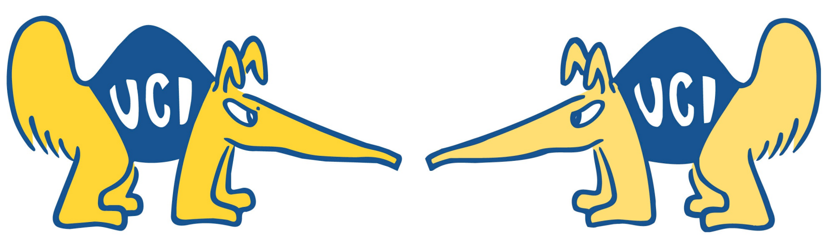
Neutrino Mixing from Modular Flavor Symmetries

Mu-Chun Chen, University of California at Irvine



Neutrinos in Cosmology and Astrophysics, TRIUMF, 3/6/2024

Where Do We Stand?

NuFIT 5.2 (2022)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.4$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
with SK atmospheric data	$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.012}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
	$\sin^2\theta_{23}$	$0.451^{+0.019}_{-0.016}$	$0.408 \to 0.603$	$0.569^{+0.016}_{-0.021}$	$0.412 \to 0.613$
	$\theta_{23}/^\circ$	$42.2^{+1.1}_{-0.9}$	$39.7 \to 51.0$	$49.0^{+1.0}_{-1.2}$	$39.9 \to 51.5$
	$\sin^2\theta_{13}$	$0.02225^{+0.00056}_{-0.00059}$	$0.02052 \to 0.02398$	$0.02223^{+0.00058}_{-0.00058}$	$0.02048 \to 0.02416$
	$\theta_{13}/^\circ$	$8.58^{+0.11}_{-0.11}$	$8.23 \to 8.91$	$8.57^{+0.11}_{-0.11}$	$8.23 \to 8.94$
	$\delta_{\mathrm{CP}}/^\circ$	232^{+36}_{-26}	$144 \to 350$	276^{+22}_{-29}	$194 \to 344$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.507^{+0.026}_{-0.027}$	$+2.427 \to +2.590$	$-2.486^{+0.025}_{-0.028}$	$-2.570 \to -2.406$

➡ hints of $\theta_{23} \neq \pi/4$

➡ expectation of Dirac CP phase δ

Recent T2K-NOvA joint analysis: (Z. Vallari, FNAL, Feb'24)

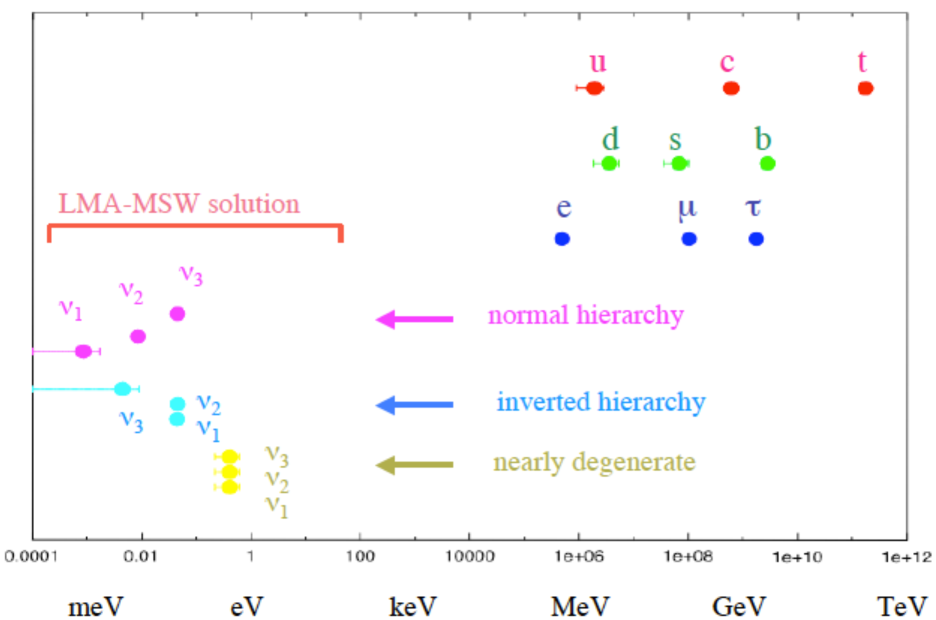
slight preference for IO; $\delta \simeq -\pi/2$; $\theta_{23} > 45^o$

T2K-NOvA-DayaBay $\Rightarrow$ NO

Open Questions - Theoretical

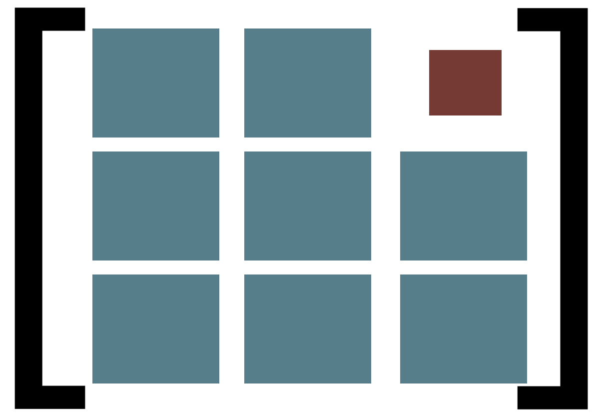
☞ **Smallness of neutrino mass:**

$$m_\nu \ll m_{e,\,u,\,d}$$

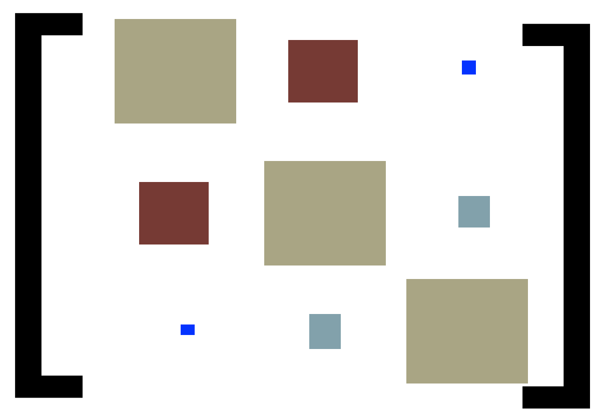


Fermion mass and hierarchy problem ➟ Dominant fraction (22 out of 28) of free parameters in SM

☞ **Flavor structure:**



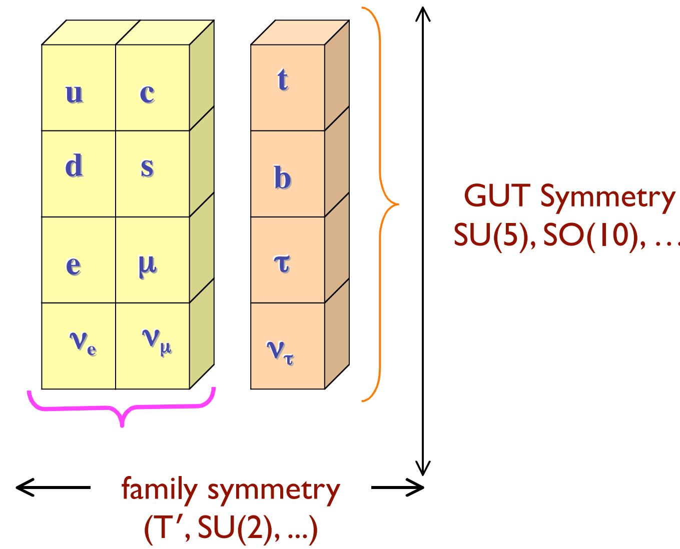
leptonic mixing

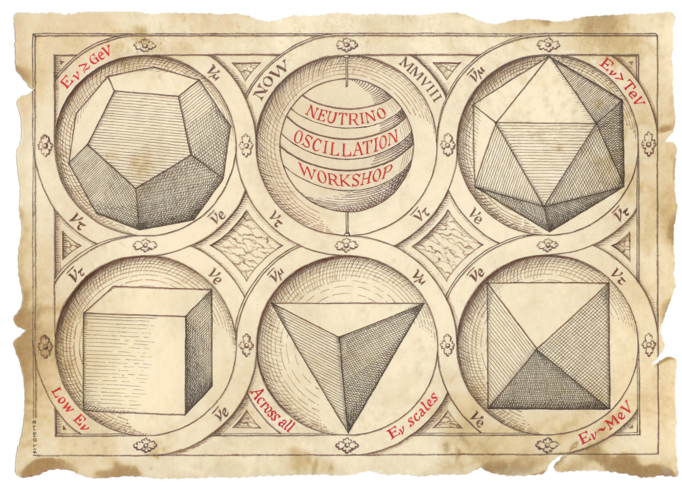


quark mixing

Non-Abelian Discrete Flavor Symmetries

- Large neutrino mixing motivates discrete flavor symmetries
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_6
  - .....



