Simultaneous Calculation of Elastic Scattering, Transfer, Breakup, and Other Direct Cross Sections for d+^{197}Au Reaction

Presentation by

Hasan Maridi

https://www.hasanmaridi.com

Nuclear Physics Group, Department of Physics & Astronomy, the University of Manchester, UK

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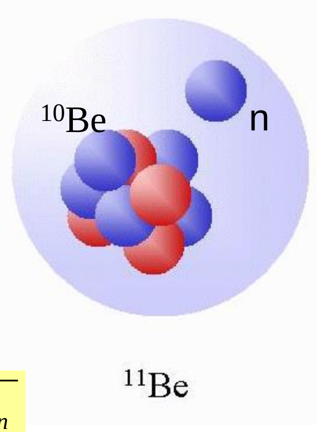
Halo nuclei and deuteron

Exotic nuclei: contain many more(fewer) neutrons than a stable isotope, far away from stability line, so short lived and rapidly decayed.

Halo nuclei are exotic nuclei with the following properties:

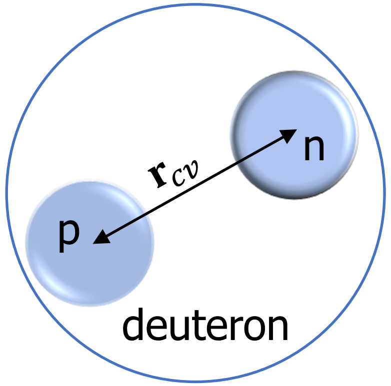
1*n*-halo nucleus
S_n=504 keV



- strong cluster structure They are described as a core plus halo neutrons.
- weakly bound with separation energy ~ 1 MeV whereas in stable nuclei is about 6 ~ 8 MeV.
- extend density Their neutron density distribution shows an extremely long tail. $\Psi(r) \propto \frac{e^{-\kappa r}}{r}$ $\kappa = \sqrt{\frac{2\mu S_{n/2n}}{\hbar^2}}$
- large root-mean-square radius and the valence neutrons are mostly located far from the core.
- short lived They have decay lifetime in order of ms~s.

For more details: I. Tanihata & B. Jonson, Handbook of Nuclear Physics, Halo Nuclei (2023) https://doi.org/10.1007/978-981-19-6345-2_63

Nucleus	J^π	$T_{1/2}$ (s)	Core	Sep. en. (MeV)
^{2}H	1^+	stable	p	2.225
^{6}He	0^+	0.807	^{4}He	0.975
^{11}Be	½$^+$	13.810	^{10}Be	0.502
^{15}C	½$^+$	2.449	^{14}C	1.218
^{19}C	½$^+$	0.046	^{18}C	0.580



Is the deuteron a halo nucleus?

A new two-cluster approach

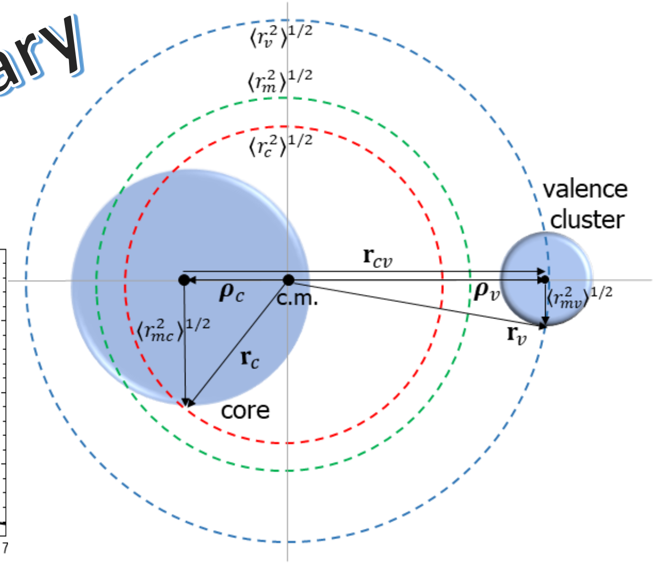
- We present a new simplified expression for the distance between two clusters in weakly-bound nuclei.

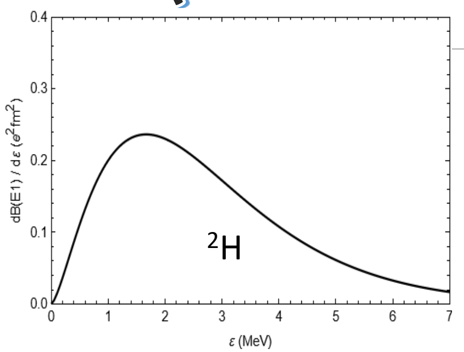
$$r_{cv}^{rms} = \sqrt{\frac{A}{A_c A_v}\left(A\langle r_m^2\rangle - A_c\langle r_{mc}^2\rangle - A_v\langle r_{mv}^2\rangle\right)}$$

- We present ground-state wave functions, $u_{lj}(r)$, as a combination of *s* & *d* harmonic oscillator states with a size related to the separation energy.
- This combination is applied successfully to the reproduce *dB(E1)/dε* data by fitting the *sd* mixing.

