Dark photon conversions in the presence of multiple resonances

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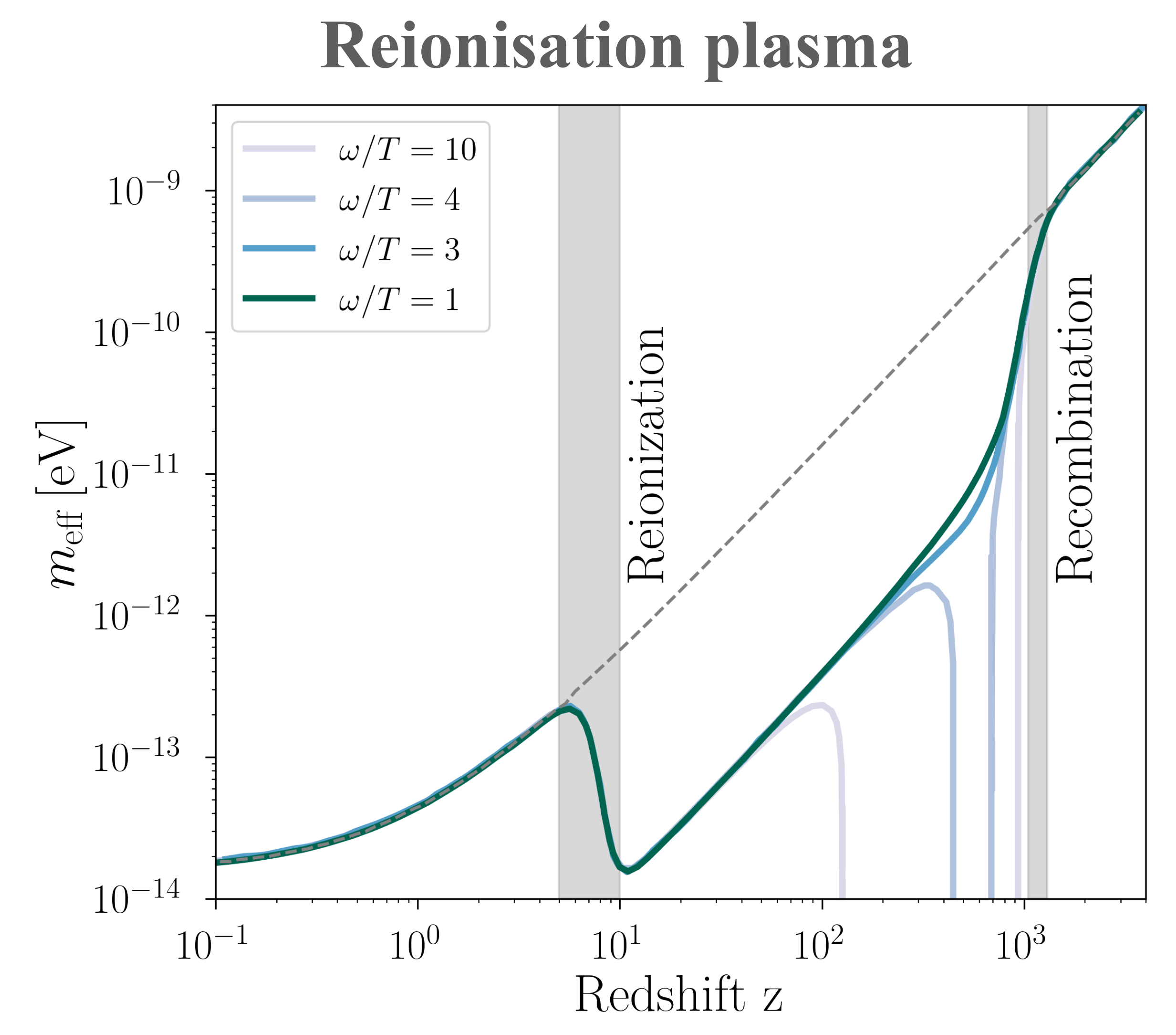
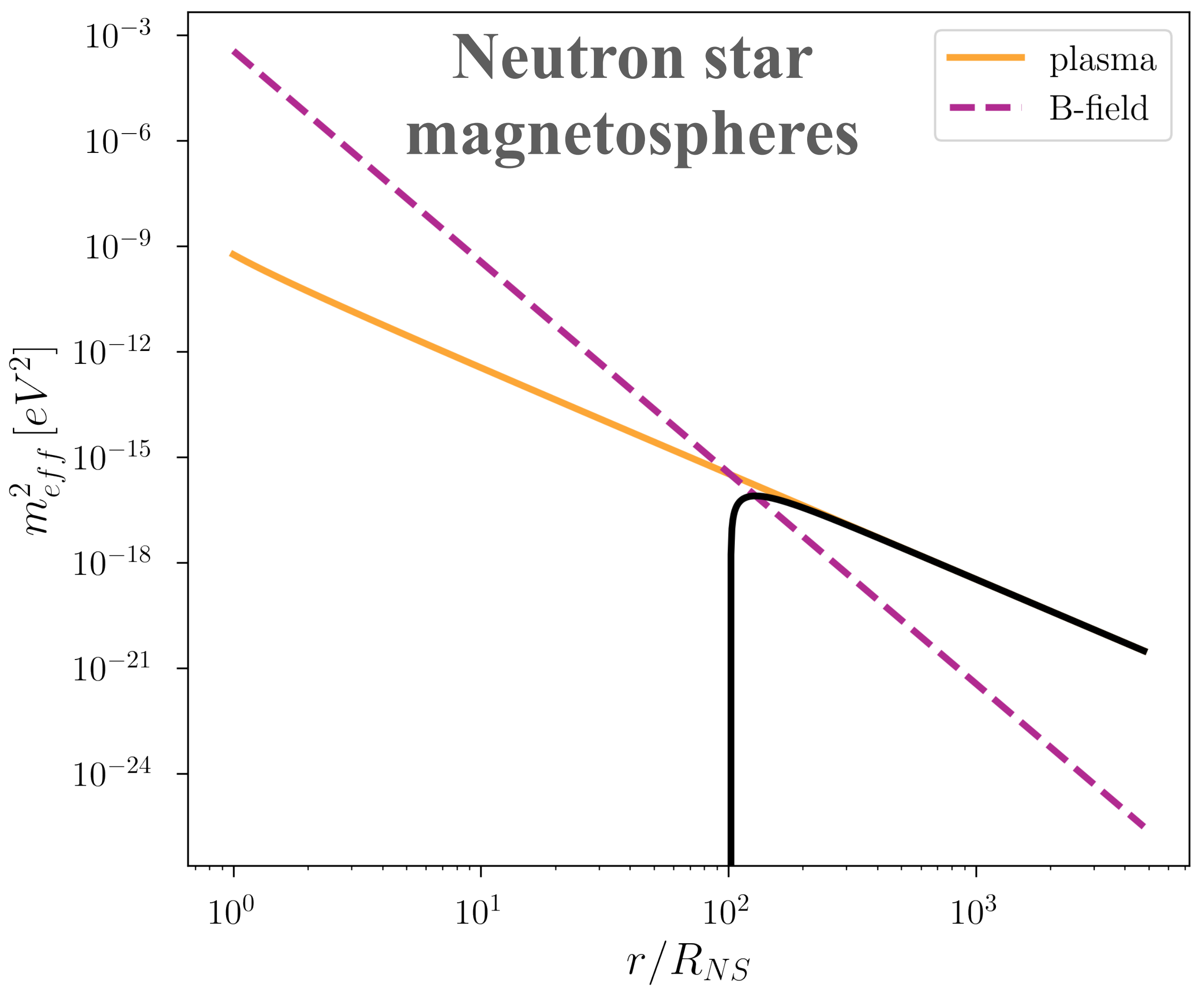
- Photons can acquire an effective (non-zero) mass in the presence of a medium. This can heavily modify the mixing properties.
- Moreover, this induced effective mass may not be constant and can vary with space and time.





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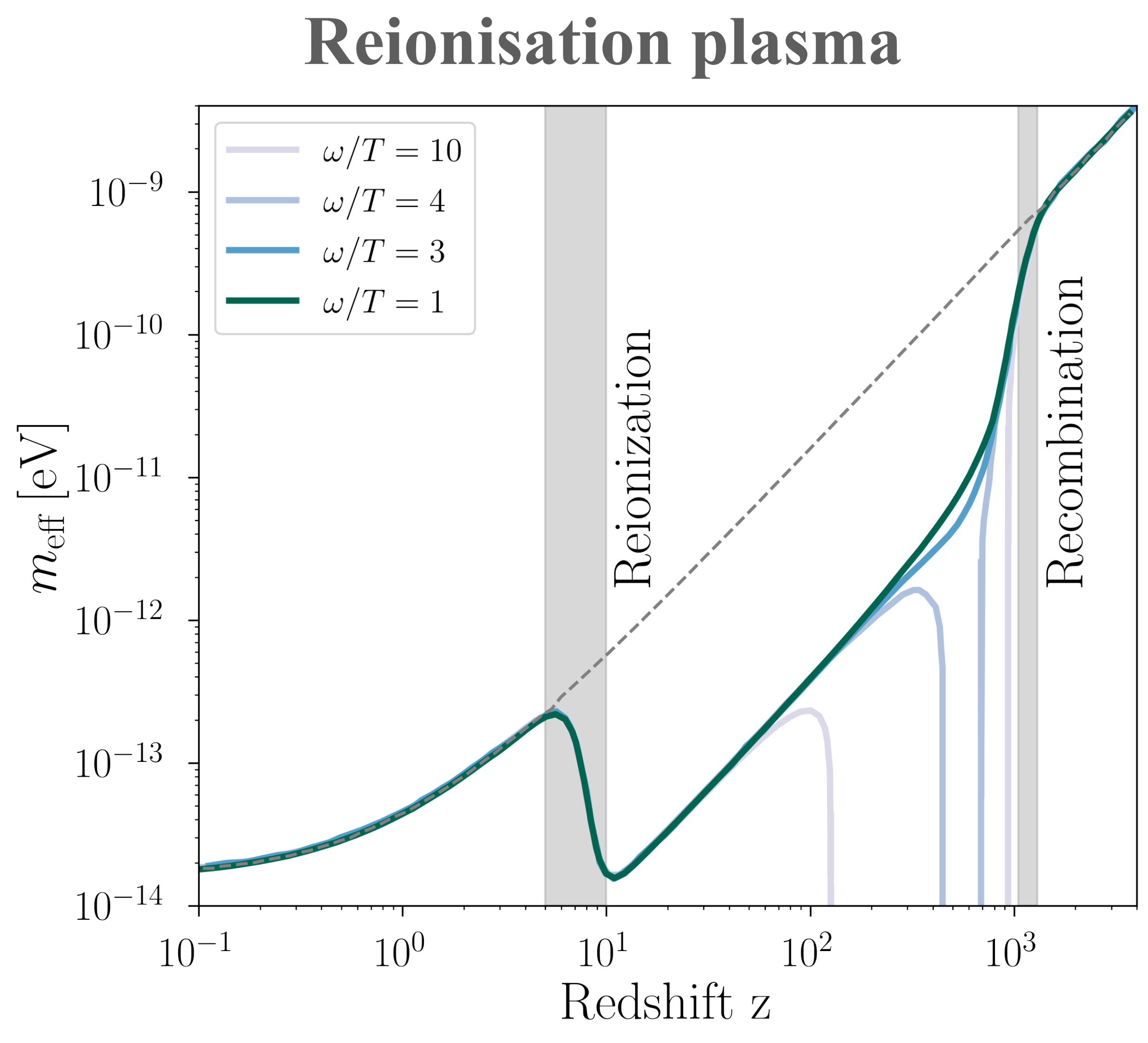
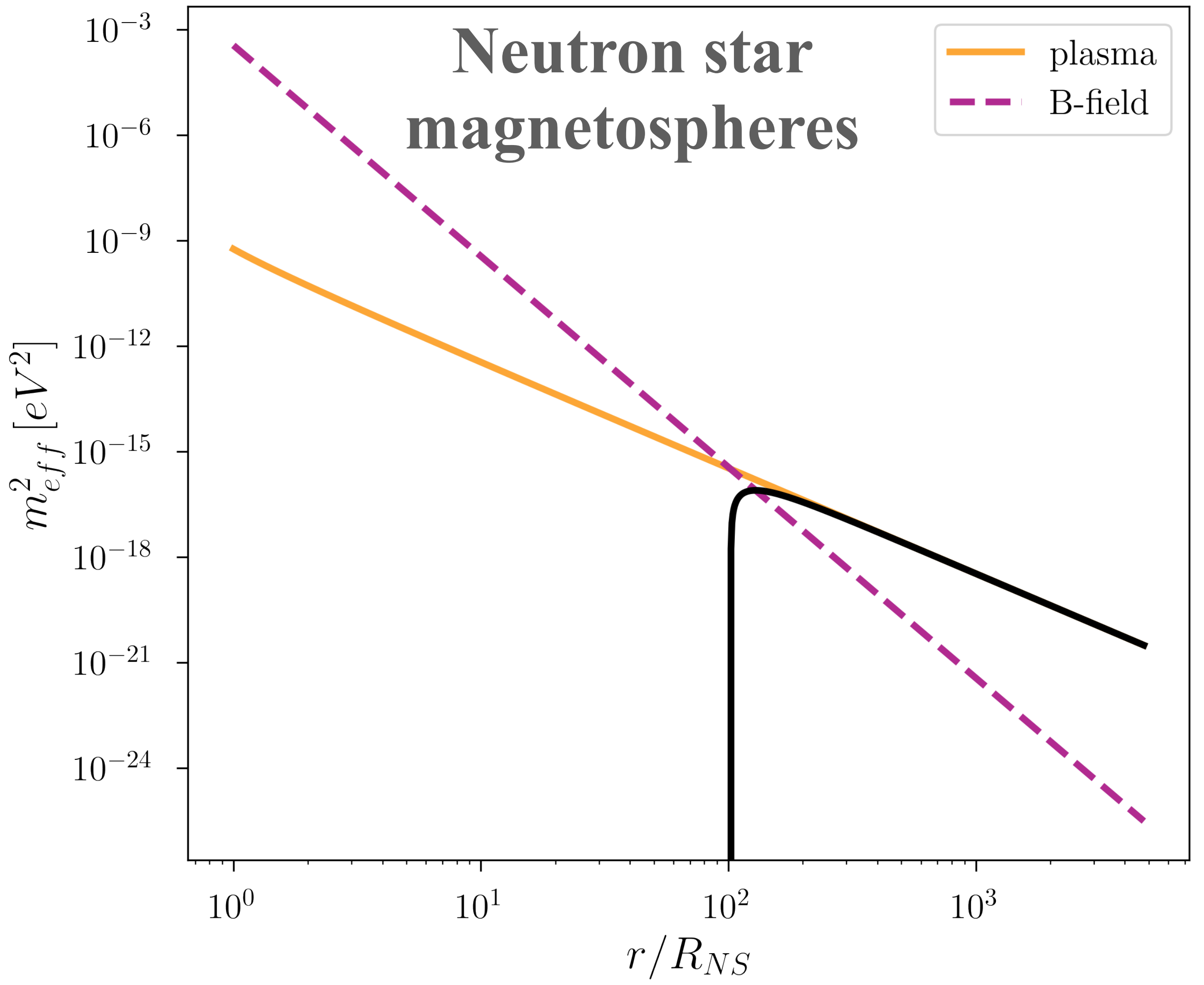
- This kinetic mixing can give rise to dark photon to photon conversions which make dark photons potentially detectable. These inter-conversions are central to lot of the current bounds.
- Photons can acquire an effective (non-zero) mass in the presence of a medium. This can heavily modify the mixing properties.
- Moreover, this induced effective mass may not be constant and can vary with space and time.
- Hence, a careful treatment of dark photon-photon oscillations in such potential profiles is important to accurately put bounds.





Photon-dark photon Lagrangian

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}\epsilon F_{\mu\nu}X^{\mu\nu} + \frac{1}{2}m_{\gamma'}^2 A'_\mu A'^\mu + eJ^\mu A_\mu$$

A^μ : photon field

A'^μ : dark photon field

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"Kinetic mixing term"

Dark Photon oscillation

$$\mathscr{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + \frac{1}{2}\begin{pmatrix} A_1^\mu & A_2^\mu \end{pmatrix}\begin{pmatrix} m_\gamma^2 & 0 \\ 0 & m_{\gamma'}^2 \end{pmatrix}\begin{pmatrix} A_{1\mu} \\ A_{2\mu} \end{pmatrix} + eJ^\mu\left(A_{1\mu} + \epsilon A_{2\mu}\right)$$

$$A_1^\mu = A^\mu - \epsilon A'^\mu$$

"Mass eigenbasis"

$$A_2^\mu = A'^\mu$$

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$A_a^\mu = A_1^\mu + \epsilon A_2^\mu$: active state

"Interaction eigenbasis"

$A_s^\mu = A_1^\mu - \epsilon A_2^\mu$: sterile state

Schrodinger equation

$$i\partial_z \begin{pmatrix} A_a \\ A_s \end{pmatrix} = H \begin{pmatrix} A_a \\ A_s \end{pmatrix}$$

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Diagonal

$$H_1 = \frac{1}{2\omega} \begin{pmatrix} 0 & \epsilon m_{\gamma'}^2 \\ \epsilon m_{\gamma'}^2 & 0 \end{pmatrix}$$

Off-diagonal

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$$\Phi(z) = \int_{z_i}^{z} dz' \left(\frac{m_{\gamma'}^2}{2\omega} - \frac{m_{\mathrm{eff}}^2}{2\omega} \right)$$

"Accumulated relative phase"

Dark photon phase

Photon phase

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"Accumulated relative phase"

- In vacuum, the photon state is massless and we have $m_{eff}^2 = 0$

$$\langle P_{\gamma\leftrightarrow\gamma'}^{vac} \rangle = 2\epsilon^2$$

Resonance and stationary phase approximation

$$P_{\gamma\leftrightarrow\gamma'} = \epsilon^2 \left| \int_{z_i}^{z} dz' \frac{m_{\gamma'}^2}{2\omega} e^{-i\Phi(z')} \right|^2$$

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- Except at stationary points, $\Phi' = 0 \longrightarrow m_{eff} = m_{\gamma'}$ "MSW effect"
- Integral gets most of it's contribution from stationary points

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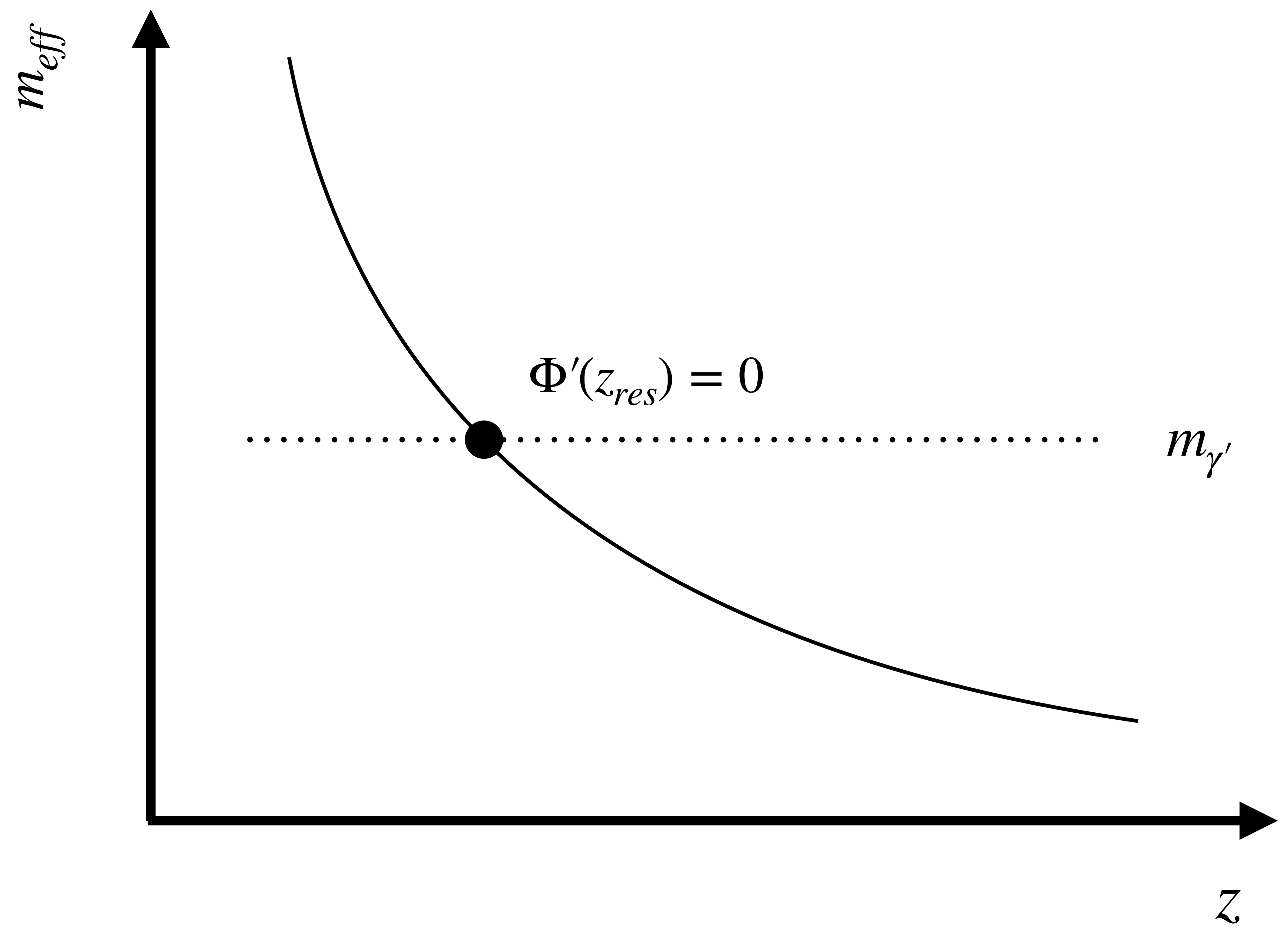
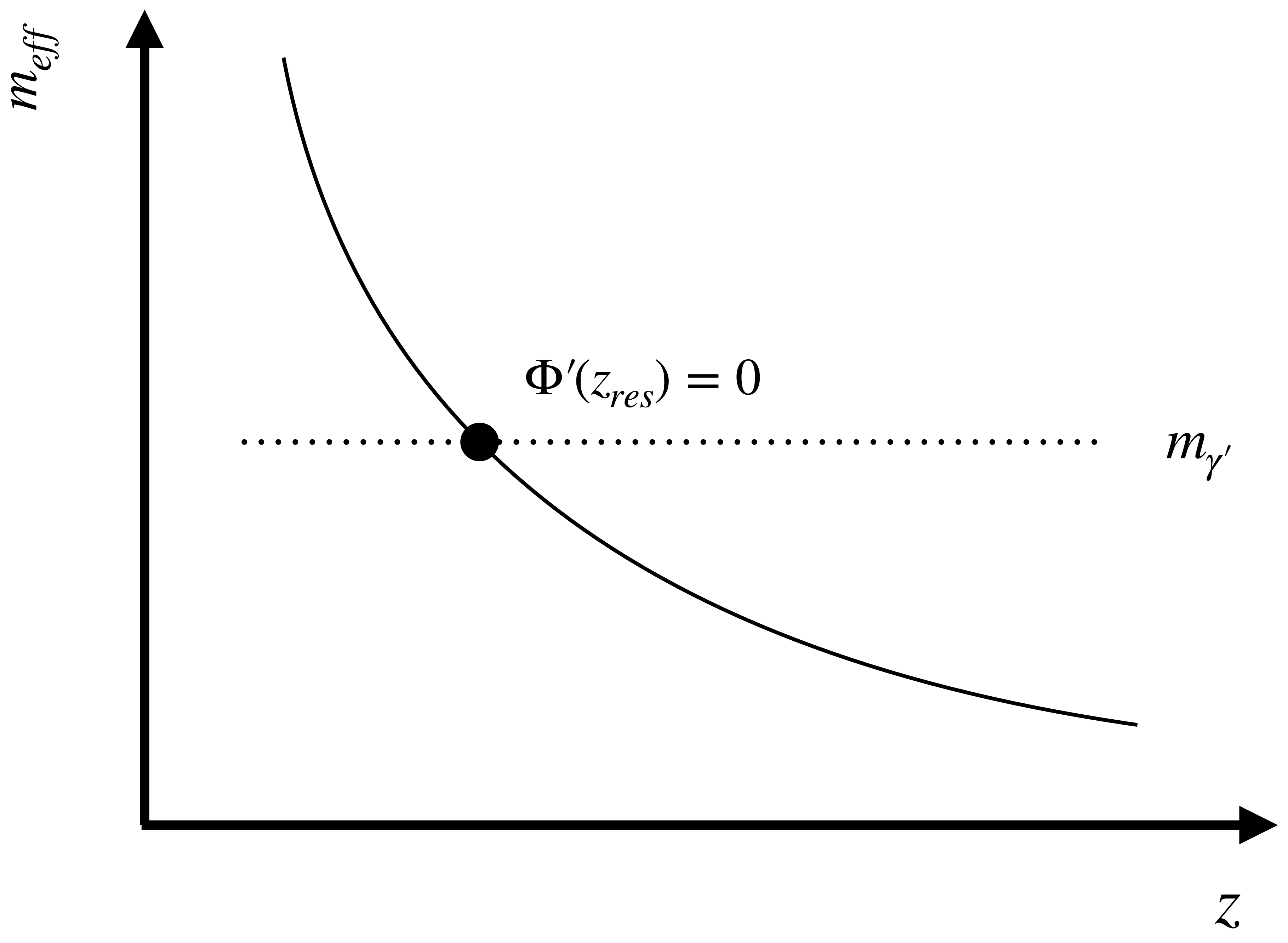


