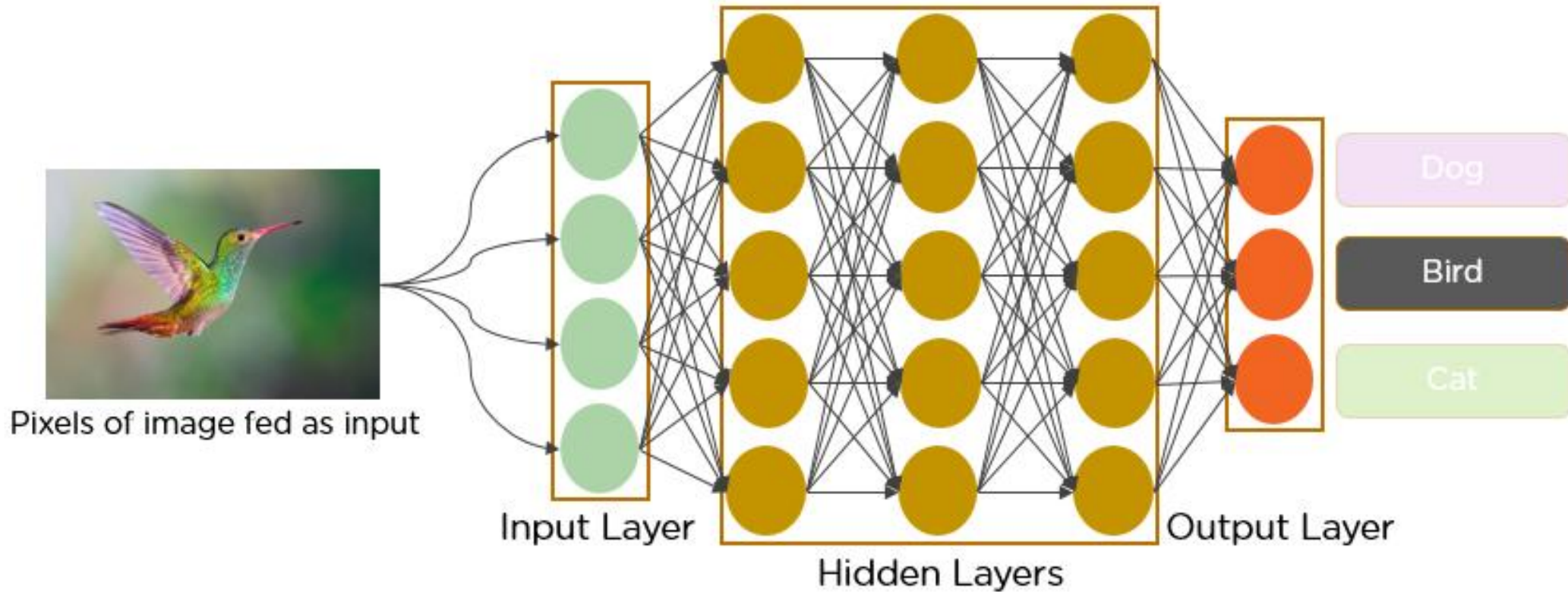
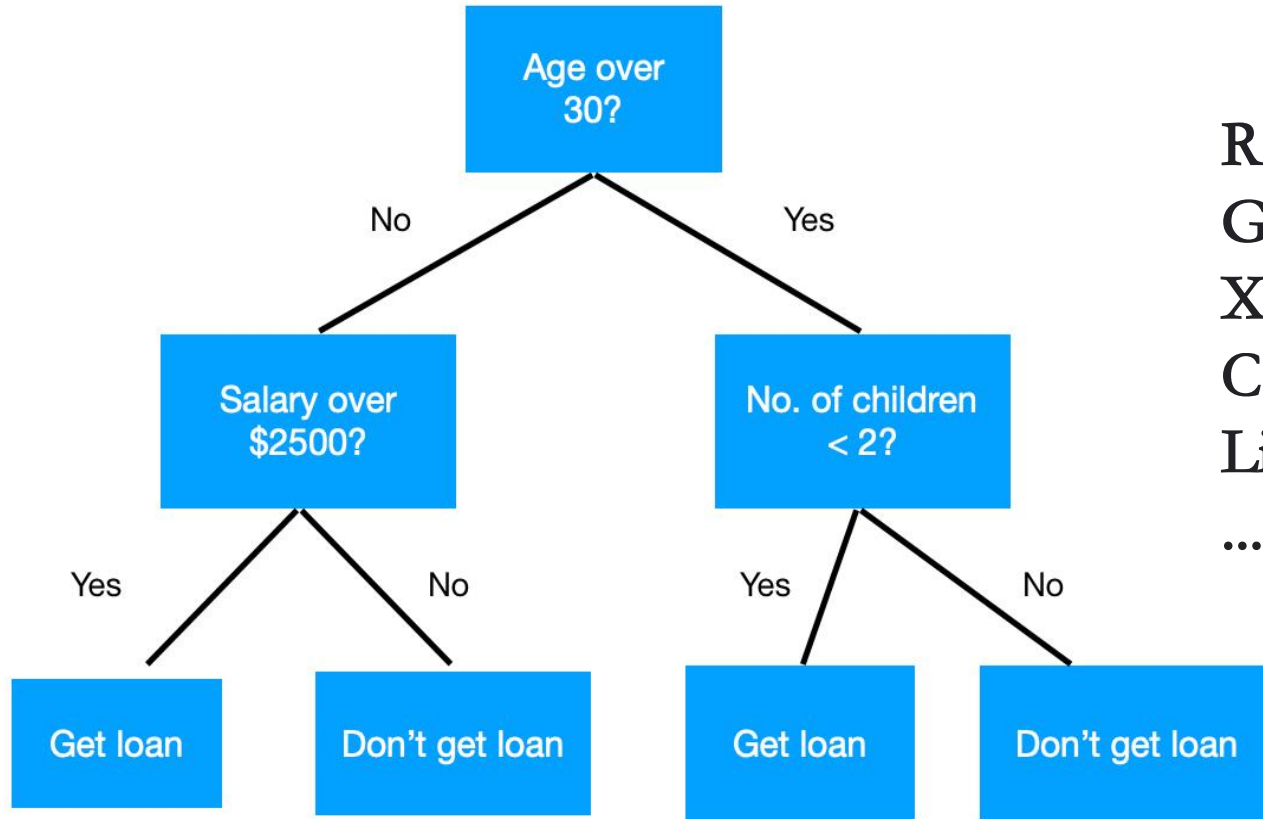


Image-like data, deep and complex structure, huge number of parameter, time-consuming, low explainability and high generalizability

Bayesian Neural Network (BNN), PointNet, Recurrent Neural Network (RNN) ...