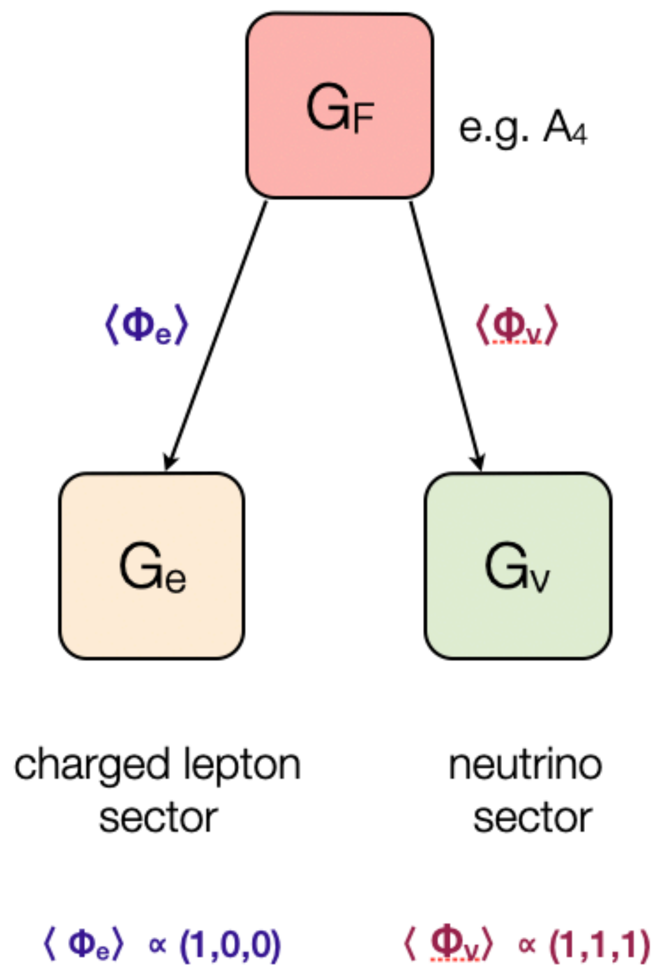


[Eligio Lisi for NOW2008]

Neutrino Mass Matrix from A4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

- Imposing A4 flavor symmetry on the Lagrangian
- A4 spontaneously broken by flavon fields



Neutrino Mass Matrix from A4

- Imposing A4 flavor symmetry on the Lagrangian

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

- A4 spontaneously broken by flavon fields

2 free parameters

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

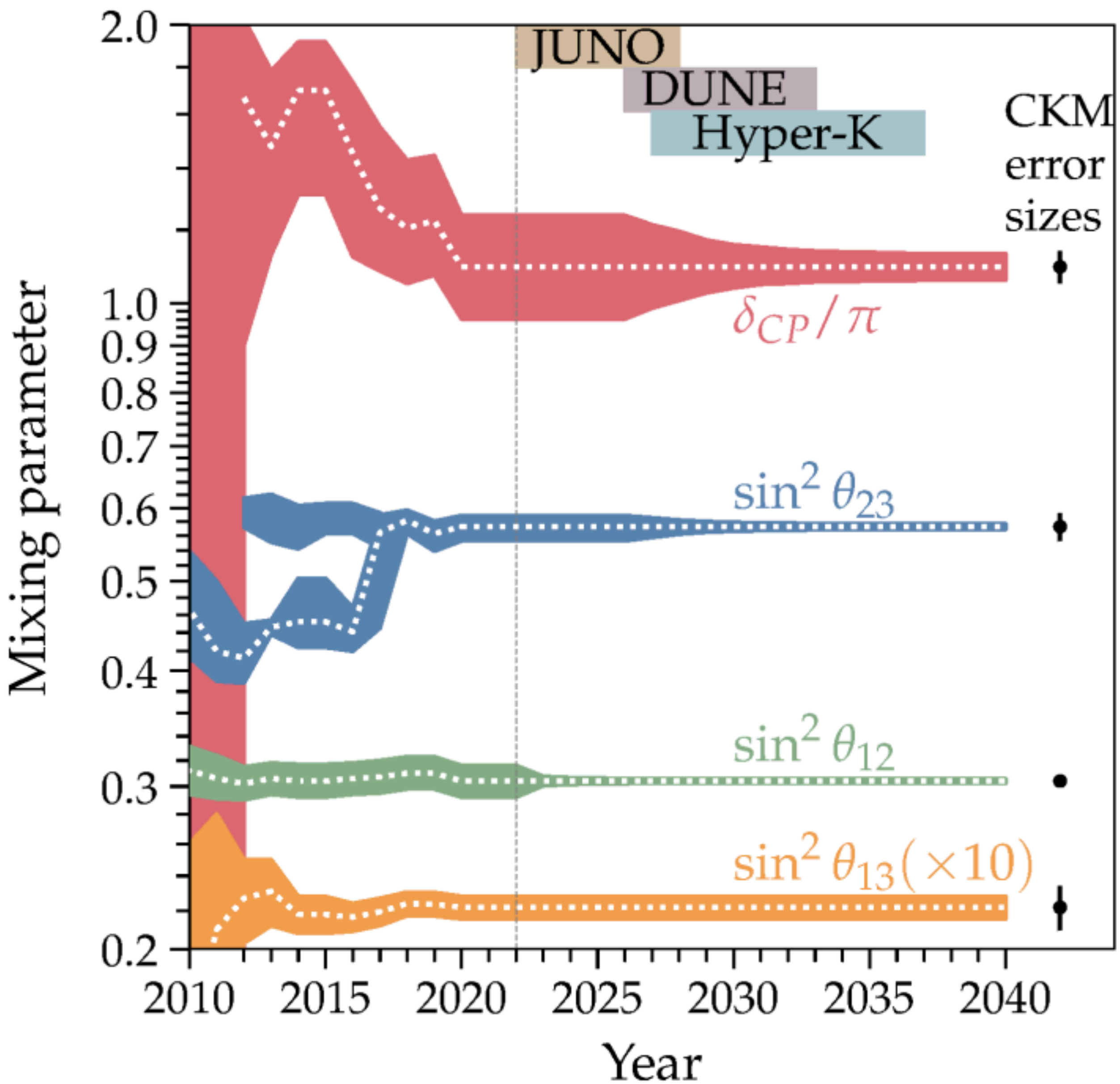
relative strengths $\Rightarrow$ CG's

- always diagonalized by TBM matrix, independent of the two free parameters

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Neutrino Mixing Angles from Group Theory

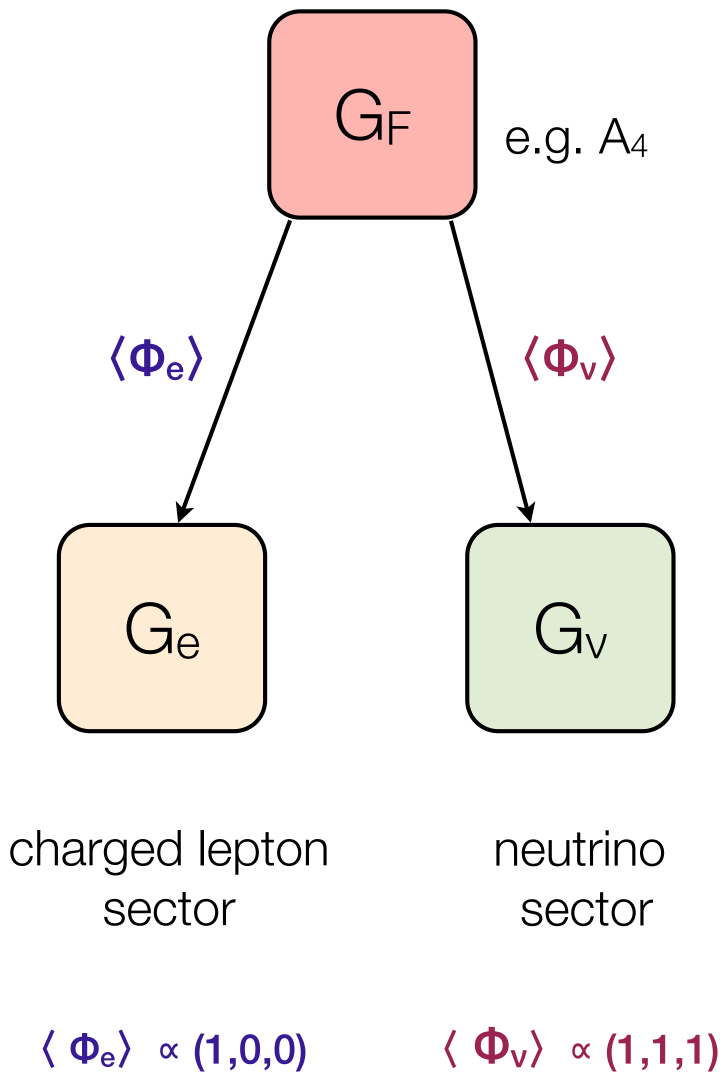
Experimental Precision



Are precisions in model predictions compatible with experimental precisions?