Resonance and stationary phase approximation

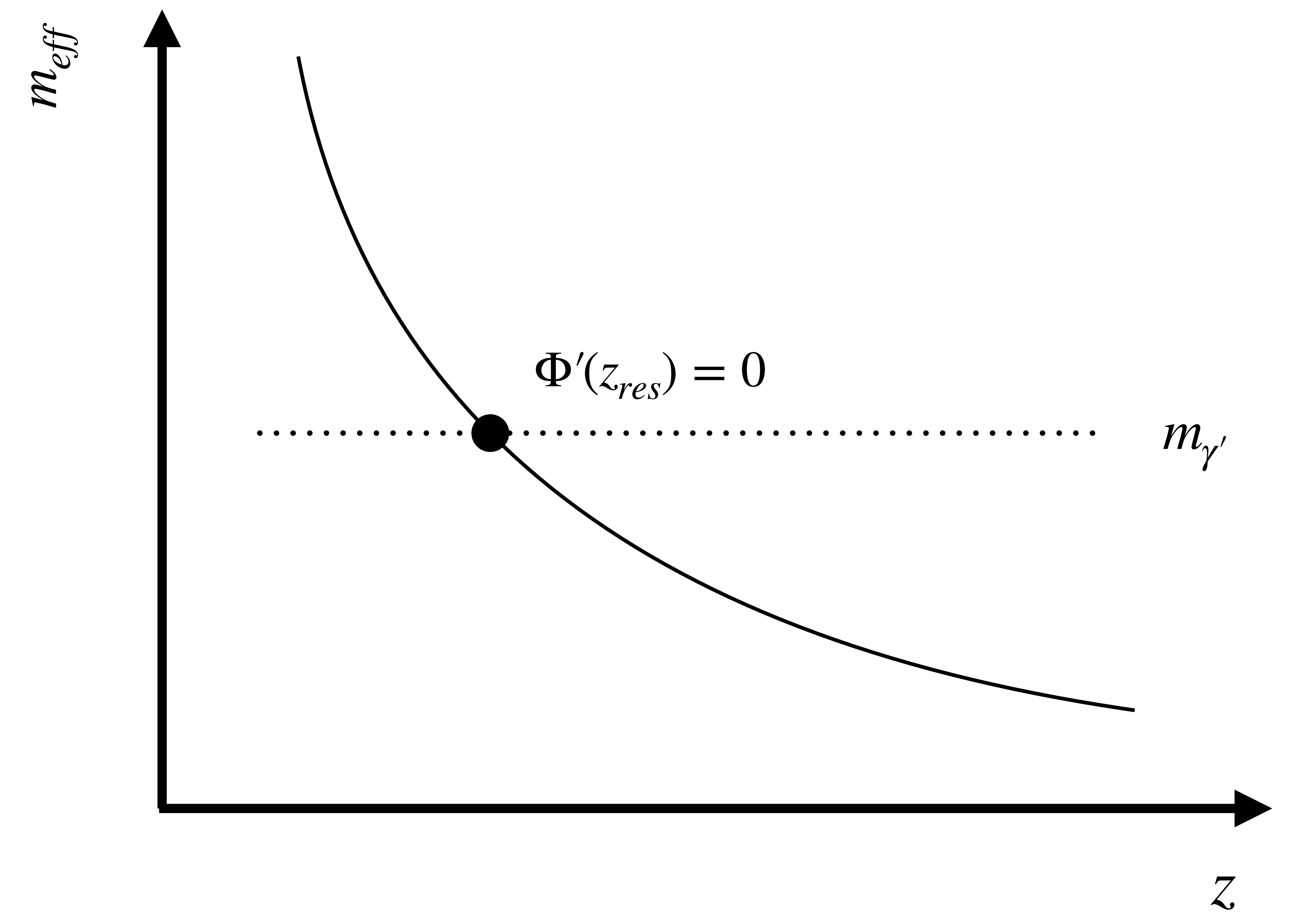
$$P_{\gamma\leftrightarrow\gamma'} \approx \epsilon^2 \left| \sqrt{\frac{2\pi}{\left|\Phi^{(2)}(z_{res})\right|}} \frac{m_{\gamma'}^2}{2\omega} e^{-i\Phi(z_{res})} \right|^2$$



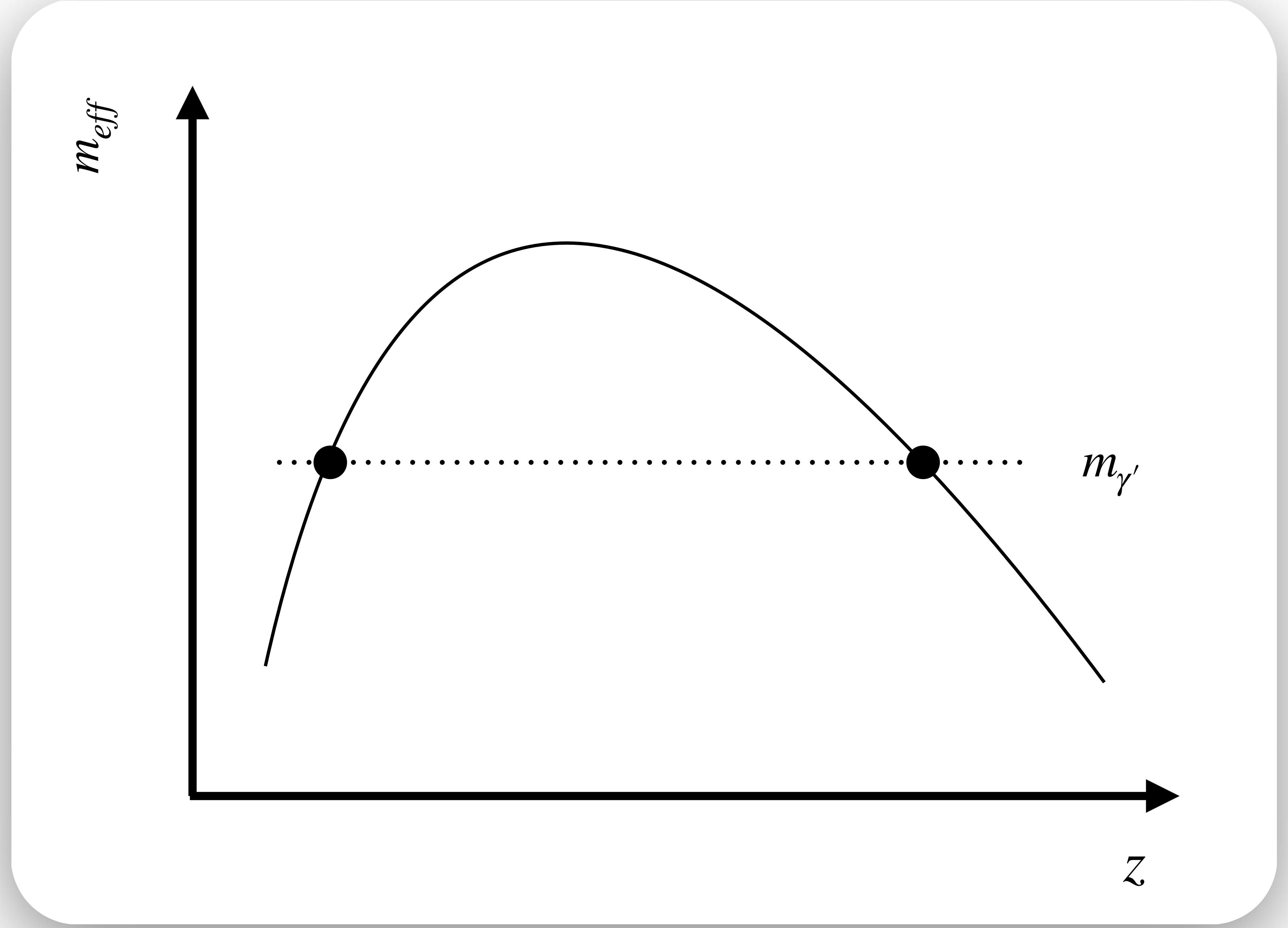
Resonance and stationary phase approximation

$$P_{\gamma\leftrightarrow\gamma'} \approx \epsilon^2 A^2 \quad \text{with} \quad A \equiv \sqrt{\frac{2\pi}{\left|\Phi^{(2)}(z_{res})\right|}}\left(\frac{m_{\gamma'}^2}{2\omega}\right)$$

"Landau-Zener"



Non-monotonic profiles and multiple resonances



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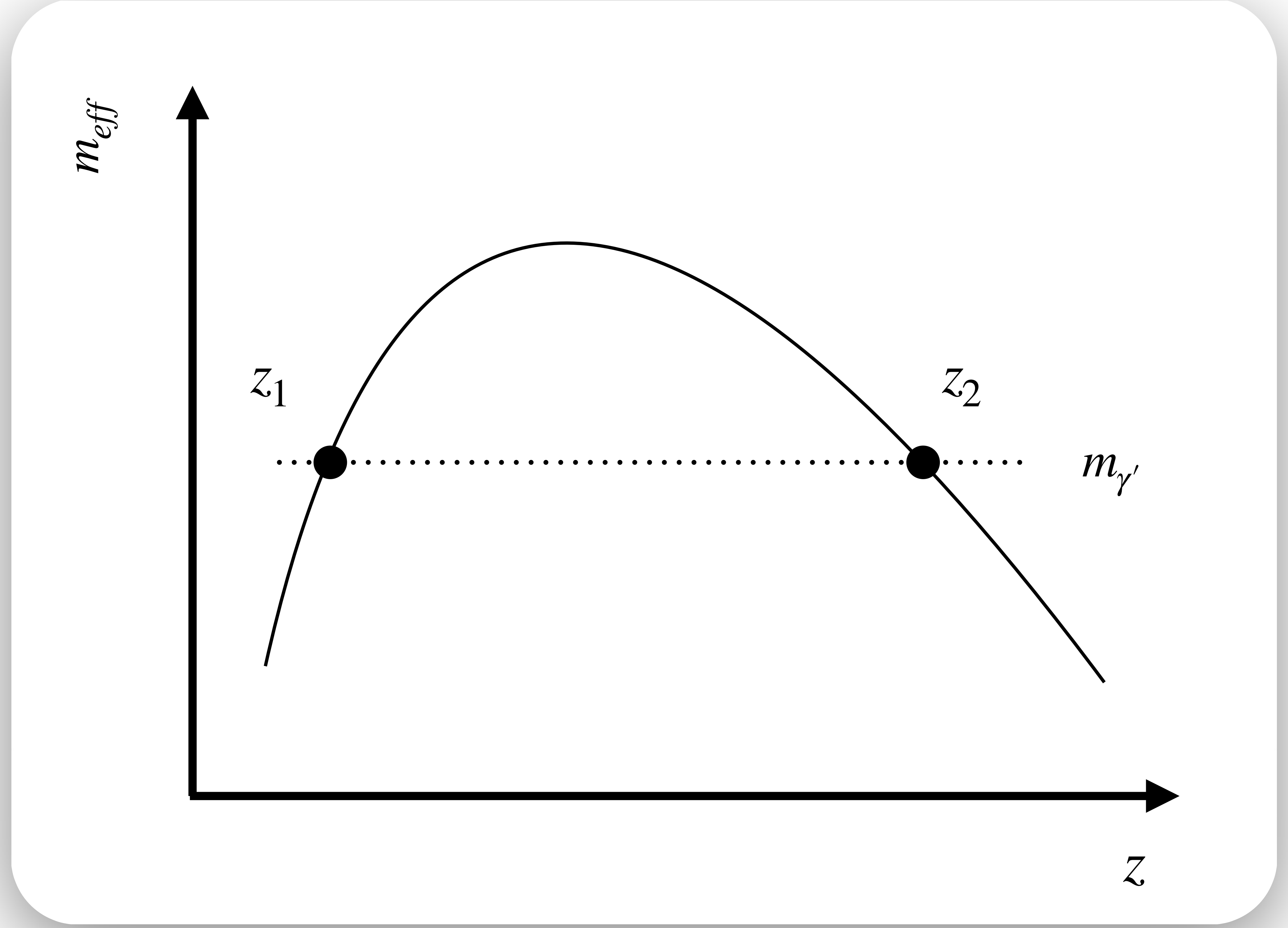

Non-monotonic profiles and multiple resonances

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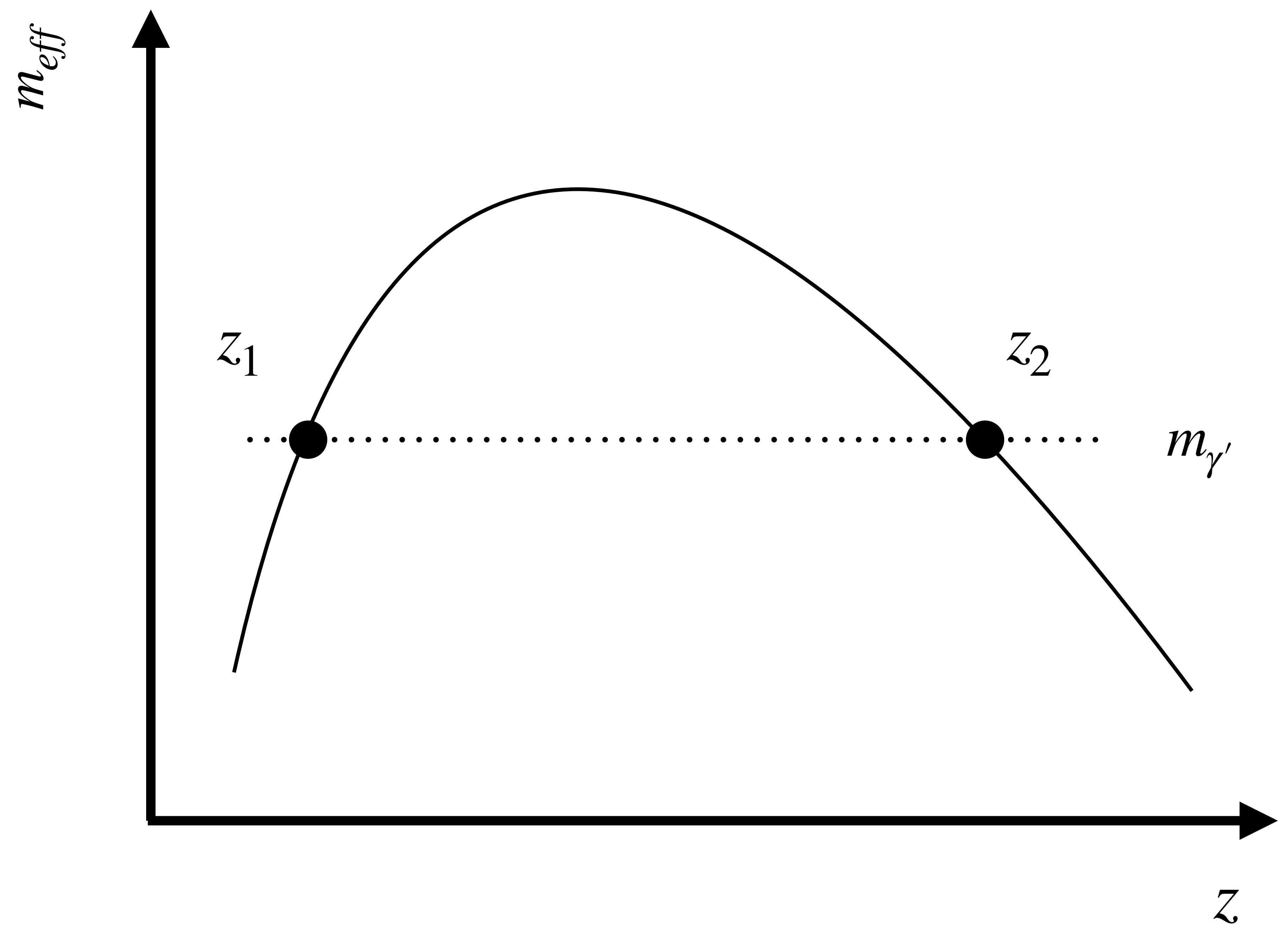
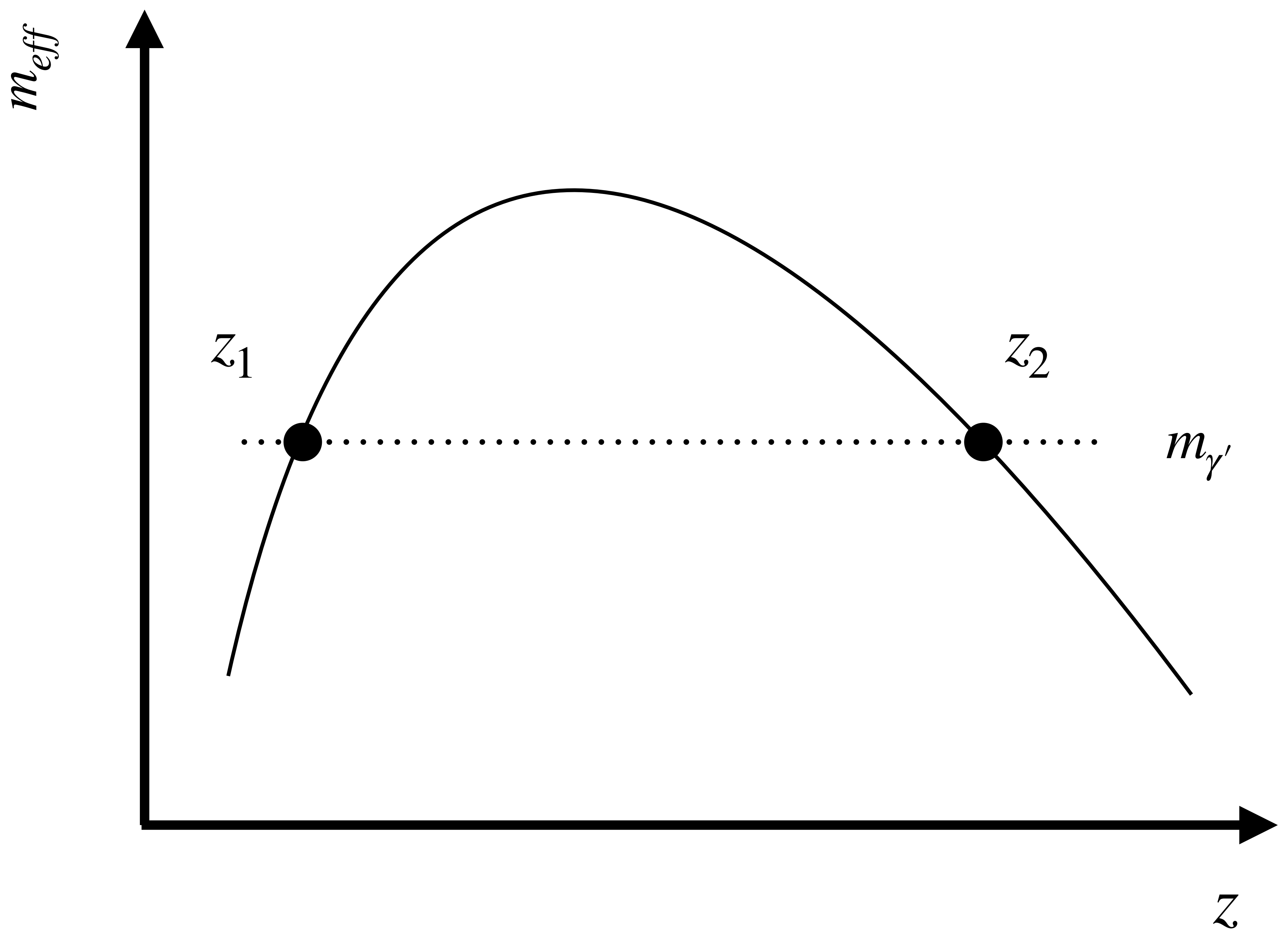


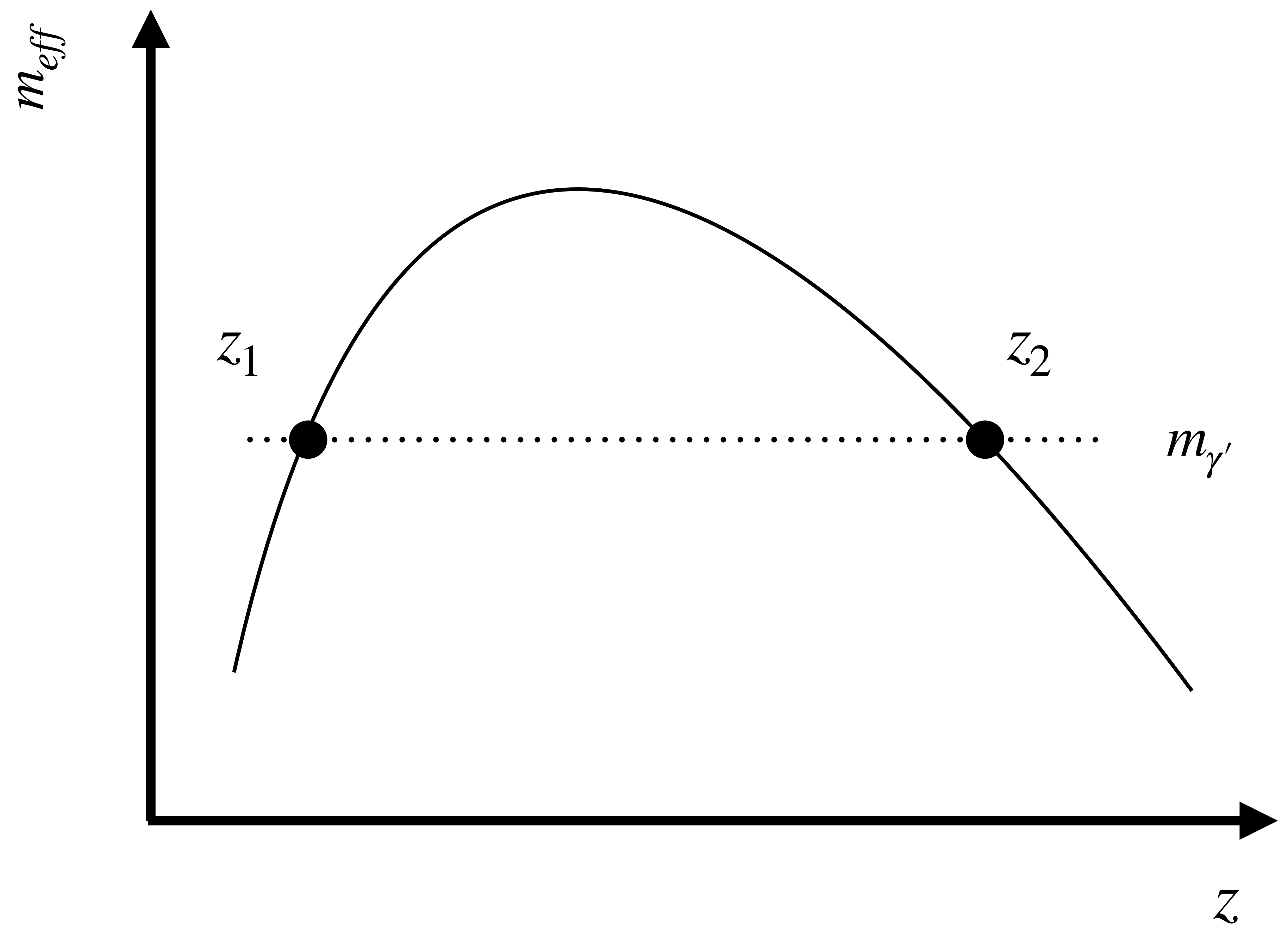
Non-monotonic profiles and multiple resonances

$$P_{\gamma\leftrightarrow\gamma'} \approx \epsilon^2 \left| \sum_n \sqrt{\frac{2\pi}{\left|\Phi^{(2)}(z_n)\right|}} \frac{m_{\gamma'}^2}{2\omega} e^{-i\Phi(z_n)} \right|^2$$