Machine learning

Decision-tree based algorithm



Random Forest

Gradient Boosting Decision Trees (GBDT)

XGBoost (eXtreme Gradient Boosting)

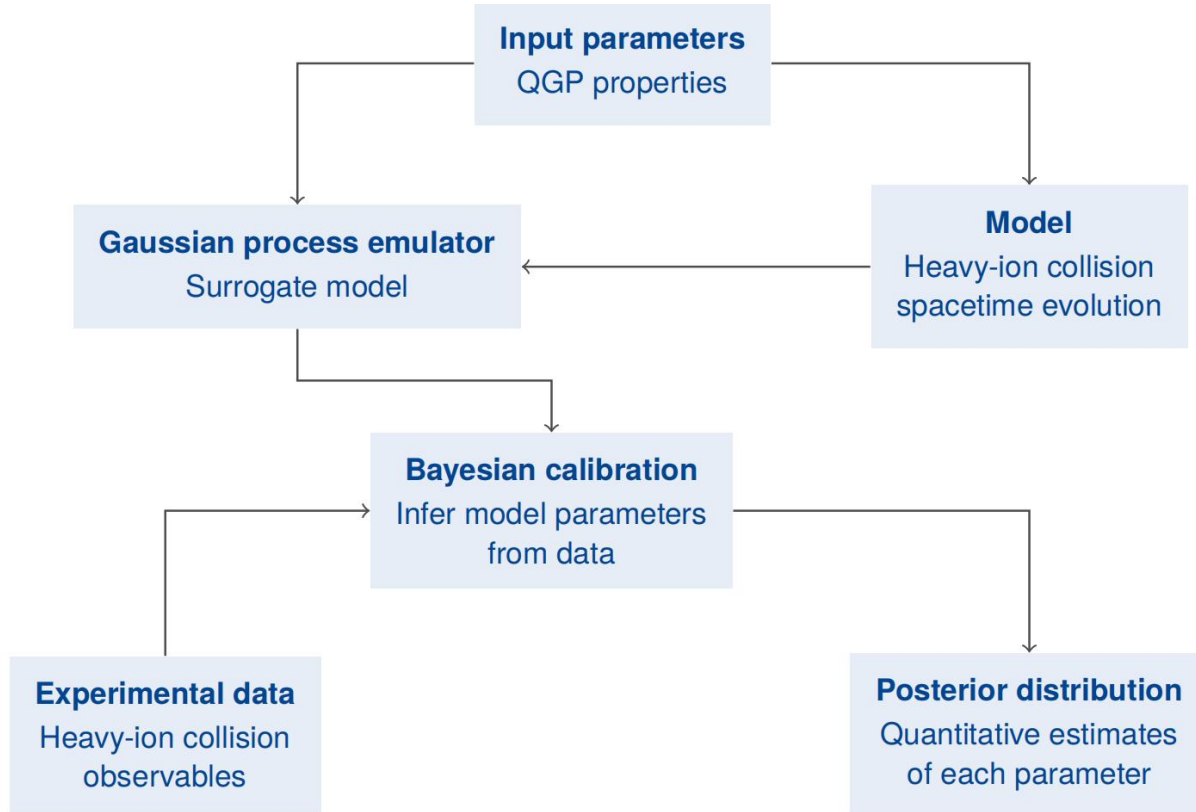
CatBoost

LightGBM (Light Gradient Boosting Machine)

.....

Feature data, white-box algorithm, faster training speed and higher efficiency, lower memory usage, capable of handling large-scale data.

Machine learning



Bernhard J E, Moreland J S, Bass S A. Bayesian estimation of the specific shear and bulk viscosity of quark–gluon plasma[J]. Nature Physics, 2019, 15(11): 1113-1117.

Constraining parameters from mutli observables.

Bayesian inference

$$\mathcal{P}(\boldsymbol{\theta} | \mathbf{y}_{\text{exp}}) \propto \mathcal{P}(\mathbf{y}_{\text{exp}} | \boldsymbol{\theta}) \mathcal{P}(\boldsymbol{\theta}).$$

$\mathcal{P}(\boldsymbol{\theta} | \mathbf{y}_{\text{exp}})$ ← Posterior distribution
 $\mathcal{P}(\mathbf{y}_{\text{exp}} | \boldsymbol{\theta})$ ← likelihood function
 $\mathcal{P}(\boldsymbol{\theta})$ ← Prior distribution

likelihood function

$$\Delta \mathbf{y}(\boldsymbol{\theta}) = \mathbf{y}(\boldsymbol{\theta}) - \mathbf{y}_{\text{exp}}$$

$$\ln[\mathcal{P}(\mathbf{y}_{\text{exp}} | \boldsymbol{\theta})] = -\frac{1}{2} \Delta \mathbf{y}(\boldsymbol{\theta})^T \Sigma^{-1} \Delta \mathbf{y}(\boldsymbol{\theta}) - \frac{1}{2} \ln[(2\pi)^n \det \Sigma].$$

Physics Letters B 833 (2022) 137348

Physics Letters B 799 (2019) 135045

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Bayesian inference

$S\pi$ RIT+ImQMD

Constraining the symmetry energy with heavy-ion collisions and Bayesian analyses

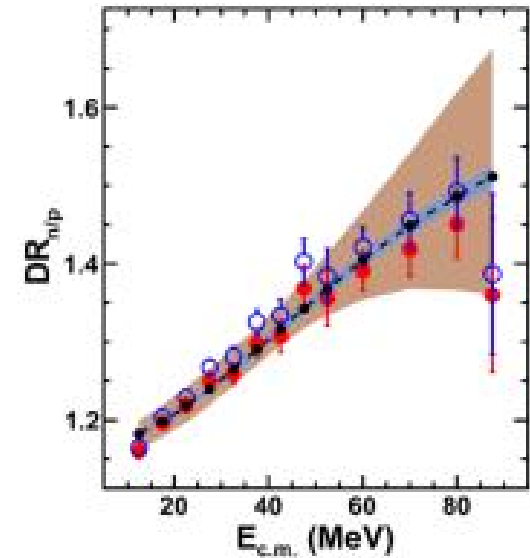
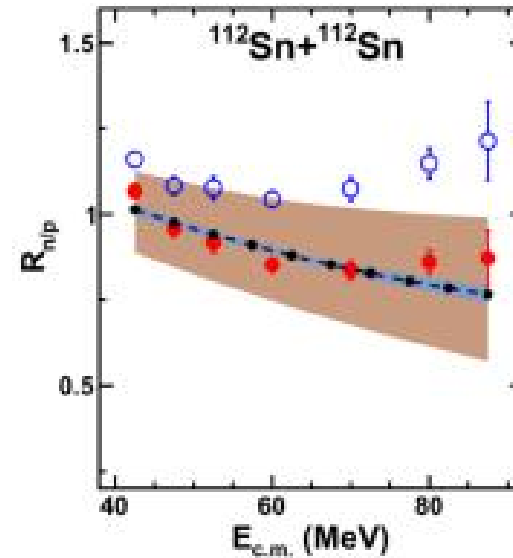
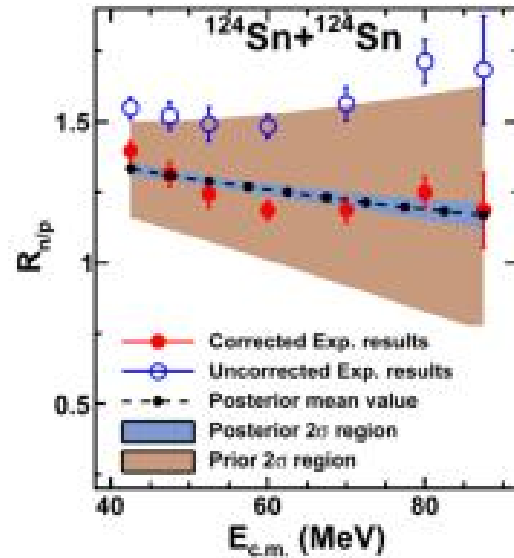


