RUB

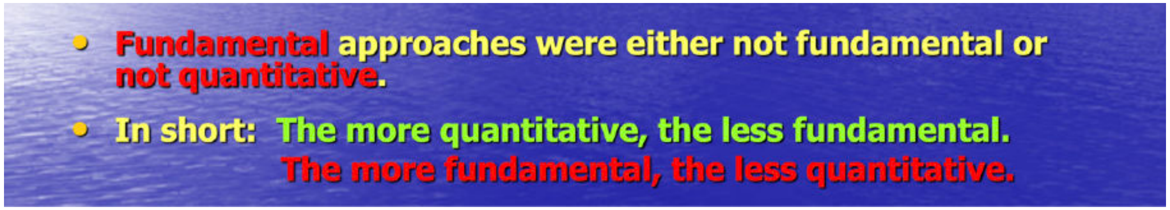
Evgeny Epelbaum, Ruhr University Bochum

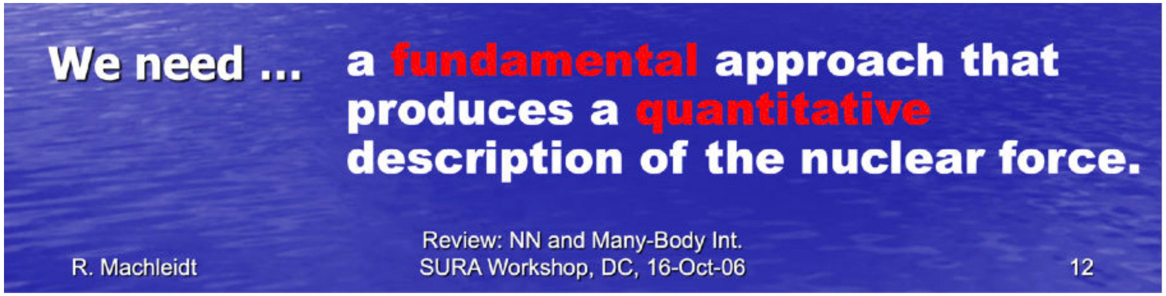
PAINT2024: Workshop on Progress in Ab Initio Nuclear Theory
TRIUMF, Vancouver, 27. February — 1. March, 2024

Precision theory for charge radii of light nuclei

In collaboration with: ***Arseniy Filin***, Daniel Möller, Vadim Baru, Christopher Körber, Hermann Krebs, Andreas Nogga and Patrick Reinert

In memory of
Ruprecht Machleidt





European Research Council
Bundesministerium für Bildung und Forschung
DFG Deutsche Forschungsgemeinschaft
Ministerium für Kultur und Wissenschaft des Landes Nordrhein-Westfalen
ERATO Exploratory Research for Advanced Technology

Experimental data

^{3}He
$r = 1.97007(94)\,\mathrm{fm}$
0.05 %
CREMA 2023
arXiv:2305.11679

^{4}He
$r = 1.6786(12)\,\mathrm{fm}$
0.07 %
Krauth et al. 2021
Pachucki:2022tgl

^{1}H
$r_p = 0.8406(4)\,\mathrm{fm}$
0.05 %
Antognini et al. 2013
Pachucki:2022tgl

^{2}H
$r_d = 2.12758(78)\,\mathrm{fm}$
0.04 %
CREMA:2016idx
Pachucki:2022tgl

^{3}H
$r = 1.7xxx(\quad 2)\,\mathrm{fm}$
TREX Mainz
[in progress]
$r = 1.7550(860)\,\mathrm{fm}$
Amroun et al. 1994
5 %

n
$r_n^2 = -0.1155(17)\,\mathrm{fm}^2$
1.5 %
PDG 2022

Number of protons Z

Number of neutrons N

Pohl:2013yb,
CREMA:2016idx,
Pohl:2016glp,
Schmidt:2018kjc,
Krauth:2021foz
...

Data from isotope shift

$(r_d^2 - r_p^2) = 3.82070(31) fm^2$
0.01 %

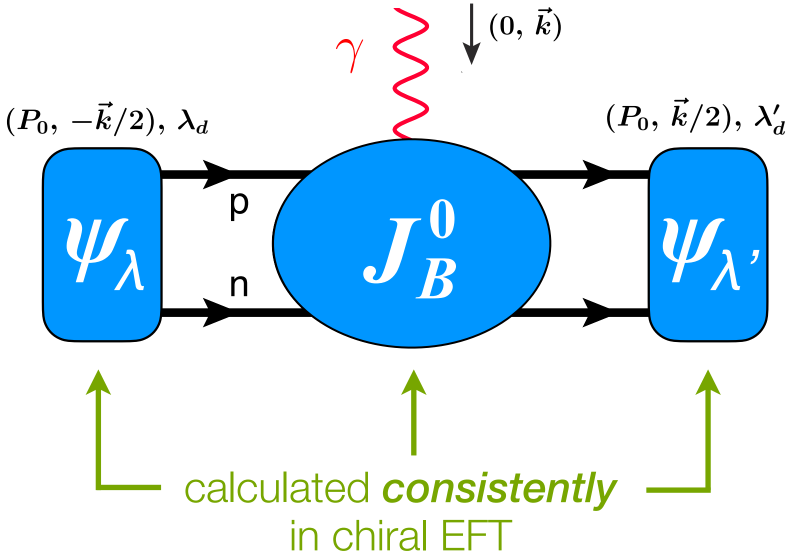
Pachucki et al. 2018
Jentschura et al. 2011

Theory in a nutshell

- **Chiral EFT** for the nuclear Hamiltonian H and J^μ

- Use **1N FFs** to avoid reliance on χEFT for J^μ_{1N} (i.e., re-summations)

- **Error analysis** (statistical uncertainties of few-N LECs, πN LECs, EFT truncation, parametrizations of the 1N FFs)

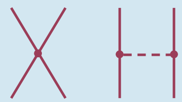
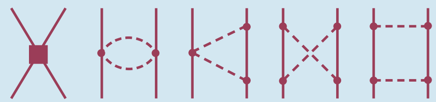
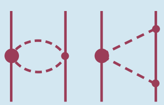
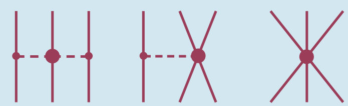
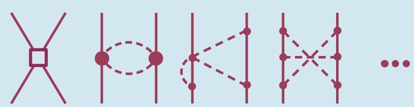
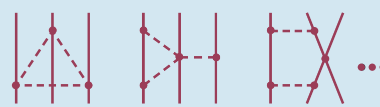
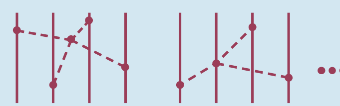
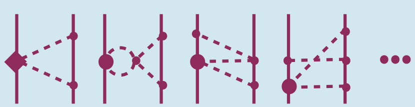
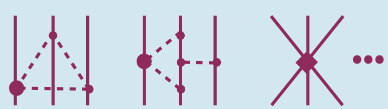


- Regularization and symmetries (3NF, MECs)
  → *gradient flow method (talk by Hermann)*