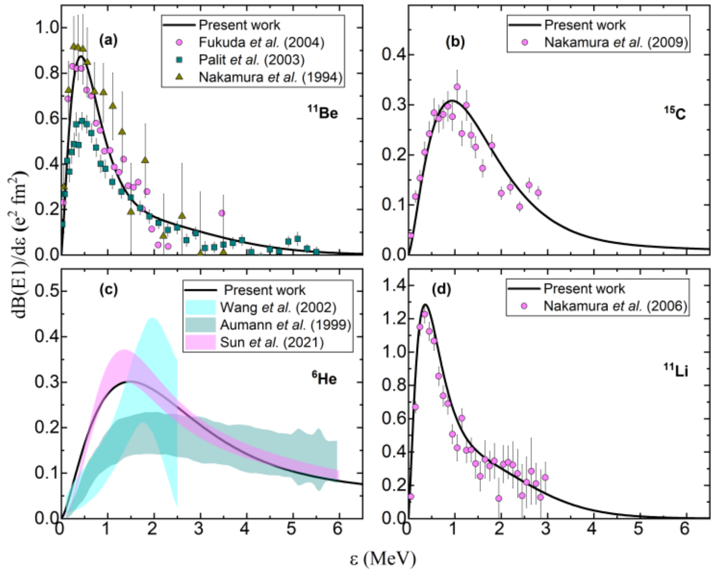
H.M. Maridi, J. Singh, N.R. Walet, D.K. Sharp, arXiv: 2407.03044

preliminary



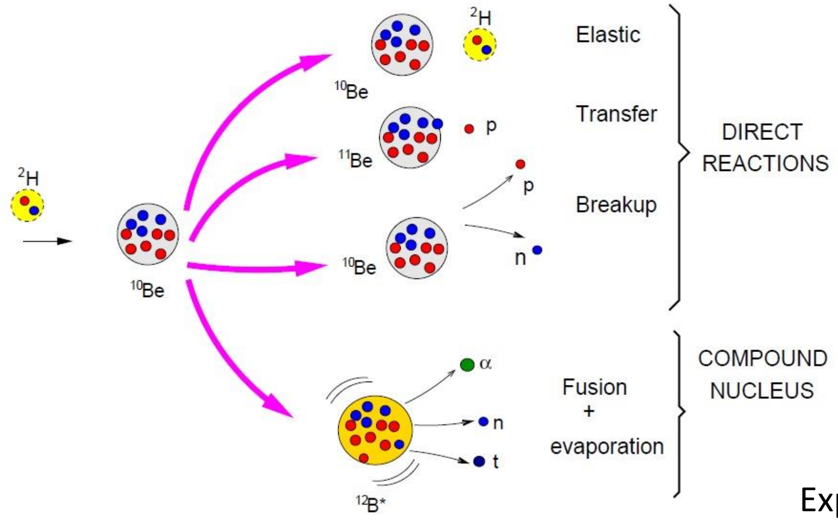


Nucleus	Core	Sep. en. (MeV)	r_{cm} (fm)	r_m (fm)	r_{cv} (fm)	exp. r_{cv} (fm)	B(E1) e^2fm^2	exp. B(E1) e^2fm^2
^{2}H	p	2.225	0.84	1.98	3.60	3.94(1)	0.72	
^{6}He	^{4}He	0.975	1.57	2.48	3.79	3.36(39),3.9(2)	1.53	1.2(2), 1.6(2)
^{11}Be	^{10}Be	0.502	2.39	2.91	6.15	5.77(16),6.1(5)	1.19	1.05(6),1.3(3)
^{11}Li	^{9}Li	0.369	3.12	2.53	4.94	5.01(32)	1.73	1.78(22)
^{15}C	^{14}C	1.218	2.43	2.60	4.36	4.15(50),4.5(2)	0.73	0.77(7)
^{19}C	^{18}C	0.580	2.75	3.0	6.06	5.5(3), 6.6(5)	0.86	0.71(7)



Reactions of deuteron

- Nuclear reactions for deuteron can occur through:
  - ✓ Elastic scattering: A+B → A+ B (Optical model)
  - ✓ Inelastic scattering: A+B → A+ B* (Coupled Channel (CC))
  - ✓ Break-up: A+B → A+C+d (CDCC)
  - ✓ Transfer reactions: A+B → C+D (*DWBA; CRC)*
  - ✓ Fusion reactions: (completely or incompletely) (CC)



Elastic scattering and Optical model (OM)

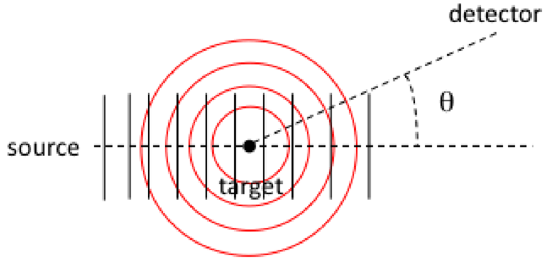
- Optical model (OM) describes the elastic scattering between two nuclei by solving a one-body Schrödinger equation.
- The system can be considered as two-body interacting via a complex mean-field potential called optical potential, deduced from the data.
- The optical potential: Coulomb + complex Nuclear, the imaginary term describes flux loss due to non-elastic channels.

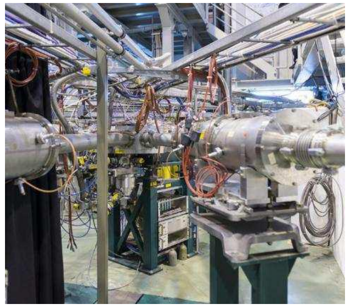
$$V_N(r) = \frac{V_0}{1+e^{(r-R_V)/a_V}} + \frac{iW_0}{1+e^{(r-R_W)/a_W}}$$

$$R_{V(W)} = r_{V(W)}\left(A_P^{1/3}+A_T^{1/3}\right)$$

$$r_{V(W)} \approx 1.1-1.3\,\text{fm}, a_{V(W)} \approx 0.5-0.7\,\text{fm}$$

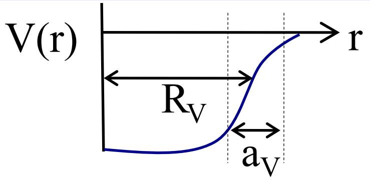
Experiment





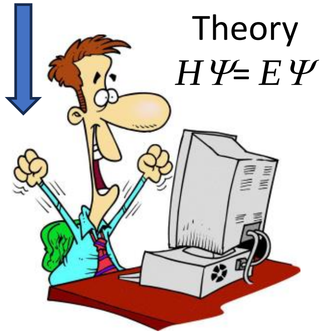
The angular distribution of emitted particles reaches the detector is

$$\frac{d\sigma}{d\Omega} = \frac{\text{flux of scattered particles through } dA = r^2 d\Omega}{\text{incident flux}}$$