Figure from Song, Li, Argüelles, Bustamante, Vincent (2020)

Flavor Model Structure



- interplay between the symmetry breaking patterns in two sectors lead to lepton mixing (BM, TBM, ...)
- symmetry breaking achieved through flavon VEVs
- each sector preserves different residual symmetry
- full Lagrangian does not have these residual symmetries

⇒ Corrections to model predictions

Corrections to Kinetic Terms

- Corrections to the kinetic terms induced by family symmetry breaking generically are present, should be properly included   Leurer, Nir, Seiberg (1993); Dudas, Pokorski, Savoy (1995); Dreiner, Thomeier (2003)
  - can be along different directions than RG corrections
  - dominate over RG corrections (no loop suppression, copious heavy states)
  - could be sizable for neutrino mass models based on discrete family symmetries, e.g. A_4 M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)
    - nontrivial flavor structure can be induced
    - non-zero CP phase can be induced
    - Presence of additional undetermined parameters

Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- Superpotential: holomorphic

$$\mathscr{W}_{\text{leading}} = \frac{1}{\Lambda}(\Phi_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{\Lambda\,\Lambda_\nu}(\Phi_\nu)_{gf}\, L^g\, H_u\, L^f\, H_u$$

$$\longrightarrow \mathscr{W}_{\text{eff}} = (Y_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{4}\kappa_{gf}\, L^g\, H_u\, L^f\, H_u$$

order parameter: $\frac{\text{< flavon vev >}}{\Lambda} \sim \theta_c$

- Kähler potential: non-holomorphic

$$K = K_{\text{canonical}} + \Delta K$$

- Canonical Kähler potential

$$K_{\text{canonical}} \supset \left(L^f\right)^\dagger \delta_{fg}\, L^g + \left(R^f\right)^\dagger \delta_{fg}\, R^g$$

- Corrections

$$\Delta K = \left(L^f\right)^\dagger (\Delta K_L)_{fg}\, L^g + \left(R^f\right)^\dagger (\Delta K_R)_{fg}\, R^g$$

- can be induced by flavon VEVs
- important for order parameter $\sim \theta_c$
- can lead to non-trivial mixing

Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- Consider infinitesimal change, x :

$$K \;=\; K_{\text{canonical}} + \Delta K \;=\; L^\dagger\,(1 - 2x\,P)\,L$$

- rotate to canonically normalized L′:

$$L \;\rightarrow\; L' \;=\; (1 - x\,P)\,L$$

⇒ corrections to neutrino mass matrix

$$\begin{aligned} \mathcal{W}_\nu &= \frac{1}{2}(L\cdot H_u)^T\kappa_\nu(L\cdot H_u) \\ &\simeq \frac{1}{2}[(\mathbb{1}+xP)L'\cdot H_u]^T\kappa_\nu[(\mathbb{1}+xP)L'\cdot H_u] \\ &\simeq \frac{1}{2}(L'\cdot H_u)^T\kappa_\nu L'\cdot H_u + x(L'\cdot H_u)^T(P^T\kappa_\nu + \kappa_\nu P)L'\cdot H_u \end{aligned}$$

with

$$\kappa\cdot v_u^2 \;=\; 2m_\nu$$

Kähler Corrections

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Consider infinitesimal change, x :

$$K \; = \; K_{\text{canonical}} + \Delta K \; = \; L^\dagger \, (1 - 2x\, P)\, L$$

- rotate to canonically normalized L′:

$$L \; \rightarrow \; L' \; = \; (1 - x\, P)\, L$$

⇒ corrections to neutrino mass matrix

$$m_\nu(x) \; \simeq \; m_\nu + x\, P^T\, m_\nu + x\, m_\nu\, P$$

⇒ differential equation

$$\frac{\mathrm{d}m_\nu}{\mathrm{d}x} \; = \; P^T\, m_\nu + m_\nu\, P$$

- same structure as the RG evolutions for neutrino mass operator
- size of Kähler corrections can be substantially larger (no loop suppression)

Back to A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Kähler corrections due to flavon field:
  - possible to forbid some contributions (linear in an individual flavor) with additional symmetries
  - quadratic in flavon

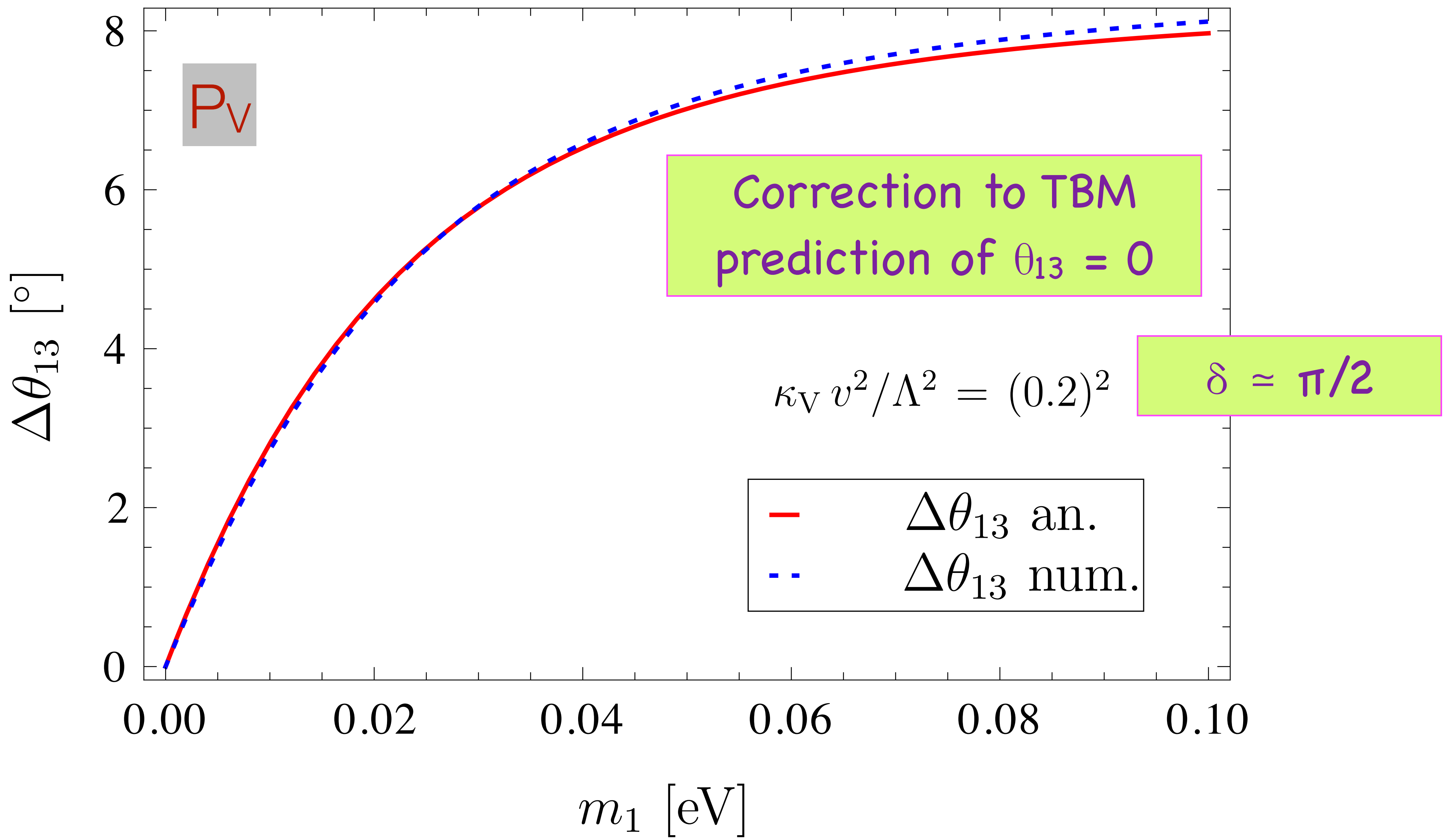
$$\Delta K^{\text{quadratic}}_{\phi^{(\prime)}} \supset \frac{1}{\Lambda^2} \sum_X^6 \kappa^X_{\phi^{(\prime)},\text{quadratic}} \left(L\phi^{(\prime)}\right)^\dagger_X \left(L\phi^{(\prime)}\right)_X + \text{h.c.}$$

$$(L\Phi_\nu)^\dagger (L\Phi_\nu) \quad \text{and} \quad (L\Phi_e)^\dagger (L\Phi_e)$$