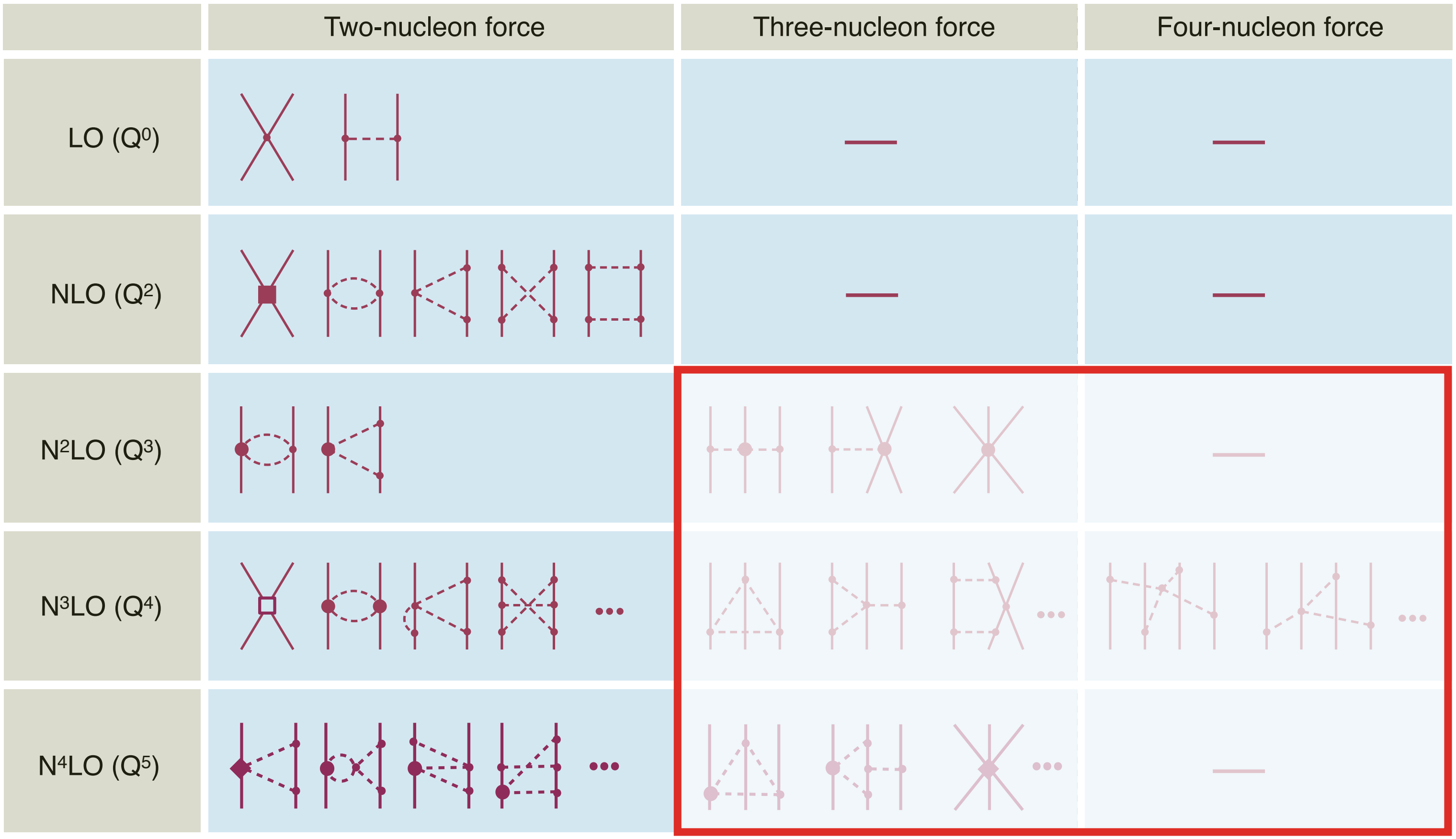
- Starting from N³LO (Q⁴), one has to worry about:
  - relativistic corrections
  - isospin breaking effects (including neutron-neutron interaction)
  - electromagnetic interactions beyond Coulomb

- **Semi-analytical results for convolution integrals** (to minimize numerical errors)

The Hamiltonian

	Two-nucleon force	Three-nucleon force	Four-nucleon force
LO (Q^0)		—	—
NLO (Q^2)		—	—
N^2LO (Q^3)			—
N^3LO (Q^4)			
N^4LO (Q^5)			—