Non-monotonic profiles and multiple resonances

$$P_{\gamma\leftrightarrow\gamma'} = \epsilon^2 \left(\sum_n A_n^2 + 2 \sum_{n<k} A_n A_k \cos \Phi_{nk} \right)$$



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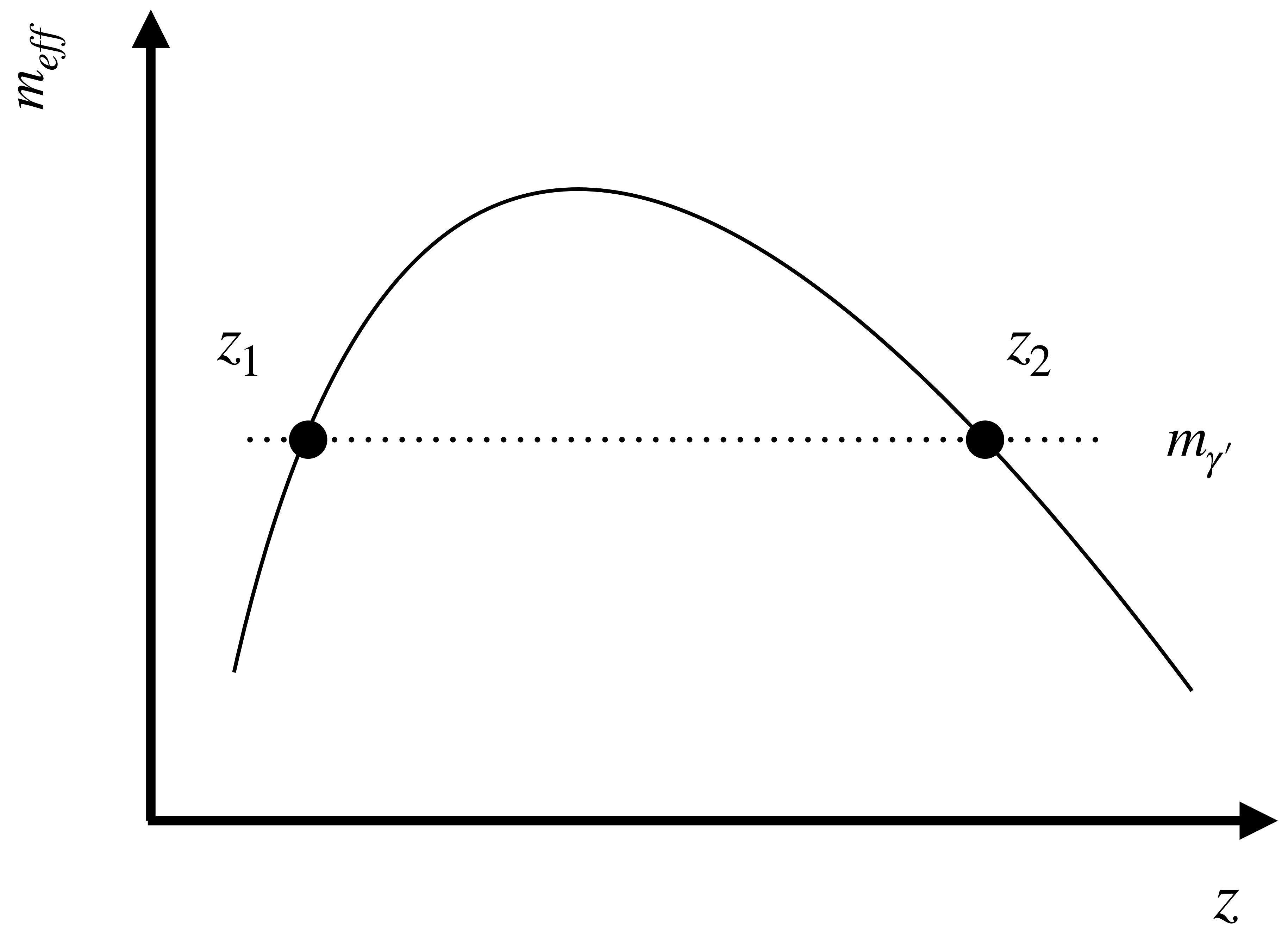
"Sum of LZ"



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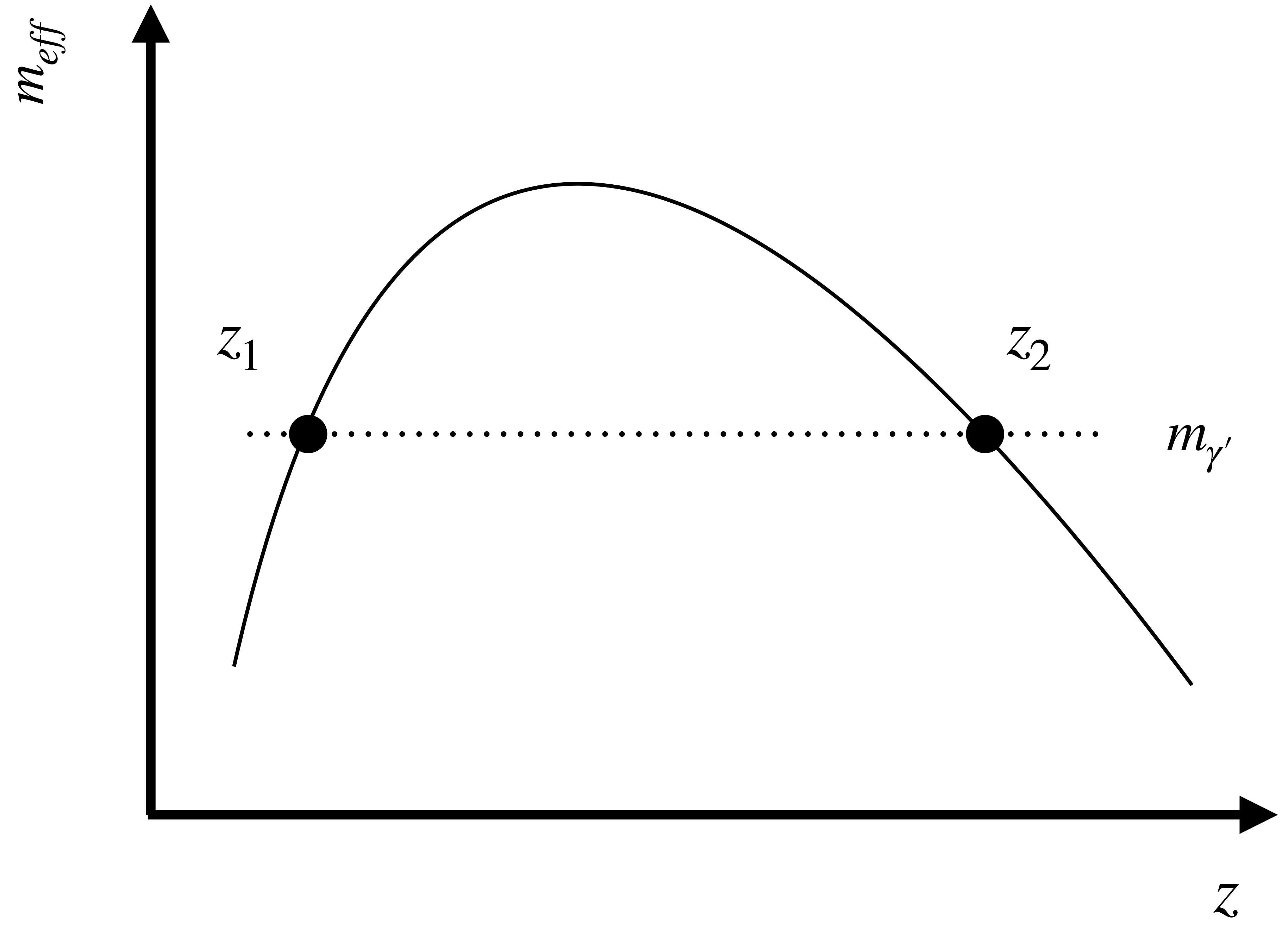
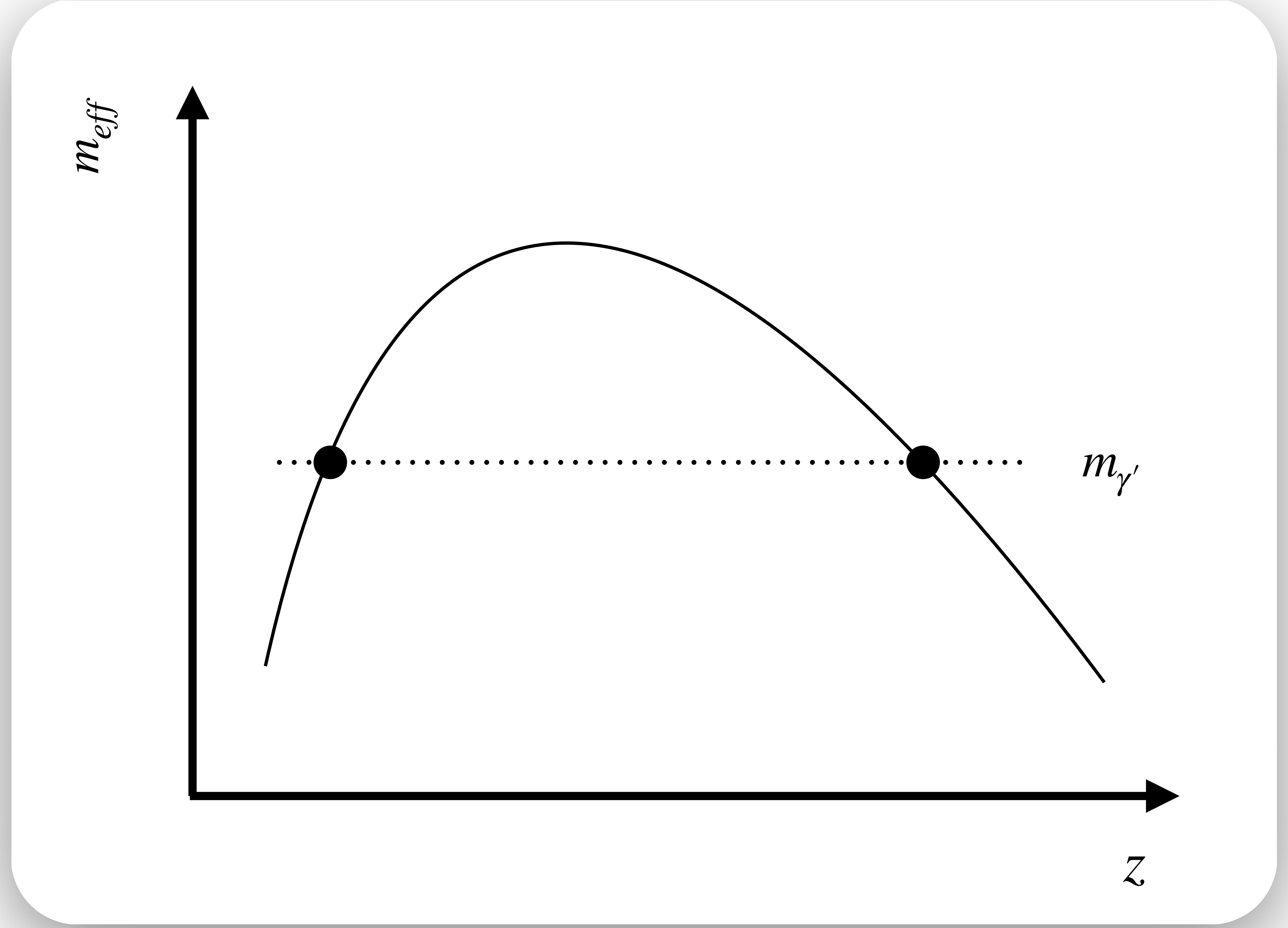
"Phase effects"

Dashgupta & Dighe (2007)



Breakdown of LZ



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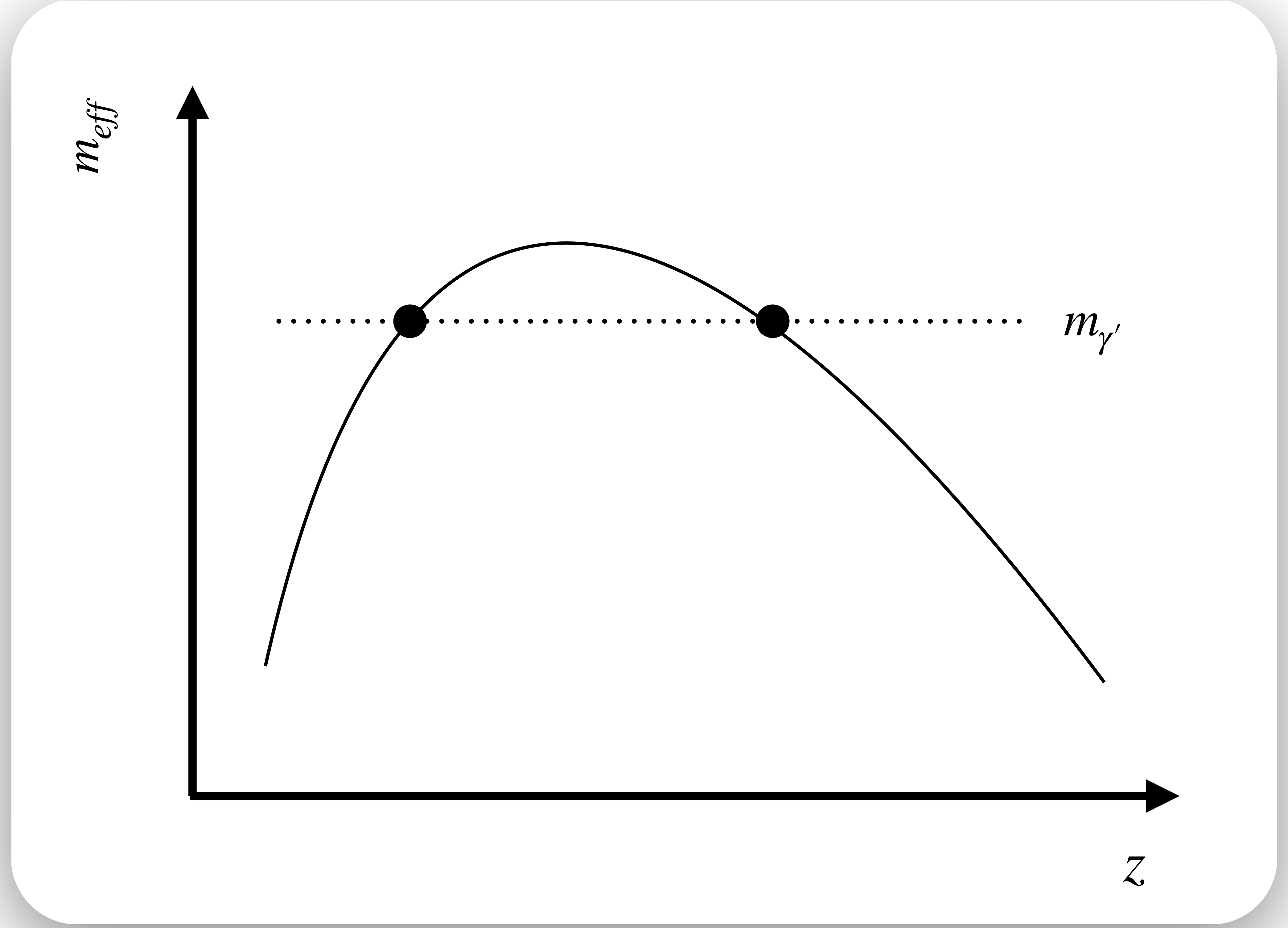
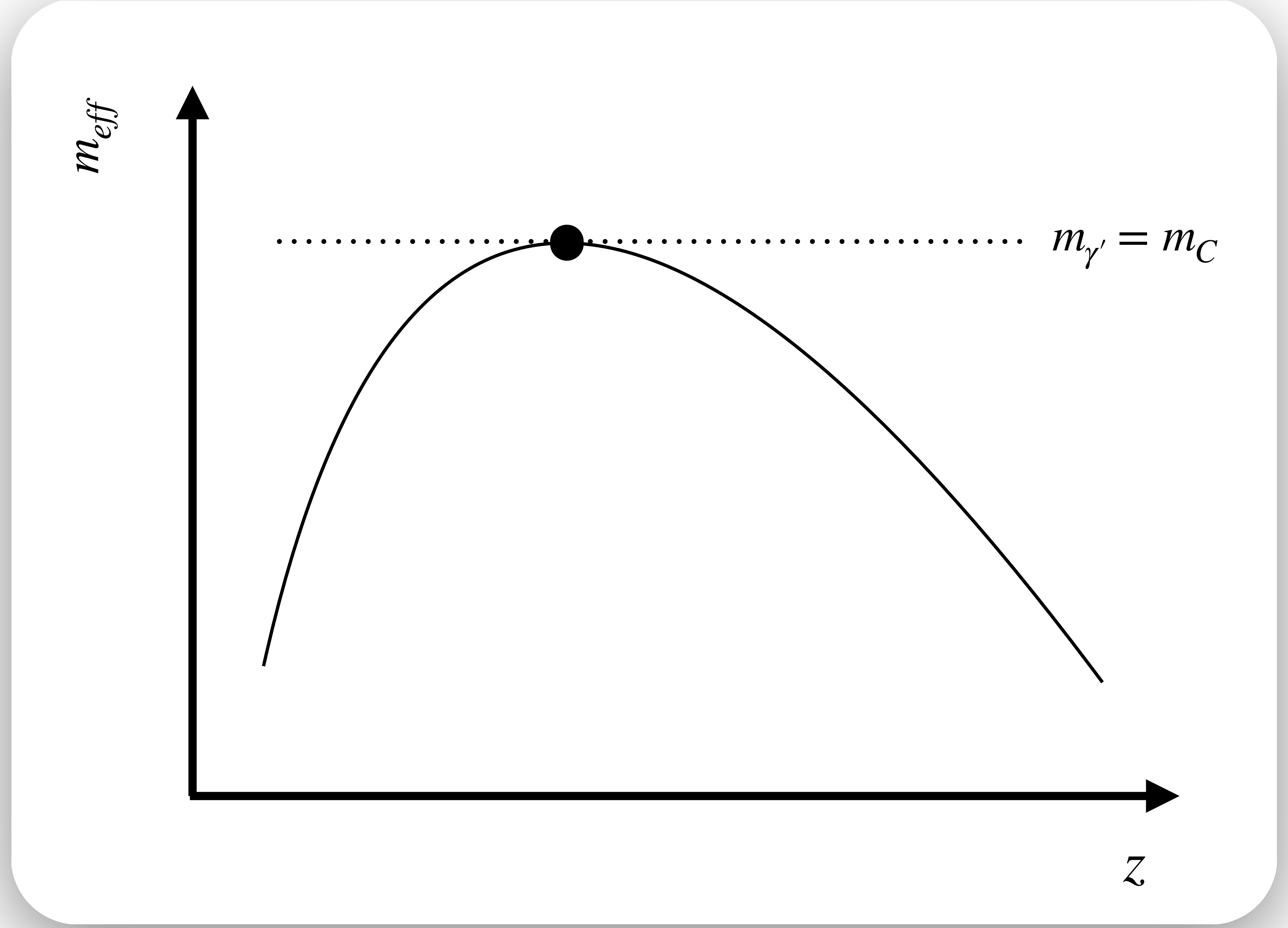


Breakdown of LZ

"Critical point"



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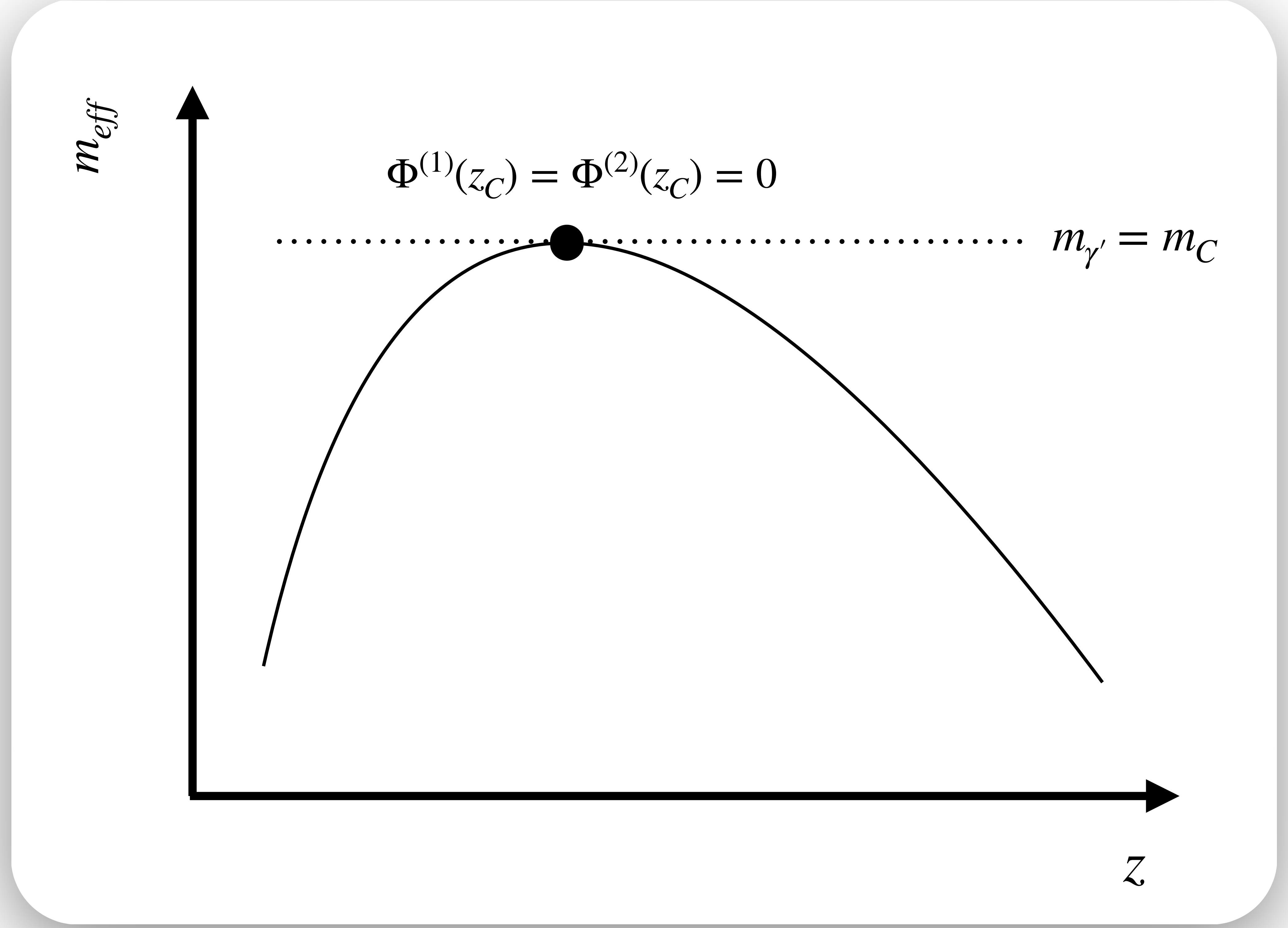
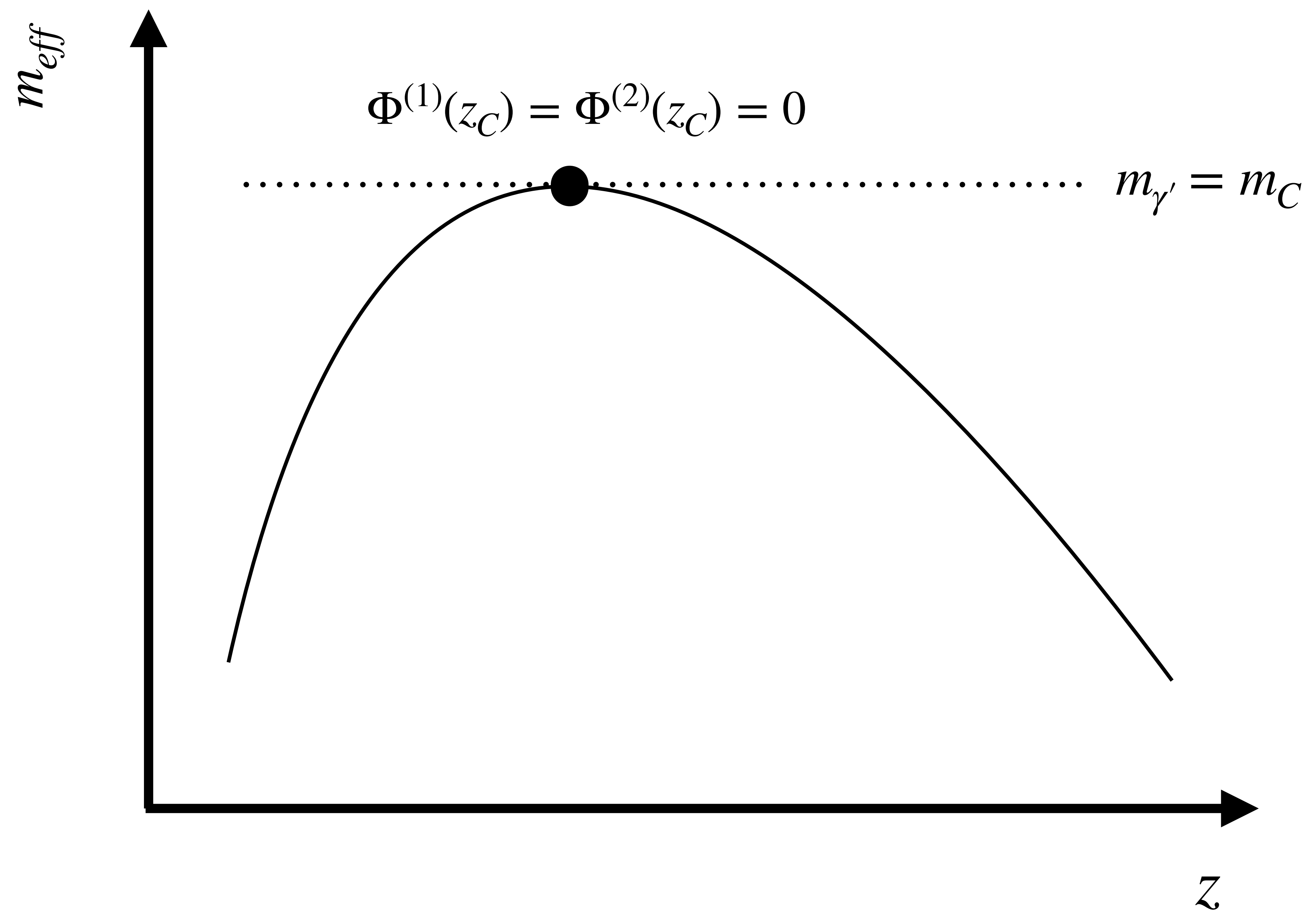