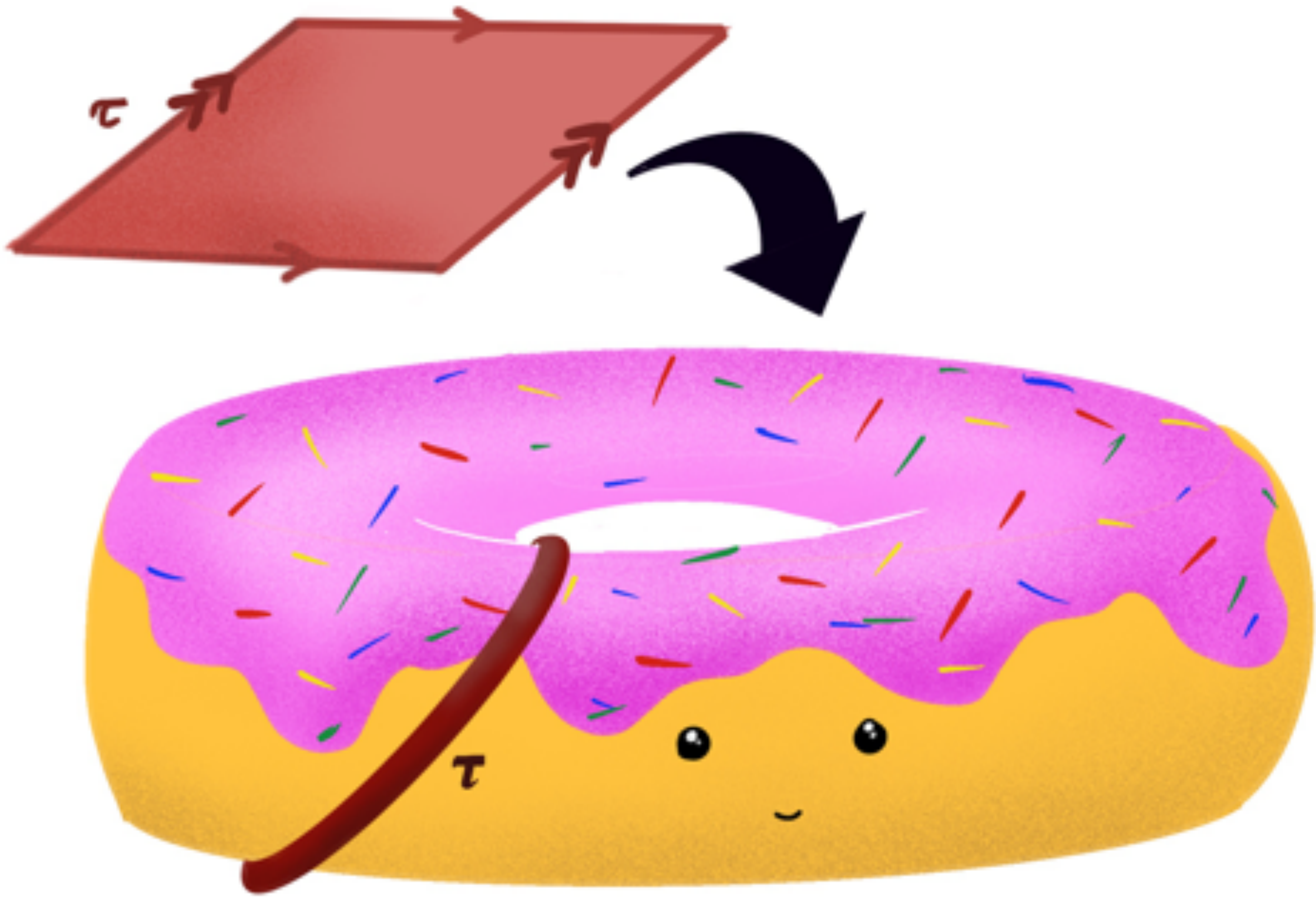
  - such terms cannot be forbidden by any (conventional) symmetry
  - Kähler corrections once flavon fields attain VEVs
  - additional parameters $\kappa^X_{\phi^{(\prime)}}$ diminish predictivity of the scheme

An Example: Enhanced θ_{13} in A_4

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

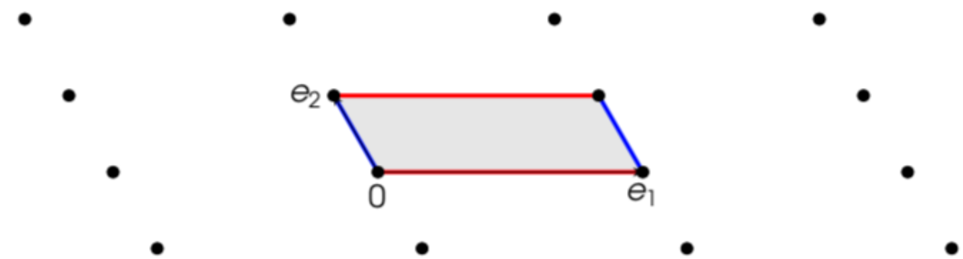


Modular Flavor Symmetries



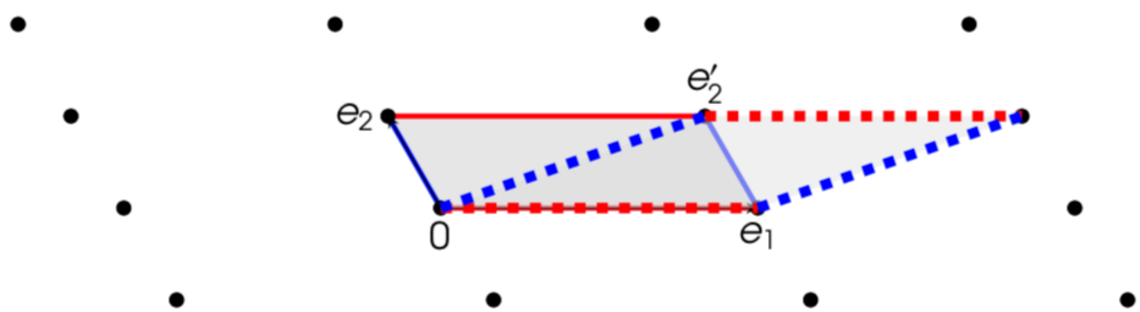
Artwork by Shreya Shukla

Modular Symmetries



edges ⇒ lattice basis vectors

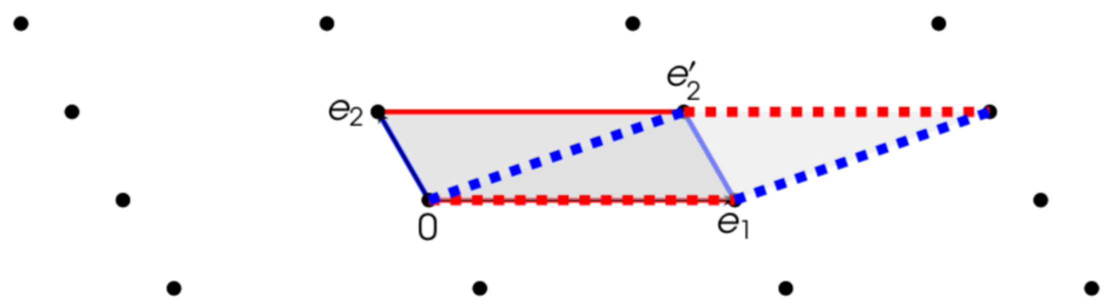
points in plane identified if differ by a lattice translation