P. Morfouace^{a,*}, C.Y. Tsang^a, Y. Zhang^b, W.G. Lynch^a, M.B. Tsang^a, D.D.S. Coupland^a, M. Youngs^a, Z. Chajecki^c, M.A. Famiano^c, T.K. Ghosh^e, G. Jhang^a, Jenny Lee^d, H. Liu^f, A. Sanetullaev^a, R. Showalter^a, J. Winkelbauer^a

Table 1

Model parameter values for prior distribution. 49 sets of calculation have been performed within this 4D model space using a Latin hyper-cube sampling.

Parameter range
$25.7 \leq S_0 \leq 36$ (MeV)
$32 \leq L \leq 120$ (MeV)
$0.6 \leq m_s^*/m_N \leq 1.0$
$0.6 \leq m_v^*/m_N \leq 1.2$



Editors' Suggestion

Open Access

Access by Huzhou

QCD Equation of State of Dense Nuclear Matter from a Bayesian Analysis of Heavy-Ion Collision Data

Manjunath Omana Kuttan, Jan Steinheimer, Kai Zhou, and Horst Stoecker
Phys. Rev. Lett. **131**, 202303 – Published 16 November 2023

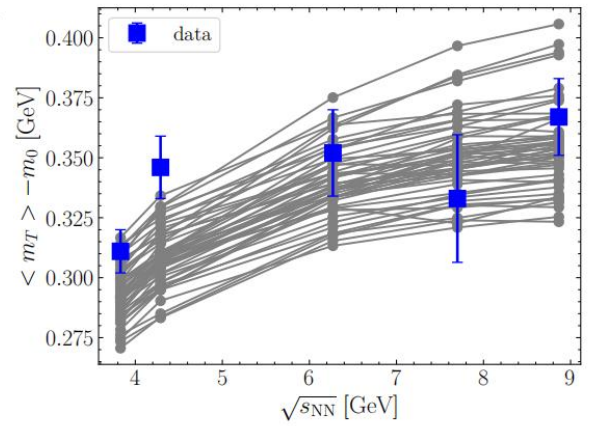
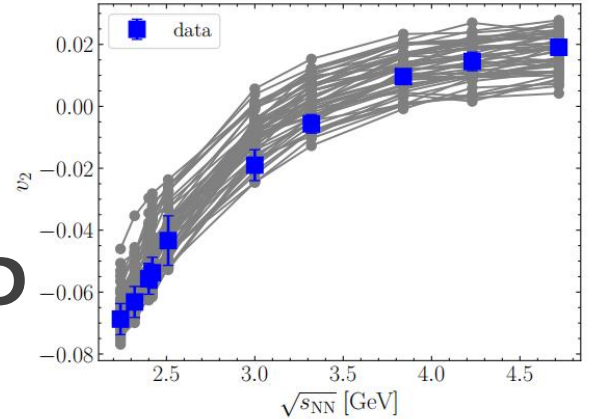
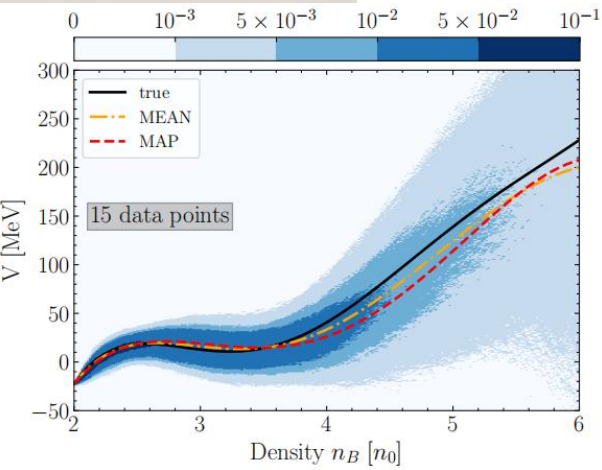
E895, NA49, STAR+UrQMD

Article References Citing Articles (1) Supplemental Material PDF HTML Export Citation



ABSTRACT

Bayesian methods are used to constrain the density dependence of the QCD equation of state for dense nuclear matter using the data of mean transverse kinetic energy and elliptic flow from heavy ion collisions (HICs), in the beam energy range $\sqrt{s_{NN}} = 2-10$ GeV. The tight constraints on the density dependent EOS up to 4 times the nuclear saturation extracted EOS yields good agreement with other observables measured in HIC experiments and constraints from astrophysical observations both of which were not used in the inference. The robustness of the inference to the choice of observables is also discussed.



3.1 Nuclear symmetry energy

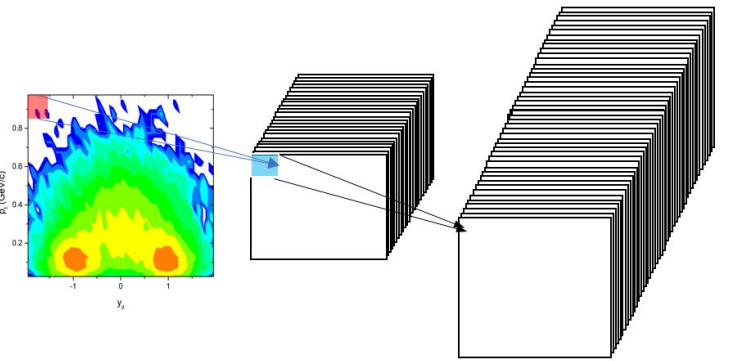


Physics Letters B 822 (2021) 136669

Finding signatures of the nuclear symmetry energy in heavy-ion collisions with deep learning

