A fluid dynamic perspective on high energy nucleus collisions

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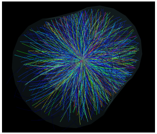
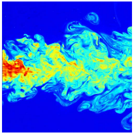
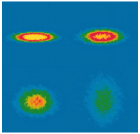
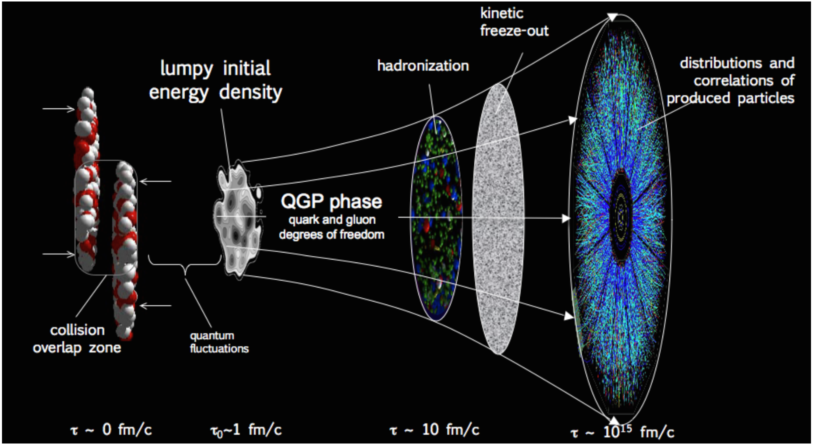
Fluid dynamics

- long distances, long times or strong enough interactions
- quantum fields form a fluid!
- needs **macroscopic** fluid properties
  - thermodynamic equation of state $p(T, \mu)$
  - shear and bulk viscosity η, ζ
  - heat conductivity
  - relaxation times
  - heavy quark diffusion coefficient κ_n
- fixed by **microscopic** properties encoded in Lagrangian $\mathscr{L}_{\text{QCD}}$

High energy nuclear collisions



Relativistic fluid dynamics

Energy-momentum tensor and conserved current

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu + (p + \pi_{\text{bulk}})\Delta^{\mu\nu} + \pi^{\mu\nu}$$
$$N^\mu = n\, u^\mu + \nu^\mu$$

- tensor decomposition using fluid velocity u^μ, $\Delta^{\mu\nu} = g^{\mu\nu} + u^\mu u^\nu$
- thermodynamic equation of state $p = p(T, \mu)$

Covariant conservation laws $\nabla_\mu T^{\mu\nu} = 0$ and $\nabla_\mu N^\mu = 0$ imply evolution equations for

- energy density ϵ

$$u^\mu \partial_\mu \epsilon + (\epsilon + p + \pi_{\text{bulk}})\nabla_\mu u^\mu + \pi^{\mu\nu}\nabla_\mu u_\nu = 0$$

- fluid velocity u^μ

$$(\epsilon + p + \pi_{\text{bulk}})u^\mu \nabla_\mu u^\nu + \Delta^{\nu\mu}\partial_\mu(p + \pi_{\text{bulk}}) + \Delta^\nu{}_\alpha \nabla_\mu \pi^{\mu\alpha} = 0$$

- particle number density n

$$u^\mu \partial_\mu n + n\nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

Constitutive relations

[Israel & Stewart]

Second order relativistic fluid dynamics:

- equation for shear stress $\pi^{\mu\nu}$

$$\tau_{\text{shear}} \, P^{\rho\sigma}{}_{\alpha\beta} \, u^\mu \nabla_\mu \pi^{\alpha\beta} + \pi^{\rho\sigma} + 2\eta \, P^{\rho\sigma\alpha}{}_{\beta} \, \nabla_\alpha u^\beta + \ldots = 0$$

  with shear viscosity $\eta(T, \mu)$

- equation for bulk viscous pressure π_{bulk}

$$\tau_{\text{bulk}} \, u^\mu \partial_\mu \pi_{\text{bulk}} + \pi_{\text{bulk}} + \zeta \, \nabla_\mu u^\mu + \ldots = 0$$