Equivalent TORI related by Modular Symmetries

Modular Symmetries

- TORI: fundamental domain not unique



- Basis Vectors are related: $\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \overset{\gamma}{\mapsto} \begin{pmatrix} e_2' \\ e_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$

$a, b, c, d \in \mathbb{Z}$

- Volume of fundamental domain the same $\Rightarrow$ $\det \gamma = 1$

Modular Symmetries

- **Finite Modular Group** (quotient group): $\Gamma_N := \Gamma/\Gamma(N)$ where principal congruence group Γ(N) is

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2 ; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

Γ

- Generators of the quotient group Γ_N satisfy

$$S^2 = 1, \quad (ST)^3 = 1, \quad T^N = 1$$

- Some examples

$$\Gamma_2 \simeq S_3, \quad \Gamma_3 \simeq A_4, \quad \Gamma_4 \simeq S_4, \quad \Gamma_5 \simeq A_5$$

Modular Symmetries

Feruglio (2017)

- Imposing modular symmetry Γ on the Lagrangian:

$$\mathscr{L} \supset \sum Y_{i_1,\, i_2,\, \dots\dots\, i_n} \Phi_{i_1} \Phi_{i_2} \cdots \Phi_{i_n}$$

$$\tau \xmapsto{\gamma} \gamma\tau := \frac{a\tau + b}{c\tau + d}\,,$$

$$\Phi_j \xmapsto{\gamma} (c\tau + d)^{k_j} \rho_{\mathbf{r}_j}(\gamma)\, \Phi_j\,, \quad \text{where } \gamma := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

k_i : integers

representation matrix of Γ_N

- Yukawa Couplings = Modular Forms at level "N" w/ weight "k"

$$f_i(\gamma\tau) = (c\tau + d)^{-k} [\rho_N(\gamma)]_{ij} f_j(\tau)$$

$k = k_{i_1} + k_{i_2} + \dots + k_{i_n}$

representation matrix of Γ_N

A Toy Modular A_4 Model

Feruglio (2017)

- Weinberg Operator $$\mathcal{W}_\nu = \frac{1}{\Lambda}[(H_u \cdot L)\ Y\ (H_u \cdot L)]_1$$
- Traditional A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Flavon VEVs (A_4 triplet, 6 real parameters)

$$Y \rightarrow \langle \phi \rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \Longrightarrow m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2a & -c & -b \\ -c & 2b & -a \\ -b & -a & 2c \end{pmatrix}$$

- Modular A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Modular Forms (A4 triplet, 2 real parameters)

$$Y \rightarrow \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \Longrightarrow m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

Modular Forms

Feruglio (2017)

- Level (N) = 3, Weight (k) = 2, in terms of Dedekind eta-function

$$
\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}-\frac{27\eta'(3\tau)}{\eta(3\tau)}\right] \\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega^2\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \\
Y_3(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega^2\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \quad .
\end{aligned}
$$

$$
\eta(\tau)=q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad\qquad q\equiv e^{i2\pi\tau}
$$

A Toy Modular A_4 Model

Feruglio (2017)

- **Input Parameters:**

$$\tau = 0.0111 + 0.9946i$$

$$v_u^2/\Lambda$$

- **Predictions:**

$$\frac{\Delta m^2_{sol}}{|\Delta m^2_{atm}|} = 0.0292$$

$$\sin^2\theta_{12} = 0.295 \qquad \sin^2\theta_{13} = 0.0447 \qquad \sin^2\theta_{23} = 0.651$$

$$\frac{\delta_{CP}}{\pi} = 1.55 \qquad \frac{\alpha_{21}}{\pi} = 0.22 \qquad \frac{\alpha_{31}}{\pi} = 1.80 \quad .$$

$$m_1 = 4.998 \times 10^{-2}\ eV \qquad m_2 = 5.071 \times 10^{-2}\ eV \qquad m_3 = 7.338 \times 10^{-4}\ eV$$

Kähler Corrections in Modular A4 Model

Feruglio (2017)

- Particle Content

	(E_1^c, E_2^c, E_3^c)	L	H_d	H_u	φ
$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$	$\mathbf{1}_1$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{1/2}$	$\mathbf{1}_0$
Γ_3	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
k	$(k_{E_1}, k_{E_2}, k_{E_3})$	k_L	k_{H_d}	k_{H_u}	k_φ

- Weinberg Operator

$$\mathscr{W}_\nu = \frac{1}{\Lambda}[(H_u \cdot L)\ Y\ (H_u \cdot L)]_\mathbf{1}$$

- Superpotential for Charged Leptons: couple to φ $\Rightarrow$ diagonal mass matrix

Kähler Corrections in Modular A4 Model

- Minimal Kähler Potential for charged leptons

$$K_L = (-\mathrm{i}\tau + \mathrm{i}\bar{\tau})^{-1} L^\dagger L$$

- Additional terms allowed in Kähler Potential

MCC, Ramos-Sánchez, Ratz (2019)

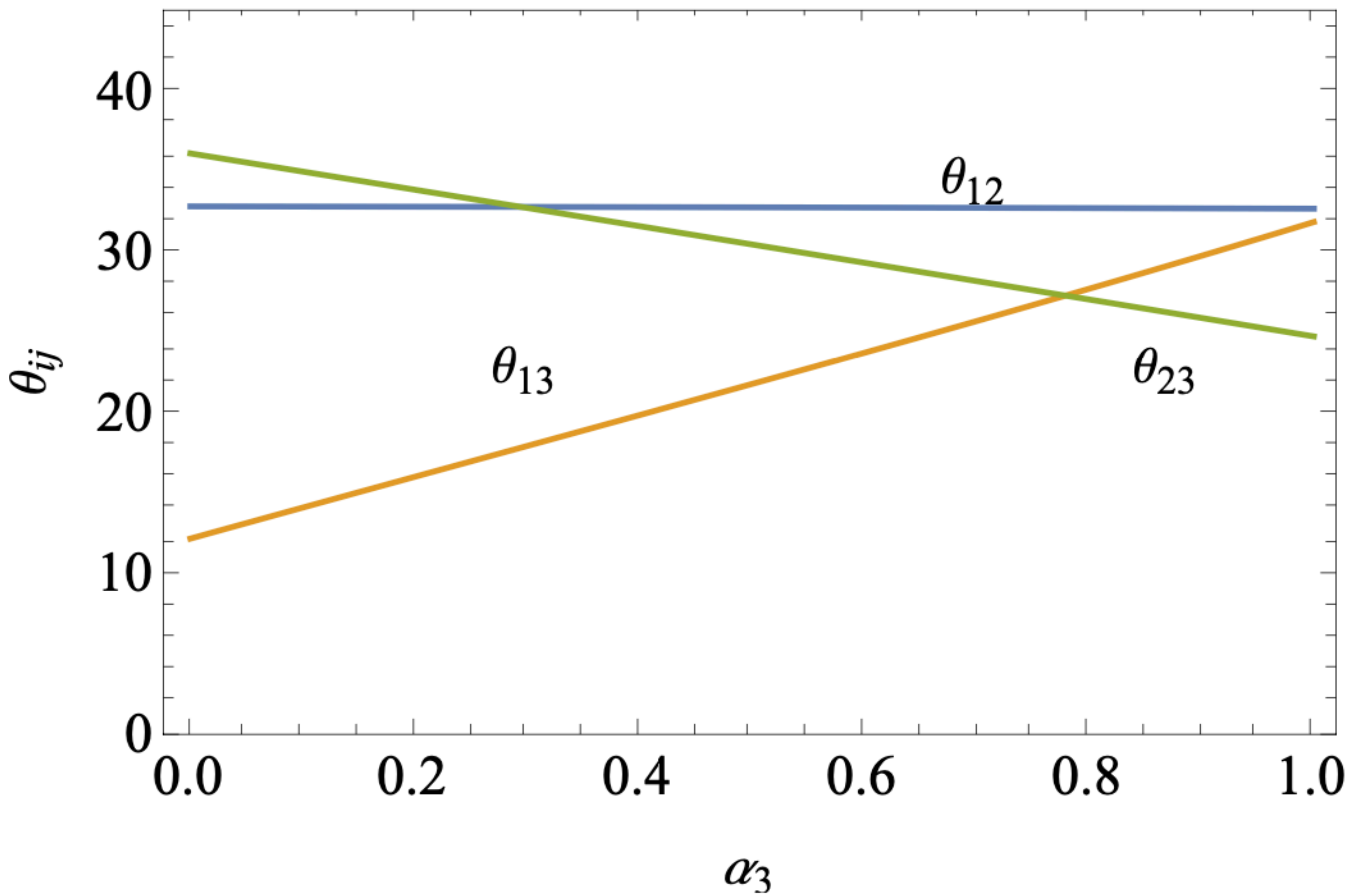
$$K = \alpha_0 (-\mathrm{i}\tau + \mathrm{i}\bar{\tau})^{-1} \left(\overline{L}\, L\right)_{\mathbf{1}} + \sum_{k=1}^{7} \alpha_k (-\mathrm{i}\tau + \mathrm{i}\bar{\tau}) \left(Y\, L\, \overline{Y}\, \overline{L}\right)_{\mathbf{1},\,k} + \dots\, .$$

$$\Delta K = \alpha_1 \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(1)}} (Y\, L)_{\mathbf{3}^{(1)}} + \alpha_2 \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(2)}} (Y\, L)_{\mathbf{3}^{(2)}} + \alpha_3 \left[\left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(1)}} (Y\, L)_{\mathbf{3}^{(2)}} + \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(2)}} (Y\, L)_{\mathbf{3}^{(1)}}\right] + \dots$$