Theory

$H\Psi = E\Psi$



For weakly-bound nuclei, we need a *long-range absorption*:
imaginary OP with large a_W, adding polarization potentials (DPP), or using CDCC

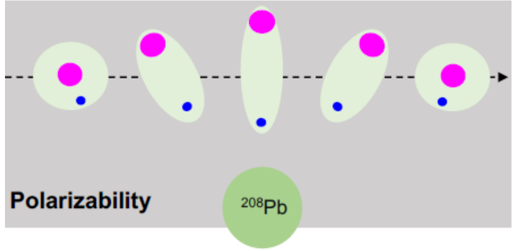
Coulomb Dynamical Polarization Potential (CDPP)

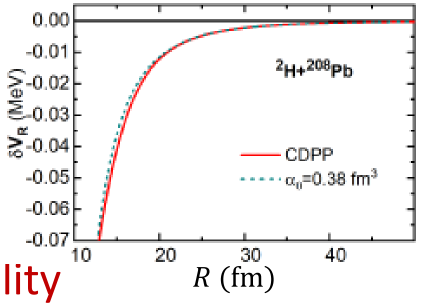
- When projectiles interact with light targets at energies below the Coulomb barrier the deviation from Rutherford scattering may be represented by a short-range nuclear potential.
- For heavy ion targets, the weakly-bound projectile may become polarized due to the electric field of the target inducing an additional long-range interaction called the Coulomb dynamical polarization potential (CDPP).
- The weakly-bound projectile may break up, giving rise to a strong Coulomb dipole excitation to the low-lying continuum.
- We presented a CDPP expression by solving Schrodinger eq. [H.M. Maridi, K. Rusek, N. Keeley, Phys. Rev. C 104, 024614 (2021)].

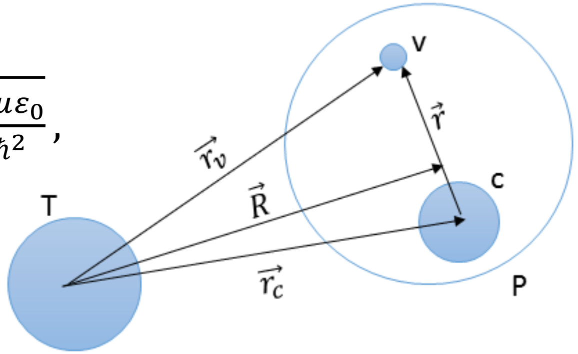
$$\delta U_C(R) = \varepsilon_0 \left[\frac{Q + Q^2 H_0' F_0'}{H_0 F_0} - 1\right]$$

ε_0 is the separation energy, H_0 and F_0 are the Coulomb functions in $\rho = kR$, $\eta = \frac{m_c^2}{\mu}\frac{Z_P Z_T e^2}{\hbar^2 k}$, $Q = \frac{\mu}{m_c}\frac{k}{\kappa_0}$, $\kappa_0 = \sqrt{\frac{2\mu\varepsilon_0}{\hbar^2}}$,

$$\mu = \frac{m_c m_v}{m_c + m_v}, \quad k \approx \sqrt{\frac{2m_c^2}{\mu\hbar^2}(V_C(R) + \varepsilon_0)}$$







The electric dipole polarizability

We presented a method to determine the electric dipole polarizability, α_0, by equating real CDPP to the classical adiabatic expression $\delta V_C = -\frac{1}{2}\alpha_0 \frac{Z_T^2 e^2}{R^4 R}$

$$\alpha_0 \approx \max\left[\frac{-2\sqrt{2}\pi^2}{16}\frac{R^4}{Z_T^2 e^2}\delta V_C(R)\right] = \frac{1}{16}\frac{\hbar^2}{\mu}\left(\frac{N_v Z_P e}{A_P}\right)^2 \frac{1}{\varepsilon_0^2}$$

This CDPP was used to study breakup & transfer effects of ^{6,8}He+ ^{208}Pb [H.M. Maridi, K. Rusek, N. Keeley, Eur. Phys. J. A 58, 49 (2022)]

proj.	Config.	$-\varepsilon_0$ (MeV)	Our α_0 (fm^3)	Previous α_0 (fm^3)
^{2}H	^{1}H+n	2.225	0.38	0.32, 0.42, 0.62, 0.56, 0.7
^{6}He	^{4}He+2n	0.975	1.32	1.00±14, 1.2, 1.3, 1.88, 1.99±40, 1.07
^{11}Li	^{9}Li+2n	0.369	5.03	5.7, 5.18
^{11}Be	^{10}Be+n	0.502	2.17	2.5, 2.66