Breakdown of LZ

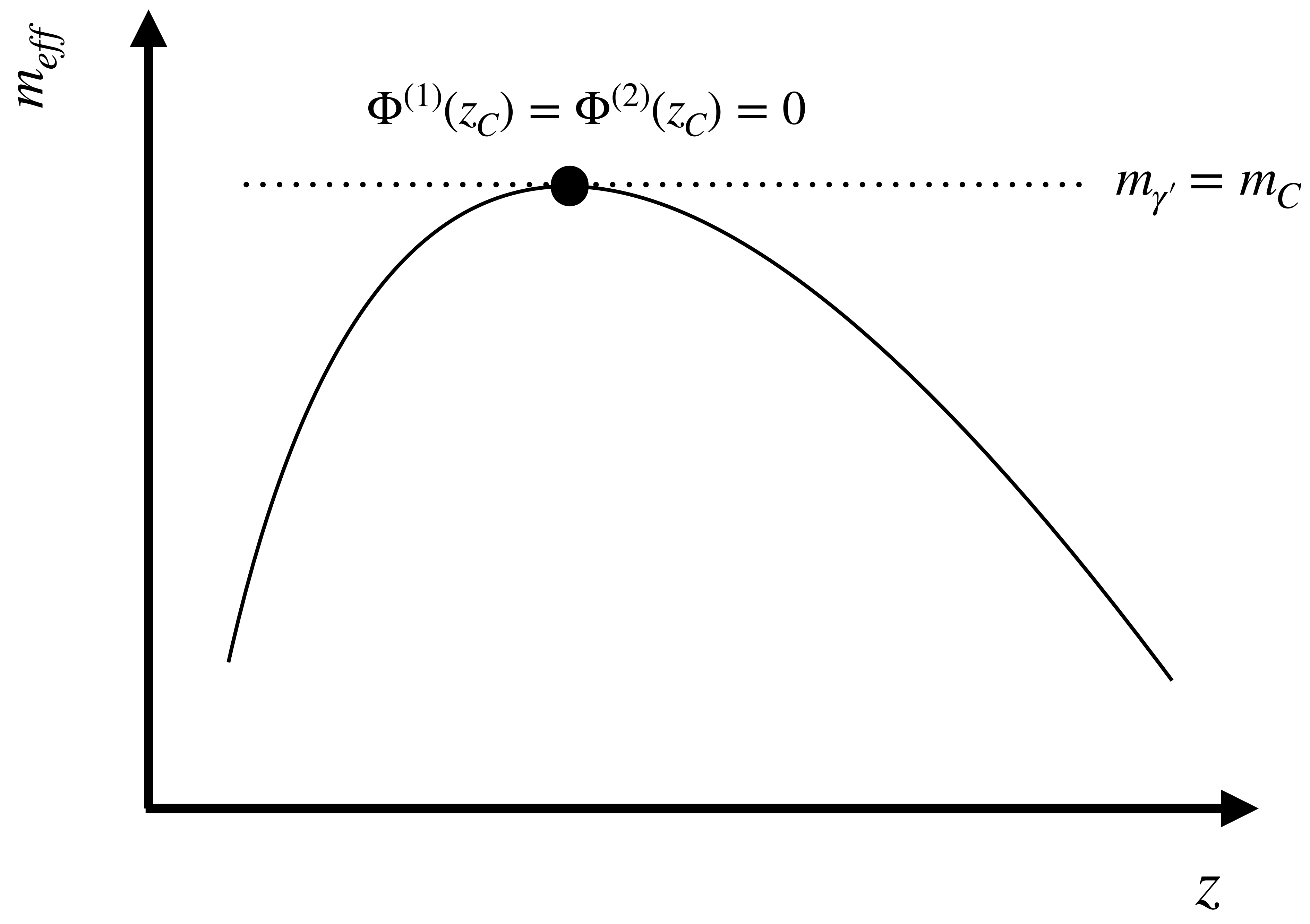
$$A_n \equiv \sqrt{\frac{2\pi}{\left|\Phi^{(2)}(z_n)\right|}}\left(\frac{m_{\gamma'}^2}{2\omega}\right)$$



Breakdown of LZ

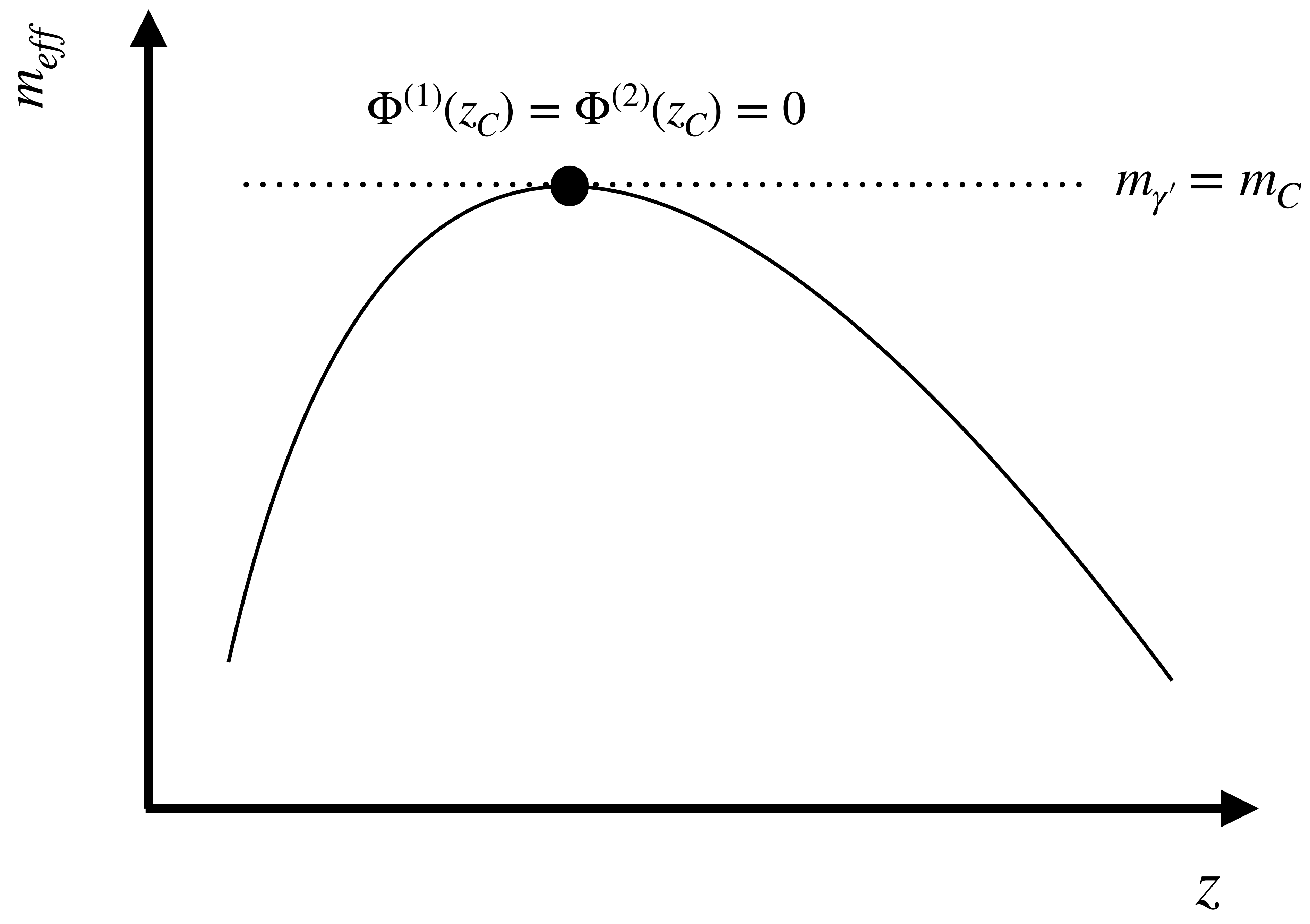
$$A_n \equiv \sqrt{\frac{2\pi}{\left|\Phi^{(2)}(z_n)\right|}} \left(\frac{m_{\gamma'}^2}{2\omega}\right) \longrightarrow \infty$$

"Breakdown of LZ"



Breakdown of LZ

$$P_{\gamma\leftrightarrow\gamma'} = \epsilon^2 \left| \int_{z_i}^{z} dz' \frac{m_{\gamma'}^2}{2\omega} e^{-i\Phi(z')} \right|^2$$

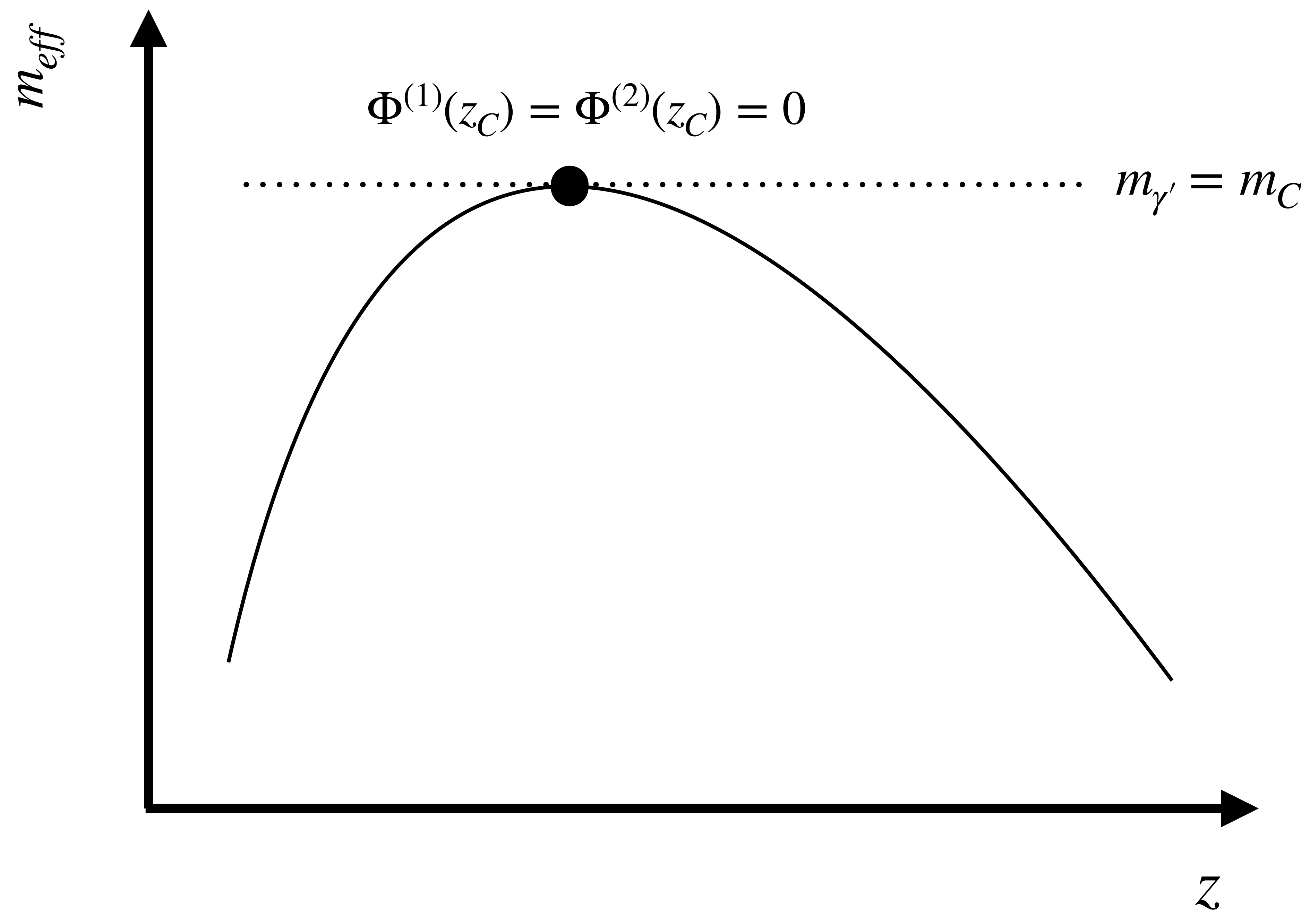


Coalescing saddle points

$$P_{\gamma\leftrightarrow\gamma'} \approx \epsilon^2 \left| 2\pi \left(\frac{2}{|\Phi^{(3)}(z_C)|} \right)^{1/3} \frac{m_{\gamma'}^2}{2\omega} \left(\mathsf{Ai}\left(-\zeta\right) + i\,\#\,\mathsf{Ai}^{(1)}(-\zeta) \right) \right|^2$$

Ai → Airy function

$$\zeta \sim \left(\frac{2}{|\Phi^{(3)}|} \right)^{1/3} \Phi^{(1)}$$



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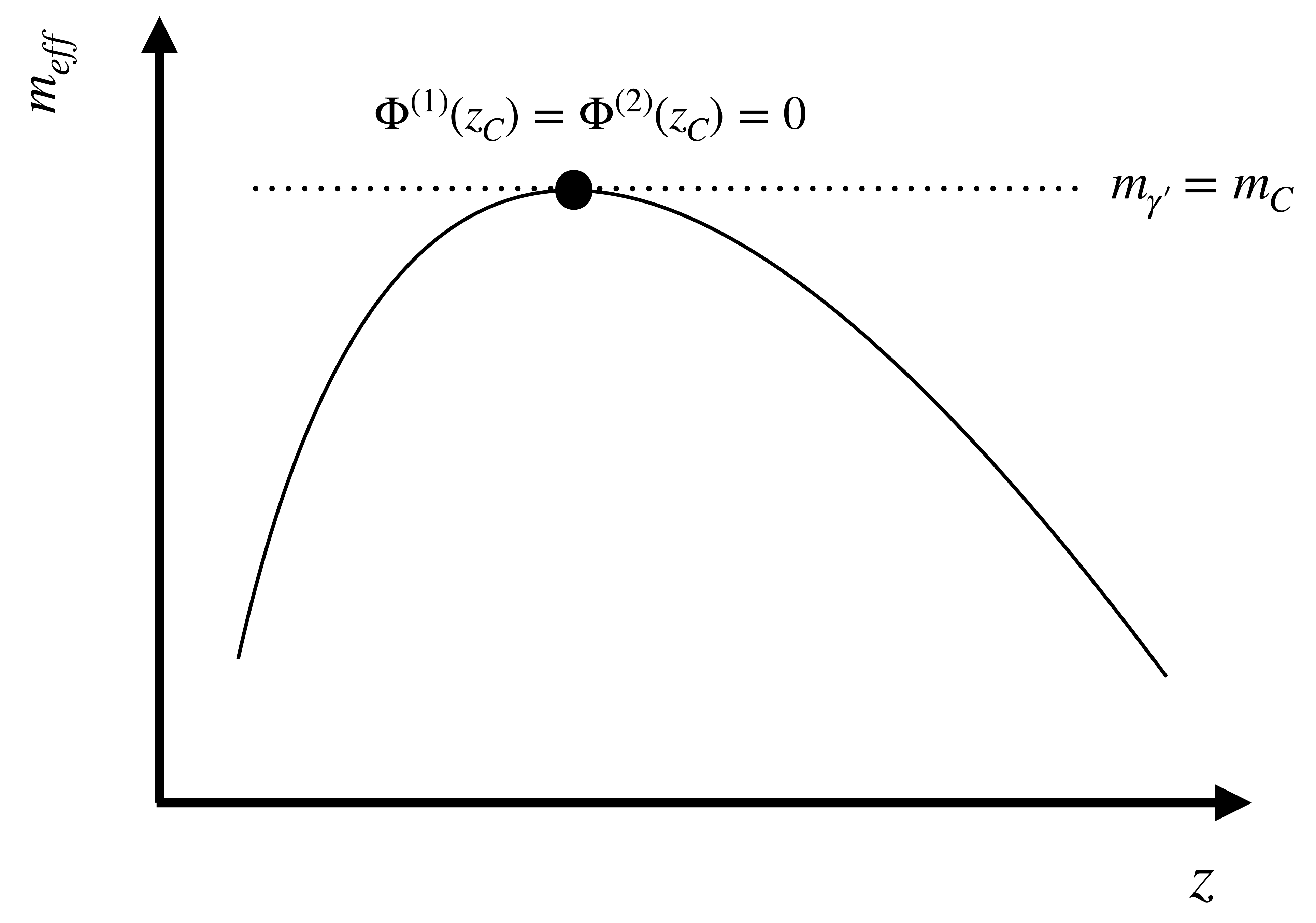
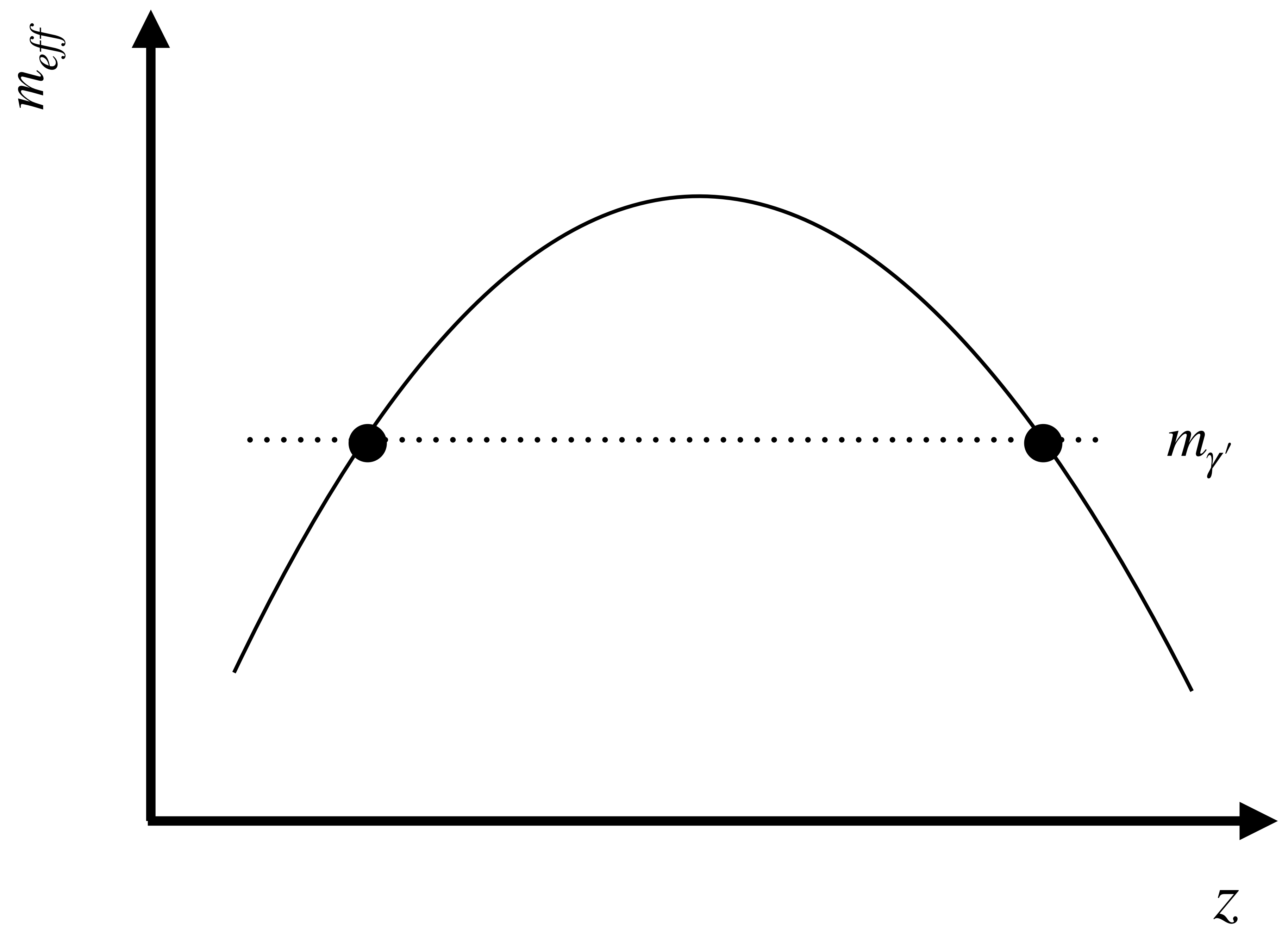


NB, Asher Berlin, Katelin Schutz (PRD 2023)