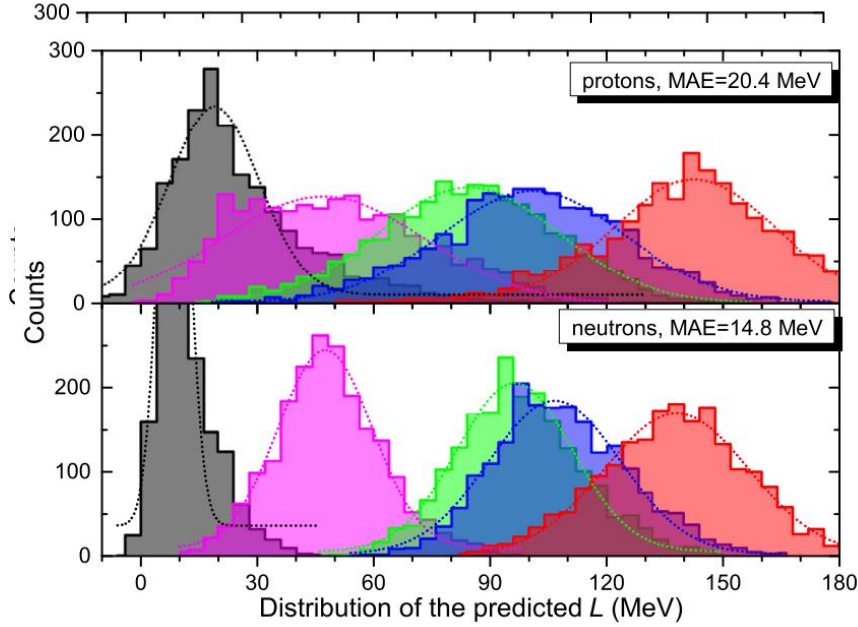
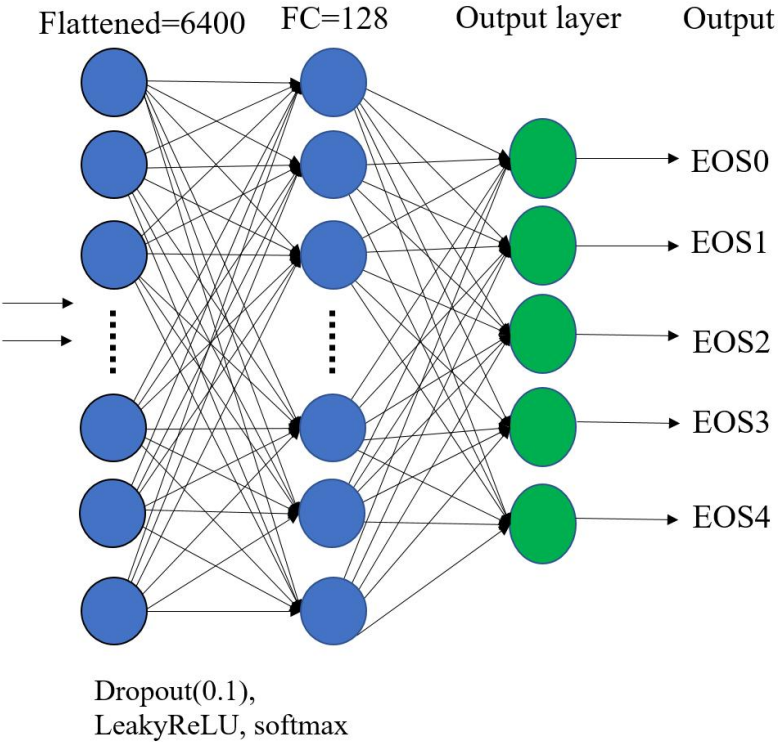
Yongjia Wang^{a,*}, Fupeng Li^{a,b}, Qingfeng Li^{a,c,**}, Hongliang Lü^d, Kai Zhou^e

Particle Spectra 20*40 pixels	64 features 20*40	128 features 10*20
----------------------------------	----------------------	-----------------------



■ 5*5 conv,64
 Dropout(0.1), BN,
 LeakyReLU, avgpool

■ 5*5 conv,128
 Dropout(0.1),
 LeakyReLU, avgpool



Fingerprints of $E_{sym}(\rho)$ on the transverse momentum and rapidity distributions of protons and neutrons can be identified by convolutional neural network algorithm.