Coulomb Dissociation at High Energies

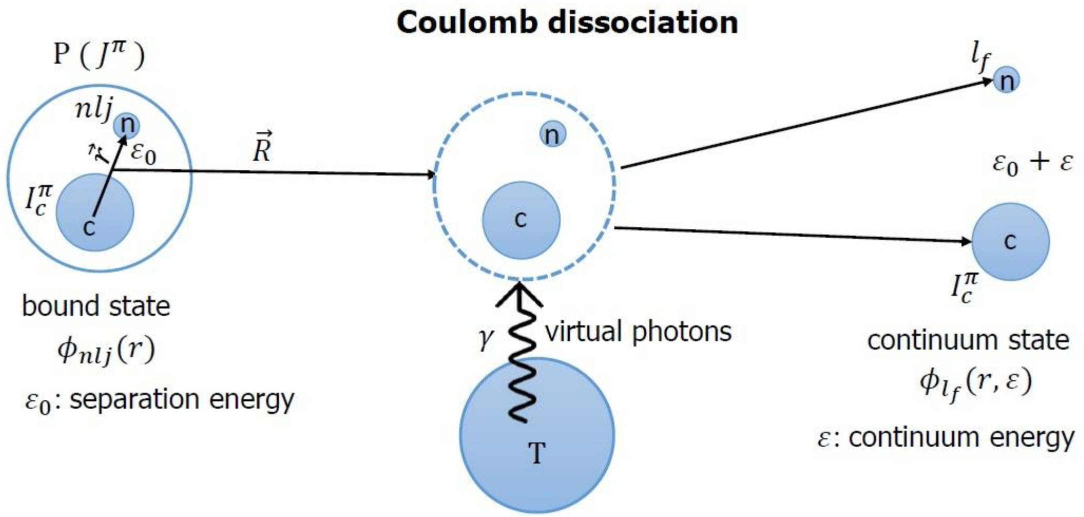
- The Coulomb dissociation can be taken place when a projectile moving with high energy (several hundred of MeV/nucleon) passes a heavy ion target
- It may be excited by absorbing virtual photons from the Coulomb field and the electromagnetic excitation is dominated by dipole excitations
- The core is assumed here as a spectator so the core state in a projectile remains after neutron removal
- Coulomb dissociation can be used to determine the electromagnetic transitions properties and astrophysical S(E) factor for radiative capture reactions b(x,γ)a
- Recently, we present a new method of calculation as

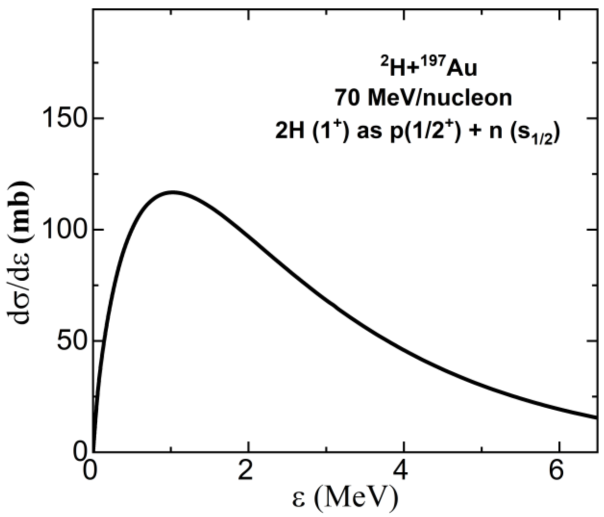
$$k \approx \sqrt{\frac{2m_c^2}{\mu\hbar^2}(V_C(R)+\varepsilon_0)} \rightarrow \sqrt{\frac{2m_c^2}{\mu\hbar^2}(V_C(R)+\varepsilon_0+\varepsilon)} \;\; \& \;\; \delta U_C(R) \rightarrow \delta U_C(\mathrm{R},\varepsilon)$$

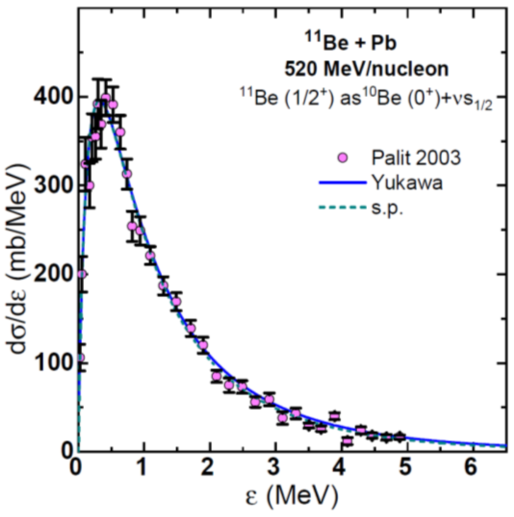
$$\frac{d\sigma}{d\varepsilon} = -\frac{2}{\hbar v}\sum_{lj} \rho^{E\lambda, I_C^\pi}_{\ell_0 j_0 \rightarrow \varepsilon\ell j}(\varepsilon) \left\langle \psi_K^+(R) \middle| \delta W_C^{I_C^\pi}(R,\varepsilon) \middle| \psi_K^+(R) \right\rangle$$

$$\rho^{E\lambda, I_C^\pi}_{\ell_0 j_0 \rightarrow \varepsilon\ell j}(\varepsilon) = \left\langle \varphi_{\varepsilon\ell j}(r,\varepsilon) | P_\lambda(Cos(\theta_r)) | \varphi^{I_C^\pi}_{\ell_0 j_0}(r,\varepsilon) \right\rangle$$

[H.M. Maridi, K. Rusek, N. Keeley, Phys. Rev. C 106, 054613 (2022)]







OM Simultaneously calculations at low energies

- However, the CDPP is usually insufficient completely to explain the long-range interactions in exotic systems. For this, we add a long-range nuclear dynamical polarization potential (NDPP) to factor in nuclear breakup and transfer.
- NDPP is usually taken as a Woods-Saxon type characterized by large radius and/or diffuseness parameters.
- **The total projectile-target optical potential** is given as:

$$U_{OP}(R) = U_C(R) + U_N(R) + \delta U_C(R) + \delta U_N(R)$$

- ❑ $U_C(R) = V_C(R)$ is the usual real Coulomb potential

$$V_C(R) = \begin{cases} \frac{Z_P Z_T e^2}{2r_C}\left(3 - \frac{R^2}{r_C^2}\right) & \text{if } R \le r_C \\ \frac{Z_P Z_T e^2}{R} & \text{if } R \ge r_C \end{cases}$$