  with bulk viscosity $\zeta(T, \mu)$

- equation for baryon diffusion current ν^μ

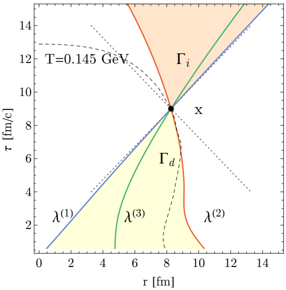
$$\tau_{\text{heat}} \, \Delta^\alpha{}_\beta \, u^\mu \nabla_\mu \nu^\beta + \nu^\alpha + \kappa \left[\frac{nT}{\epsilon + p} \right]^2 \Delta^{\alpha\beta} \partial_\beta \left(\frac{\mu}{T} \right) + \ldots = 0$$

  with heat conductivity $\kappa(T, \mu)$

- non-hydrodynamic degrees of freedom are needed for relativistic causality!

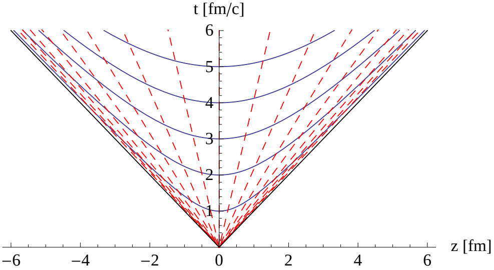
Causality

[Floerchinger & Grossi, JHEP 08 (2018) 186]



- dissipative fluid equations *can* be of hyperbolic type
- characteristic velocities depend on fluid fields
- need $|\lambda^{(j)}| < c$ for relativistic causality
- works when relaxation times are large enough

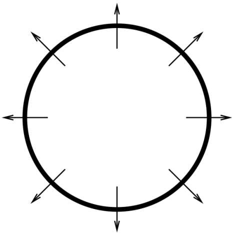
Bjorken boost invariance



How does the fluid velocity look like?

- Bjorkens guess: $v_z(t, x, y, z) = z/t$
- leads to an invariance under Lorentz-boosts in the z-direction
- use coordinates $\tau = \sqrt{t^2 - z^2}$, x, y, $\eta = \text{arctanh}(z/t)$
- Bjorken boost symmetry is reasonably accurate close to mid-rapidity $\eta \approx 0$

Transverse expansion



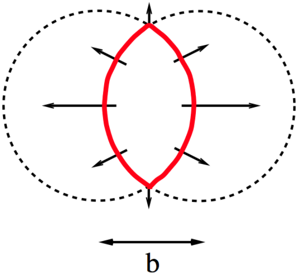
- for central collisions ($r = \sqrt{x^2 + y^2}$)

$$\epsilon = \epsilon(\tau, r)$$

- initial pressure gradient leads to **radial flow**

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} f(\tau, r)$$

Non-central collisions



- pressure gradients larger in reaction plane
- leads to larger fluid velocity in this direction
- more particles fly in this direction
- can be quantified in terms of elliptic flow v_2
- particle distribution

$$\frac{dN}{d\phi} = \frac{N}{2\pi}\left[1 + 2\sum_m v_m \cos\left(m\left(\phi - \psi_R\right)\right)\right]$$

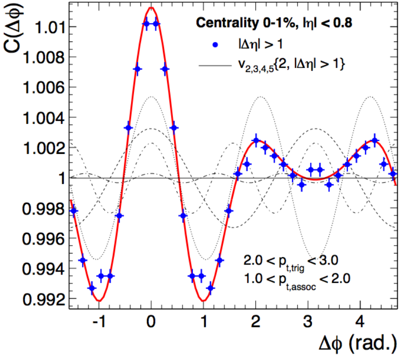
- symmetry $\phi \to \phi + \pi$ implies $v_1 = v_3 = v_5 = \ldots = 0$.