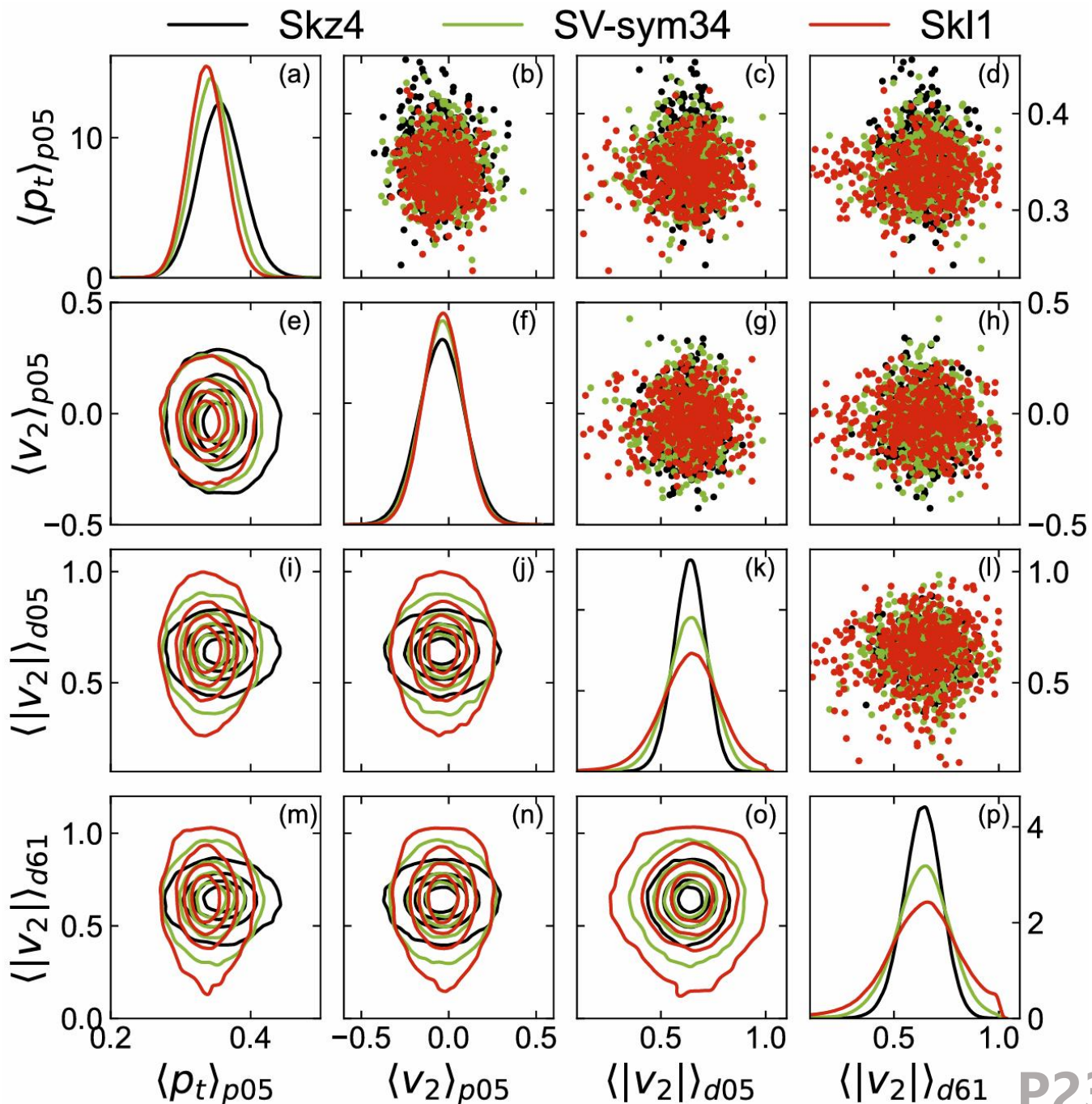
Physics Letters B 835 (2022) 137508

Decoding the nuclear symmetry energy event-by-event in heavy-ion collisions with machine learning

Yongjia Wang^a, Zepeng Gao^{a,b}, Hongliang Lü^c, Qingfeng Li^{a,d,*}

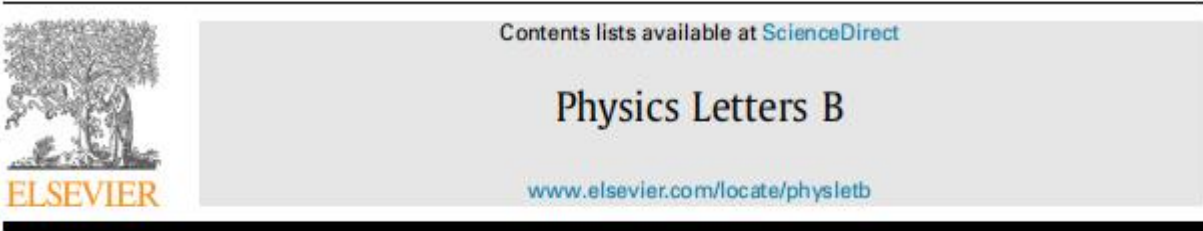
Particles	Rapidity window	Feature	Description	Particles	Rapidity window	Feature
Free protons	$ y_0 < 0.5$	$\langle p_t \rangle_{p05}$	Mean value of p_t	Deuterons	$ y_0 < 0.5$	$\langle p_t \rangle_{d05}$
		$\langle p_x \rangle_{p05}$	Mean value of $ p_x $			$\langle p_x \rangle_{d05}$
		$\langle p_y \rangle_{p05}$	Mean value of $ p_y $			$\langle p_y \rangle_{d05}$
		$\langle v_2 \rangle_{p05}$	Mean value of v_2			$\langle p_z \rangle_{d05}$
		$\langle v_1 \rangle_{p05}$	Mean value of $ v_1 $			$\langle v_2 \rangle_{d05}$
		$\langle v_2 \rangle_{p05}$	Mean value of $ v_2 $			$\langle v_1 \rangle_{d05}$
		$\langle v_3 \rangle_{p05}$	Mean value of $ v_3 $			$\langle v_2 \rangle_{d05}$
Free protons	$0.6 < y_0 < 1.0$	$\langle p_t \rangle_{p61}$	Mean value of p_t	Deuterons	$0.6 < y_0 < 1.0$	$\langle p_t \rangle_{d61}$
		$\langle p_x \rangle_{p61}$	Mean value of $ p_x $			$\langle p_x \rangle_{d61}$
		$\langle p_y \rangle_{p61}$	Mean value of $ p_y $			$\langle p_y \rangle_{d61}$
		$\langle v_2 \rangle_{p61}$	Mean value of v_2			$\langle p_z \rangle_{d61}$
		$\langle v_1 \rangle_{p61}$	Mean value of $ v_1 $			$\langle v_2 \rangle_{d61}$
		$\langle v_2 \rangle_{p61}$	Mean value of $ v_2 $			$\langle v_1 \rangle_{d61}$
		$\langle v_3 \rangle_{p61}$	Mean value of $ v_3 $			$\langle v_2 \rangle_{d61}$
						$\langle v_3 \rangle_{d61}$

30 event-by-event observables related to momenta of protons and deuterons.



3.1 Nuclear symmetry energy

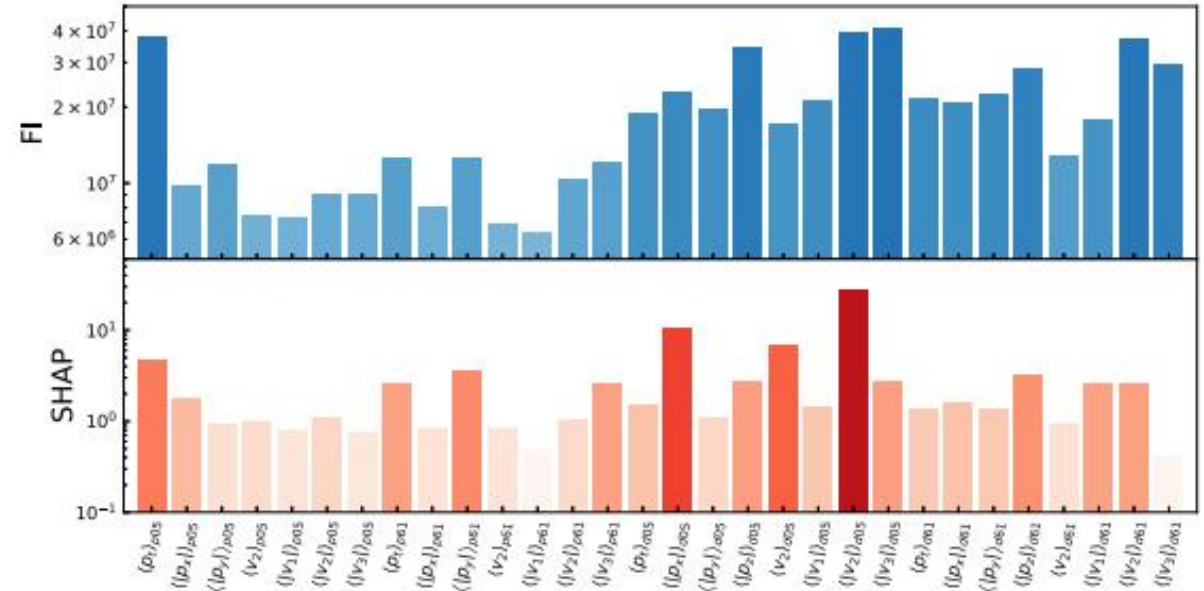
Feature importance



Physics Letters B 835 (2022) 137508

Decoding the nuclear symmetry energy event-by-event in heavy-ion collisions with machine learning

Yongjia Wang^a, Zepeng Gao^{a,b}, Hongliang Lü^c, Qingfeng Li^{a,d,*}



Good generalizability.