- ❑ $\delta U_C(R) = \delta V_C(R) + i\,\delta W_C(R)$ is the long-range CDPP

$$\delta V_C(R) = \varepsilon_0\left[\frac{QG_0F_0 + Q^2G_0F_0G_0'F_0' + Q^2F_0^2F_0'^2}{F_0^4 + F_0^2G_0^2} - 1\right]$$

$$\delta W_C(R) = \varepsilon_0\left[\frac{Q^2F_0F_0' - QF_0^2}{F_0^4 + F_0^2G_0^2}\right]$$

- ❑ $U_N(R) = V_N(R) + iW_N(R)$ is the bare short–range nuclear potential (Woods–Saxon or double–folding)
- ❑ $\delta U_N(R) = \delta V_N(R) + i\,\delta W_N(R)$ is the long–range NDPP

$$\delta U_N(R) = -(V_L + iW_L)\frac{e^{(R-R_L)/a_L}}{[1 + e^{(R-R_L)/a_L}]^2}$$

From the semi-classical approx. Bonaccorso & Carstoiub, Nucl. Phys. A 706, 322 (2002), $a_L = 1/\sqrt{8\mu\varepsilon_0/\hbar^2}$, $R_L = 1.4\,(A_P^{1/3} + A_T^{1/3})$ the strong absorption radius, V_L & W_L vary to fit the data. V_L can be ignored.

One- or two- free parameters

- **The cross sections** are given by:

$$\sigma_i = \sum_\ell \sigma_{i,\ell} = -\frac{2}{\hbar v}\frac{4\pi}{k^2}\sum_{\ell=1}^{\infty}(2\ell+1)\int dR\,|\chi_\ell(R)|^2 W_i(R)$$

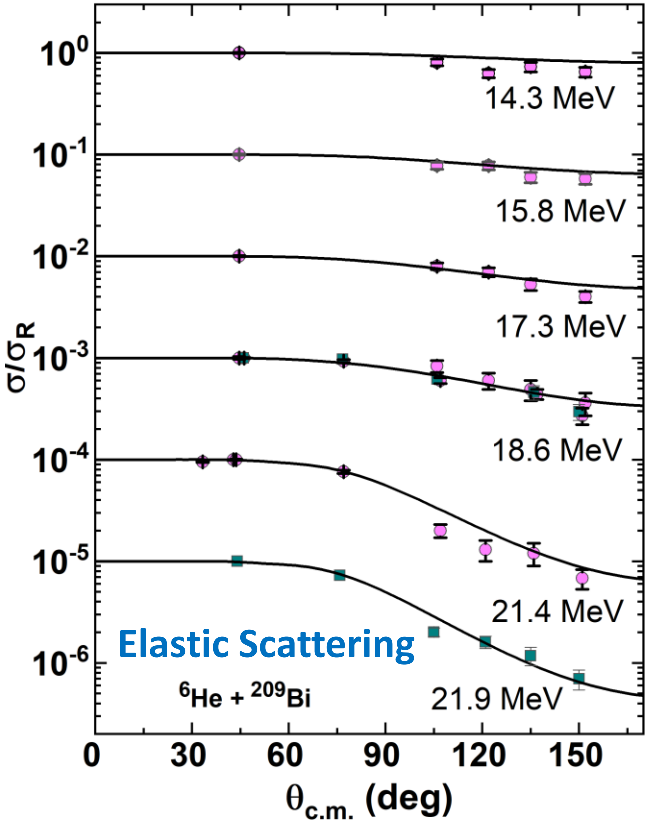
i = Fus, DC, DN

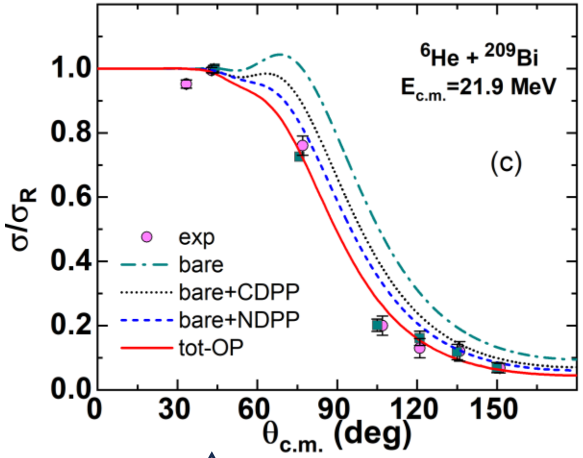
H.M. Maridi, N. Keeley, K. Rusek, Phys. Rev. C 109, 034601 (2024).

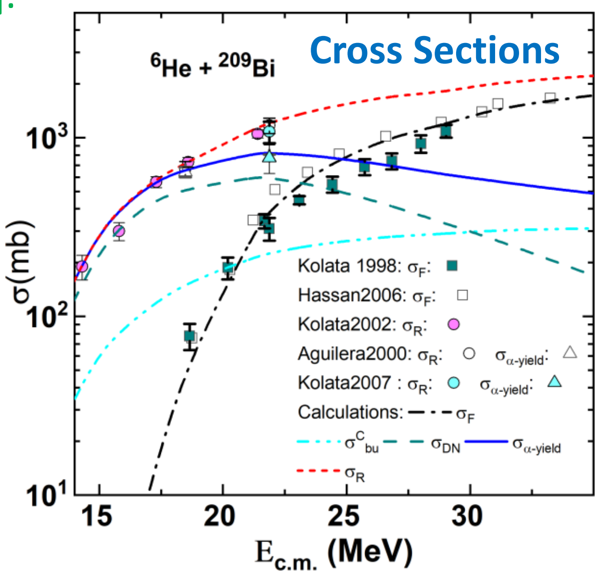
- ✓ Fus → fusion $[W_N(R)]$
- ✓ DN → direct nuclear (transfer & nuclear breakup) $[\delta W_N(R)]$
- ✓ DC → direct Coulomb breakup $[\delta W_C(R)]$