Two-particle correlation function

- normalized two-particle correlation function

$$C(\phi_1, \phi_2) = \frac{\left\langle \frac{dN}{d\phi_1} \frac{dN}{d\phi_2} \right\rangle_{\text{events}}}{\left\langle \frac{dN}{d\phi_1} \right\rangle_{\text{events}} \left\langle \frac{dN}{d\phi_2} \right\rangle_{\text{events}}} = 1 + 2 \sum_m v_m^2 \cos(m(\phi_1 - \phi_2))$$

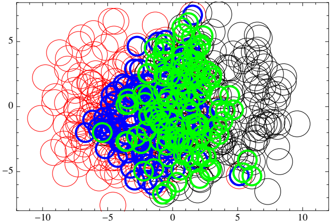
- surprisingly v_2, v_3, v_4, v_5 and v_6 are all non-zero!

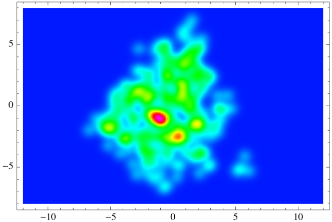


[ALICE 2011, similar results from CMS, ATLAS, Phenix, Star]

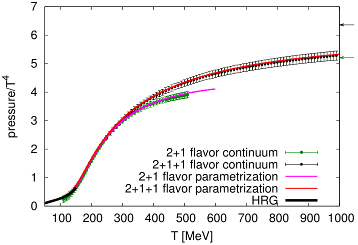
Event-by-event fluctuations

- deviations from symmetric initial energy density distribution from event-by-event fluctuations
- one example is Glauber model

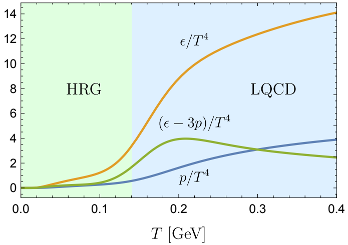




Thermodynamics of QCD



[Borsányi *et al.* (2016), similar Bazavov *et al.* (2014)]

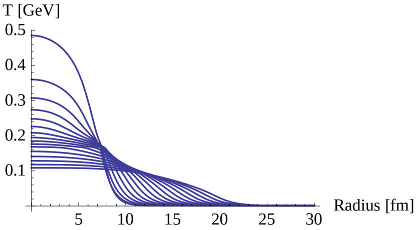


[Floerchinger, Grossi, Lion (2019)]

- equation of state at vanishing chemical potential is well known now
- at large temperature lattice QCD
- at small temperature hadron resonance gas approximation
- extensions to non-zero chemical potentials e. g. by Taylor expansion

Fluid dynamic description of heavy ion collisions

Fluid*u*M: Fluid dynamics of heavy ion collisions with Mode expansion
[Floerchinger & Wiedemann, PLB 728, 407 (2014), PRC 88, 044906 (2013), 89, 034914 (2014)]
[Floerchinger, Grossi & Lion, PRC 100, 014905 (2019)]

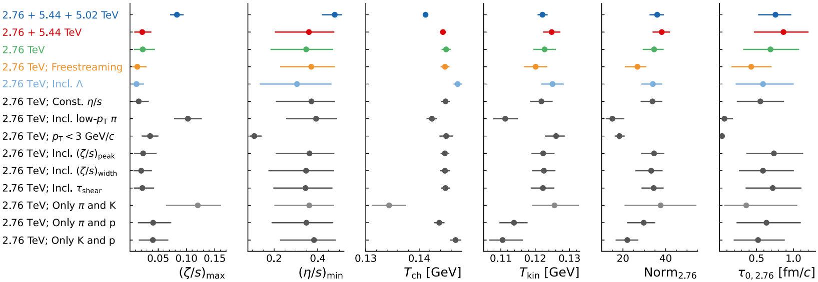


- background-fluctuation splitting + mode expansion
- resonance decays included
  [Mazeliauskas, Floerchinger, Grossi & Teaney, EPJC 79, 284 (2019)]
- allows fast and precise comparison between theory and experiment