Table 1
The mean values of predicted $L(\rho_0)$ and their standard deviation σ obtained with Gaussian fit. All units are in MeV.

$L^{\text{true}}(\rho_0)$	Testdata1 (MAE=29.6)		Testdata2 (MAE=29.4)		Testdata3 (MAE=29.4)		Testdata4 (MAE=27.8)	
	$\langle L^{\text{pred}}(\rho_0) \rangle$	σ	$\langle L^{\text{pred}}(\rho_0) \rangle$	σ	$\langle L^{\text{pred}}(\rho_0) \rangle$	σ	$\langle L^{\text{pred}}(\rho_0) \rangle$	σ
Skz4	5.8	16.8	43.3	16.1	38.4	16.4	48.0	17.0
SLy230a	44.3	19.4	51.3	17.5	47.3	19.0	58.7	20.2
SV-sym32	57.0	25.1	69.1	23.2	66.6	25.3	82.9	25.8
SV-sym34	81.2	27.2	76.6	24.8	73.9	27.2	93.0	27.6
SkI2	106.4	27.9	79.6	25.7	77.7	28.1	98.6	28.2
SkI1	159.0	29.7	110.3	29.8	109.7	31.5	140.8	22.6

- Fingerprints of $E_{\text{sym}}(\rho)$ can be decoded from a large set of observables in HICs on an event-by-event basis by the trained machine learning algorithm.
- With feature attribution methods, the most important features that drive predictions can be identified.

3.2 Bayesian Inference of the in-medium nucleon-nucleon cross section

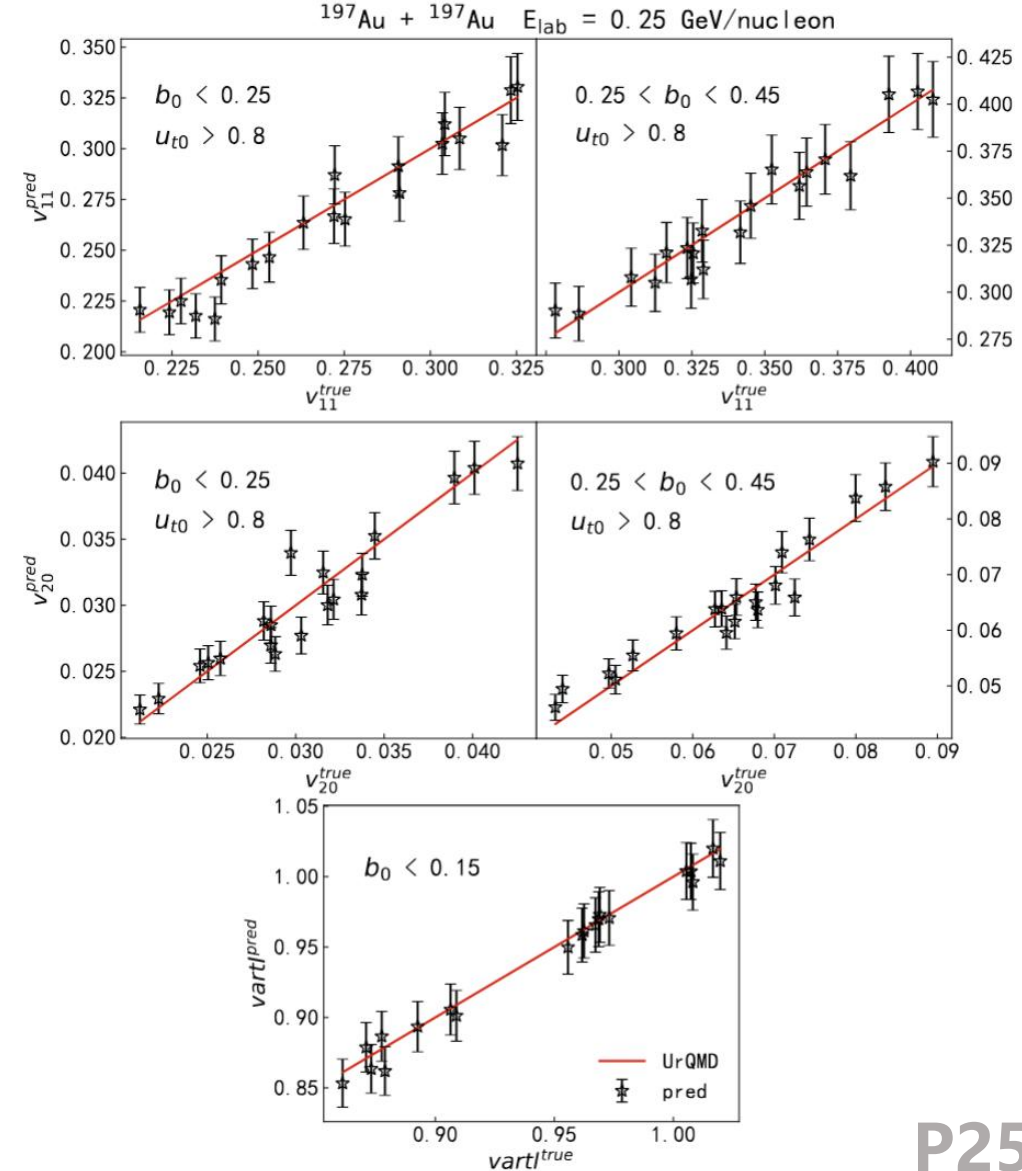
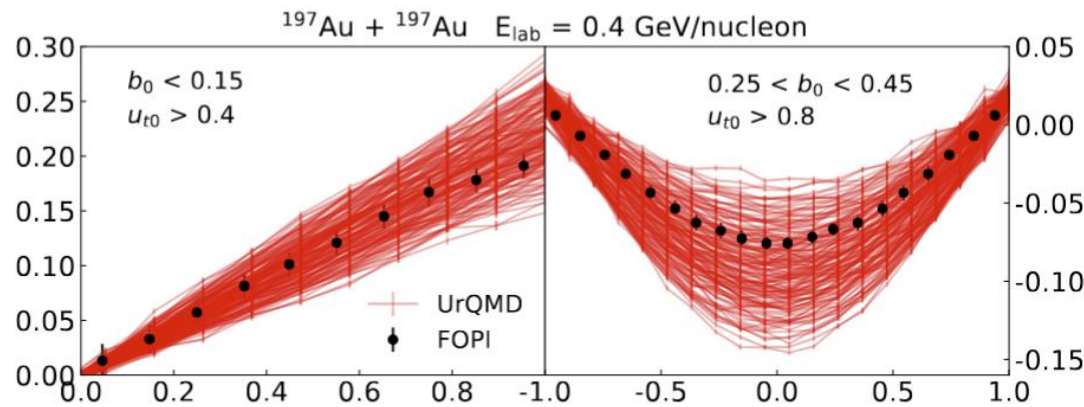
Table I. List of observables used in the analysis in $^{197}\text{Au} + ^{197}\text{Au}$ collisions with a beam energy of 0.25 GeV/nucleon