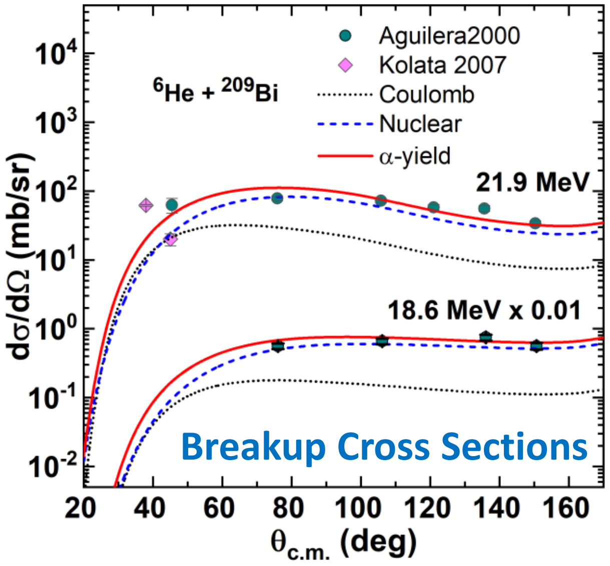
OM Simultaneously calculations of $^{6}He+^{209}Bi$

[H.M. Maridi, N. Keeley, K. Rusek, Phys. Rev. C 109, 034601 (2024)].









$$U_{OP}(R) = U_C(R) + U_N(R) + \delta U_C(R) + \delta U_N(R)$$

$$U_N(R) = (1+0.78i)e^{-v^2/c^2} V_F(R)$$

$$V_F(R) = \int \rho_P(r_P)\rho_T(r_T)v_{nn}(s)\, d\vec{r}_P d\vec{r}_T$$

$$\delta U_N(R) = -(V_L + iW_L)\frac{e^{(R-R_L)/a_L}}{[1+e^{(R-R_L)/a_L}]^2}$$

$V_L \sim 0$

One free parameter can fit:

- ✓ Elastic scattering Angular distribution
- ✓ Breakup cross sections
- ✓ Fusion cross sections
- ✓ Alpha-yield cross sections
- ✓ Total reaction cross sections
- ✓ Angular distribution of breakup cross sections.

Data from:
Aguilera *et al.*, Phys. Rev. Lett. 84, 5058 (2000)
& Phys. Rev. C 63, 061603(R) (2001);
Kolata *et al.*, Phys. Rev. C 75, 031302(R) (2007)
& Phys. Rev. Lett. 81, 4580 (1998)
& Eur. Phys. J. A 13, 117 (2002);
Hassan *et al.*, Bull. Rus. Acad. Sci. Phys. 70, 1785 (2006).

d-^{197}Au elastic scattering data and OM calculation

System	E_{sep} (MeV)	d_I (fm)	d_S (fm)
d+ ^{197}Au	2.224	2.49(7)	1.30(2)
^{11}Li + ^{208}Pb	0.369	5.2(4)	1.59(4)
^{6}He + ^{208}Pb	0.973	2.20(5)	1.589(7)
^{8}He + ^{208}Pb	2.140	2.24(7)	1.718(6)
^{6}Li + ^{208}Pb	1.474	1.95(4)	1.521(5)
^{7}Li + ^{208}Pb	2.467	1.74(2)	1.491(3)
^{9}Be + ^{208}Pb	1.655	1.86(2)	1.540(4)
^{12}C + ^{208}Pb	7.275	1.66(1)	1.491(2)
^{16}O + ^{208}Pb	7.162	1.64(1)	1.498(2)

the Boltzmann-type exponential function

$$\frac{d\sigma}{d\sigma_R} = \frac{p_1}{1+e^{p_2(d-p_3)}}$$

$$D = \frac{Z_p Z_T e^2}{2E_{c.m.}}\left(1+\frac{1}{Sin(\theta_{c.m.})}\right)\frac{1}{(A_P^{1/3}+A_T^{1/3})}$$

the reduced critical interaction distance d_I and the reduced strong-absorption distance d_S are defined at 0.98 and 0.25 of $d\sigma_{el}/d\sigma_R$, respectively.

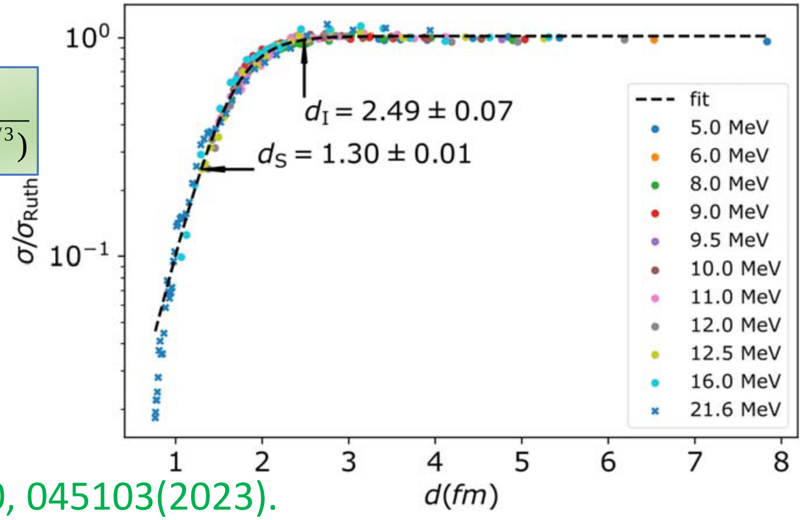


Table and data from T Giudice *et al.*, J. Phys. G 50, 045103(2023).