Parameter estimation from theory-experiment comparisson

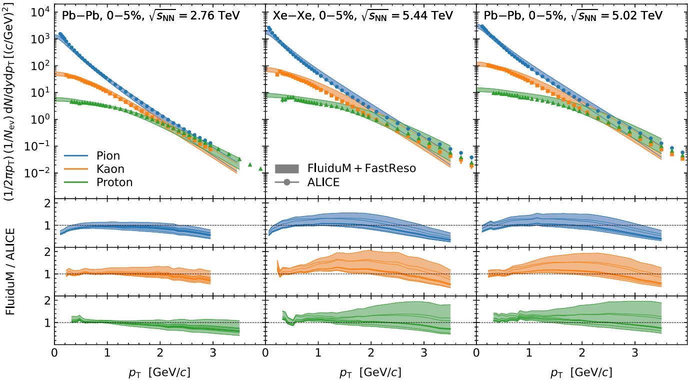
[Vermunt, Seemann, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, Selyuzhenkov, PRC 108, 064908 (2023)]

- fluid models have parameters
- can be determined with Bayesian analysis from data
- here based on transverse momentum spectra of pions, kaons, protons
- data from Pb-Pb (2.76 TeV), Pb-Pb (5.02 TeV), Xe-Xe (5.44 TeV)



Particle production at the Large Hadron Collider

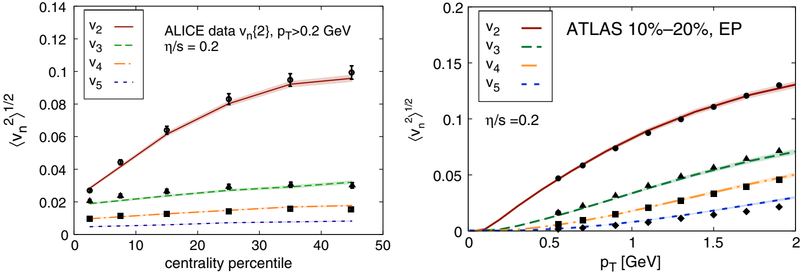
[Vermunt, Seemann, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, Selyuzhenkov, PRC 108, 064908 (2023)]



- overall good description
- some deviations for pions at small p_T

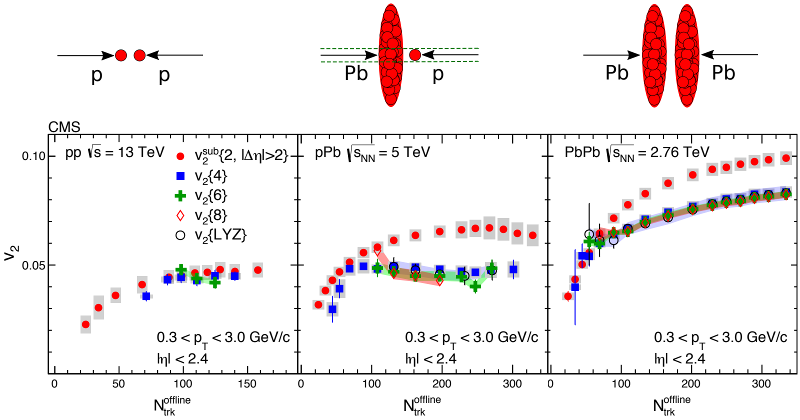
Event-by-event fluid simulations

[Gale, Jeon, Schenke, Tribedy, Venugopalan (2013)]



- fluctuating initial conditions from different models
- overall good agreement with experimental data reached

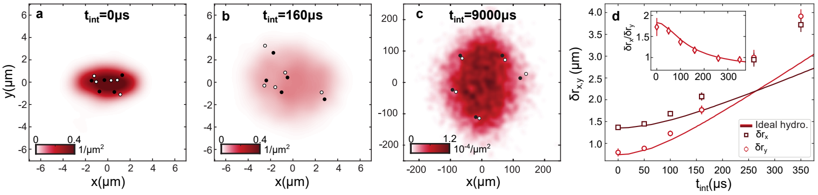
Collective behavior in large and small systems