Observable	b_0	u_{t0}	value
v_{11}	$b_0 < 0.25$	$u_{t0} > 0.8$	0.23 ± 0.01
	$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.37 ± 0.01
$-v_{20}$	$b_0 < 0.25$	$u_{t0} > 0.8$	0.026 ± 0.001
	$0.25 < b_0 < 0.45$	$u_{t0} > 0.8$	0.046 ± 0.005
$vartl$	$b_0 < 0.15$	None	0.891 ± 0.041

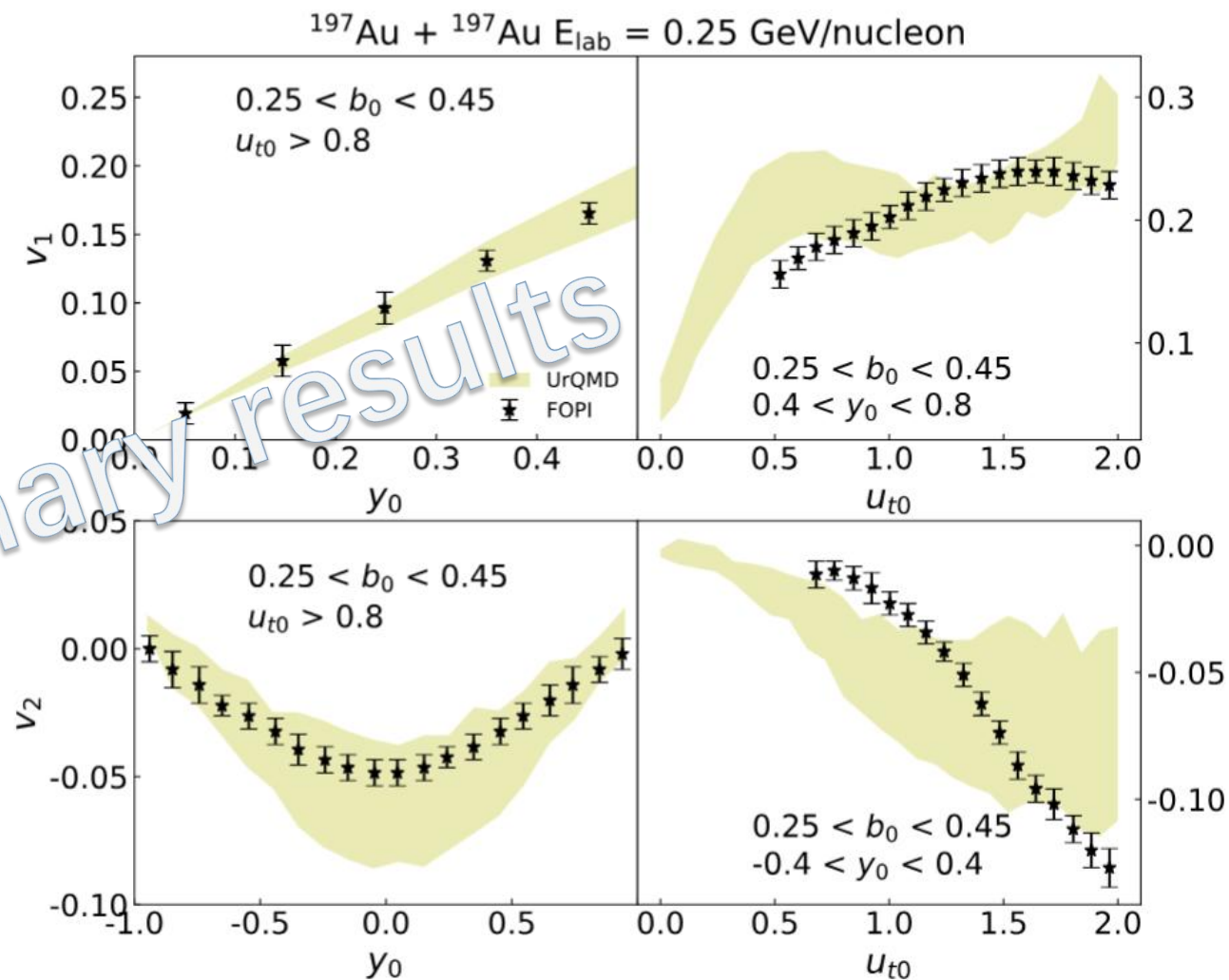
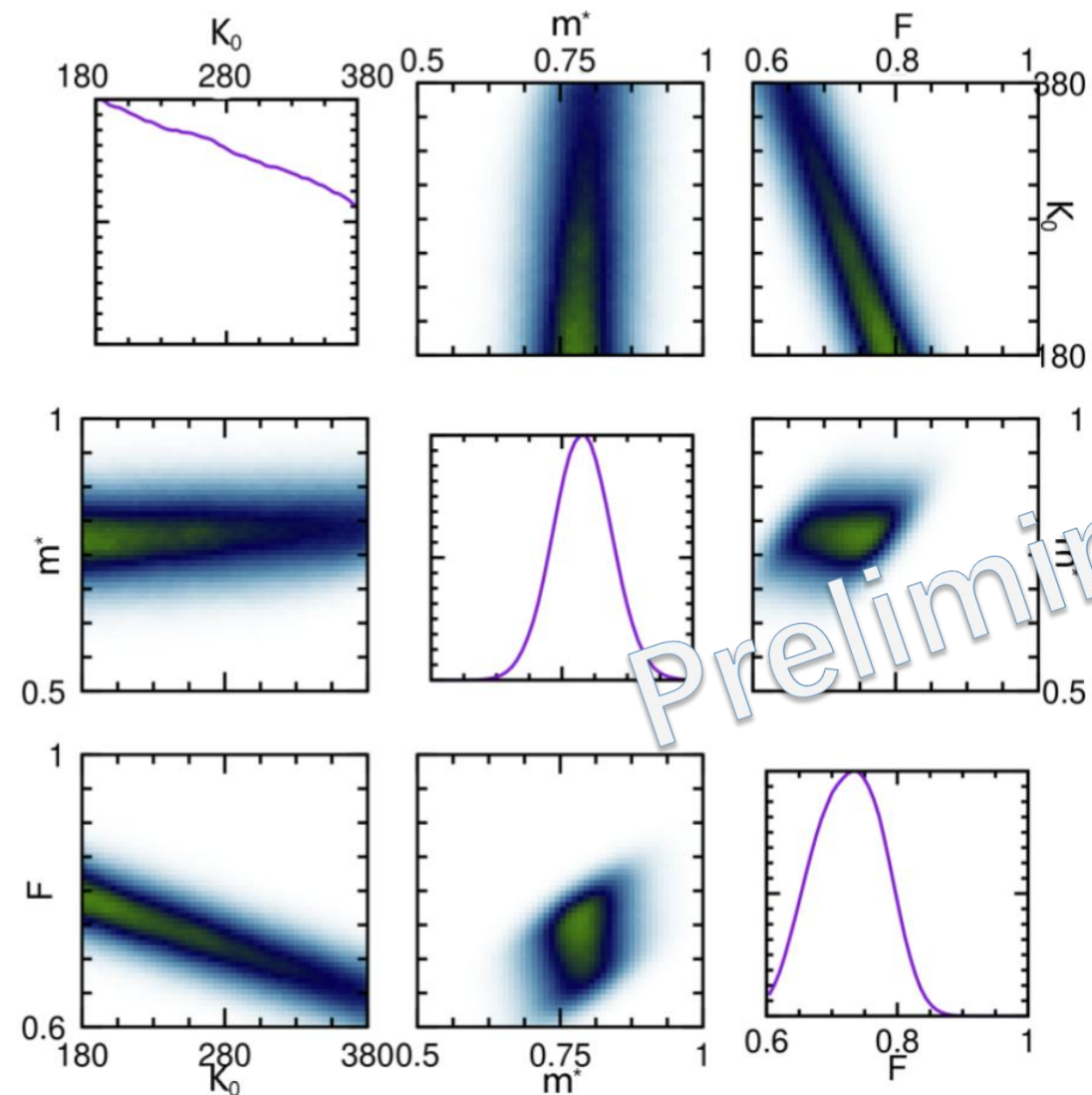
Table III. Parameters used in the present work