- "Leading terms" and "corrections" are compatible
- Back to Canonical Basis ➜ sizable corrections to mixing parameters

Kähler Corrections in Modular A4 Model

M.-C.C., Ramos-Sánchez, Ratz (2019)



Quasi-Eclectic Modular Symmetry

- Quasi-eclectic setup:

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)

$$G_{\text{quasi-eclectic}} = G_{\text{traditional}} \times G_{\text{modular}} = A_4 \times \Gamma_3$$

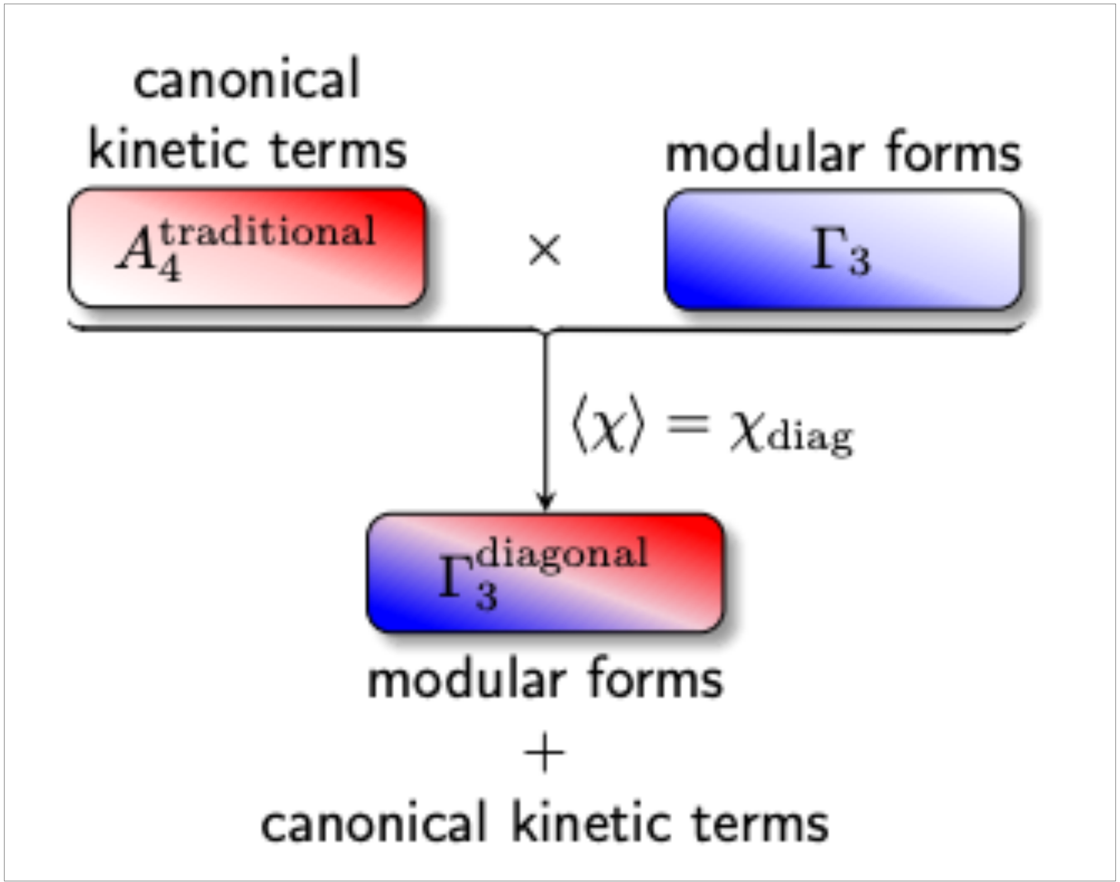
- Field Content:

	$(E_1^{\mathcal{C}}, E_2^{\mathcal{C}}, E_3^{\mathcal{C}})$	L	H_d	H_u	χ	φ	S_χ	S_φ	Y
$\mathrm{SU}(2)_{\mathrm{L}} \times \mathrm{U}(1)_Y$	$\mathbf{1}_1$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{1/2}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
$A_4^{\text{traditional}}$	$(\mathbf{1}_0, \mathbf{1}_2, \mathbf{1}_1)$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
$\mathbb{Z}_3^\chi$	0	0	0	1	1	0	1	0	0
$\mathbb{Z}_3^\varphi$	1	0	1	0	0	1	0	1	0
Γ_3	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$
k	$(k_{E_1}, k_{E_2}, k_{E_3})$	k_L	k_{H_d}	k_{H_u}	k_χ	k_φ	k_S	k_S	k_Y
modular weights	$(1, 1, 1)$	-1	0	0	0	0	0	0	2

Quasi-Eclectic Modular Symmetry

- Symmetry Breaking

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)



- VEVs pattern resulting from vacuum alignment

$$\langle \chi_i^a \rangle = v_1 \, \mathbb{1}_3$$

$$\langle \varphi_i \rangle = v_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Quasi-Eclectic Modular Symmetry

- After Symmetry Breaking: diagonal Γ_3

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)

  - Neutrino Sector: $$\mathscr{W}_\nu = \frac{1}{\Lambda^2}\left[(H_u \cdot L)\ \chi\ (H_u \cdot L)\ Y\right]_{\mathbf{1}_0}$$

$$\rightarrow \quad m_\nu = \frac{v_u^2 \varepsilon_1}{\sqrt{3}\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

  - Charged lepton sector:

$$\mathscr{W}_e = \frac{\widetilde{y}_e}{\Lambda} H_d (L\varphi E_1^{\mathcal{C}})_{\mathbf{1}_0} + \frac{\widetilde{y}_\tau}{\Lambda} H_d (L\varphi E_2^{\mathcal{C}})_{\mathbf{1}_0} + \frac{\widetilde{y}_\mu}{\Lambda} H_d (L\varphi E_3^{\mathcal{C}})_{\mathbf{1}_0}$$

$$\rightarrow \quad m_e = v_d \frac{v_2}{\Lambda} \operatorname{diag}\left(\widetilde{y}_e, \widetilde{y}_\tau, \widetilde{y}_\mu\right)$$

Quasi-Eclectic Modular Symmetry

- After Symmetry Breaking: diagonal Γ_3

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)

- Kähler Corrections: $$K_L = L^\dagger L + \mathcal{O}(\varepsilon_1^2) + \mathcal{O}(\varepsilon_2^2)$$

  - Corrections involving only Y: absent to all orders, due to traditional A4 symmetry (corrections in modular setup)

  - Corrections involving flavon $\langle \varphi_i \rangle$: suppressed (corrections in traditional setup)

$$\Delta K_L = \varepsilon_2^2 \left(C_1 \mathbb{1}_3 + \frac{2C_2}{3} \operatorname{diag}(2,-1,-1) + \frac{2C_3}{\sqrt{3}} \operatorname{diag}(0,1,-1) \right)$$

$$\varepsilon_2^2 = v_2^2/\Lambda^2 \gtrsim y_\tau^2$$

$$\Delta\theta_{12} \simeq C_i \left(\frac{\varepsilon_2}{0.03}\right)^2 \cdot \begin{cases} 0\,, & \text{if } i = 1\,, \\ -0.05\,, & \text{if } i = 2\,, \\ 0.01\,, & \text{if } i = 3\,. \end{cases}$$