- flow coefficients from higher order cumulants $v_2\{n\}$ agree:
  $\rightarrow$ collective behavior
- elliptic flow signals also in pPb and pp collisions
- this conference: hints for v_2 even in high-multiplicity e^+e^- collisions
  [Chris McGinn, Talk on Monday]

Elliptic flow for a few interacting atoms

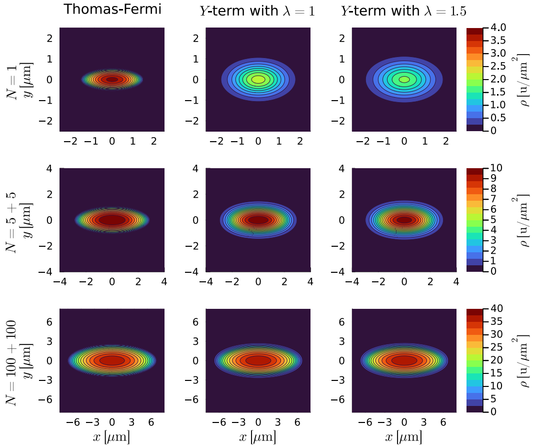
[S. Floerchinger, G. Giacalone, L. H. Heyen, L. Tharwat, PRC 105, 044908 (2022)]
[S. Brandstetter, P. Lunt, C. Heintze, G. Giacalone, L. H. Heyen, M. Gałka, K. Subramanian, M. Holten, P. M. Preiss, S. Floerchinger, S. Jochim, 2308.09699]



- elliptic flow of 5+5 strongly interacting fermionic atoms released from anisotropic trap
- qualitative agreement with ideal fluid dynamics

Quantum corrections at second order in derivatives

[Heyen, Giacalone, Floerchinger, 2408.06104]



- nonrelativistic superfluid with bulk pressure and shear stress

$$\pi_{\text{bulk}} = \lambda \frac{\hbar^2}{8 m n_s} \left[n_s \boldsymbol{\nabla}^2 n_s + (\boldsymbol{\nabla} n_s)^2 \right]$$

$$\pi_{jk} = \lambda \frac{\hbar^2}{4 m n_s} \left[(\partial_j n_s)(\partial_k n_s) - \frac{1}{2} \delta_{jk} (\boldsymbol{\nabla} n_s)^2 \right]$$

Fluid equations of motion for charm

- net heavy quark number current $N^\mu_- = N^\mu_Q - N^\mu_{\bar{Q}}$ conserved in QCD
- averaged quark number current $N^\mu_+ = (N^\mu_Q + N^\mu_{\bar{Q}})/2$ approximately conserved for small temperatures $T \ll m_Q$
- decompose

$$N^\mu = N^\mu_+ = n u^\mu + \nu^\mu$$

- conservation law

$$\nabla_\mu N^\mu = u^\mu \partial_\mu n + n \nabla_\mu u^\mu + \nabla_\mu \nu^\mu = 0$$

- constitutive relation for diffusion current

$$\tau_n \Delta^\rho{}_\sigma u^\lambda \nabla_\lambda \nu^\sigma + \nu^\rho + \kappa_n \Delta^{\rho\sigma} \partial_\sigma \left(\frac{\mu}{T} \right) = 0$$

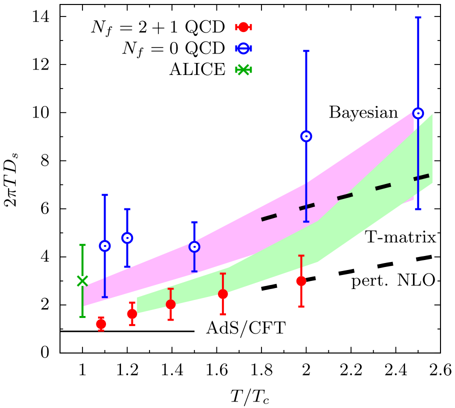
- chemical potential μ conjugate to heavy quark number
- heavy quark diffusion coefficient $\kappa_n = D_s n$
- relaxation time τ_n