The Hamiltonian

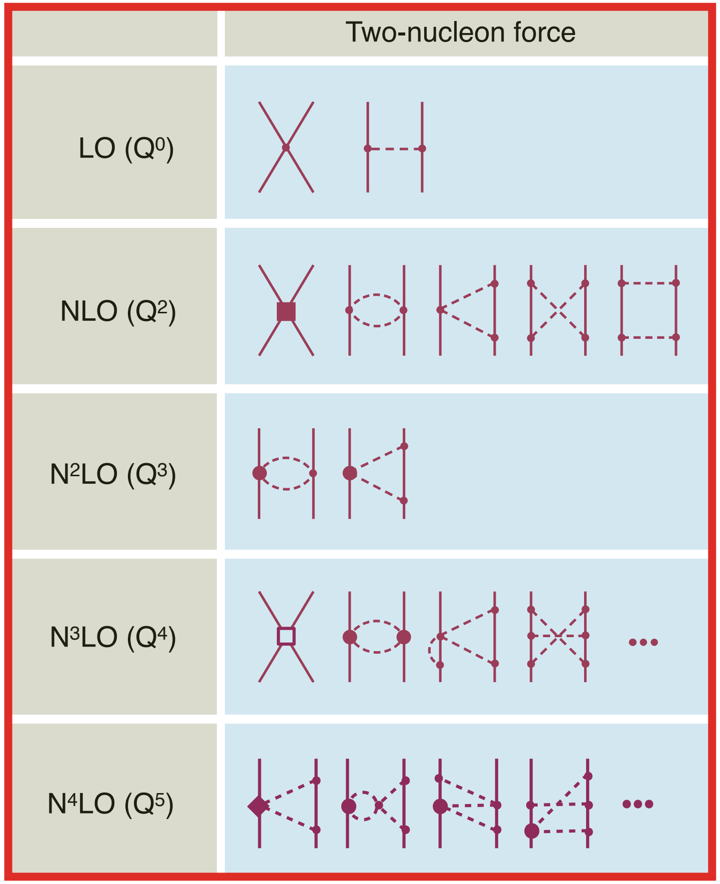


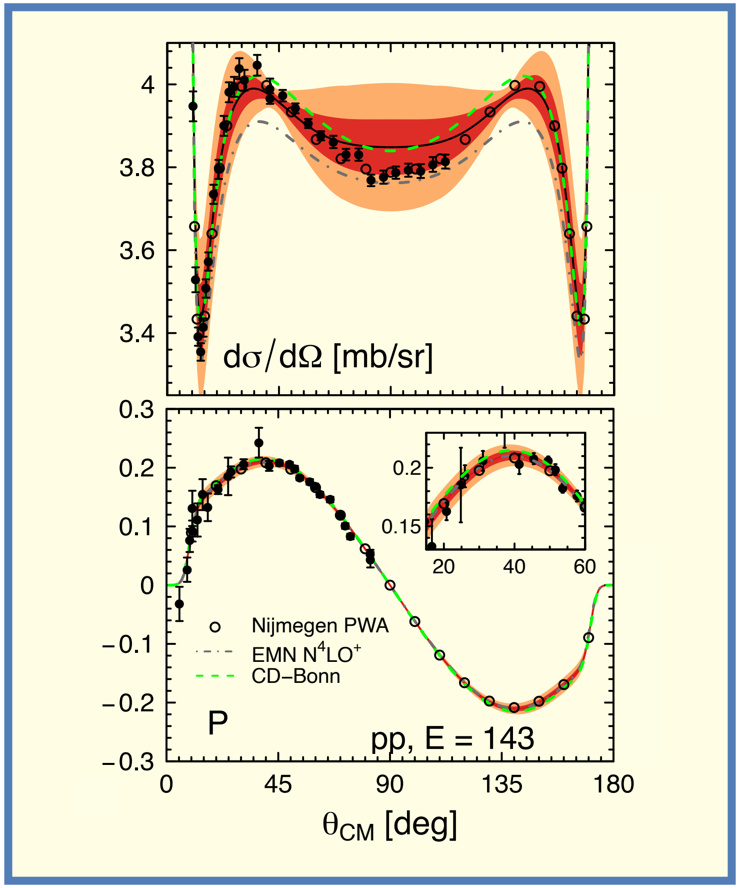
mixing DimReg with Cutoff regularization violates χ-symmetry

$\Rightarrow$ re-derive using Gradient Flow regulator

— talk by Hermann —

The Hamiltonian





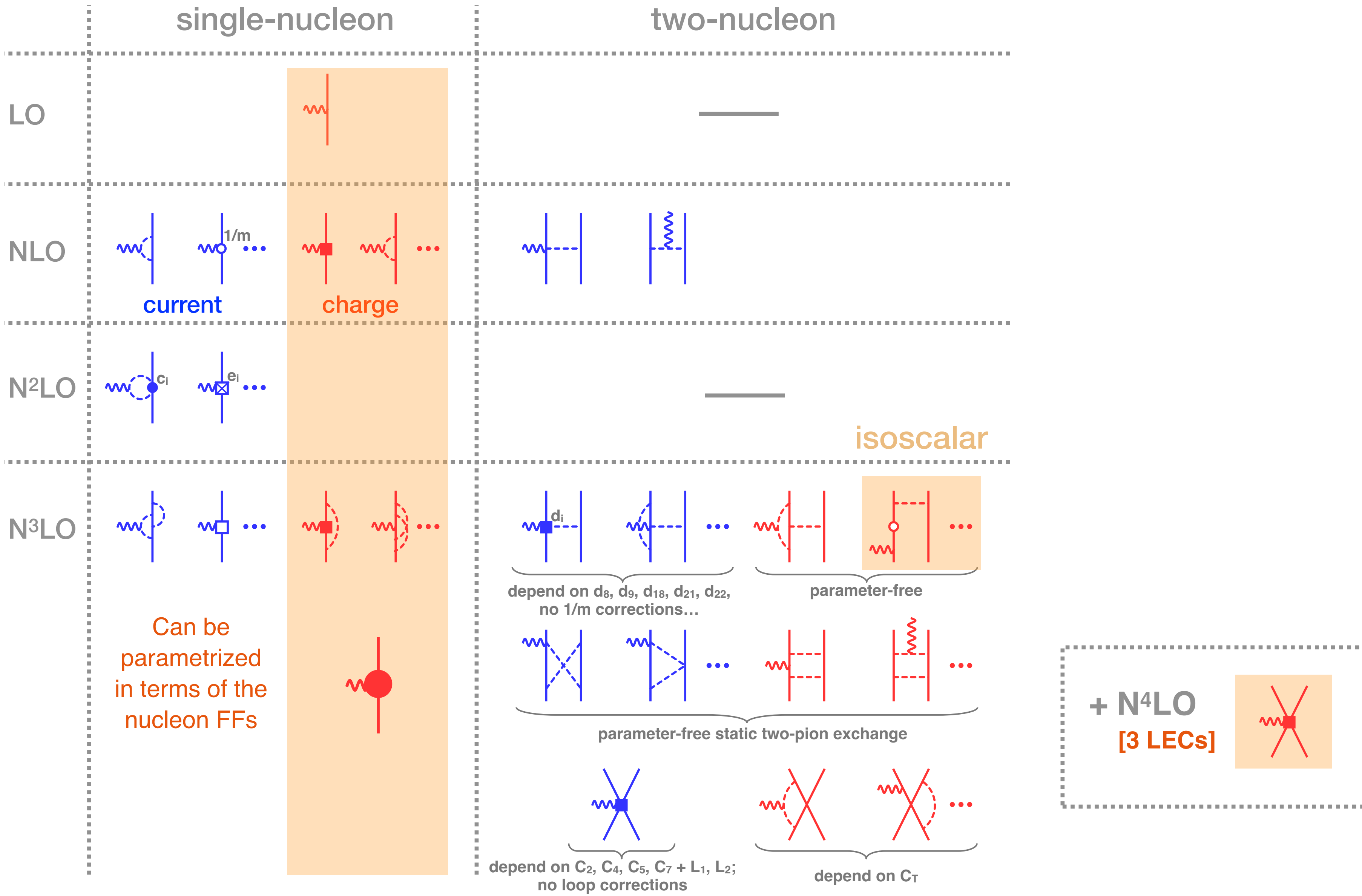
The newest Bochum NN interactions Reinert, Krebs, EE, EPJA 54 (2018) 86; PRL 126 (2021) 092501

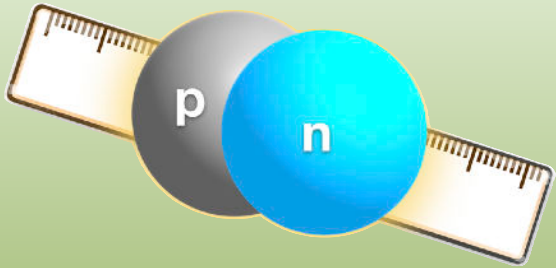
$$V_{1\pi}(q) = \frac{\alpha}{\vec{q}^2 + M_\pi^2} \; e^{-\frac{\vec{q}^2 + M_\pi^2}{\Lambda^2}} + \text{subtraction}, \qquad V_{2\pi}(q) = \frac{2}{\pi}\int_{2M_\pi}^{\infty} d\mu\,\mu\,\frac{\rho(\mu)}{\vec{q}^2 + \mu^2} \; e^{-\frac{\vec{q}^2 + \mu^2}{2\Lambda^2}} + \text{subtractions}$$

+ nonlocal (Gaussian) cutoff for contacts

Electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001; Krebs, EE, Meißner, FBS 60 (2019) 31