Toy model

$$m_{eff}^2(z) = b^2\left[1-\left(\frac{z}{a}-1\right)^2\right]$$



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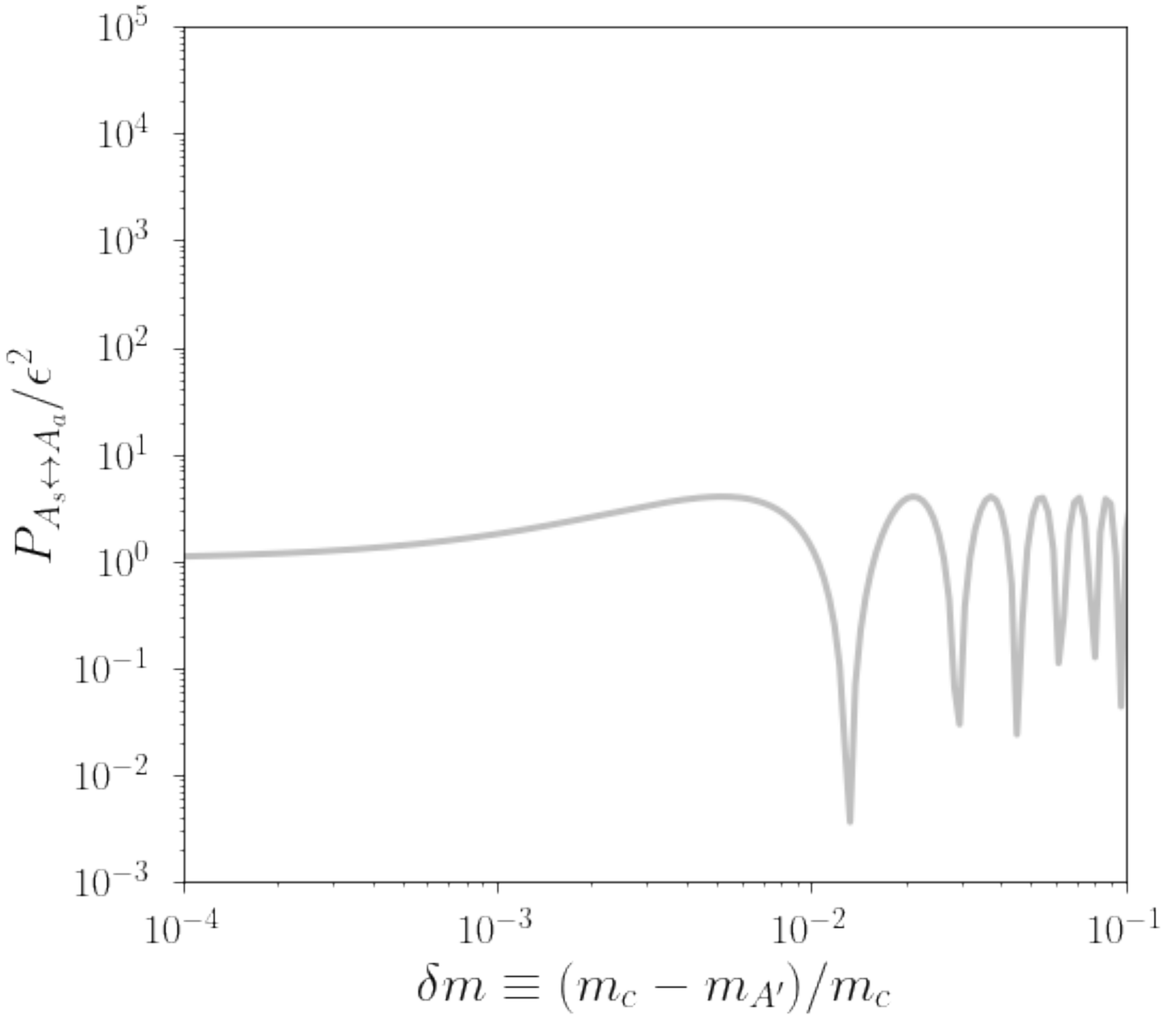

vacuum

$a = 2000, b = 10$

Toy model

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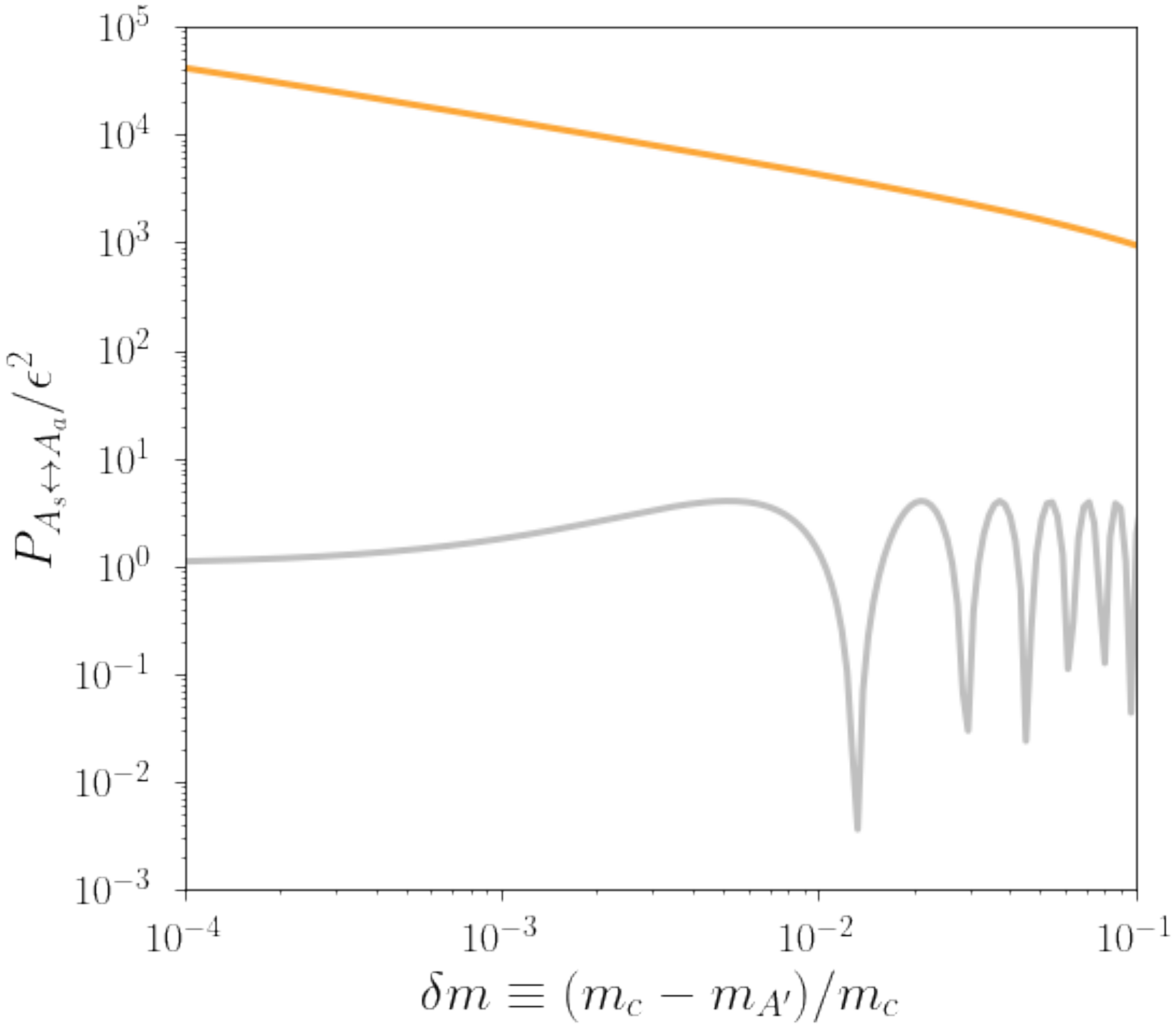

vacuum LZ

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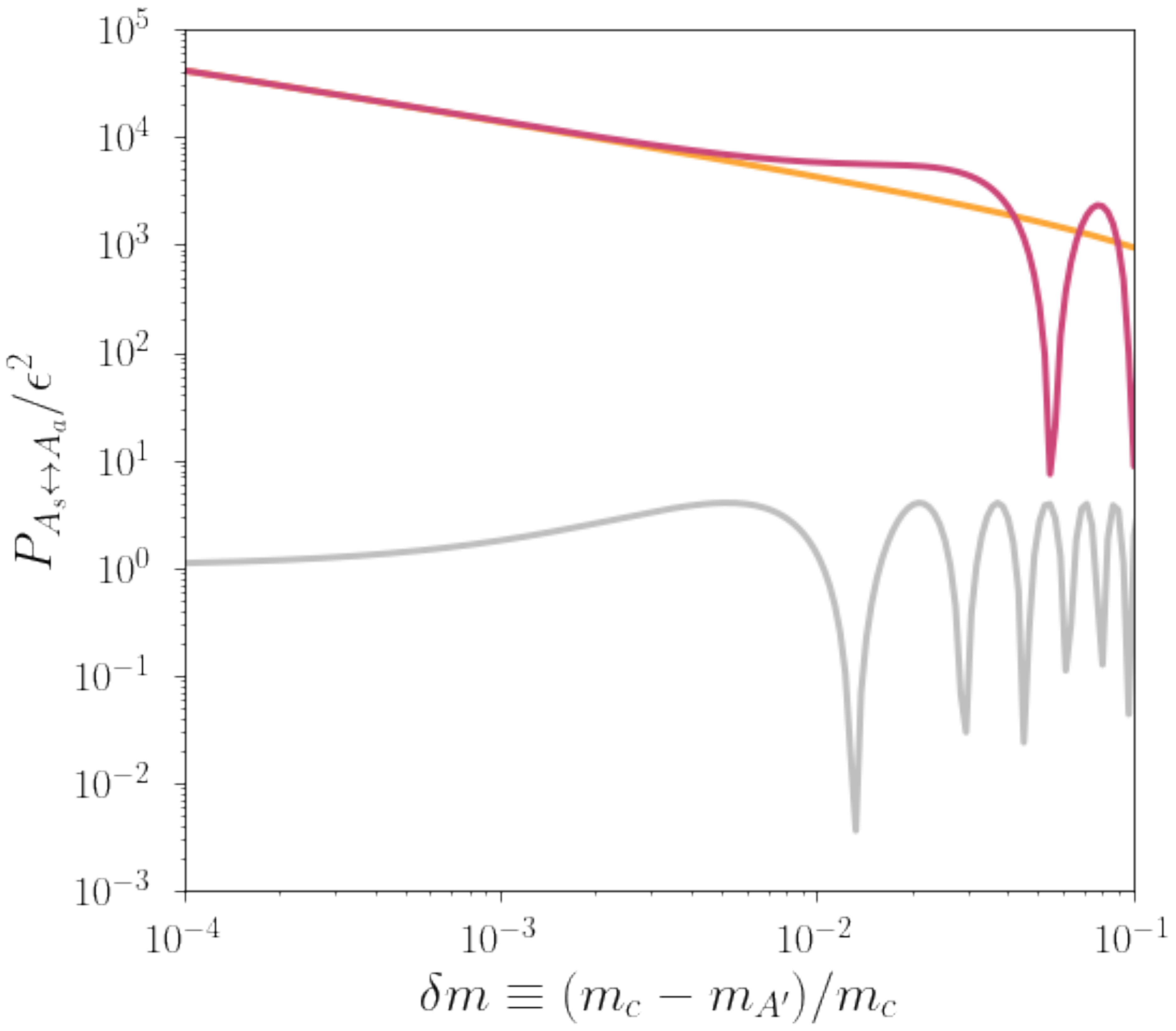

vacuum LZ Phase

$a = 2000, b = 10$

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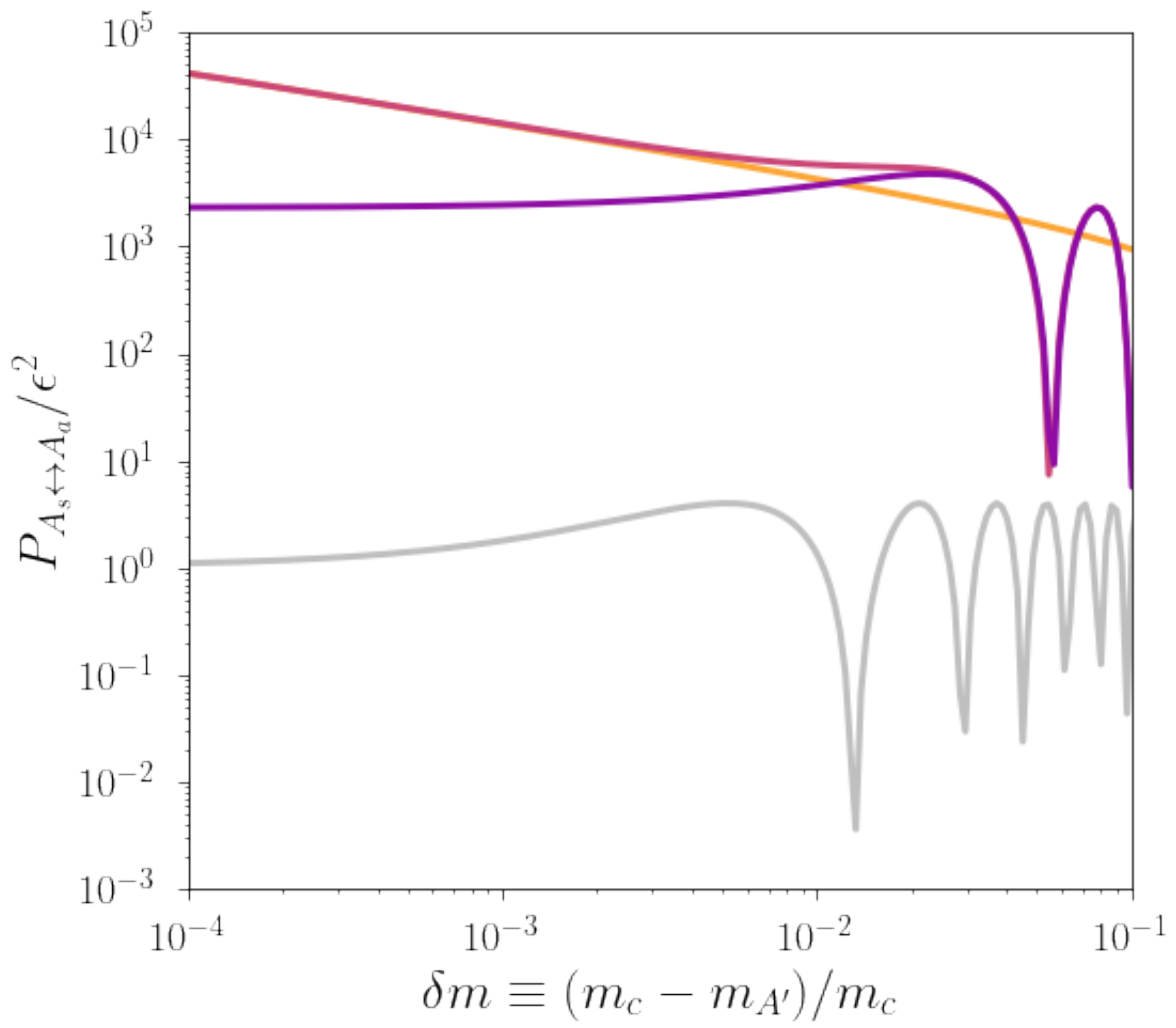

vacuum LZ Phase **This work**

$a = 2000, b = 10$

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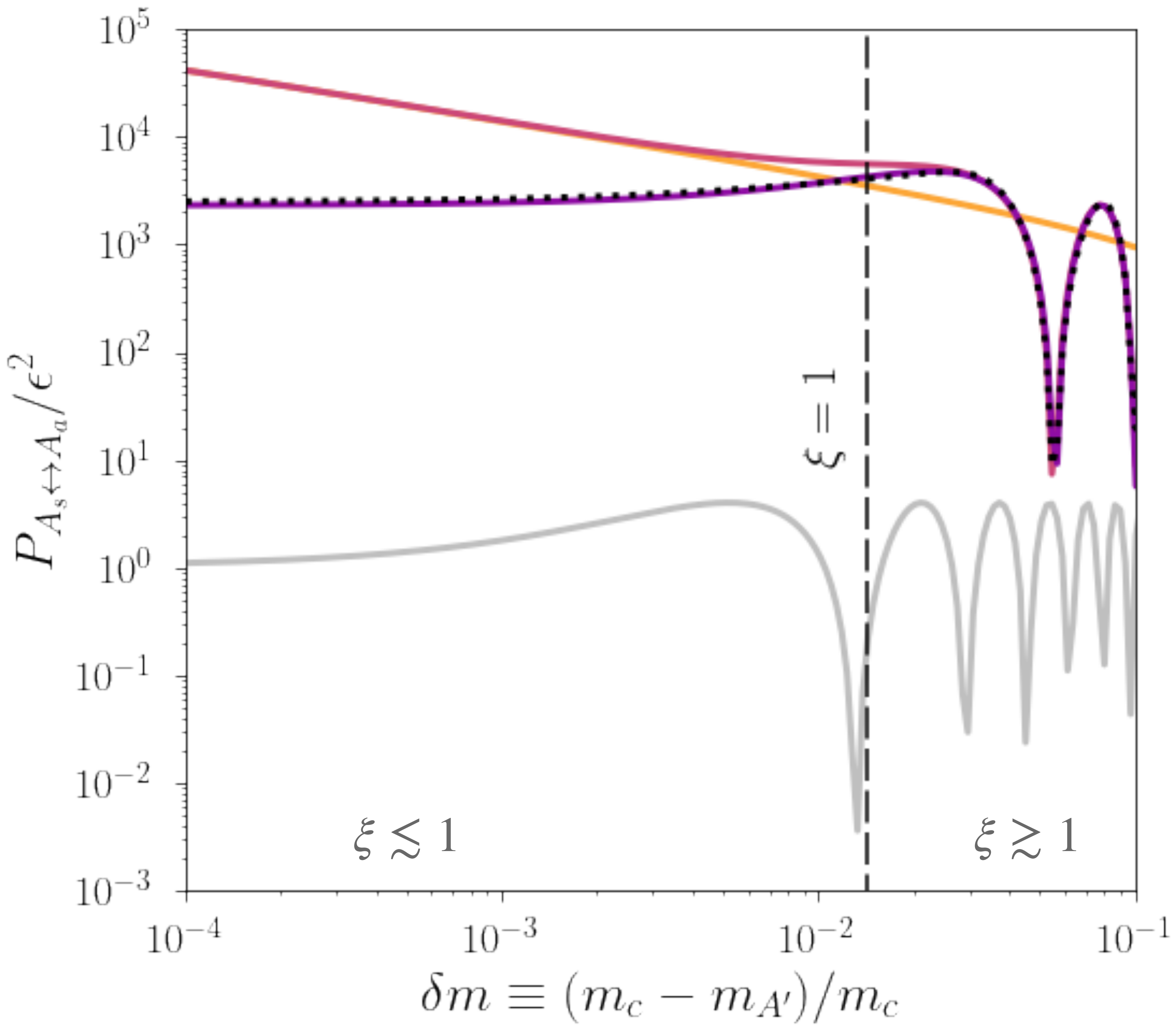


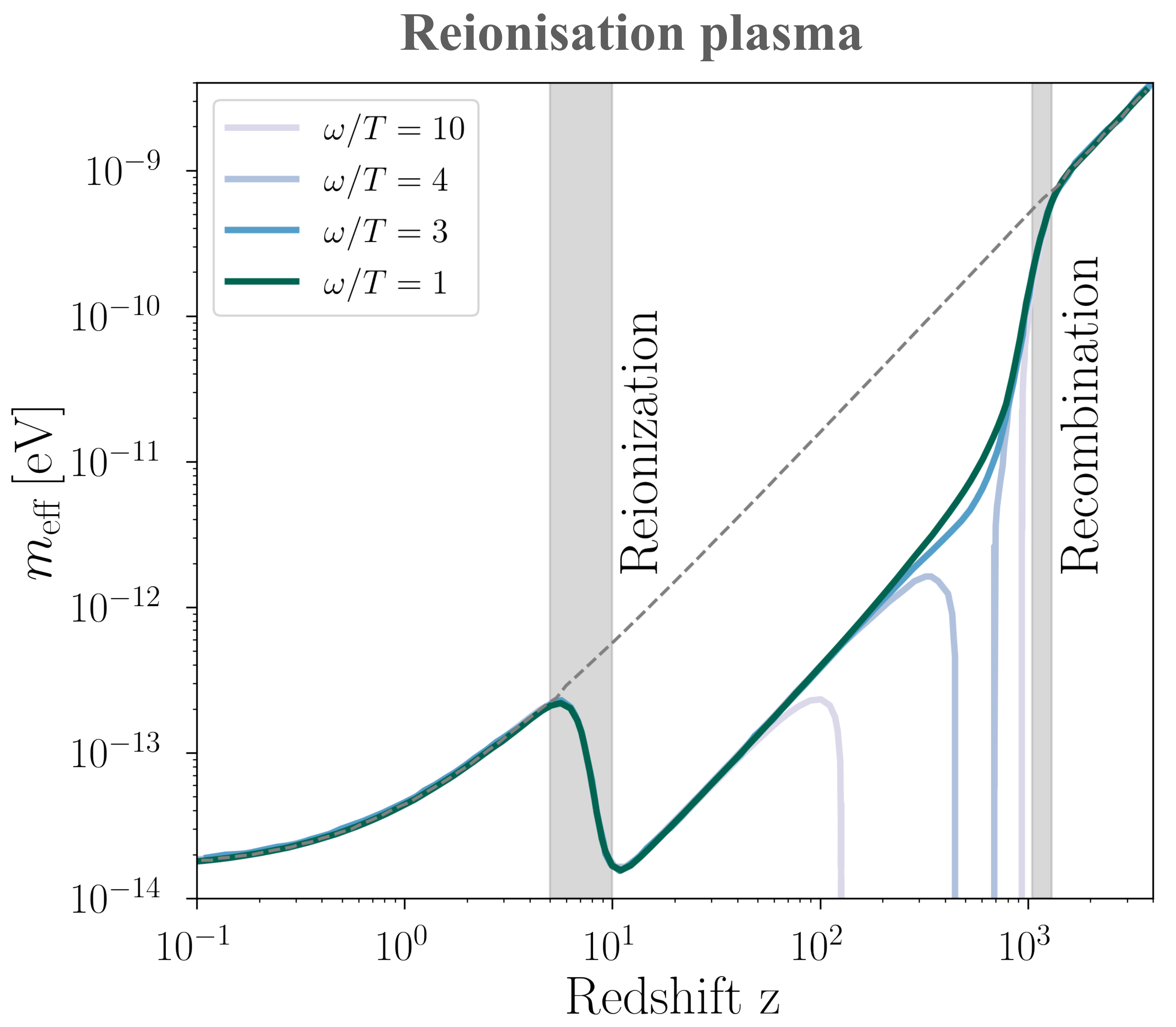
$$\xi \sim \frac{|\Phi^{(2)}(z_C)|}{|\Phi^{(3)}(z_C)|^{2/3}}$$