- fluid dynamics needs a thermodynamic equation of state
- we use a hadron resonance model approximation with sum over all measured charmed states

$$n(T,\mu) = \frac{T}{2\pi^2} \sum_{i \in \mathsf{HRGc}} q_i M_i^2 \exp\left(\frac{q_i \mu}{T}\right) K_2\left(\frac{M_i}{T}\right)$$

- lattice results would be nice to have

Constraints on charm quark diffusion on the lattice

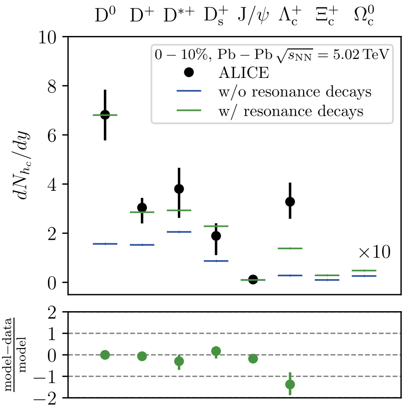


- latest lattice results for heavy quark diffusion coefficient for $N_f = 2+1$ flavor QCD indicate small D_s [HotQCD, PRL 130, 231902 (2023)]
- supports fast hydrodynamization of heavy quarks

Yields of charmed hadrons

[Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108, 116011 (2023)]

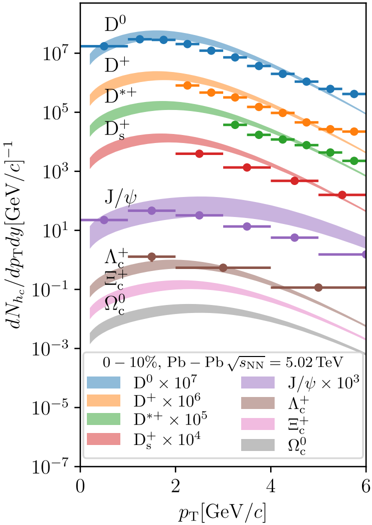
[related work by Andronic, Braun-Munzinger, Redlich, Stachel et al.]



- resonance decays from FastReso sizeable
- yield of Λ_c^+ underpredicted, possibly missing higher resonances in PDG list?
- prediction for Ξ_c^+ and Ω_c^0

Transverse momentum spectra of charmed hadrons

[Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108, 116011 (2023)]



- good agreement for D-mesons up to $p_T \approx 4-5$ GeV
- some deviations for J/Ψ (dissipative correction at freezeout?)

Photon and dilepton production rate in local thermal equilibrium

- photon production rate per unit volume and momentum

$$p^0 \frac{dR}{d^3p} = \frac{1}{(2\pi)^3} n_{\mathsf{B}}(\omega)\rho(\omega)$$

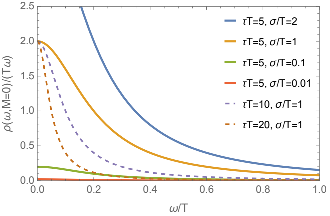
- frequency in the fluid rest frame $\omega = -u_\mu p^\mu$
- Bose-Einstein factor

$$n_{\mathsf{B}}(\omega) = \frac{1}{e^{\omega/T} - 1}$$

- electromagnetic spectral function $\rho(\omega)$
- similar for dileptons

Electromagnetic spectral function

[S. Floerchinger, C. Gebhardt, K. Reygers, PLB 837 (2023) 137647]