The deuteron (A = 2)

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, Phys. Rev. Lett. 124 (2020) 082501;

Phys. Rev. C103 (2021) 024313

$$\rho_{\rm 1N}^{\rm DF} = -\,e\frac{\mathbf{k}^2}{8m_N^2}G_E(\mathbf{k}^2)$$

$$G(Q^2) = G^{\rm Main}(Q^2) + G^{\rm DF}(Q^2) + G^{\rm SO}(Q^2) + G^{\rm Boost}(Q^2) + G^{1\pi}(Q^2) + G^{\rm Cont}(Q^2)$$

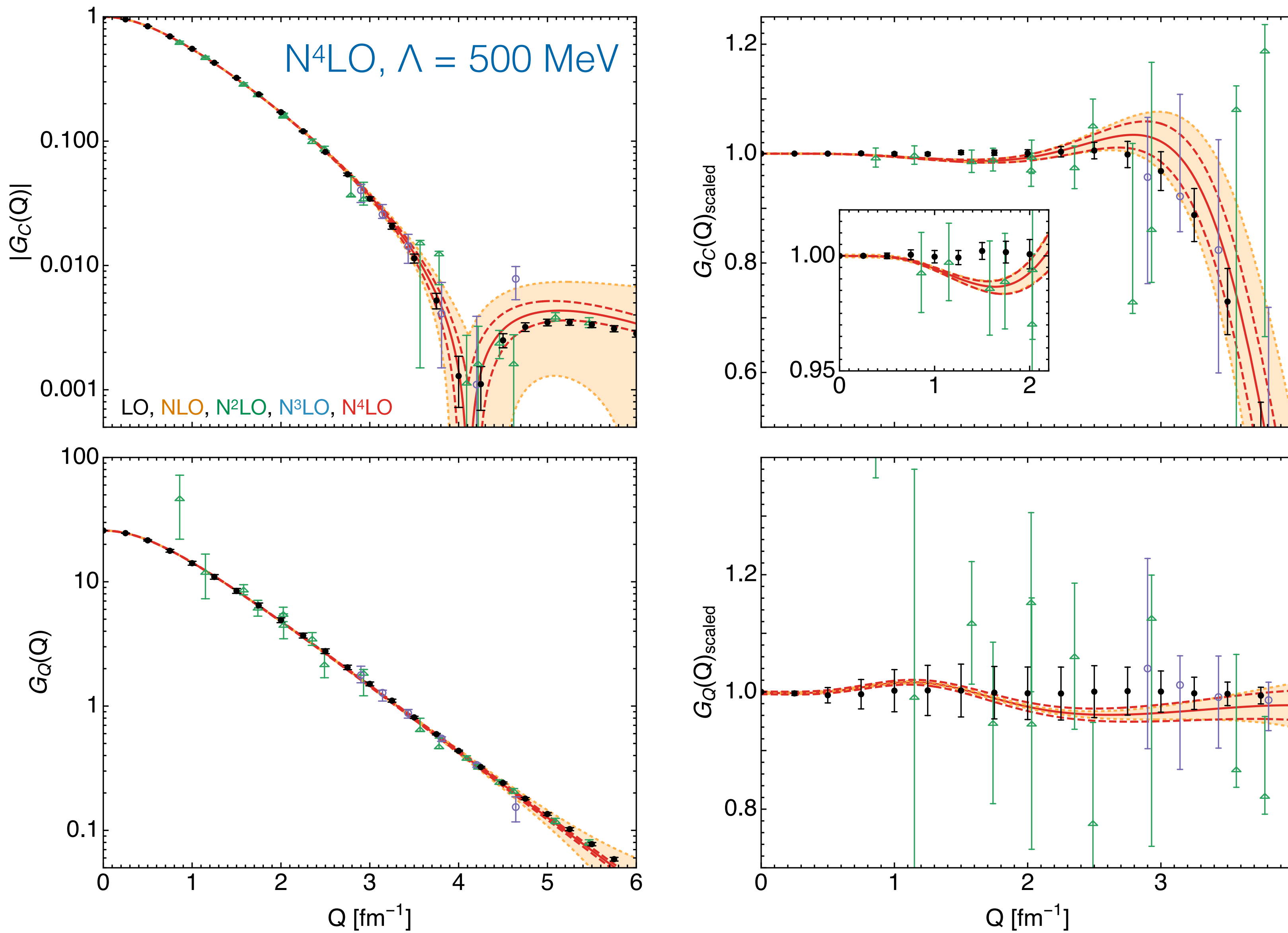
$$\rho_{\rm 1N}^{\rm Main} = eG_E(\mathbf{k}^2) \qquad \rho_{\rm 1N}^{\rm SO} = ie\frac{2G_M(\mathbf{k}^2)-G_E(\mathbf{k}^2)}{4m_N^2}\sigma\cdot\mathbf{k}\times\mathbf{p}$$

— Both the nuclear force and the 2N charge density are available to N^4LO

— Simple numerics

The charge and quadrupole FFs of ^{2}H

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313



The charge and quadrupole FFs of ^2H

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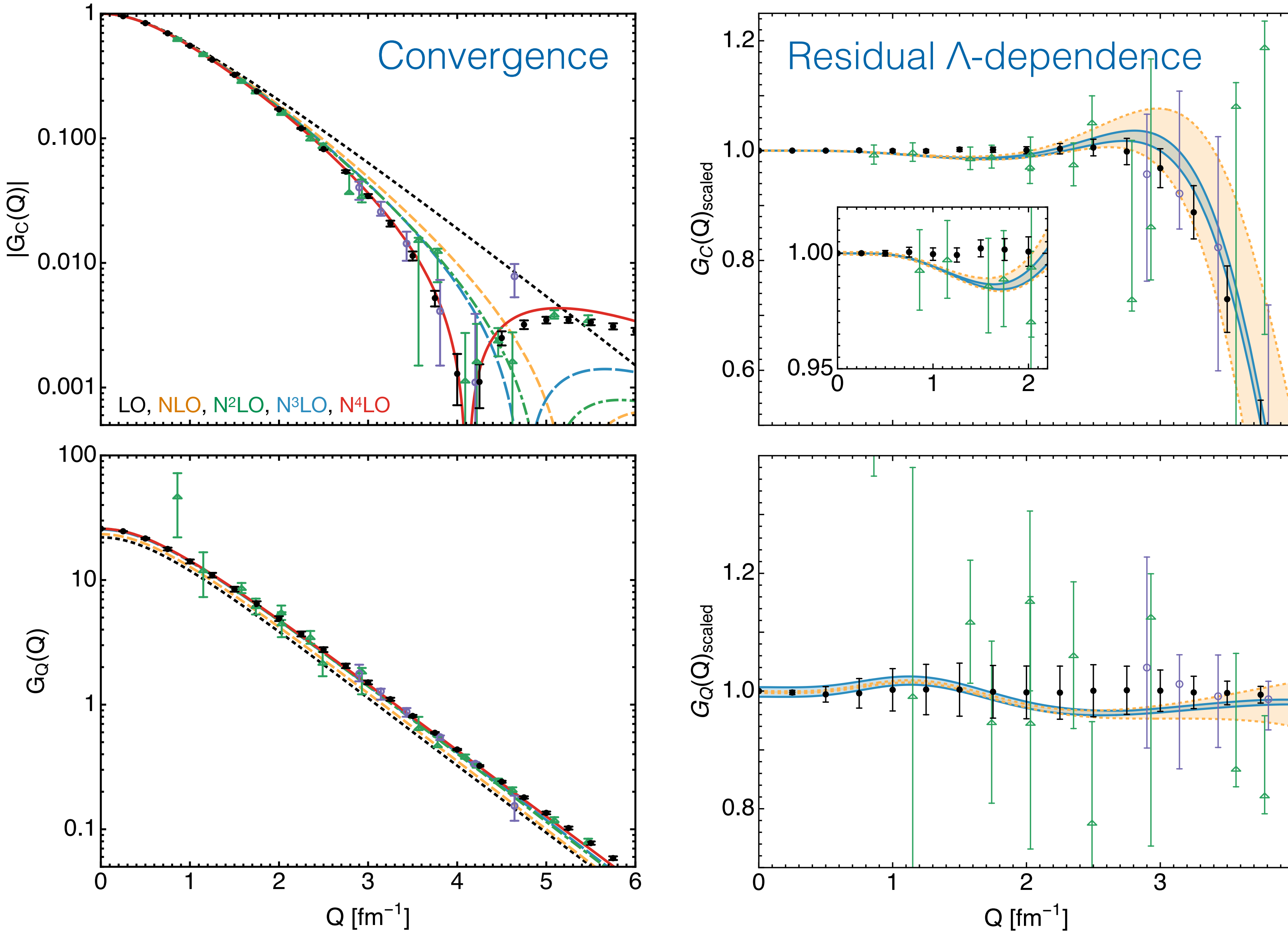