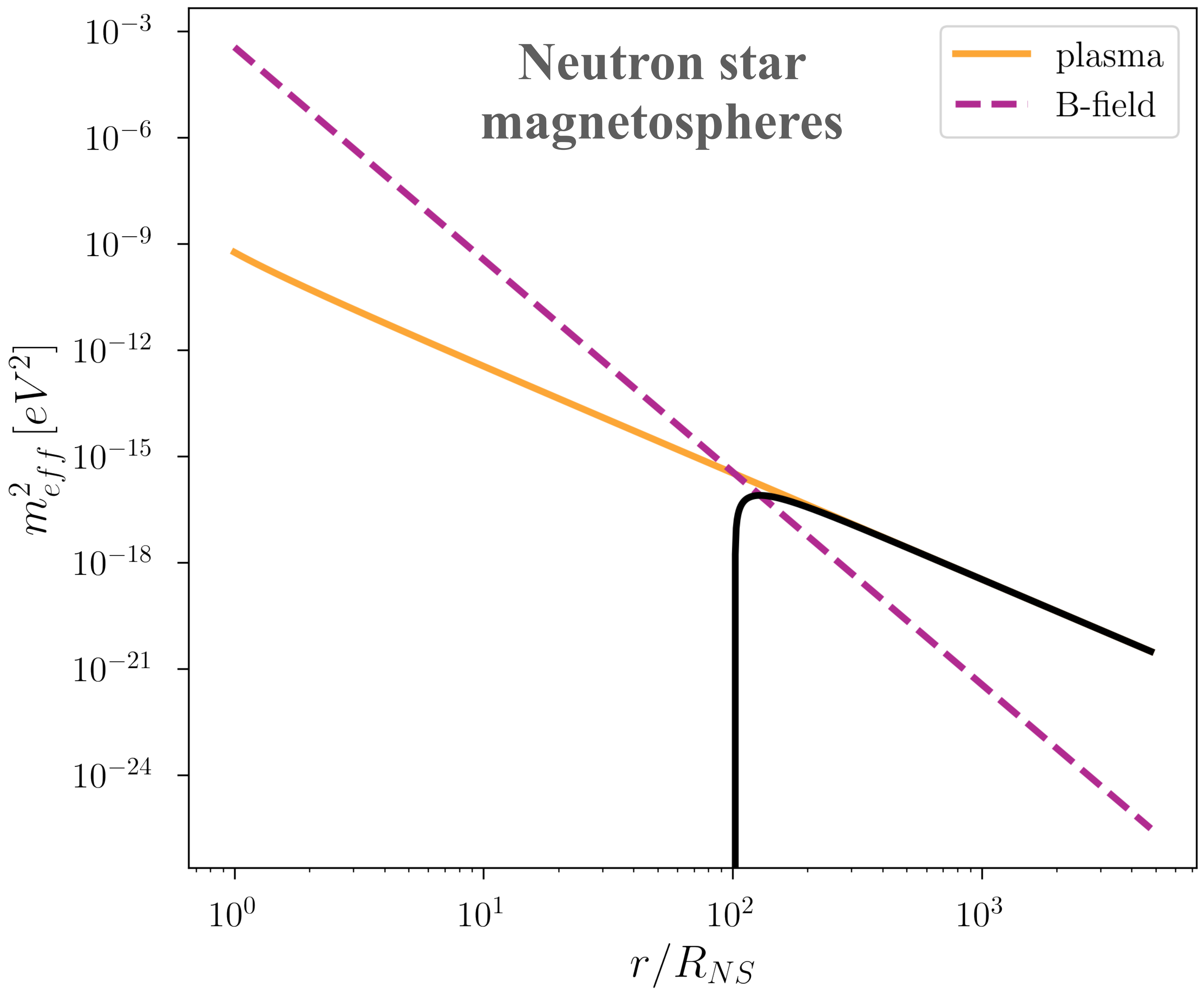
Vacuum LZ Phase **This work** Numerical

$a = 2000, b = 10$

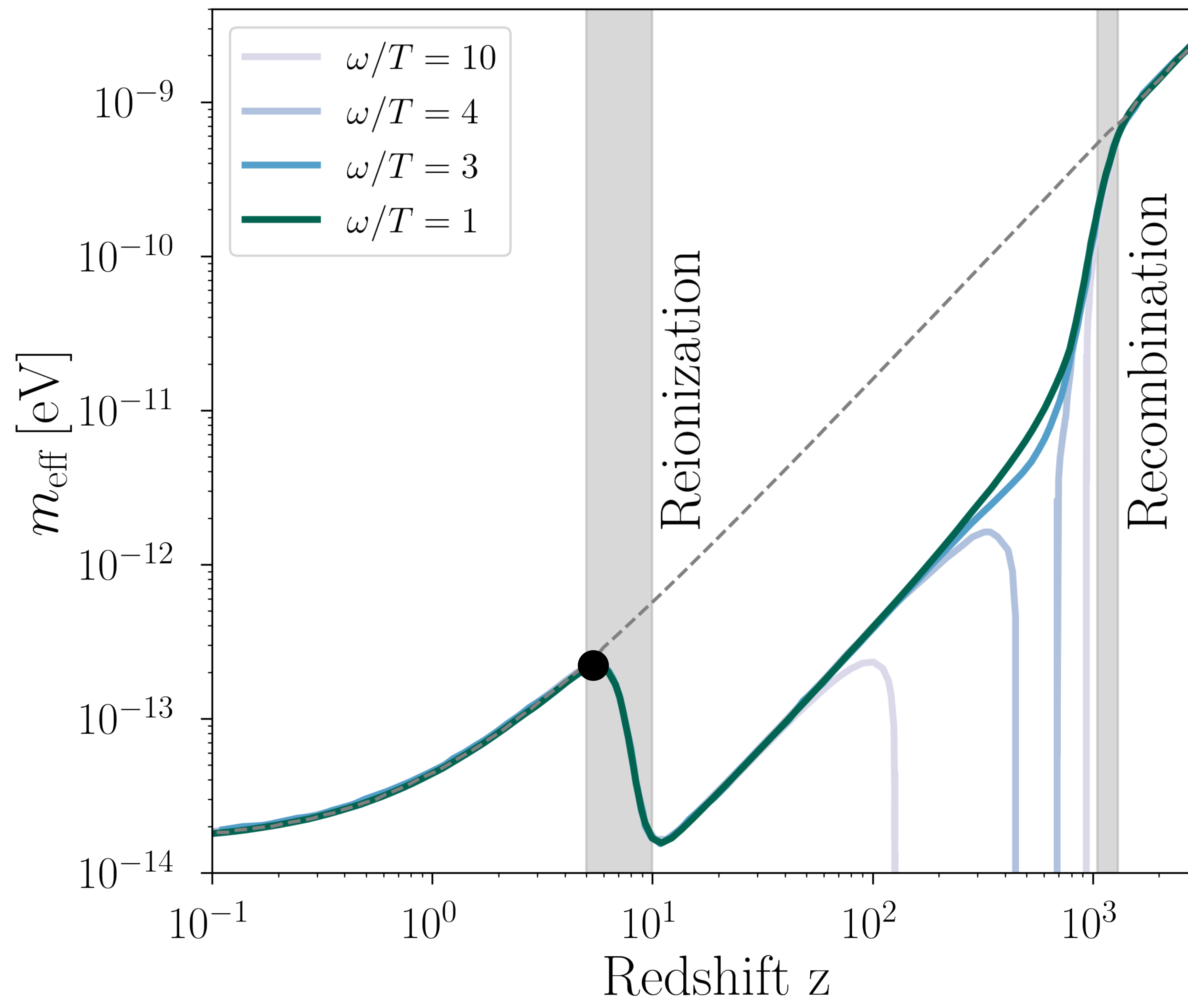
NB, Asher Berlin, Katelin Schutz (PRD 2023)

Astrophysical examples



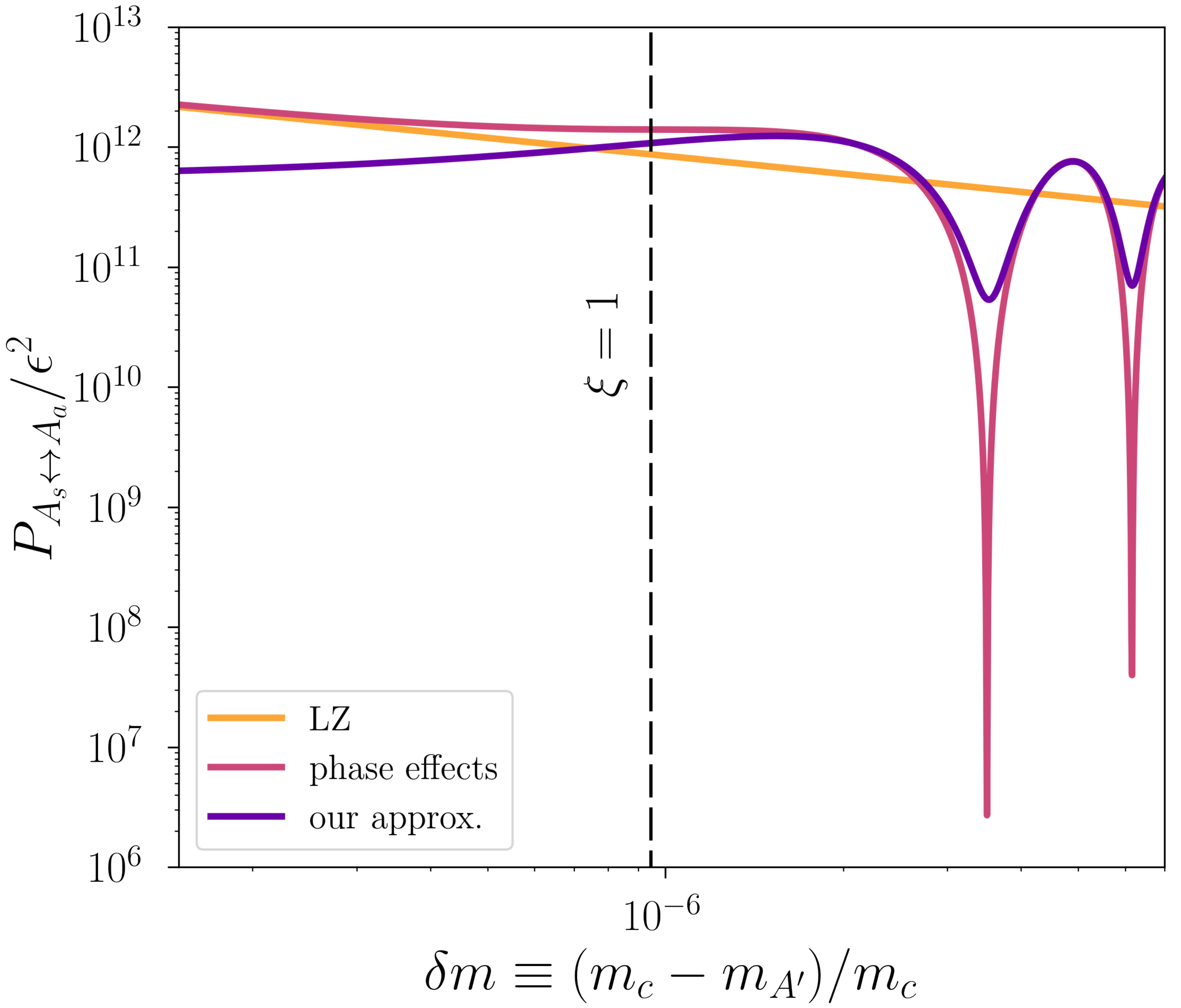


Reionisation plasma



Mirizzi et al. (2009), Caputo et al. (2020),
NB, Asher Berlin, Katelin Schutz (PRD 2023)

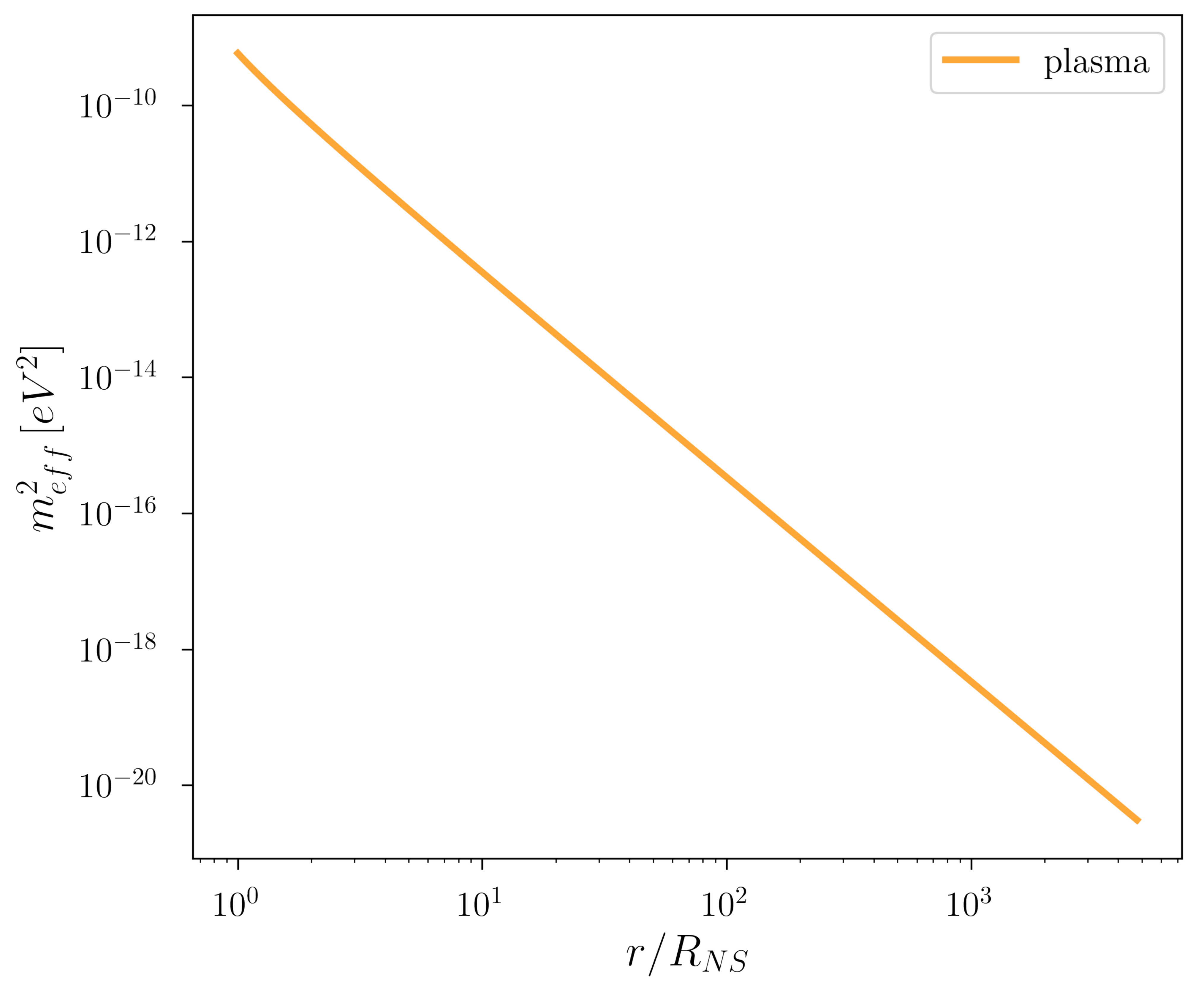
Reionization plasma



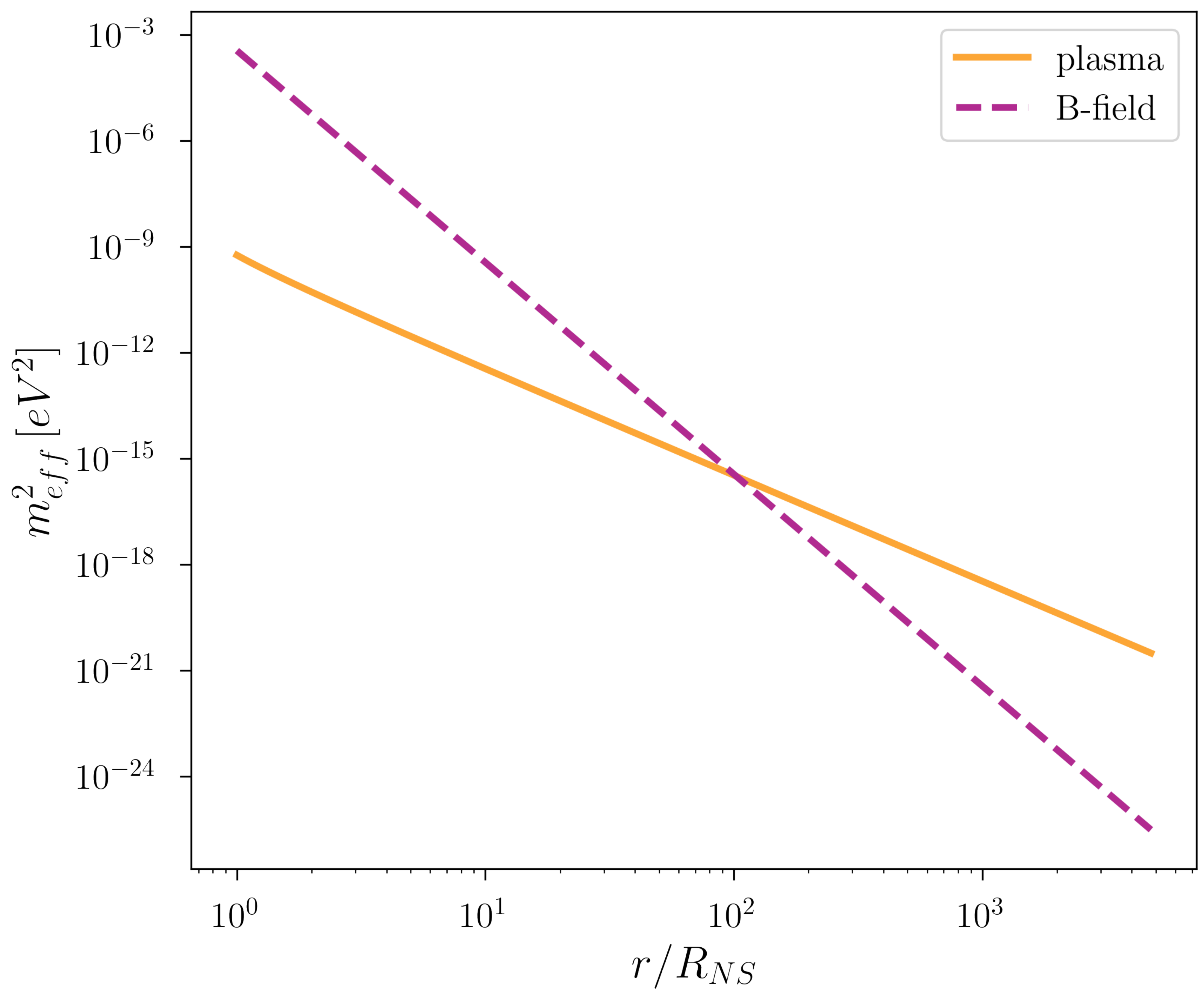
NB, Asher Berlin, Katelin Schutz (PRD 2023)

Neutron star magnetospheres

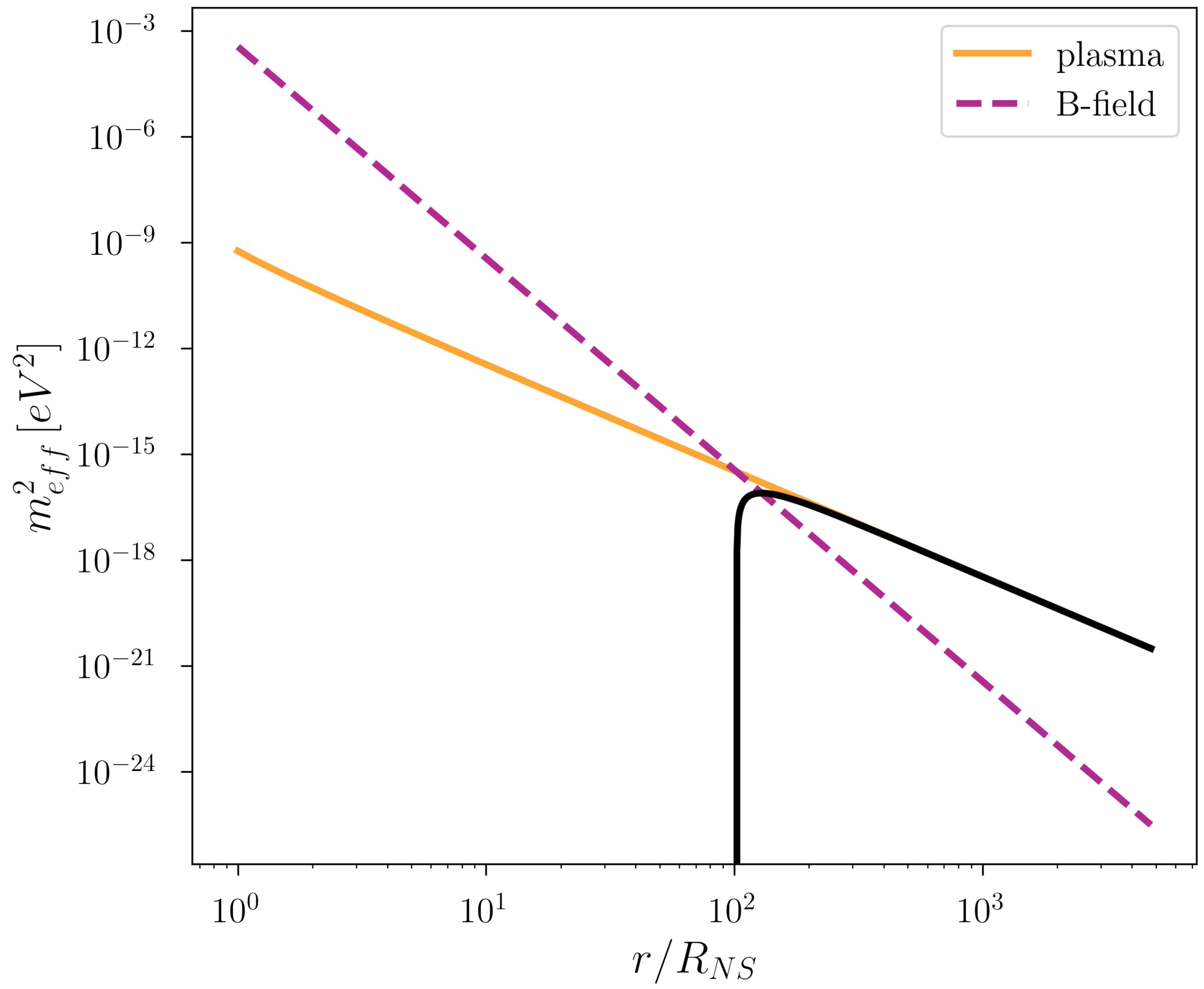
Neutron star magnetospheres



Neutron star magnetospheres

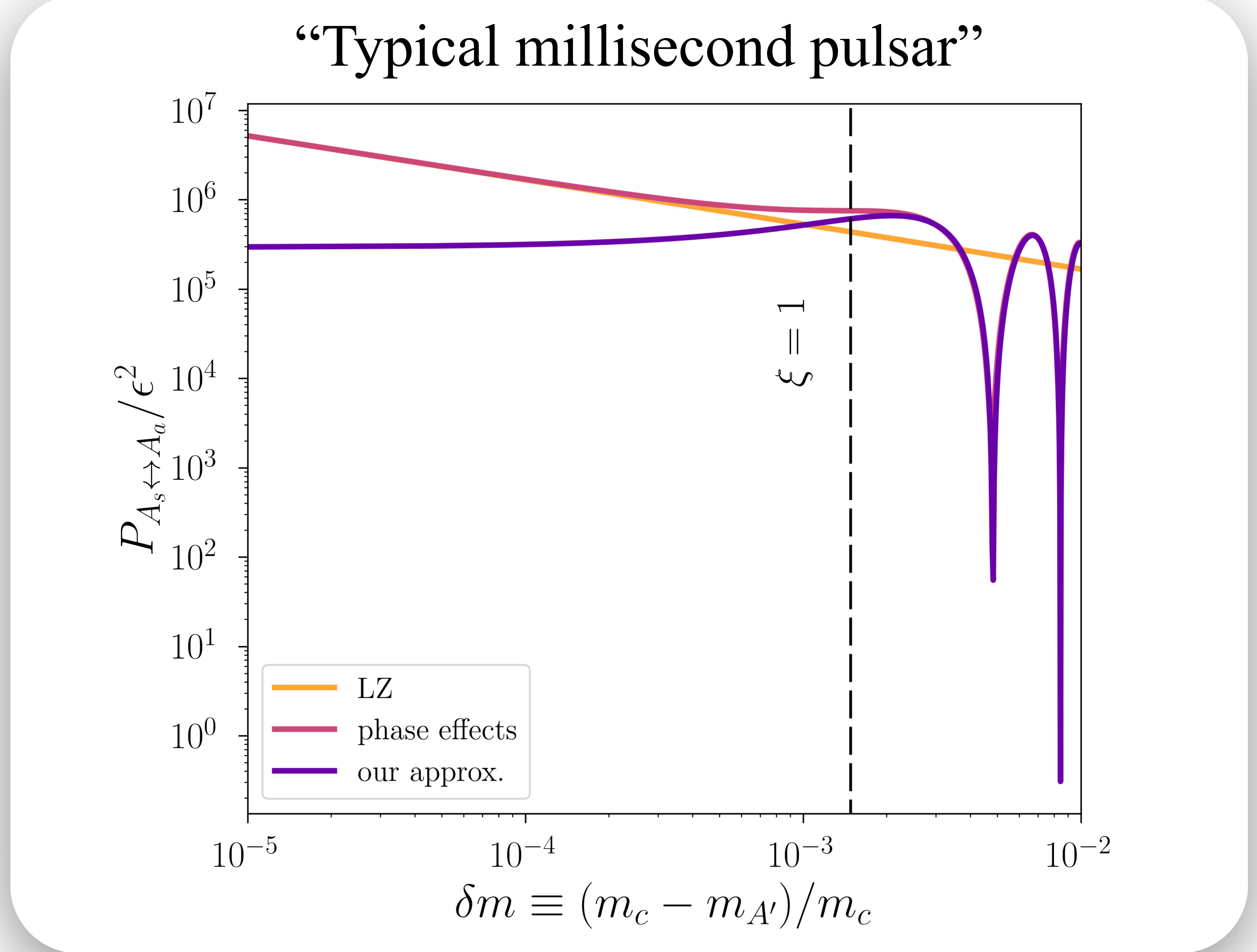


Neutron star magnetospheres



$B_0/B_{crit} = 1, P = 1\text{sec}, \omega = 1\text{eV}$

Neutron star magnetospheres



LZ Phase **This work**

$B_0/B_{crit} \sim 10, P \sim 1$ ms, $\omega \sim 0.1$ eV

NB, Asher Berlin, Katelin Schutz (PRD 2023)