Para. Name	Description	Prior ranges
K_0	Incompressibility	[180, 380]
m^*	Isoscalar effective mass	[0.6, 0.95]
F	In-medium correction factor	[0.5, 1.0]

Gaussian process (GP) model is trained as an emulator of UrQMD model to interpolate the simulation results in the parameter space.



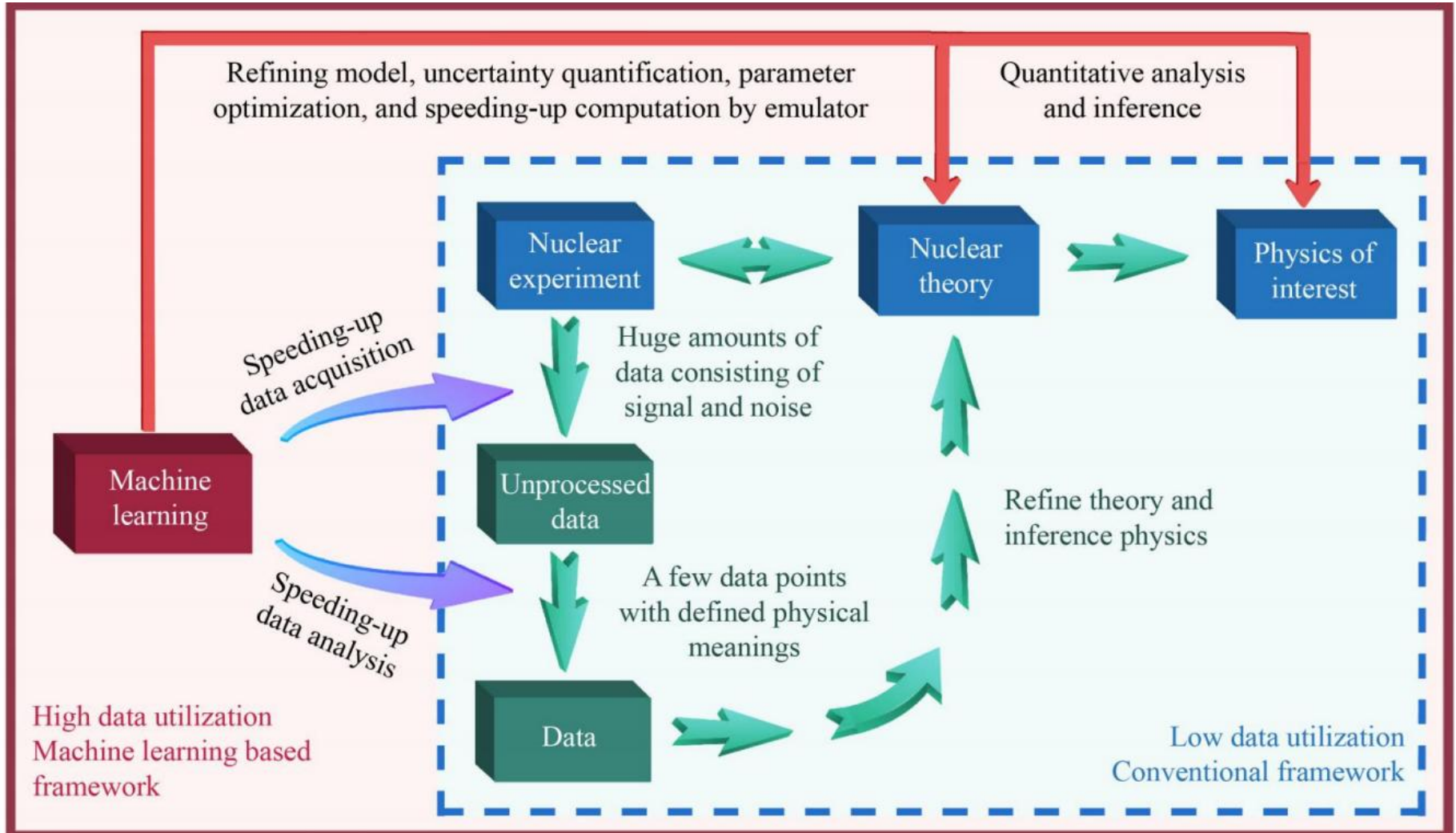
3.2 Bayesian Inference of the in-medium nucleon-nucleon cross section



Summary and outlook

Machine learning transforms the inference of the nuclear equation of state

Yongjia Wang¹, Qingfeng Li^{1,2,3,†}



Summary and Outlook



01

Developing more sophisticated models, or using different models to generate data

Improving the quality of data

Introducing physical information into ML algorithms

02

Using input features with defined physical meanings or by considering physical symmetries and laws when constructing architectures of ML algorithms

03

Using experiences of ML applications in other fields

Condensed matter physics and particle physics.

04

Introducing the latest developments of ML into tools for studying nuclear physics

A diverse array of ML algorithm has been developed and continue to be refined to cover a wide variety of data types and tasks, this is a sufficiently large and diverse pool of tools feasible to study heavy-ion physics

Thanks!