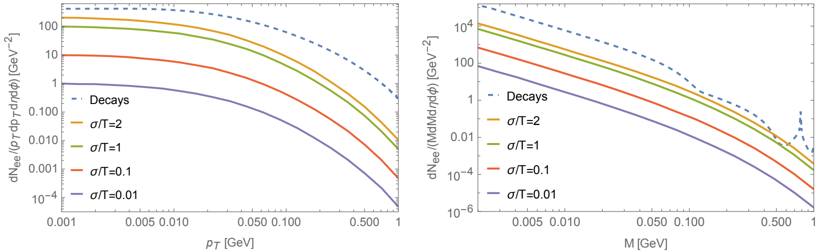
- fluid dynamics for electric current yields spectral function at small frequencies and momenta

$$\rho(\omega, \mathbf{p}) = \frac{\sigma\omega(\omega^2 - \mathbf{p}^2)}{(\tau\omega^2 - D\mathbf{p}^2)^2 + \omega^2} + 2\frac{\sigma\omega}{\tau^2\omega^2 + 1}$$

- electric conductivity σ
- diffusion coefficient $D = \sigma/\chi$
- charge susceptibility $\chi = (\partial n/\partial\mu)|_T$
- relaxation time τ constrained by causality $\tau > D = \sigma/\chi$

Dielectron transverse momentum and mass spectra

[S. Floerchinger, C. Gebhardt, K. Reygers, PLB 837 (2023) 137647]



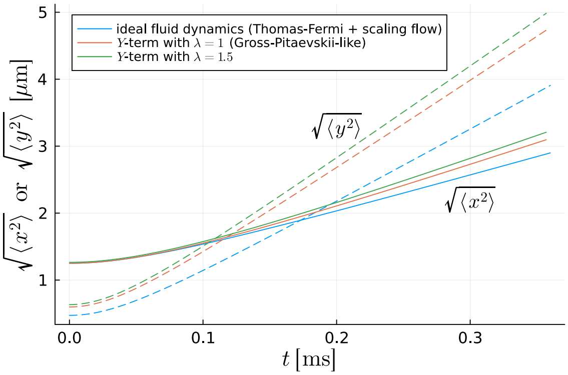
- integration over fireball volume at LHC energies
- background contribution from resonance decays dominates
- Hanbury Brown-Twiss correlations could help

Conclusions

- fluid dynamics approximately describes quantum fields out-of-equilibrium
- based on thermodynamic and transport properties of QCD
- can explain many aspects of high-energy heavy ion collisions
- seems applicable even for relatively small systems
- finds applications also for heavy quarks
- photon and dilepton spectra in the soft regime

Backup

Expansion with second order corrections



Fluid dynamics with several conserved quantum numbers

- fluid with conserved quantum number densities $c_m = (\epsilon, n_\mathsf{B}, n_\mathsf{C}, n_\mathsf{S}, \ldots)$
- equation of state in grand canonical ensemble in terms of Massieu potential $w(\beta, \alpha_j) = \beta p(\beta, \alpha_j)$ with $\beta = 1/T$, $\alpha_j = \mu_j/T$,

$$dw = -\epsilon d\beta + n_j d\alpha_j$$

- second derivative yields a matrix of susceptibilities with $\gamma^m = (\beta, \alpha_1, \alpha_2, \ldots)$

$$G_{mn}(\gamma) = \frac{\partial^2 w}{\partial\gamma^m \partial\gamma^n}$$

- fluid evolution equations from conservation laws

$$u^\mu \partial_\mu c_m + f_m = 0$$

- can be written with inverse susceptibility matrix as

$$u^\mu \partial_\mu \gamma^n + (G^{-1}(\gamma))^{nm} f_m = 0$$

Fluid dynamics for heavy quarks from Fokker-Planck equation

- phase-space distribution function $f(t, \mathbf{x}, p)$
- currents are moments with respect to momenta

$$N^\mu(t, \mathbf{x}) = \int \frac{d^3p}{(2\pi)^3 p^0} p^\mu f(t, \mathbf{x}, p)$$

- Boltzmann equation for time evolution

$$p^\mu \frac{\partial}{\partial x^\mu} f(t, \mathbf{x}, p) = C[f]$$

- heavy quarks get small "momentum kicks" from light partons
- Fokker-Planck approximation to collision kernel

$$C[f] = k^0 \frac{\partial}{\partial p^j} \left[A^j f + \frac{\partial}{\partial p^k} \left[B^{jk} f \right] \right]$$