Global deuteron optical potential (dashed lines)

$$U_{OP}(r) = V_C(r) - V_r f_r(r) - iW_v f_v(r) + i4a_s W_s \frac{d}{dr} f_s(r) + \lambda_\pi^2 V_{so}\frac{1}{r}\frac{d}{dr} f_{so}(r)\sigma.l$$

$$f_i(r) = \frac{1}{1+e^{(r-R_i)/a_i}}$$

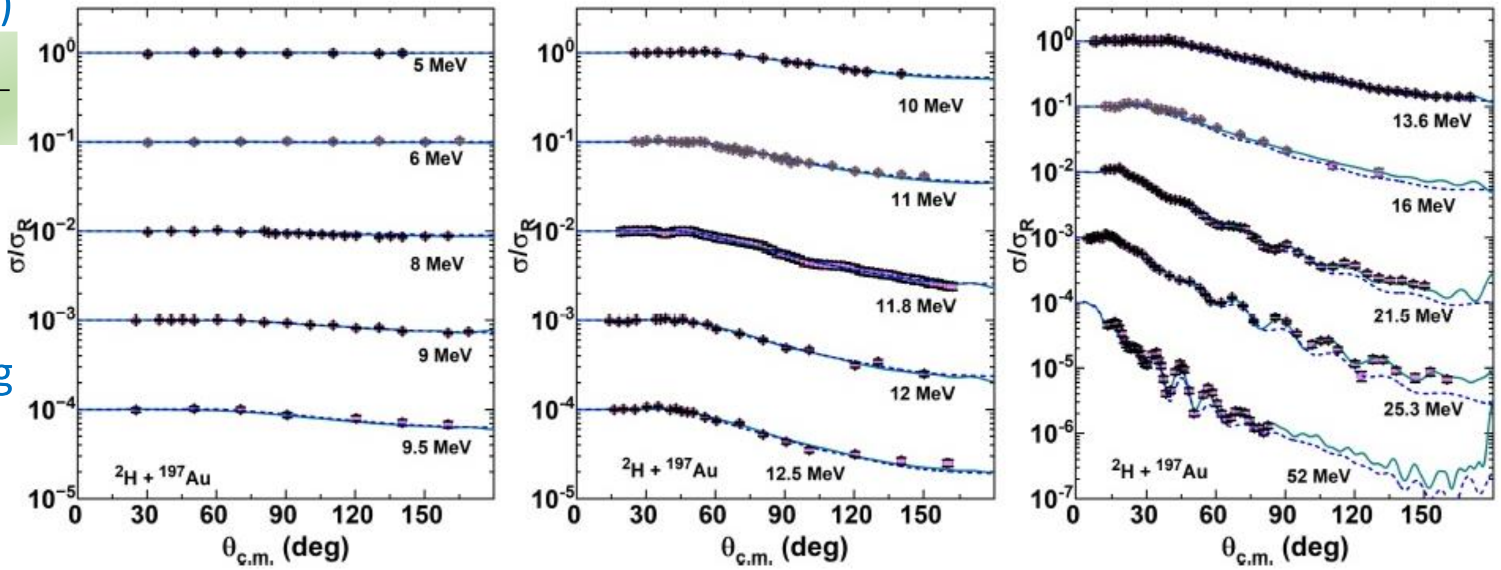
$$R_i = r_i A_T^{1/3}$$

Our optical potential (solid lines)

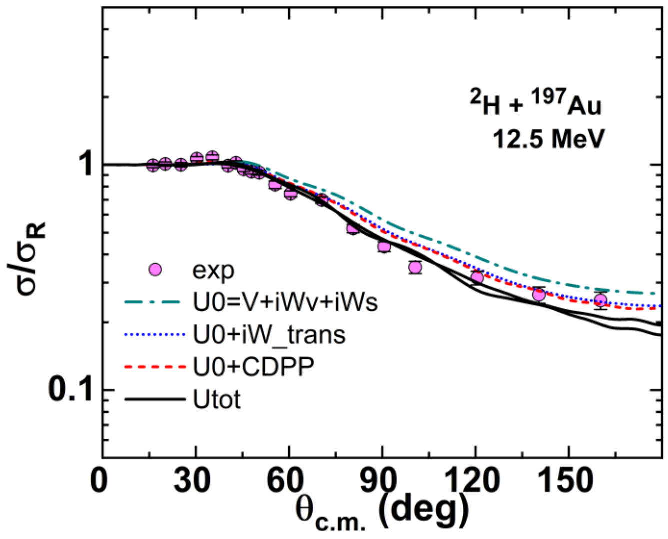
$$U_{OP}(r) = V_C(r) - V_r f_r(r) - iW_v f_v(r) + i4a_s W_s \frac{d}{dr} f_s(r) + \lambda_\pi^2 V_{so}\frac{1}{r}\frac{d}{dr} f_{so}(r)\sigma.l + \delta V_C(r) + i\delta W_C(r) + i4a_L W_L \frac{d}{dr} f_L(r)$$