Charge radius and quadrupole moment

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

Deuteron charge ***and structure*** radii: $r_d^2 = r_{\rm str}^2 + r_p^2 + r_n^2 + \frac{3}{4m_p^2}$

EFT truncation, choice of fitting range, NN, πN and γNN LECs

Our results: $r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}$ fm, $Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}$ fm^2

$Q_{\rm d}^{\rm exp} = 0.285\,699(15)(18)$ fm^2 Puchalski et al., PRL 125 (2020)

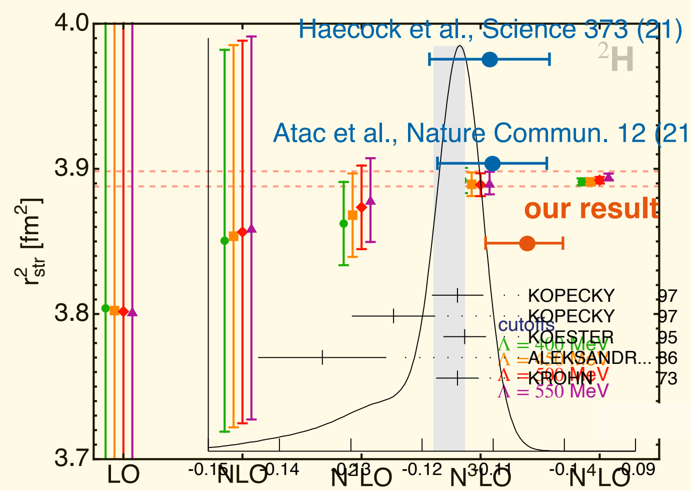
Error budget:

	central	truncation	$\rho^{\rm reg}_{\rm Cont}$	πN LECs RSA	2N and πN LECs	Q-range	total
$r^2_{\rm str}$ [fm^2]	3.8925	±0.0030	±0.0024	±0.0003	±0.0025	$^{+0.0035}_{-0.0005}$	$^{+0.0058}_{-0.0046}$
Q_d [fm^2]	0.2854	±0.0005	±0.0007	±0.0003	±0.0016	$^{+0.0035}_{-0.0005}$	$^{+0.0038}_{-0.0017}$

Combining our result for $r^2_{\rm str}$ with the [1]H-[2]H isotope shift datum $r_d^2 - r_p^2 = 3.82070(31)$ fm^2 Jentschura et al., PRA 83 (2011) leads to the prediction for the neutron radius:

Consistency check
(residual cutoff dependence):

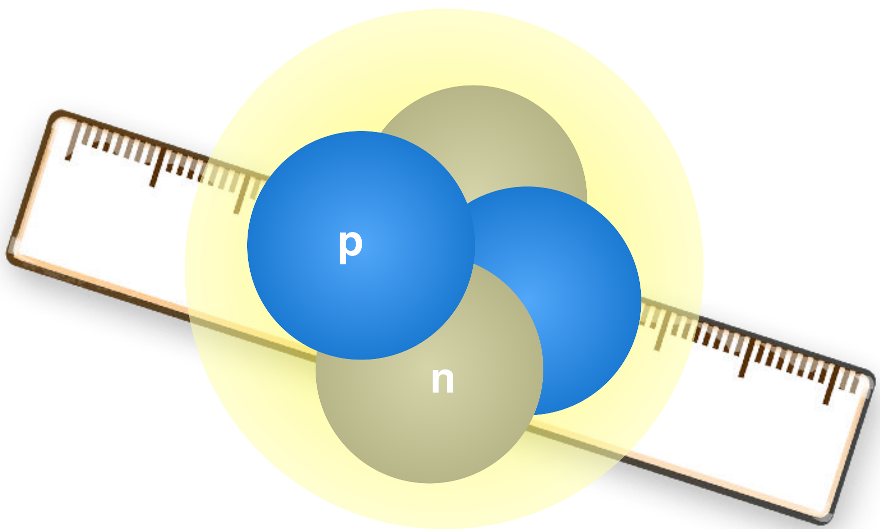
$r_n^2 = -0.105^{+0.005}_{-0.006}$ fm^2



The charge FF of ^{4}He (A = 4)

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation

3NF beyond N^{2}LO not yet available (However, no sensitivity to 3NF once the BEs are reproduced…)



Relativistic effects

- Relativistic corrections start appearing in V_{2N}, V_{3N} and ρ_{2N} at N^{3}LO
- Boosting ^{2}H, ^{3}H(e), ^{4}He to the Breit frame straightforward (effect decreases with A)
- Subtleties related to the form of the Schrödinger equation

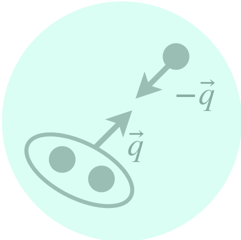
For 2N:
$$\underbrace{\left(2\sqrt{\hat{p}^2+m^2}+\tilde{V}\right)\Psi = \underbrace{2\sqrt{k^2+m^2}}_{E_{CMS}}\Psi}_{\text{equation to be used for } A>2} \quad\Rightarrow\quad \underbrace{\left(\frac{\hat{p}^2}{m}+V\right)\Psi = \frac{k^2}{m}\Psi}_{\text{equation we solve for } A=2}$$