Summary

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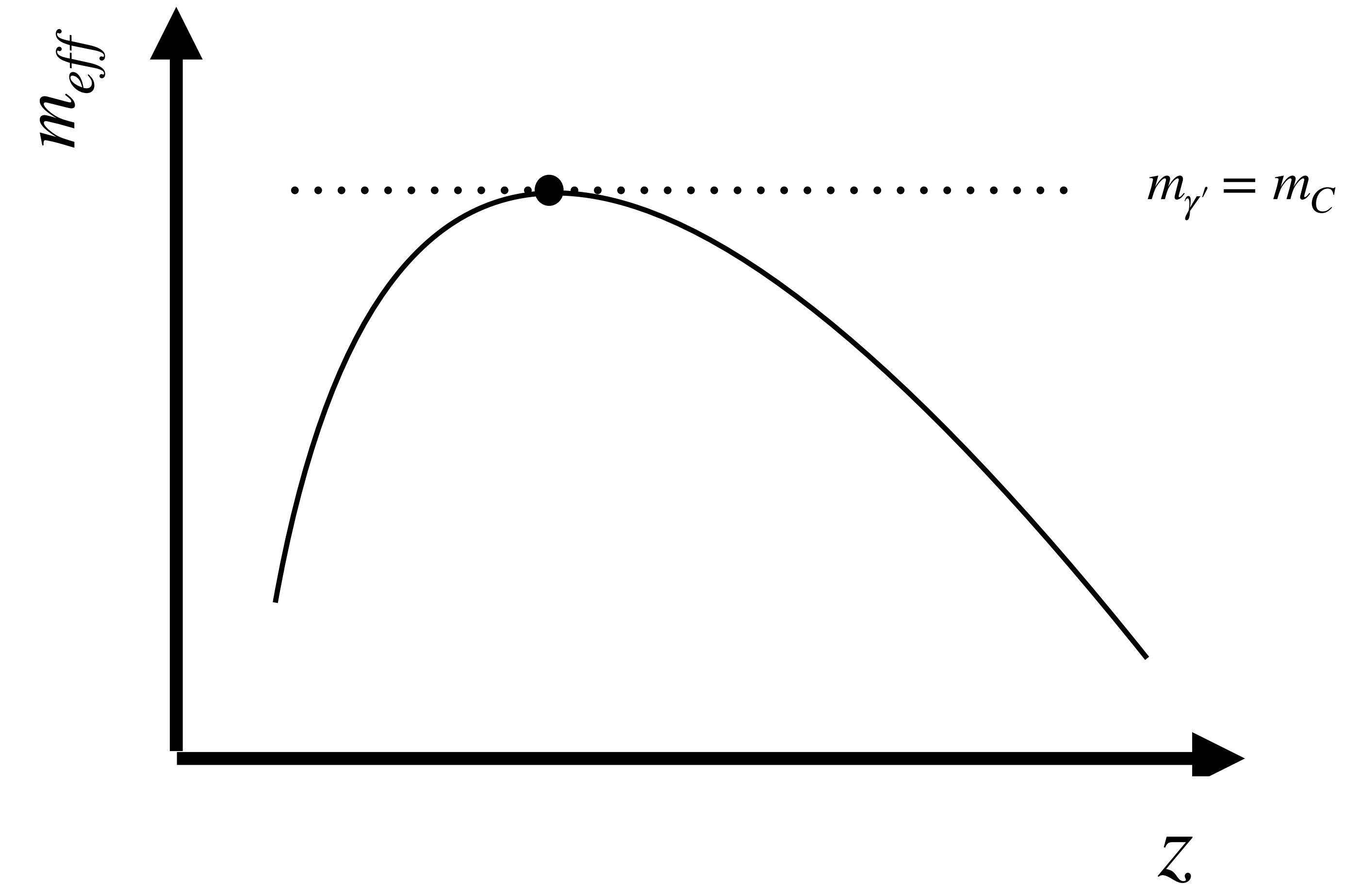
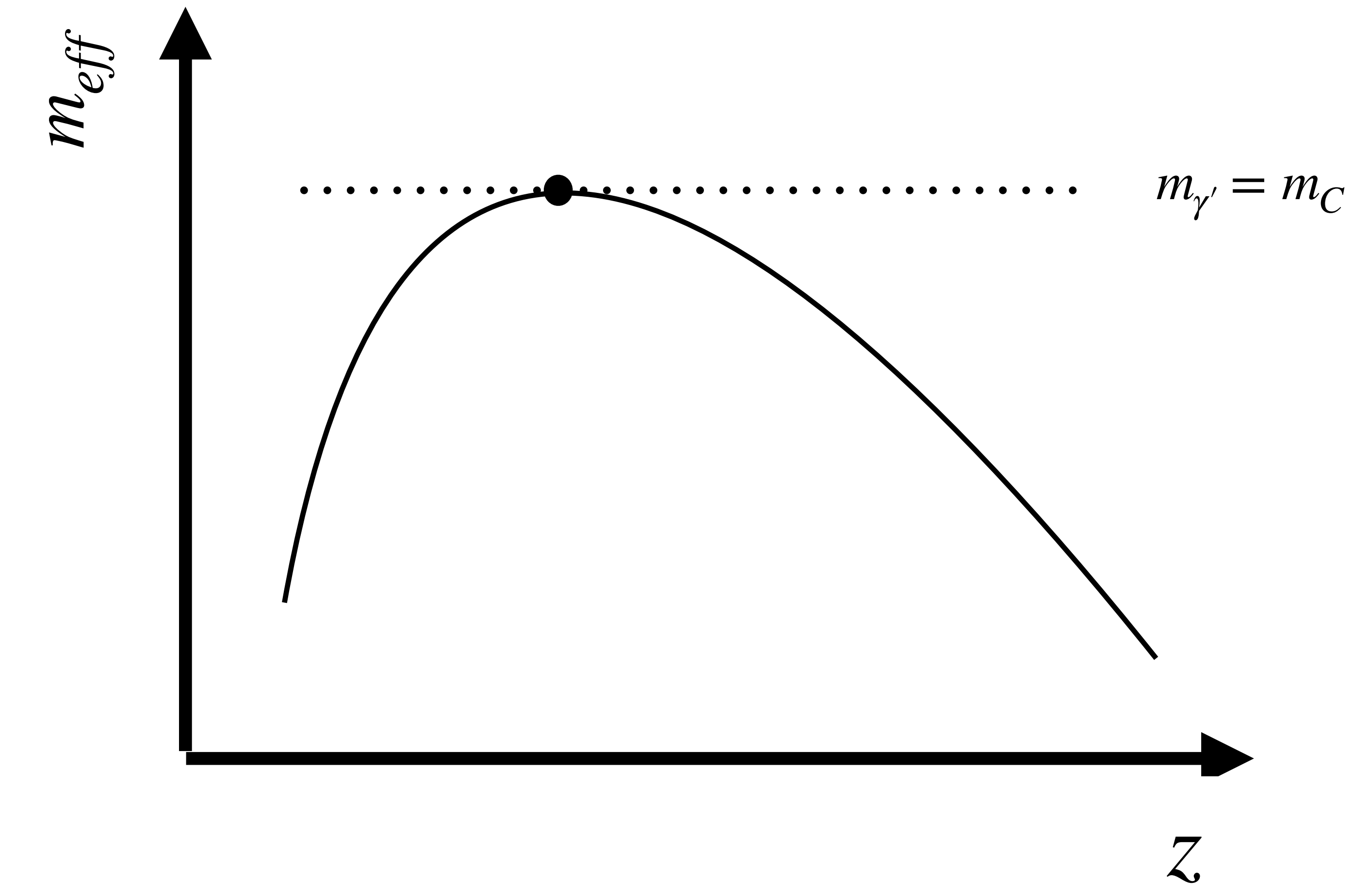
$$P_{\gamma\leftrightarrow\gamma'} \approx \epsilon^2 \left| 2\pi \left(\frac{2}{|\Phi^{(3)}(z_C)|} \right)^{1/3} \frac{m_{\gamma'}^2}{2\omega} \left(\mathrm{Ai}\,(0) + i\,\#\,\mathrm{Ai}'(0) \right) \right|^2$$



Summary

- Non-monotonic potential profiles are ubiquitous in astrophysics. More examples - supernova shockwave, solar chromosphere etc.
- The usual Landau-Zener formula breaks down near critical points.
- Our expression for coalescing saddle point provides an accurate prescription for evaluating the conversion probability.
- Moreover, it can be used for neutrino oscillations, axion-photon conversions, etc.

$$P_{\gamma\leftrightarrow\gamma'} \approx \epsilon^2 \left| 2\pi \left(\frac{2}{|\Phi^{(3)}(z_C)|} \right)^{1/3} \frac{m_{\gamma'}^2}{2\omega} \left(\mathrm{Ai}(0) + i\#\,\mathrm{Ai}'(0) \right) \right|^2$$



Thank you!

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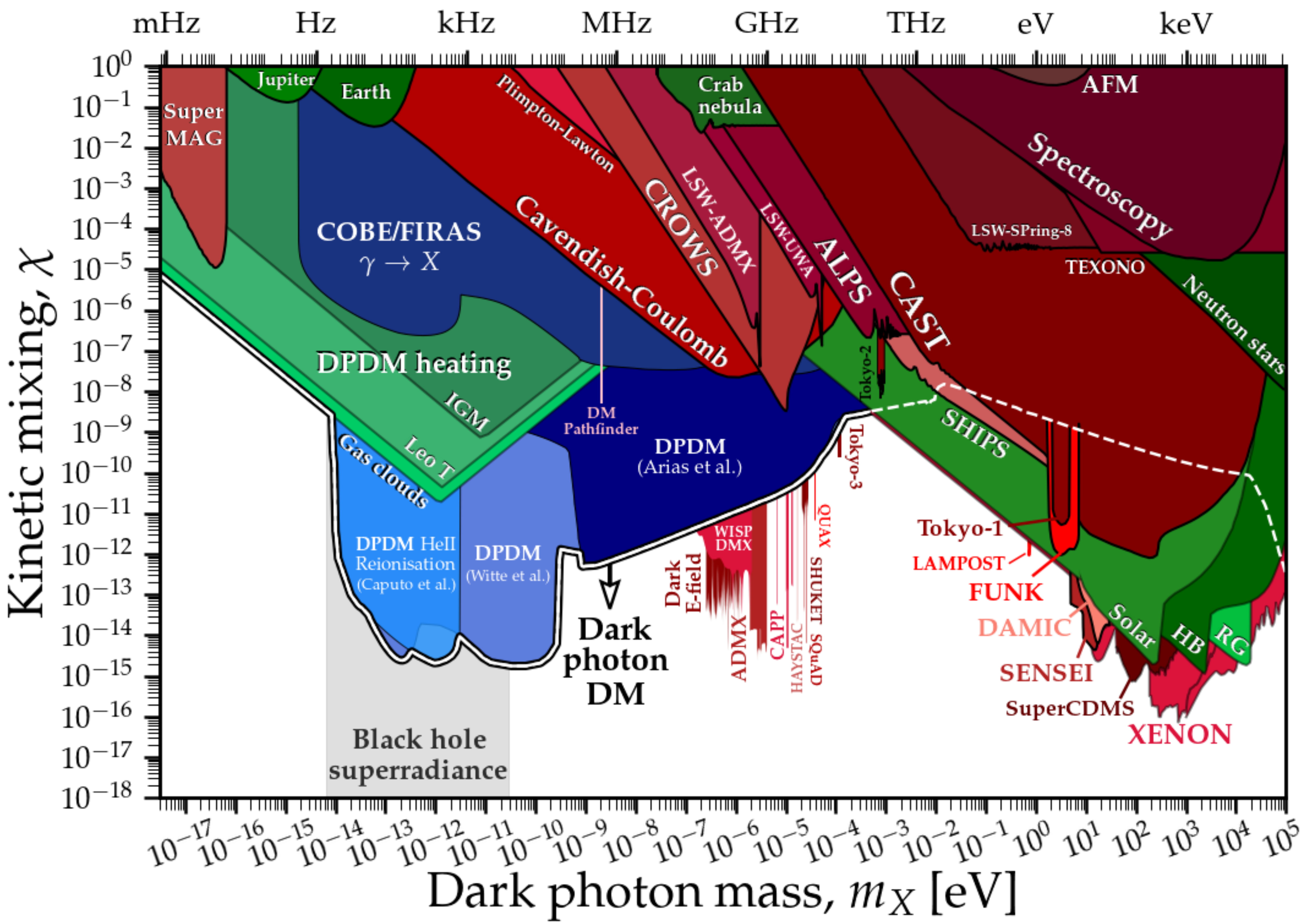
Stationary phase approximation

$$\Phi(m, z) = \Phi(m, z_0) + \Phi^{(1)}(m, z_0)(z - z_0) + \frac{1}{2!}\Phi^{(2)}(m, z_0)(z - z_0)^2 + \frac{1}{3!}\Phi^{(3)}(m, z_0)(z - z_0)^3 + \cdots$$

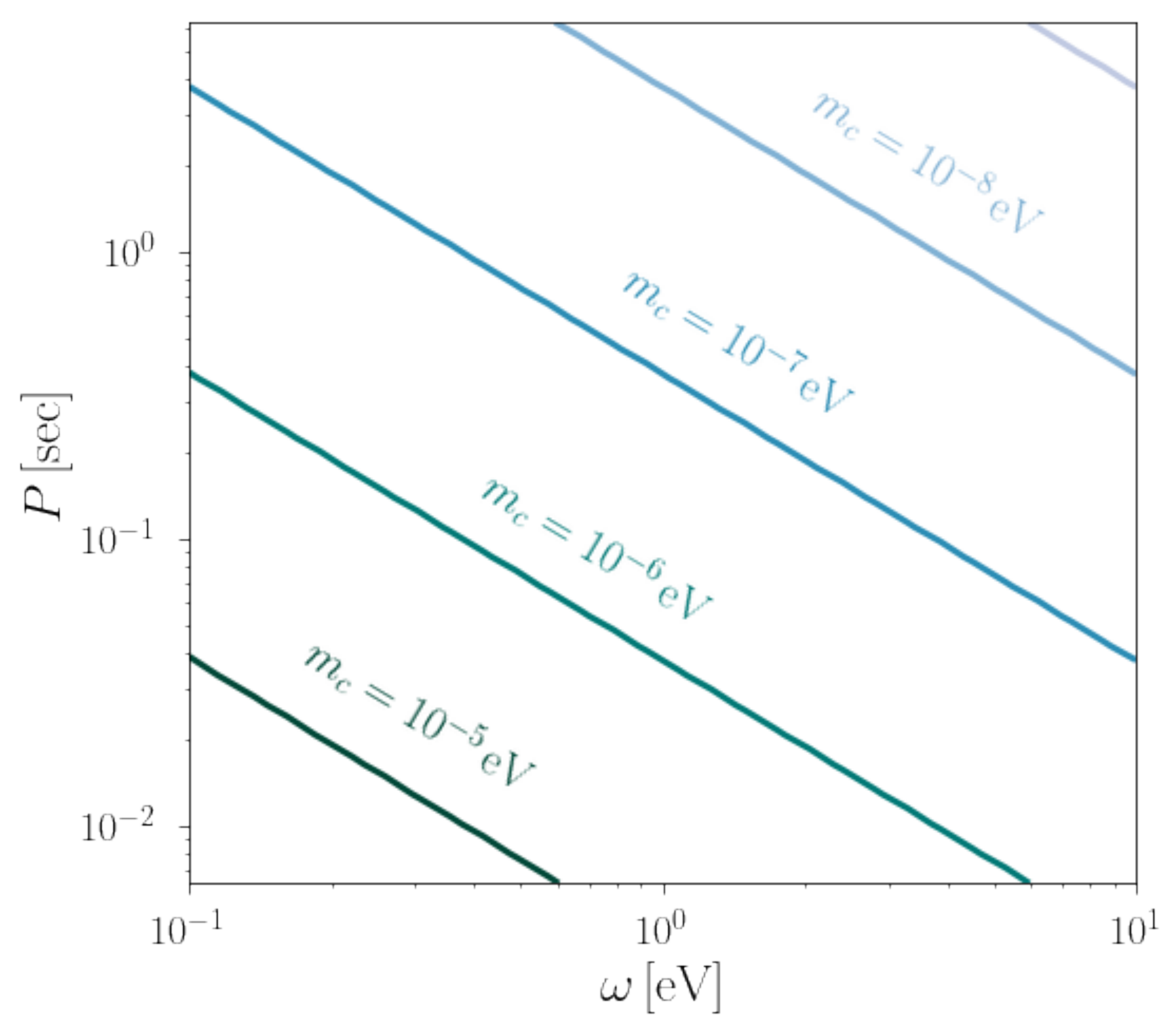
- At critical point, $m = m_C$

$$\Phi^{(1)}(m_C, z_0) = \Phi^{(2)}(m_C, z_0) = 0$$

DP parameter space and bounds



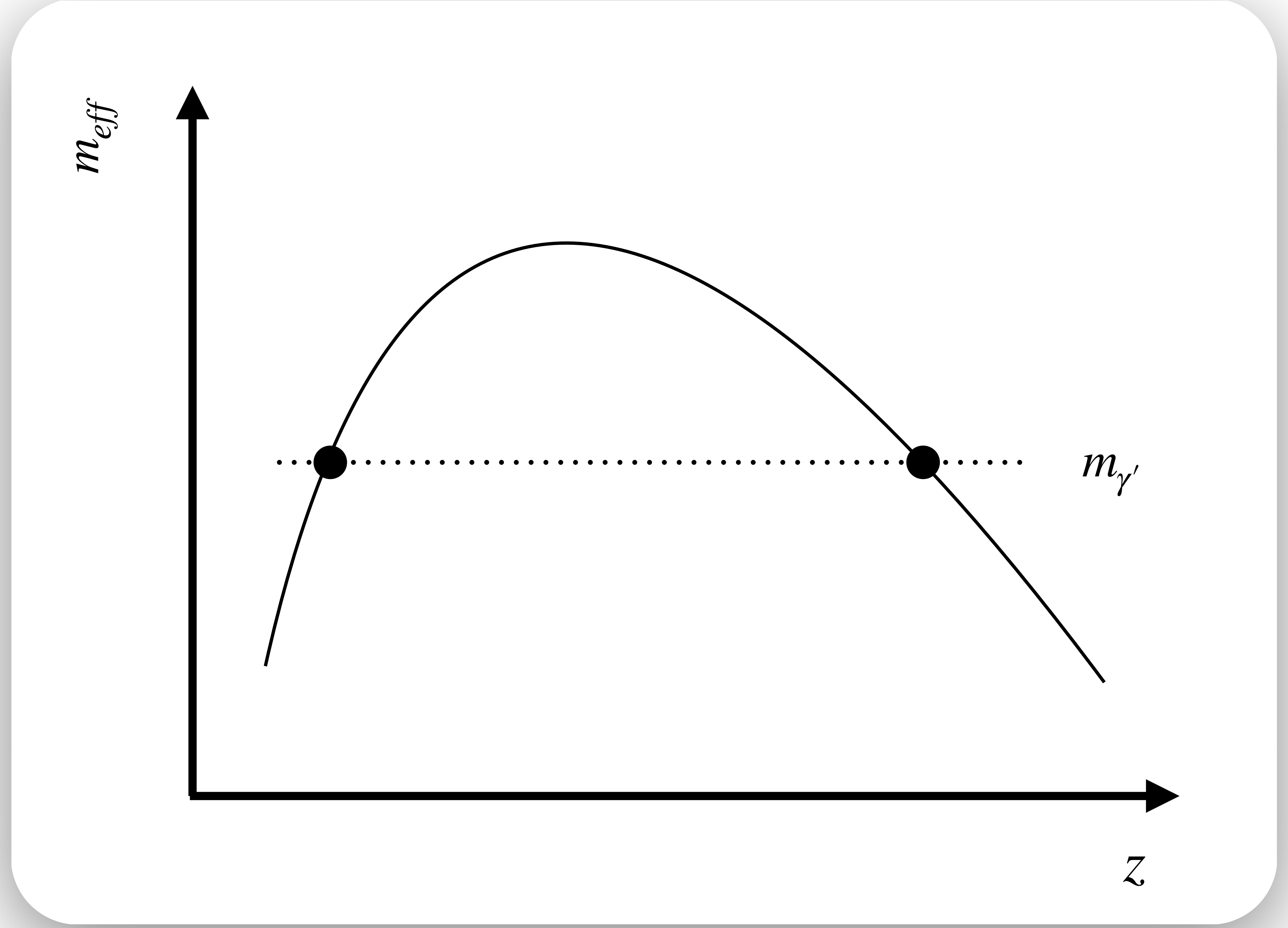
Neutron star magnetospheres



$B_0/B_{crit} = 10$

NB, Berlin, Schutz (PRD 2023)