RG Invariants: Neutrino Mass Operator

- RGE for effective neutrino mass operator κ in SM, 2HDM, MSSM

$$16\pi^2 \frac{d}{dt}\kappa = P^T\kappa + \kappa P + \alpha\kappa, \qquad P = C_e Y_e^\dagger Y_e \quad \text{(at 1-loop)}$$

In diagonal P basis:

$$\Delta\kappa_{ij} = \frac{\Delta t}{16\pi^2}\kappa_{ij}\left(P_{ii} + P_{jj} + \alpha\right)$$

- RG Invariants

Chang, Kuo (2002)

$$I_{ij} = \frac{\kappa_{ii}\,\kappa_{jj}}{\kappa_{ij}^2} \qquad (i \neq j)$$

- Wave function renormalization cancel

Renormalization Group Invariants

Chang, Kuo (2002)

- In P-diagonal basis: $P = C_e Y_e^\dagger Y_e$

$$\frac{d}{dt}\kappa = \tilde{P}\kappa\tilde{Q}^T + \tilde{Q}\kappa\tilde{P}^T + \tilde{\alpha}\kappa,$$

$$\tilde{P} = \text{diag}\left(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3\right), \quad \tilde{Q} = \text{diag}\left(\tilde{Q}_1, \tilde{Q}_2, \tilde{Q}_3\right)$$

- At 1-loop: $\tilde{P} = \frac{1}{16\pi^2}P, \ \tilde{Q} = I, \ \tilde{\alpha} = \frac{1}{16\pi^2}\alpha$

$$\dot{\kappa}_{ij} = \kappa_{ij}\left(\tilde{P}_i\tilde{Q}_j + \tilde{P}_j\tilde{Q}_i + \tilde{\alpha}\right), \quad \frac{d}{dt}I_{ij} = 2\left(\tilde{P}_i - \tilde{P}_j\right)\left(\tilde{Q}_i - \tilde{Q}_j\right)I_{ij}$$

$\Rightarrow$ I_{ij} is RG invariant

Invariants in Toy Modular A_4 Model

MCC, X.-G. Liu, X.-Q. Li, O. Medina, M. Ratz (2024)

- Mass matrix in canonical basis:

$$M_e = u\, v_d\, \mathrm{diag}(\alpha, \beta, \gamma)\ ,$$

$$m_\nu(\tau, \bar\tau) = (-\mathrm{i}\,\tau + \mathrm{i}\,\bar\tau)\frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_2(\tau) & -Y_3(\tau) \\ -Y_2(\tau) & 2Y_3(\tau) & -Y_1(\tau) \\ -Y_3(\tau) & -Y_1(\tau) & 2Y_2(\tau) \end{pmatrix} =: (-\mathrm{i}\,\tau + \mathrm{i}\,\bar\tau) v_u^2 \begin{pmatrix} \kappa_{11} & \kappa_{12} & \kappa_{13} \\ \kappa_{12} & \kappa_{22} & \kappa_{23} \\ \kappa_{13} & \kappa_{23} & \kappa_{33} \end{pmatrix}$$

- Invariants

$$I_{12}(\tau) = 4\frac{Y_1(\tau)\ Y_3(\tau)}{\big(Y_2(\tau)\big)^2}, \qquad I_{13}(\tau) = 4\frac{Y_1(\tau)\ Y_2(\tau)}{\big(Y_3(\tau)\big)^2}, \qquad I_{23}(\tau) = 4\frac{Y_2(\tau)\ Y_3(\tau)}{\big(Y_1(\tau)\big)^2}$$

- Algebraic constraint $\quad Y_2^2 + 2Y_1Y_3 = 0$

- Thus

$$I_{12}(\tau) = -2, \qquad I_{13}(\tau) = -2\Big(1 + \frac{1}{3} j_3(\tau)\Big)^3, \qquad I_{23}(\tau) = -\frac{32}{I_{23}}$$

Invariants in Toy Modular A_4 Model

- Two interesting relations: **RG invariant, independent of τ**

$$I_{12}(\tau) = -2, \qquad I_{13}(\tau) I_{23}(\tau) = -32$$

- Invariants I_{ij} : functions of physical observables

$$\left(m_1, m_2, m_3, \theta_{12}, \theta_{23}, \theta_{13}, \delta, \alpha_{12}, \alpha_{23}\right)$$

$\Rightarrow$ **sum rules among physical observables:**

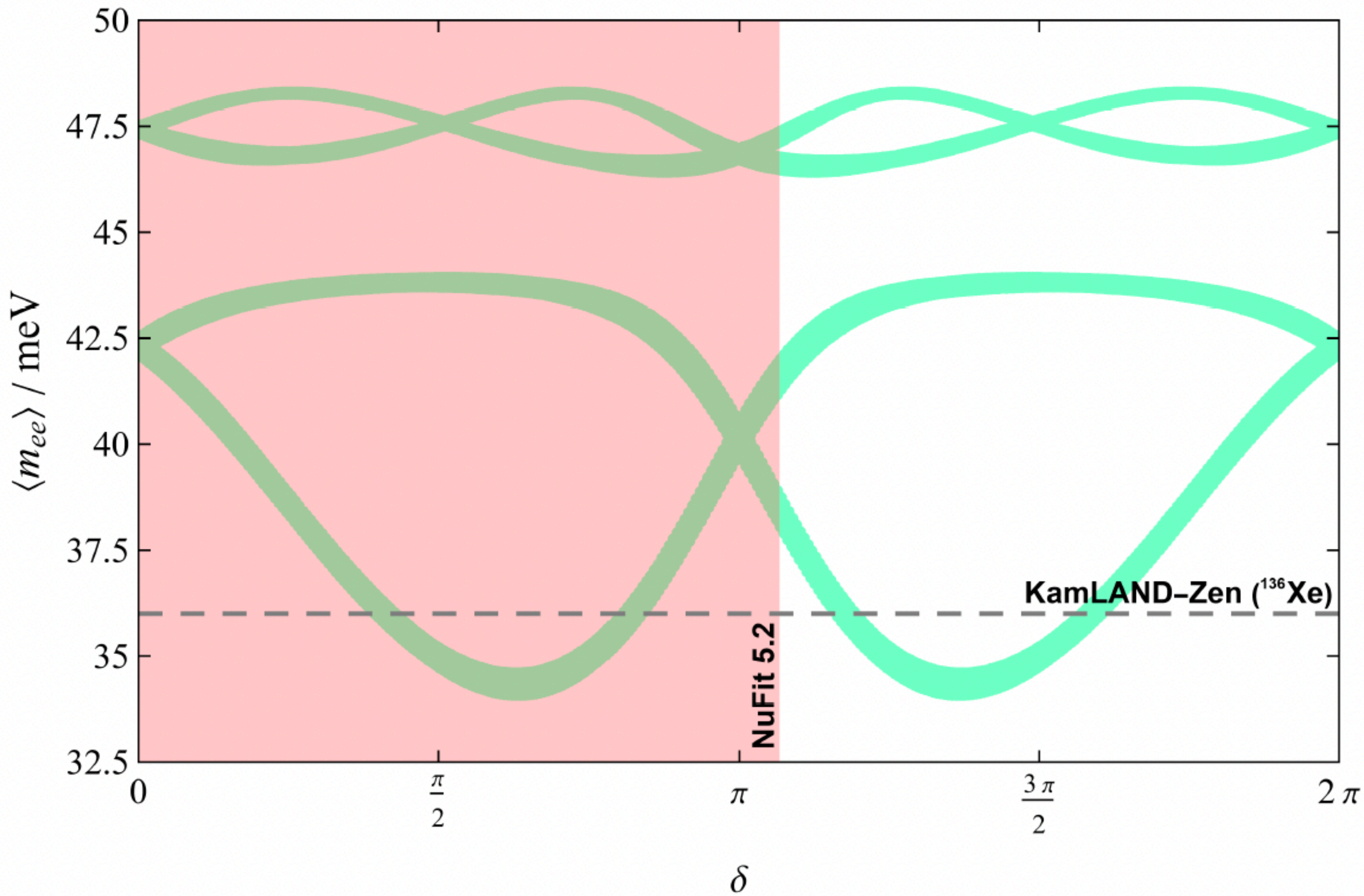
RG invariant, τ independent

MCC, X.-G. Liu, X.-Q. Li, O. Medina, M. Ratz (2024)

Invariants in Toy Modular A₄ Model

- Predictions from $I_{12} = -2$ invariant for Inverted Ordering

MCC, X.-G. Liu, X.-Q. Li, O. Medina, M. Ratz (2024)