χEFT NN potential (→ V)

Relationship between $\tilde{V}$ and V:
$$\underbrace{2\left\{\sqrt{\hat{p}^2+m^2},\,\tilde{V}\right\}+\tilde{V}^2 = 4mV}_{\text{can be solved by iterations to obtain } \tilde{V}}$$

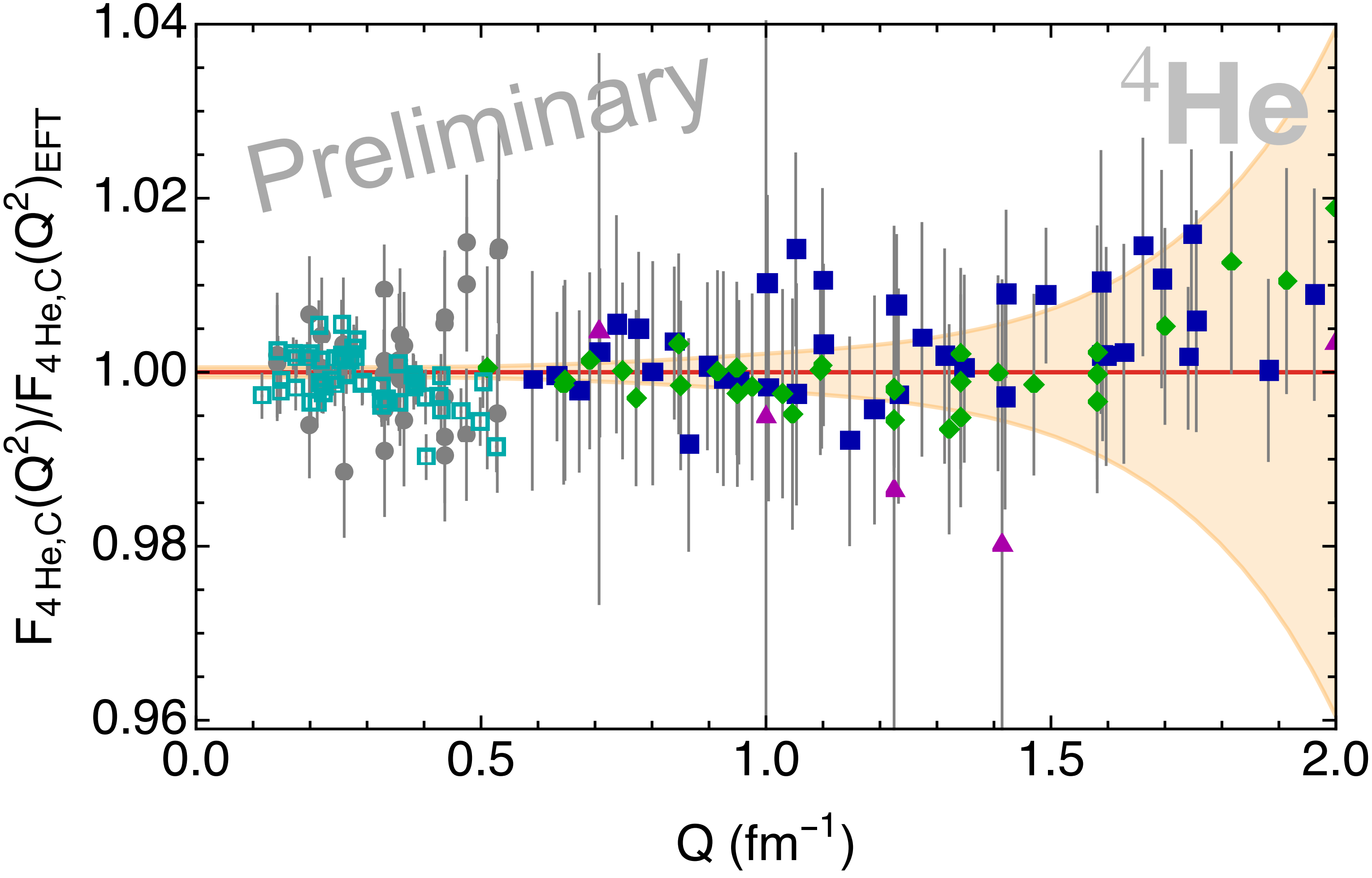
- Boosted potentials from the dynamical mass operator Polyzou et al., Few Body Syst. 49 (2011)

$$\tilde{V}_q := \sqrt{\Big(\underbrace{2\sqrt{\hat{p}^2+m^2}+\tilde{V}}_{\hat{M}_{12}(\tilde{V})}\Big)^2+q^2} - \sqrt{\Big(\underbrace{2\sqrt{\hat{p}^2+m^2}}_{\hat{M}^0_{12}}\Big)^2+q^2}$$



- Relativistic Faddeev/FY equations Witala et al. '09; Kamada et al. '19; Kamada '20; Hadizadeh et al. '20