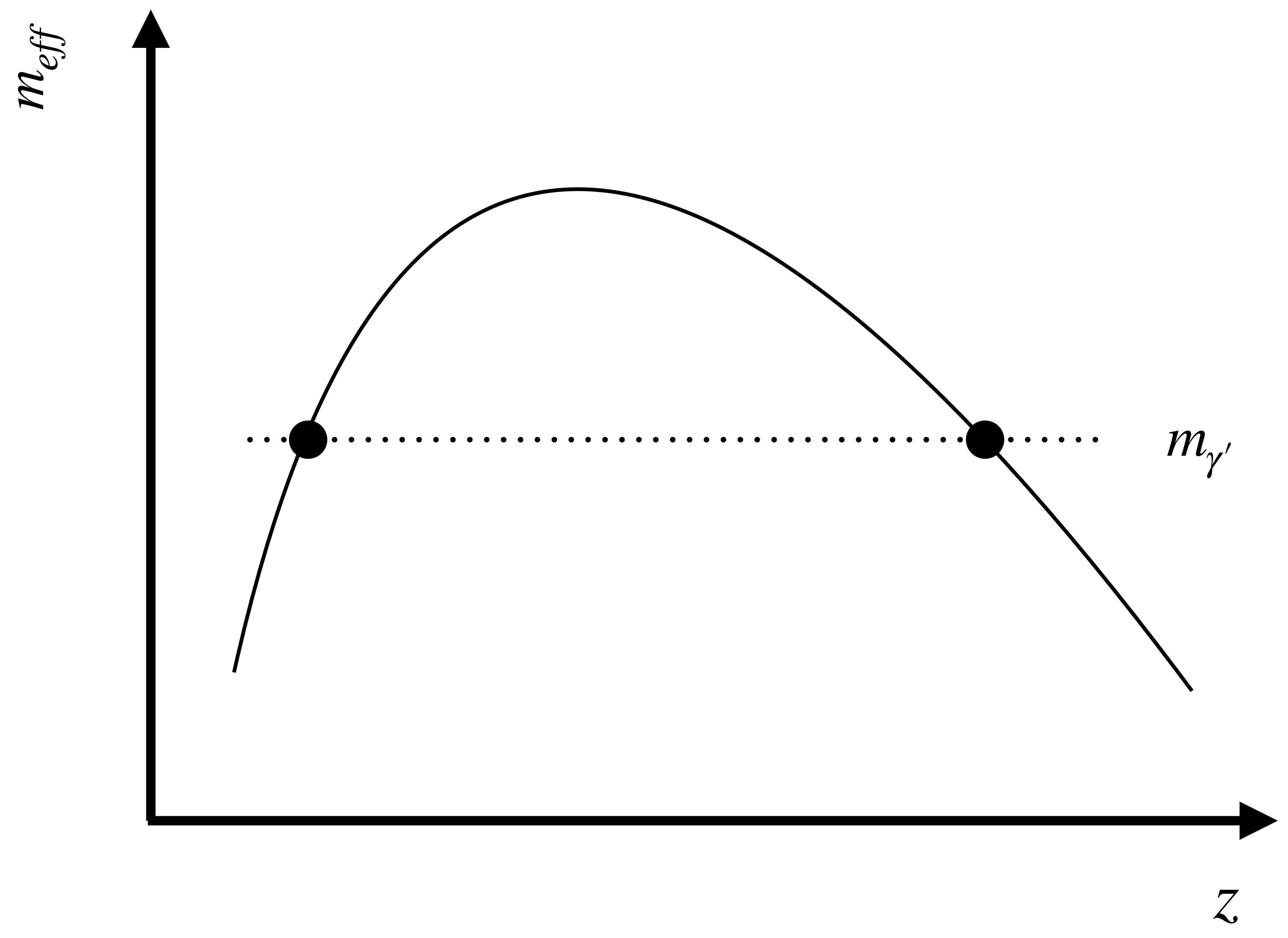
Sudden approximation

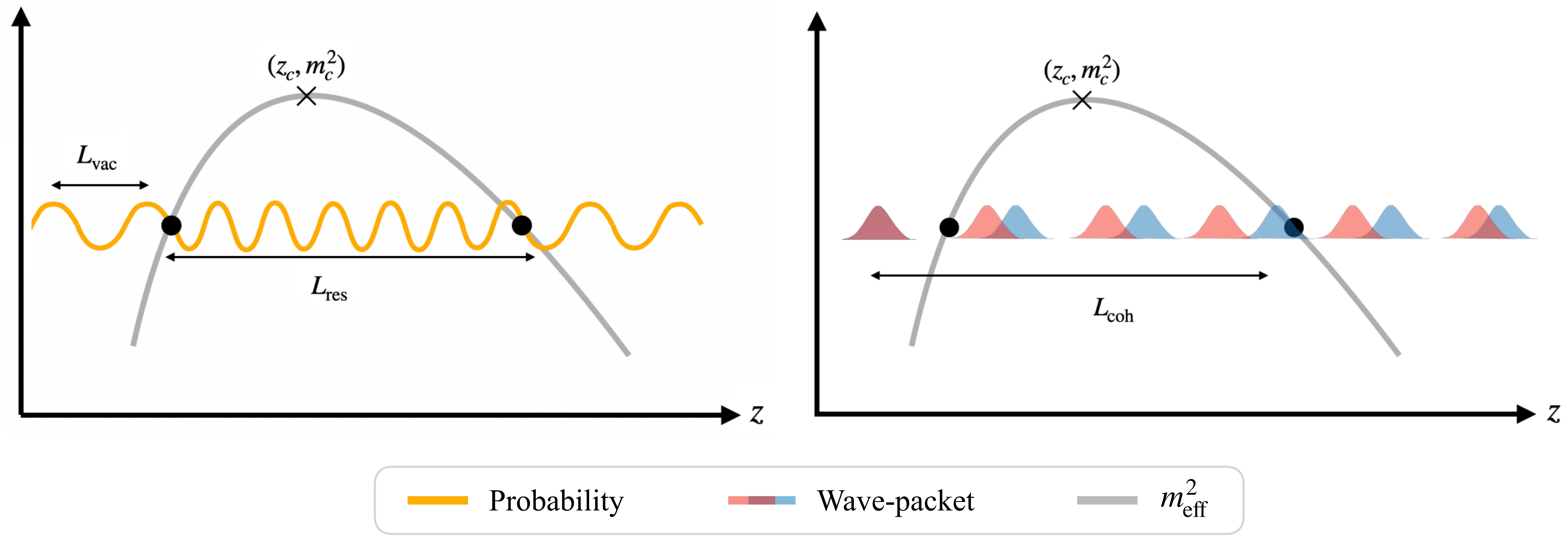


Sudden approximation

$$\mu \equiv \max(A_n)$$

"Resonance enhancement

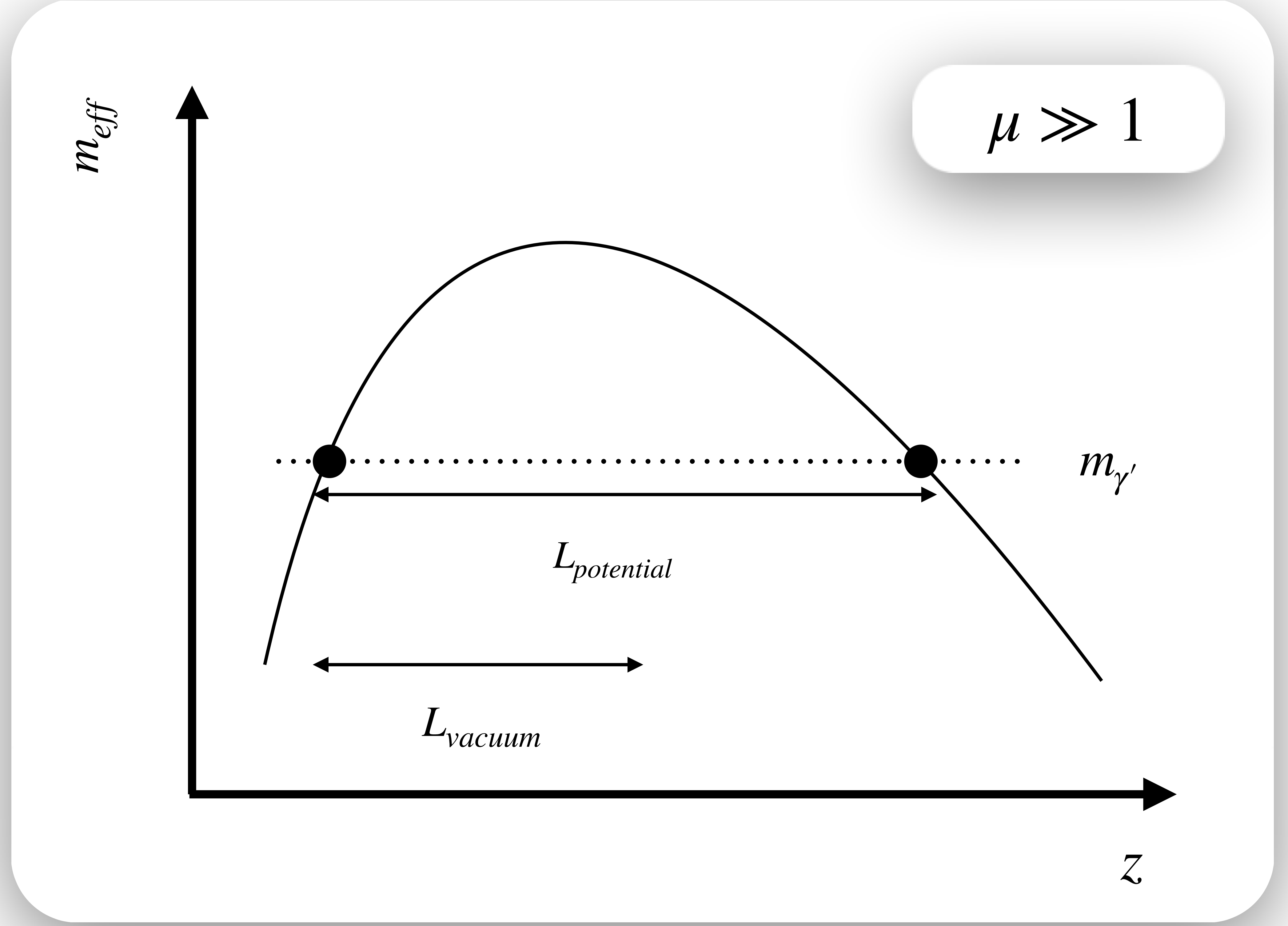




(z_c, m_c^2)
$L_{\rm vac}$
$L_{\rm res}$
z
(z_c, m_c^2)
$L_{\rm coh}$
z
Probability
Wave-packet
$m_{\rm eff}^2$

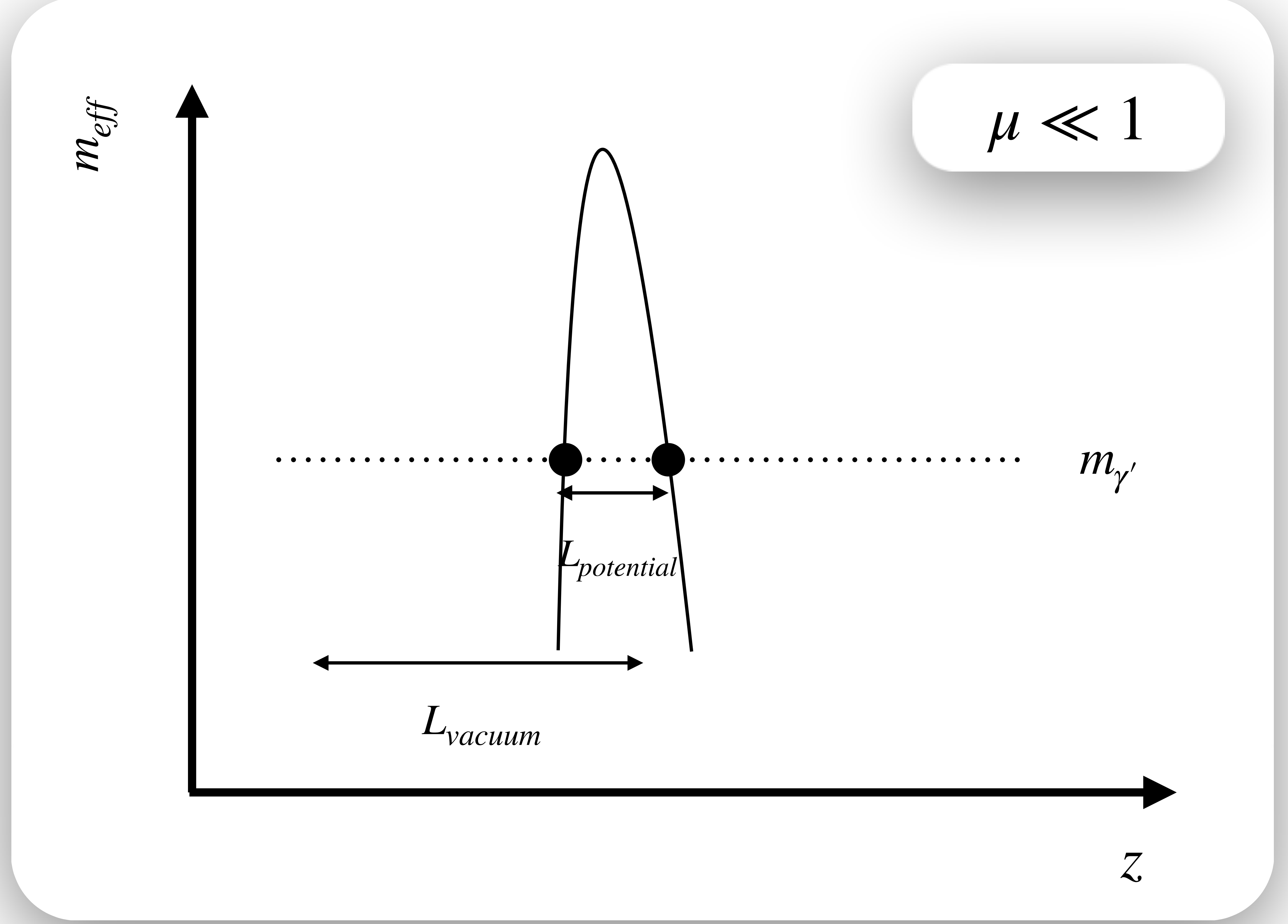
Sudden approximation

$$\mu \equiv \frac{L_{potential}}{L_{vacuum}}$$



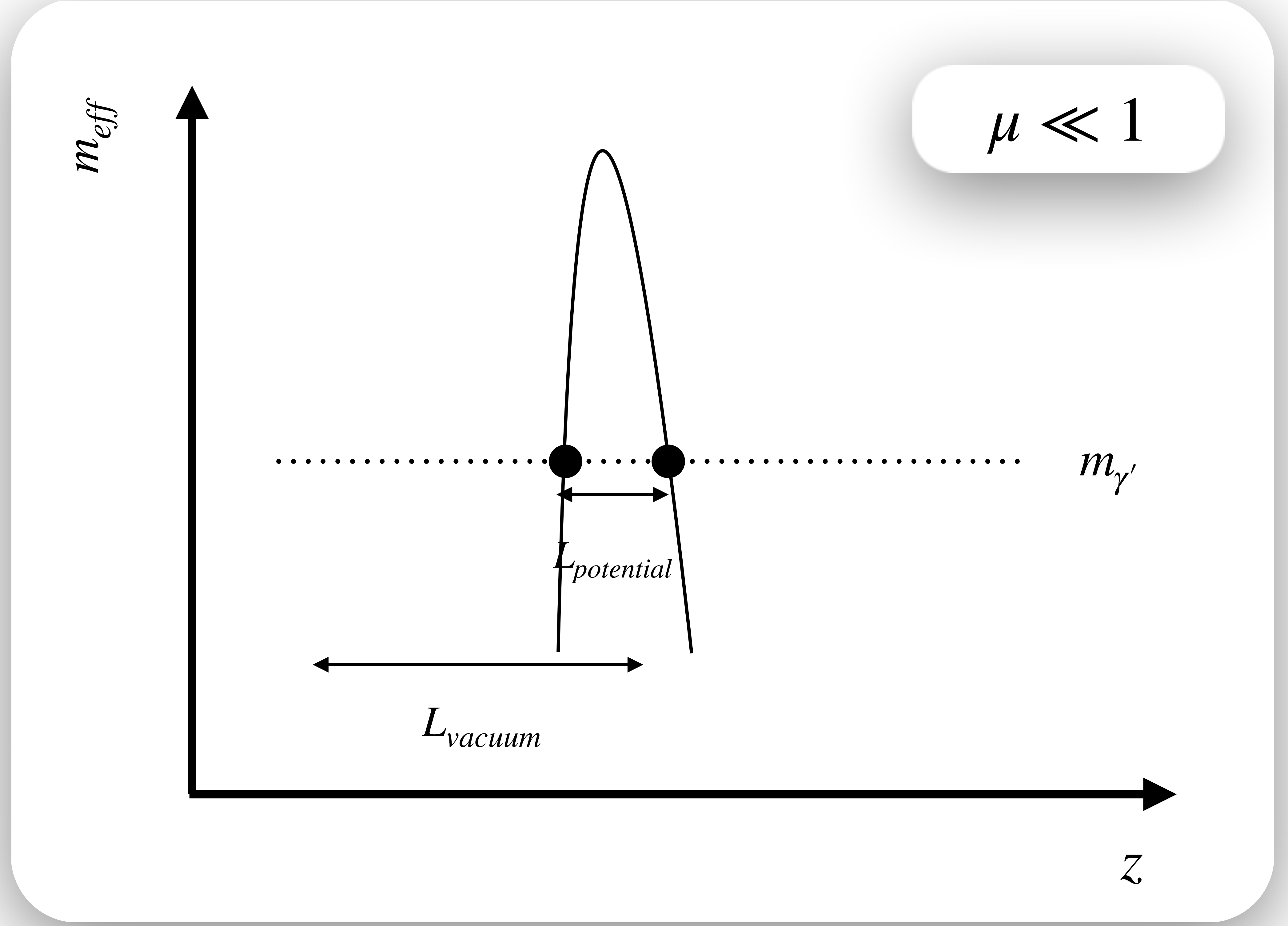
Sudden approximation

$$\mu \equiv \frac{L_{potential}}{L_{vacuum}}$$

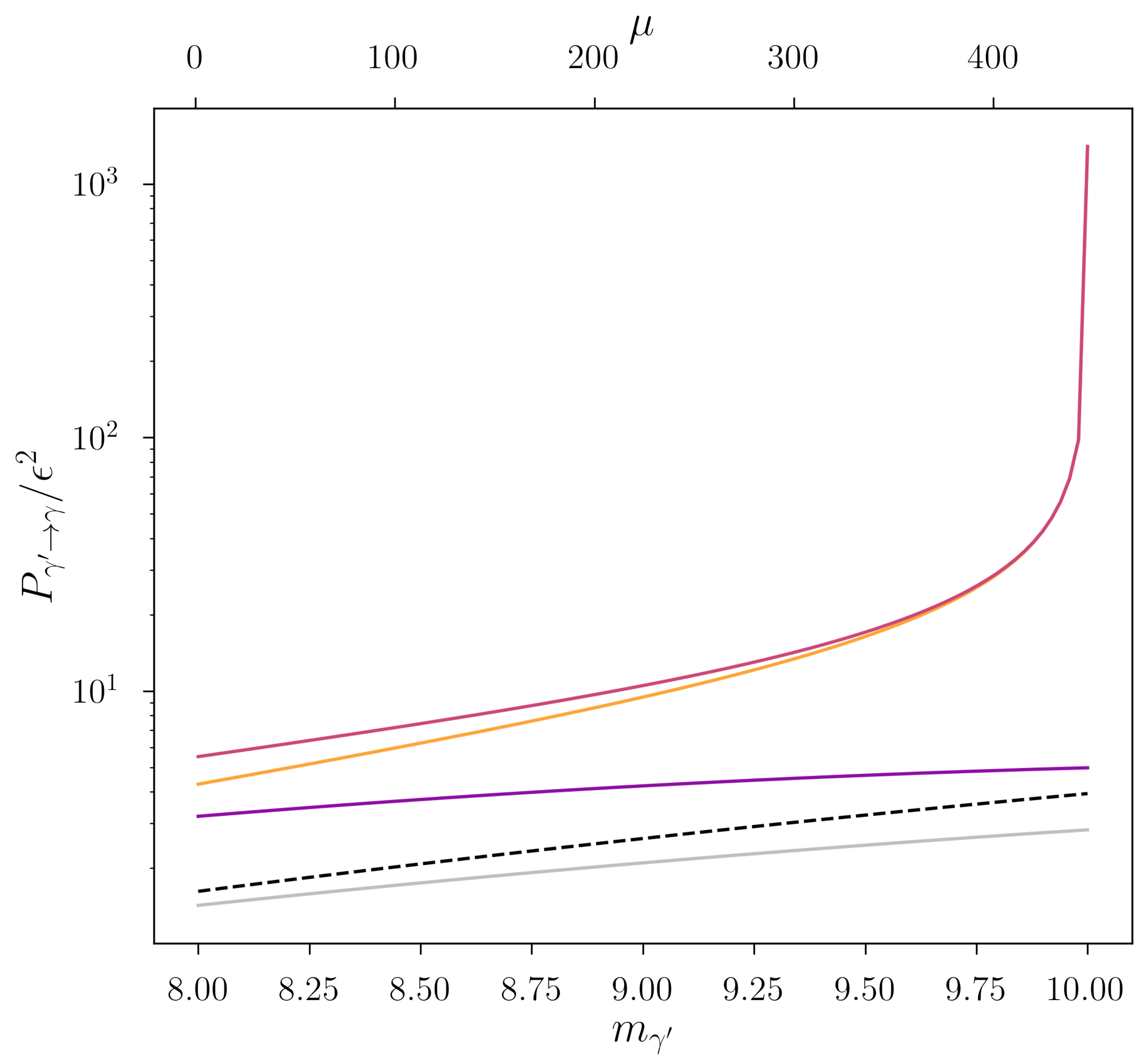


Sudden approximation

$$\mu \ll 1 \longrightarrow P_{\gamma \leftrightarrow \gamma'} = \text{vacuum osc.}$$



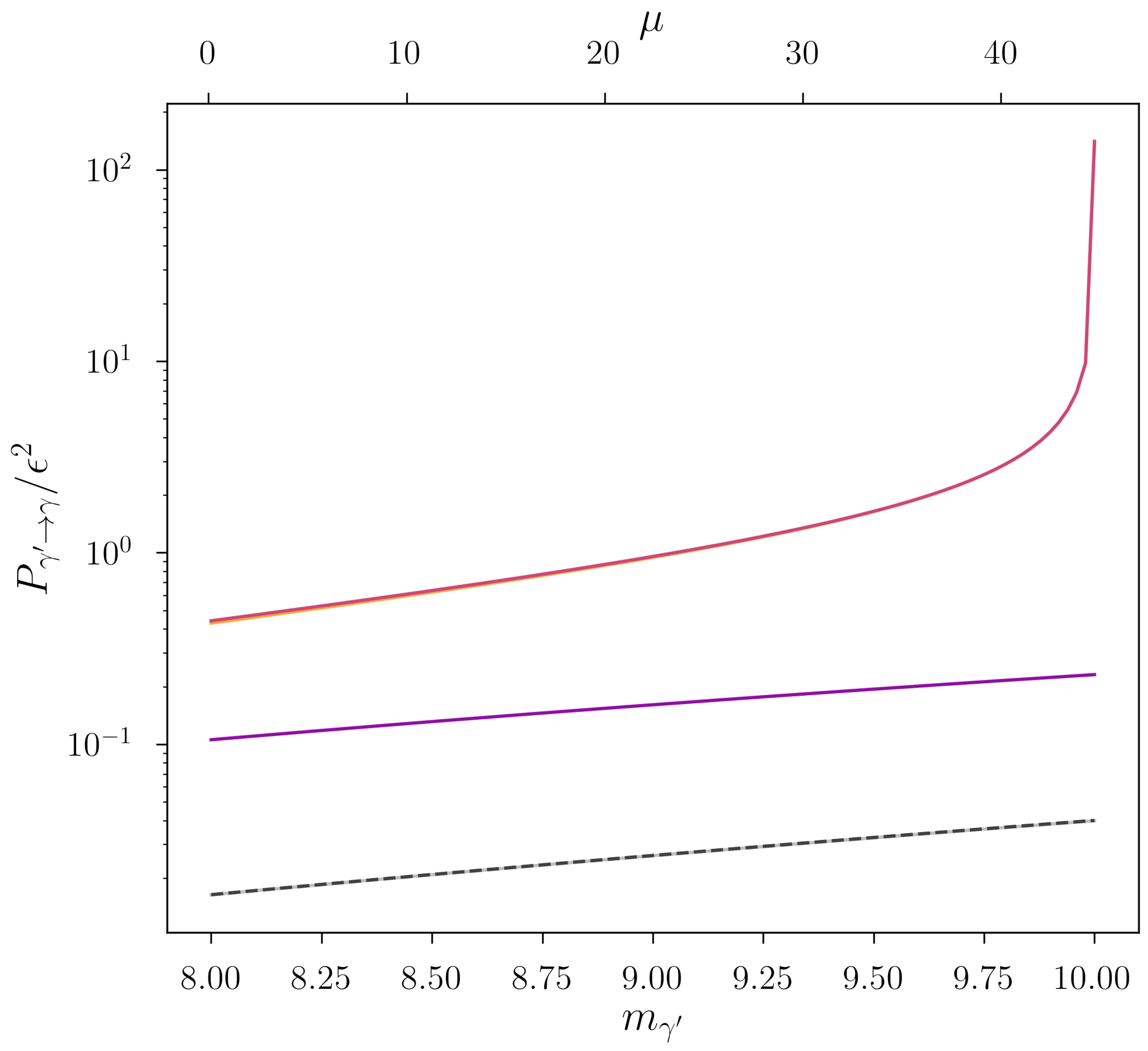
Toy model



Vacuum LZ Phase This work Numerical

$a = 20, b = 10$

Toy model



Vacuum LZ Phase This work Numerical

$a = 2, b = 10$

Neutron star magnetospheres

- Effective mass induced by plasma

$$m_{eff}^2 = \frac{4\pi\alpha\rho_{GJ}}{em_e}$$

- Effective mass induced by large external magnetic fields

$$m_{eff}^2 = -\frac{7\alpha}{45\pi}\left(\frac{B_{ext}}{B_{crit}}\right)^2 \omega^2$$

- B_{ext} is dominated by the dipole component

Non-monotonic profiles and multiple resonances

$$\left| \int_{z_i}^{z_f} dz' \Delta_{\gamma'}(z') e^{-i\Phi(m_{\gamma'}, z')} \right|^2 \approx \left| \sum_n \sqrt{\frac{2\pi}{\left| \Phi^{(2)}(m_{\gamma'}, z_n) \right|}} \Delta_{\gamma'}(z_n) e^{-i\Phi(m_{\gamma'}, z_n) - i\sigma_n \frac{\pi}{4}} \right|^2$$

$$P_{\gamma \to \gamma'}(m_{\gamma'}) \approx 4\pi^2 \epsilon^2 \Delta^2_{\gamma'}(z_C) \left(\frac{2}{|\Phi^{(3)}_C(m_{\gamma'})|} \right)^{2/3} \left\{ \textbf{Ai}\left(-\zeta\right) + i\sigma_1 \left(\frac{2}{|\Phi^{(3)}_C(m_{\gamma'})|} \right)^{1/3} \left[\frac{\omega'_C}{\omega_C} - \frac{1}{6} \frac{\Phi^{(4)}_C(m_{\gamma'})}{\Phi^{(3)}_C(m_{\gamma'})} \right] \textbf{Ai}'(-\zeta) \right\}^2$$

$$\zeta(m_{\gamma'}) = \left(\frac{2}{|\Phi^{(3)}(z_C, m)|} \right)^{1/3} \Phi^{(1)}(z_C, m)$$