Reality of I_{12} ⇒

Constraints on CP phases:

Given δ ⇒

Majorana phases α_{12}, α_{13}

Invariants in Toy Modular A_4 Model

- No simultaneous solution for I_{ij} that is consistent with data
  - Agree with previous analysis by scanning parameter space (i.e. toy modular A4 model does not fit all data)
  - Here, arrived at conclusion without the need to scan

MCC, X.-G. Liu, X.-Q. Li, O. Medina, M. Ratz (2024)

Invariants in Toy Modular A_5 Model

MCC, X.-G. Liu, X.-Q. Li, O. Medina, M. Ratz (2024)

- In a model based on modular A5:

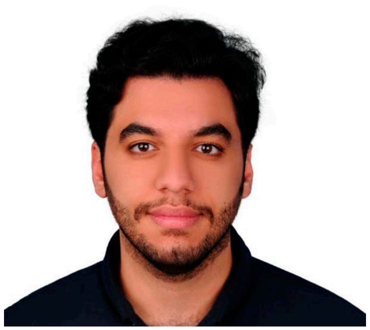
$$I_{12} = \frac{2\sqrt{6}}{3}\frac{Y_1(\tau)Y_4(\tau)}{Y_5^2(\tau)}, \qquad I_{13} = \frac{2\sqrt{6}}{3}\frac{Y_1(\tau)Y_3(\tau)}{Y_2^2(\tau)}, \qquad I_{23} = 6\frac{Y_3(\tau)Y_4(\tau)}{Y_1^2(\tau)}$$

- Algebraic relations among the invariants

$$\begin{aligned} 0 &= 4 + 18I_{12} + 18I_{13} + 9I_{12}I_{13} + I_{12}I_{13}I_{23}\,, \\ 0 &= 8 + 12I_{12} - 108I_{12}^2 + 12I_{13} + 414I_{12}I_{13} + 108I_{12}^2I_{13} - 108I_{13}^2 + 108I_{12}I_{13}^2 + 81I_{12}^2I_{13}^2 \\ &\quad - I_{12}^2I_{23} - I_{13}^2I_{23}\,. \end{aligned}$$

- Exchange symmetry: $I_{12} \leftrightarrow I_{13} \;\Rightarrow\; \mu - \tau$ symmetry built in

Acknowledgements



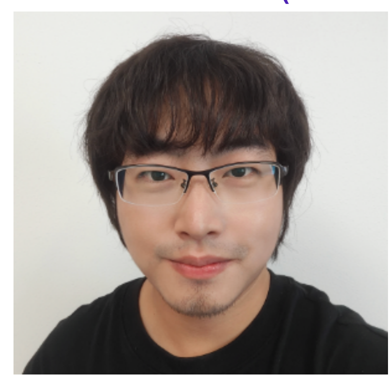
Yahya Almumin
(UCI Grad)

Víctor Knapp-Pérez
(UCI Grad)

Cameron Moffett-Smith
(UCI Grad)

Shreya Shukla
(UCI Grad)

Xueqi Li
(UCI Grad)

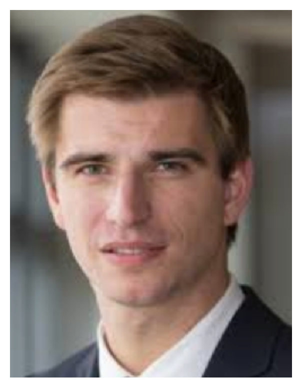


Xiang-Gan Liu
(UCI PD)

Omar Medina
(IFIC Valencia Grad; former UCI visiting student)

Mario Ramos-Hamud
(Cambridge Grad)

Maximilian Fallbacher
(former TUM Grad)



Christian Staudt
(former TUM Grad)

Saúl Ramos-Sánchez
(UNAM, Mexico)

Michael Ratz
(UCI)

Conclusions

- Modular Flavor Symmetries: Significant reduction of the number of parameters
  - Kähler Corrections: as in traditional discrete flavor symmetries
- In quasi-eclectic setup: corrections can be greatly reduced to the level compatible with experiment uncertainty
- τ-independent RG Invariants: robust sum rules among physical observables, independent of renormalization scale, model parameters
  $\Rightarrow$ need further exploration
- Top-down connection:
  - Modular flavor symmetries from strings e.g. Baur, Nilles, Trautner, Vaudrevange (2021)
  - Modular flavor symmetries from magnetized tori e.g. Almumin, MCC, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)
- Diversity drives intellectual excellence

NEUTRINO 2026

University of California, Irvine, U.S.A.

About Irvine, California

a metropolitan city located at about 40 miles (64 km) south of Los Angeles, 70 miles (112 km) north of San Diego, on the beautiful coast of the Pacific Ocean with 11,000 ft (3500 m) towering San Bernadino Mountains in its backdrop.

70th Anniversary of Neutrino Discovery

by George Cowan and Fred Reines. Fred Reines (1995 Nobel Laureate) was the founding Dean of School of Physical Sciences at UC Irvine.

Contact Us

co-Chairs: Mu-Chun Chen, Michael Smy | E-Mail: neutrino2026uci@gmail.com

Invariants in Toy Modular A_4 Model

$$I_{12} = \frac{a_0\left[\widetilde{m}_1\left(e^{i\delta}c_{23}s_{12}+c_{12}s_{13}s_{23}\right)^2+\widetilde{m}_2\left(e^{i\delta}c_{12}c_{23}-s_{12}s_{13}s_{23}\right)^2+e^{2i\delta}m_3c_{13}^2s_{23}^2\right]}{c_{13}^2\left[\widetilde{m}_1c_{12}\left(e^{i\delta}c_{23}s_{12}+c_{12}s_{13}s_{23}\right)+\widetilde{m}_2s_{12}\left(s_{12}s_{13}s_{23}-e^{i\delta}c_{12}c_{23}\right)-e^{2i\delta}m_3s_{13}s_{23}\right]^2},$$

$$I_{13} = \frac{a_0\left[\widetilde{m}_1\left(c_{12}c_{23}s_{13}-e^{i\delta}s_{12}s_{23}\right)^2+\widetilde{m}_2\left(c_{23}s_{12}s_{13}+e^{i\delta}c_{12}s_{23}\right)^2+e^{2i\delta}m_3c_{13}^2c_{23}^2\right]}{c_{13}^2\left[\widetilde{m}_1c_{12}\left(c_{12}c_{23}s_{13}-e^{i\delta}s_{12}s_{23}\right)+\widetilde{m}_2s_{12}\left(c_{23}s_{12}s_{13}+e^{i\delta}c_{12}s_{23}\right)-e^{2i\delta}m_3c_{23}s_{13}\right]^2},$$

$$I_{23} = \left[e^{2i\delta}m_3c_{13}^2s_{23}^2+\widetilde{m}_1\left(e^{i\delta}c_{23}s_{12}+c_{12}s_{13}s_{23}\right)^2+\widetilde{m}_2\left(e^{i\delta}c_{12}c_{23}-s_{12}s_{13}s_{23}\right)^2\right]$$
$$\times\frac{4\left[e^{2i\delta}m_3c_{13}^2c_{23}^2+\widetilde{m}_2\left(c_{23}s_{12}s_{13}+e^{i\delta}c_{12}s_{23}\right)^2+\widetilde{m}_1\left(c_{12}c_{23}s_{13}-e^{i\delta}s_{12}s_{23}\right)^2\right]}{\left[\widetilde{m}_1a_1+\widetilde{m}_2a_2-e^{2i\delta}m_3\sin(2\theta_{23})c_{13}^2\right]^2},$$

$$\widetilde{m}_1 := m_1\,e^{i\varphi_1}$$

$$\widetilde{m}_2 := m_2\,e^{i\varphi_2}.$$

$$a_0 := \left(\widetilde{m}_1c_{12}^2+\widetilde{m}_2s_{12}^2\right)c_{13}^2+e^{2i\delta}m_3s_{13}^2,$$

$$a_1 := \left[\left(e^{2i\delta}s_{12}^2-c_{12}^2s_{13}^2\right)\sin(2\theta_{23})-e^{i\delta}\cos(2\theta_{23})\sin(2\theta_{12})s_{13}\right],$$

$$a_2 := \left[e^{i\delta}\cos(2\theta_{23})\sin(2\theta_{12})s_{13}+\left(e^{2i\delta}c_{12}^2-s_{12}^2s_{13}^2\right)\sin(2\theta_{23})\right].$$