The charge FFs of ^{4}He

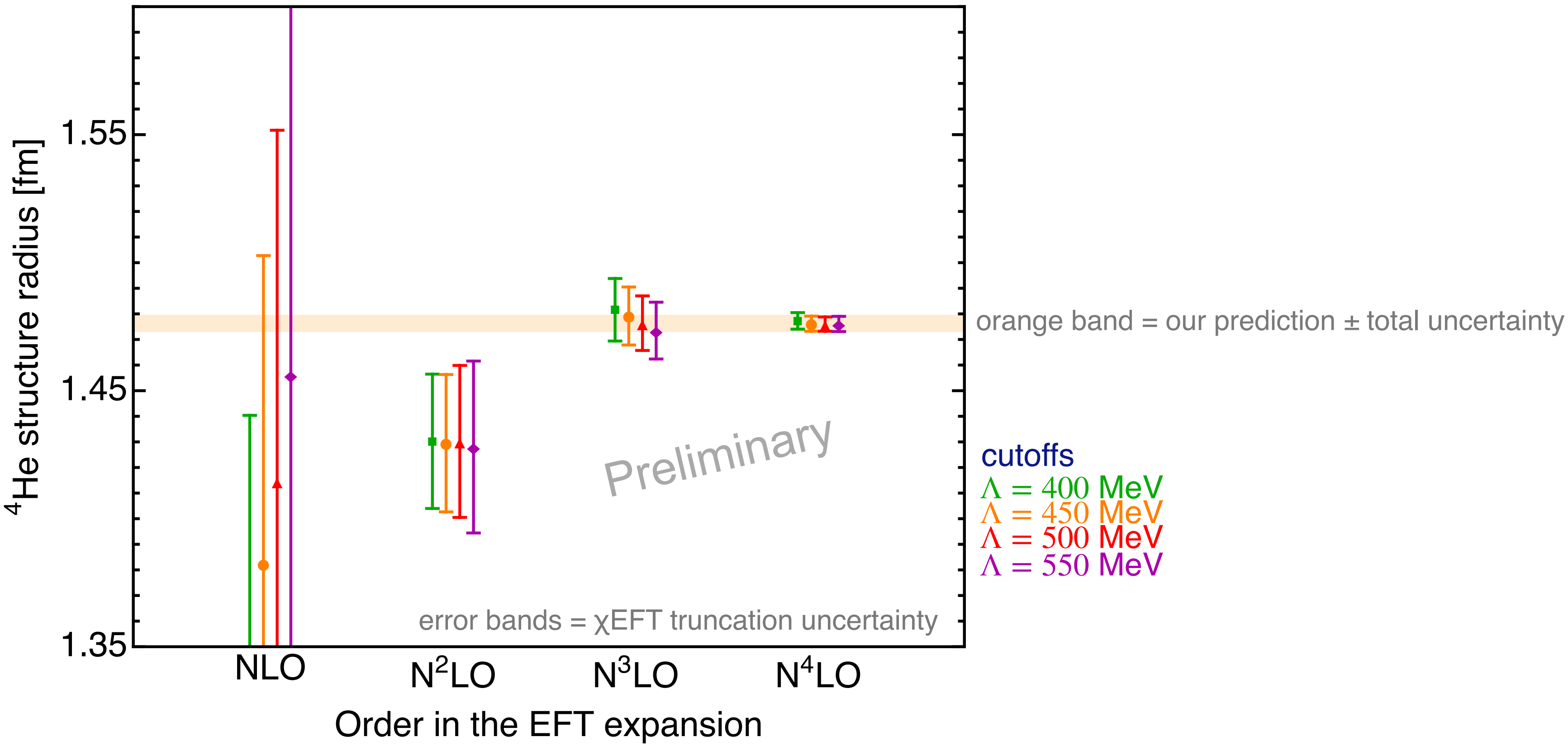


The structure radius of ^{4}He

Preliminary result for the ^{4}He structure radius:

$$r_{str}(^4\text{He}) = 1.47\text{xx} \pm 0.0028_{trunc} \pm 0.0011_{stat} \pm 0.0010_{nucl-FF} \text{ fm}$$ (Preliminary)

Consistency check (residual cutoff dependence):



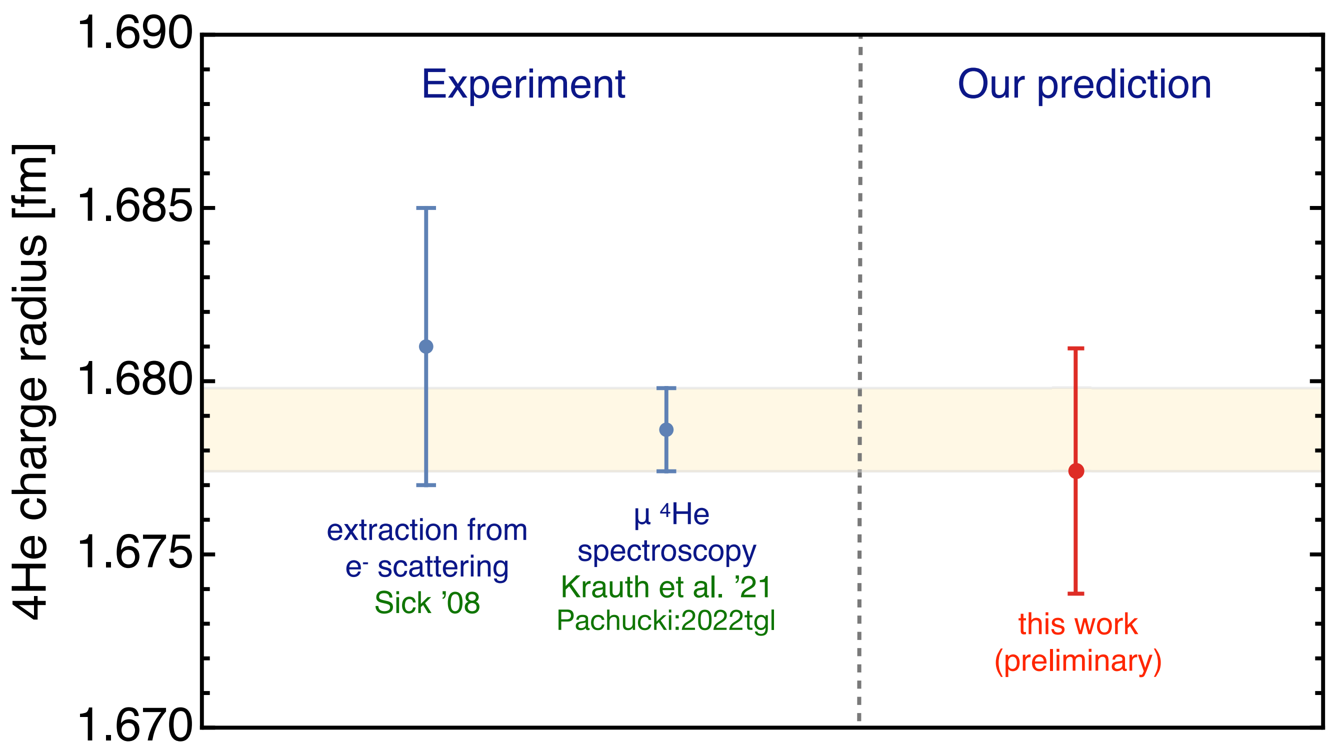
The charge radius of ^{4}He

Preliminary result for the ^{4}He charge radius:

$$r_C^2(^4\text{He}) = r_{\text{str}}^2(^4\text{He}) + \left(r_p^2 + \frac{3}{4m_p^2}\right) + r_n^2 \quad \Rightarrow \quad r_C(^4\text{He}) = (1.67\text{xx} \pm 0.0035)\ \text{fm}$$

preliminary (CODATA 2018 r_p + own determination of r_n)

Theory versus experiment:



Nucleon size from the ^{4}He charge radius

Alternatively: Nucleon size from ^{4}He radius

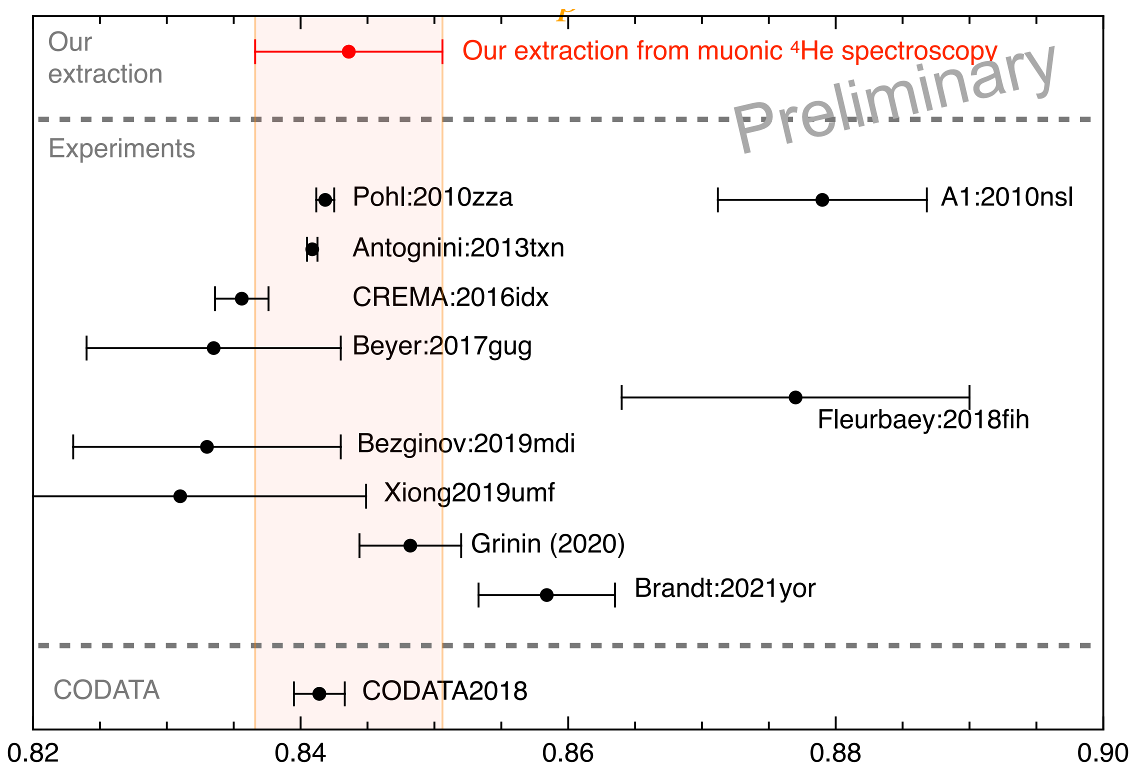
$$r_C^2(^4\mathrm{He}) = r_{\mathrm{str}}^2(^4\mathrm{He}) + \left(r_p^2 + \frac{3}{4m_p^2}\right) + r_n^2 \quad \Rightarrow$$

$$r_p^2 + r_n^2 = (0.6\mathrm{xx} \pm 0.010)\ \mathrm{fm}^2$$

$$r_p = (0.8\mathrm{xx} \pm 0.007)\ \mathrm{fm}$$

preliminary (own determination of r_n)

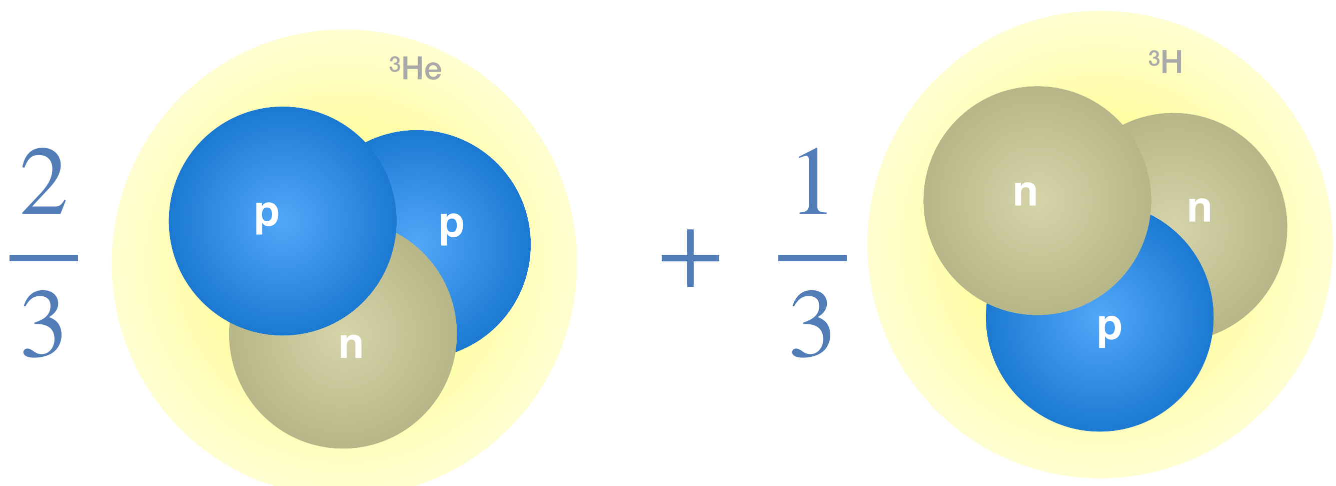
Proton charge radius



Isoscalar charge radius of A = 3 nuclei

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Andreas Nogga, Patrick Reinert, in preparation

Isovector ρ_{2N} beyond N^{2}LO not yet available $\Rightarrow$ focus on the ***isoscalar*** radius



Isoscalar charge radius of A = 3 nuclei

Predicted value of the isoscalar 3N charge radius $r_C^{\text{isoscalar}} = \sqrt{\frac{1}{3}(r_C^{3H})^2 + \frac{2}{3}(r_C^{3He})^2}$

$$r_C^{\text{isoscalar}} = (1.90\text{xx} \pm 0.0026)\ \text{fm}$$

preliminary (own determination of r_n)

Experimental value: $r_{C,\,exp}^{\text{isoscalar}} = (1.9010 \pm 0.0260)\ fm$

error dominated by the ^{3}H datum

⇒ our prediction is 10x more precise than the current experimental value

The ongoing T-REX experiment in Mainz [Pohl et al.] aims at measuring the ^{3}H charge radius within ± 0.0002 fm (i.e., 400x more precise) ⇒ the isoscalar radius will be known within ± 0.0009 fm

⇒ precision test of nuclear chiral EFT

^{3}He

$r = 1.97007(94)\text{fm}$

0.05 %

CREMA 2023
arXiv:2305.11679

^{3}H

$r = 1.7\text{xxx}(\quad 2)\text{fm}$
TREX Mainz
[in progress]

$r = 1.7550(860)\text{fm}$
Amroun et al. 1994

Summary and outlook

- Charge & quadrupole FFs of ^2H are in good shape (N^4LO, high-precision)

- Other systems and processes are limited to N^2LO accuracy due to unavailability of (consistently regularized) many-body forces & exchange currents

  ⇒ **symmetry-preserving gradient flow regularization** talk by Hermann

- Correlations between BEs and radii can be employed to obtain precise results for the charge FFs of ^4He & $^3H(e)_{isoscalar}$ already at this stage Arseniy Filin et al., in progress

- ^4He: Nuclear effects under control ⇒ new source of information about 1N radii

- $^3He/^3H$: prediction for the isoscalar 3N charge radius 10x more precise than exp!

Thank you for your attention

The 11th International Workshop on Chiral Dynamics

Aug 26 – 30, 2024
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The 11th International Workshop on Chiral Dynamics (CD2024) will take place August 26-30, 2024 at the Ruhr University Bochum, Germany. This series of workshops started at MIT in 1994 and brings together theorists and experimentalists every three years to discuss the status, progress and challenges in the physics of low-energy QCD, Goldstone Boson dynamics, meson-baryon Interactions, few-body physics, lattice QCD and ChPT. Previous workshops took place in Pisa (2015), Durham, NC (2018) and Beijing (2021).

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