- fluid dynamics from taking moments of the Fokker-Planck equation
- approximations justified for slow dynamics

Initial conditions for charm current

[Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108, 116011 (2023)]

- initial density distribution from hard scattering

$$n(\tau_0, r) = \frac{1}{\tau_0} n_{\text{coll}}(r) \frac{1}{\sigma_{pp}^{\text{in}}} \frac{d\sigma^{Q\bar{Q}}}{dy}$$

$\sigma_{pp}^{\text{in}} = 67.6$ mb, $\frac{d\sigma^{Q\bar{Q}}}{dy} = 0.463$ mb [Cacciari, Frixone, Nason, JHEP03(2001)006]

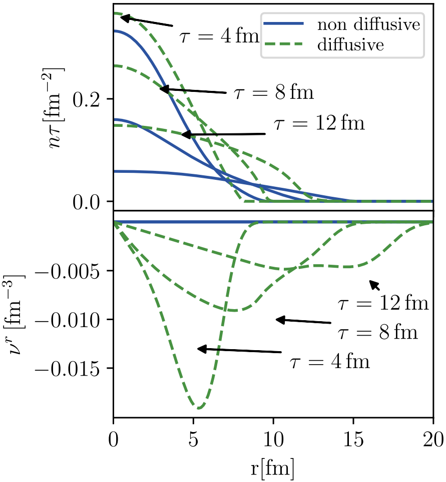
- diffusion current initially assumed to vanish

$$\nu^{\mu}(\tau_0, r) = 0$$

- leads to parameter-free model for initial charm density and current

Evolution of charm density and diffusion current

[Capellino, Dubla, Floerchinger, Grossi, Kirchner, Masciocchi, PRD 108, 116011 (2023)]



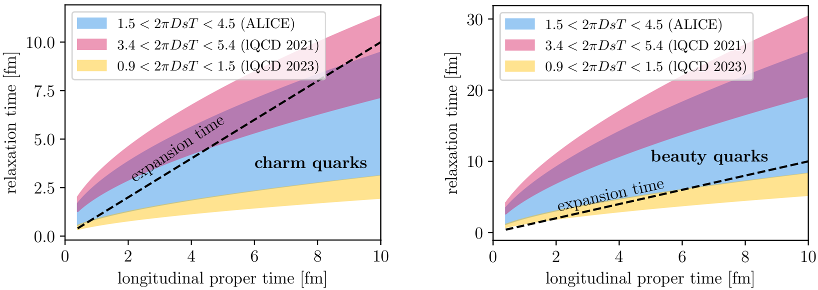
- charm density expands and dilutes like energy density
- diffusion leads to further dilution

Applicability of fluid description

[Capellino, Beraudo, Dubla, Floerchinger, Masciocchi, Pawlowski, Selyuzhenkov, PRD 106, 034021 (2022)]

- Fokker-Planck equation yields relation for relaxation time τ_n in terms of diffusion coefficient D_s
- fluid dynamics applicable when the relaxation time is small compared to the dynamics
- for initial Bjorken expansion

$$\tau_n < 1/(\nabla_\mu u^\mu) = \tau$$



Production rate for thermal dileptons

- thermal dilepton $(e^+ e^-)$ production rate per unit volume and time

$$\frac{dR}{d^4p} = \frac{\alpha}{12\pi^4}\frac{1}{M^2}\, n_{\mathsf{B}}(\omega)\, \rho(\omega, M)\left(1 + \frac{2m^2}{M^2}\right)\sqrt{1 - \frac{4m^2}{M^2}}\,\Theta(M^2 - 4m^2)$$

- momentum of the dilepton pair $p^\mu = p_1^\mu + p_2^\mu$
- invariant mass defined by $p^2 + M^2 = 0$
- lepton mass m
- electromagnetic fine structure constant $\alpha = e^2/(4\pi)$