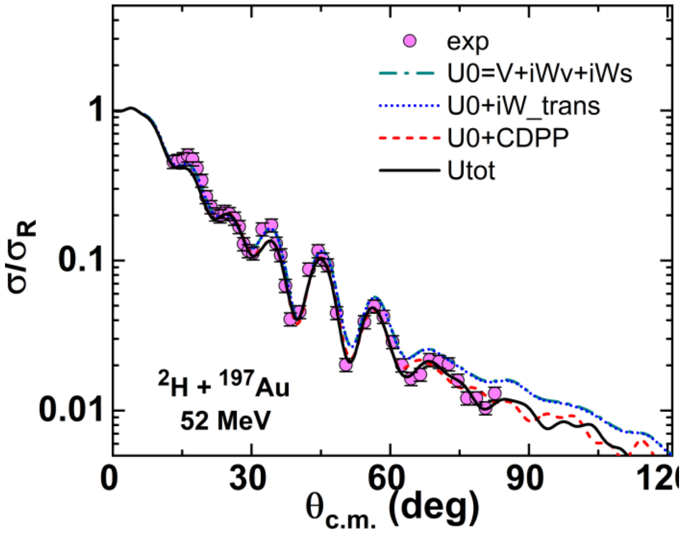
Elastic Scattering

V_B ~10 MeV



OM Simultaneously calculations of *d*-^{197}Au cross section



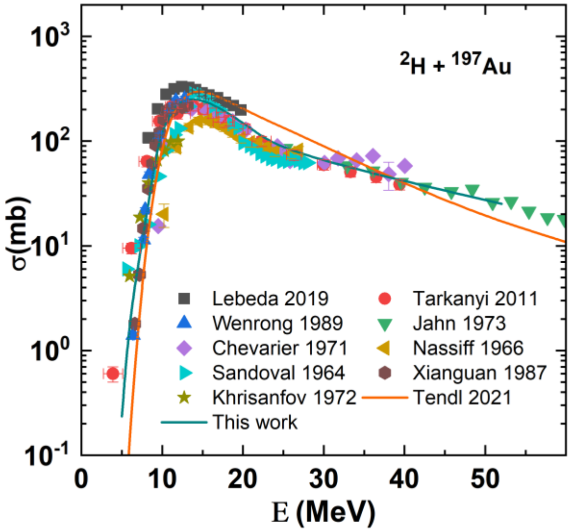


Effect of breakup and transfer on elastic scattering

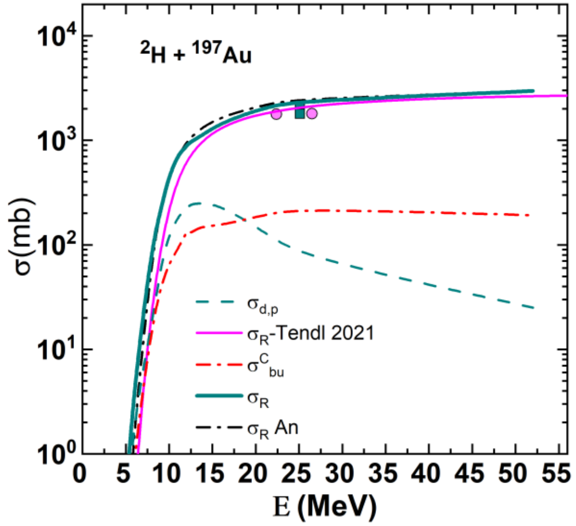
At 12.5 MeV, both breakup and transfer are important

At 52 MeV, effect of breakup is dominant

(*d*,*p*) transfer cross sections



Cross Sections



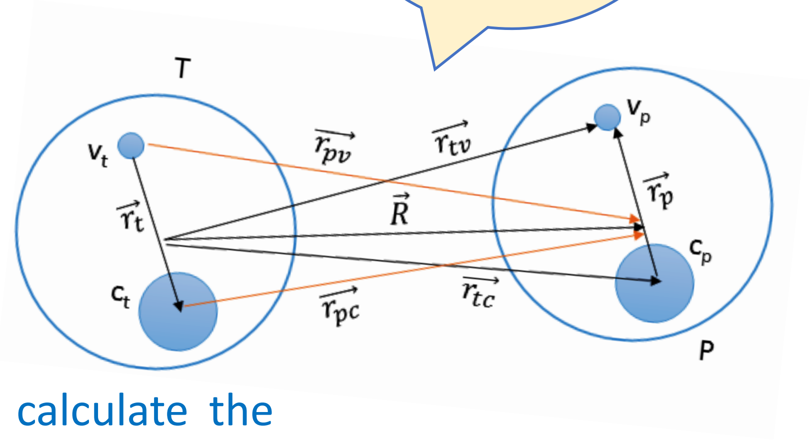
The transfer cross section and total reaction cross section are reproduced well.

Summary

<u>This work</u>

- The comparison between reactions of halo nuclei and deuteron is presented
- Presenting new expression for two-cluster distance in weakly-bound nuclei (like deuteron)
- Presenting new expressions for
  - ✓ the Coulomb dynamical polarization potential (CDPP)
  - ✓ the nuclear dynamical polarization potential (NDPP)
- The CDPP have succeeded to:
  - ✓ determine the electric dipole polarizability
  - ✓ used in a simultaneously calculation of elastic scattering, fusion, and direct cross sections and applied for $^{6}He+^{209}Bi$ and $^{2}H+^{197}Au$ reactions

Is deuteron a halo nucleus?



<u>In progress</u>

- Solving the four-body problem for colliding deuteron with halo nuclei & calculate the CDPP arising from both the projectile and target.
  $d+^{11}Li$, $d+^{6}He$, $d+^{11}Be$ scattering @ low energies.

<u>Future of this work</u>

- Improve this study to include all available data of deuteron elastic scattering and cross sections to test the effect of the breakup and transfer at wide range of energy and target mass.

<u>Publications</u>

- ✓ H.M. Maridi, D.K. Sharp, J. Lubian, In preparation (2024).
- ✓ H.M. Maridi, J. Singh, N.R. Walet, D.K. Sharp, arXiv:2407.03044 (2024).
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Acknowledgments

- D. K. Sharp Nuclear Physics Group, University of Manchester, UK
- J. Singh & N.R. Walet Theoretical Physics Group, University of Manchester, UK
- K. Rusek Heavy Ion Laboratory, University of Warsaw, Poland
- N. Keeley National Centre for Nuclear Research, Otwock, Poland
- J. Lubian Instituto de F´ısica, Universidade Federal Fluminense, R.J., Brazil
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