## What's missing? An investigation of ${}^{3}\text{He}(\alpha, \gamma){}^{7}\text{Be radiative capture}$

#### Mack C. Atkinson



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$$\sigma(E) = \frac{S_{34}(E)}{E} \exp\left\{-\frac{2\pi Z_1 Z_2 e^2}{\hbar \sqrt{2E/m}}\right\}$$

# ${}^{3}$ He $(\alpha, \gamma)$ <sup>7</sup>Be important for solar-model predictions



Adelberger et al., Rev Mod Phys 83 195 (2011)

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- Reaction rates too low at solar energies in the lab
- Current evaluations depend on both theory and experiment
- Ideally, theory will accurately predict  $S_{34}(E)$





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 $\bullet~{\sf GPU}$  speedup  $\implies~{\sf NNN}$  forces are now included

• Calculate EM transitions from  ${}^{3}\mathrm{He}{+}\alpha$  scattering state to  ${}^{7}\mathrm{Be}$  bound state

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$$\left\langle \Psi_{bs} \left(^{7} \mathrm{Be}\right) \middle| \hat{\mathcal{M}}_{\mathrm{EM}} \middle| \Psi_{sc} \left(^{3} \mathrm{He} + \alpha\right) \right\rangle$$

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  - Only E1, E2, and M1 transitions

$$\left\langle \Psi_{bs}\left(^{7}\mathrm{Be}\right)\left| \hat{\mathcal{M}}_{\mathrm{EM}} \left| \Psi_{sc}\left(^{3}\mathrm{He}+lpha
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• Need a method to calculate  $\psi_{sc}$  and  $\psi_{bs}$  simultaneously



$$\left\langle \Psi_{bs}\left(^{7}\mathrm{Be}\right)\left|\hat{\mathcal{M}}_{\mathrm{EM}}\right|\Psi_{sc}\left(^{3}\mathrm{He}+\alpha\right)
ight
angle$$

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$$\hat{H}=\hat{T}+\hat{V}_{NN}+\hat{V}_{NNN}$$
 $\hat{H}\ket{\Psi^A}=E\ket{\Psi^A}$ 

$$\left\langle \Psi_{bs}\left(^{7}\mathrm{Be}\right)\left|\hat{\mathcal{M}}_{\mathrm{EM}}\right|\Psi_{sc}\left(^{3}\mathrm{He}+\alpha\right)
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 $\left\langle \Psi_{bs}\left(^{7}\mathrm{Be}\right) \middle| \hat{\mathcal{M}}_{\mathrm{EM}} \middle| \Psi_{sc}\left(^{3}\mathrm{He} + \alpha\right) \right\rangle$ 



$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} \left| \stackrel{(A)}{\Longrightarrow} , \lambda \right\rangle + \sum_{\nu} \int d\vec{r} \, \gamma_{\nu}(\vec{r}) \, \hat{A}_{\nu} \left| \stackrel{\bullet}{\underbrace{\bullet}}_{\substack{(A-a)}} , \nu \right\rangle$$

$$\left\langle \Psi_{bs}\left(^{7}\mathrm{Be}\right)\left|\hat{\mathcal{M}}_{\mathrm{EM}}\right|\Psi_{sc}\left(^{3}\mathrm{He}+\alpha\right)
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$$\uparrow | \stackrel{7}{\text{Be}} \rangle$$

$$\mathcal{V}_{bs} (^{7}\text{Be}) | \hat{\mathcal{M}}_{\text{EM}} | \Psi_{sc} (^{3}\text{He} + \alpha) \rangle$$



$$\Psi^{(A)} = \sum_{\lambda} c_{\lambda} | \overset{\text{(a)}}{\Longrightarrow}, \lambda \rangle + \sum_{\nu} \int d\vec{r} \gamma_{\nu}(\vec{r}) \hat{A}_{\nu} | \overset{\vec{r}}{\underbrace{}_{(A-a)}}, \nu \rangle$$

$$\uparrow \qquad \uparrow \qquad \uparrow$$

$$|^{7}\text{Be} \rangle \qquad |\alpha\rangle \otimes |^{3}\text{He} \rangle$$

$$\left\langle \Psi_{bs}\left(^{7}\mathrm{Be}\right)\left|\hat{\mathcal{M}}_{\mathrm{EM}}\right|\Psi_{sc}\left(^{3}\mathrm{He}+\alpha\right)
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• Capture rate accurate only if Expt. levels reproduced

$$\begin{array}{l} {{E_{{3/2}^ - }}: - 37.1{\rm{MeV}} \to - 37.7{\rm{MeV}} \\ {{E_{{1/2}^ - }}: - 36.9{\rm{MeV}} \to - 37.2{\rm{MeV}} \end{array} \end{array}$$



• Capture rate accurate only if Expt. levels reproduced

$$\begin{array}{l} E_{3/2^-}:-37.1 {\rm MeV} \to -37.7 {\rm MeV} \\ E_{1/2^-}:-36.9 {\rm MeV} \to -37.2 {\rm MeV} \end{array}$$

$$E_{\lambda}^{NCSM} \rightarrow E_{\lambda}^{NCSM} + \epsilon$$



### Minimal effect of phenomenological shift on scattering states

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- Main impact of pheno is to alter  $\psi_{bs}$
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![](_page_34_Figure_3.jpeg)

#### Minimal effect of phenomenological shift on scattering states

- Main impact of pheno is to alter  $\psi_{\textit{bs}}$
- Phase shifts at the relevant energies do not change
- The  $3/2^-$  and  $1/2^-$  scattering channels contribute minimally to  $S_{34}(E)$ 
  - Dominated by the E1 transitions from  $1/2^+$  scattering channel

![](_page_35_Figure_5.jpeg)

#### Comparing to other theoretical predictions of $S_{34}(E)$

• Inclusion of 3N force shows marked improvement over previous NN-only

$$\begin{split} &\mathsf{NN-N3LO+3NInI}\\ &\hbar\Omega=20~\mathsf{MeV}\\ &\lambda_{\mathrm{SRG}}=2.0~\mathsf{fm}^{-1} \end{split}$$

![](_page_36_Figure_3.jpeg)

## Comparing to other theoretical predictions of $S_{34}(E)$

- Inclusion of 3N force shows marked improvement over previous NN-only
- NCSMC prediction similar to FMD (AV18-like interaction)

$$\begin{split} &\mathsf{NN}\text{-}\mathsf{N3LO}\text{+}3\mathsf{NInI}\\ &\hbar\Omega=20~\mathsf{MeV}\\ &\lambda_{\mathrm{SRG}}=2.0~\mathsf{fm}^{-1} \end{split}$$

![](_page_37_Figure_4.jpeg)

### Comparing to other theoretical predictions of $S_{34}(E)$

- Inclusion of 3N force shows marked improvement over previous NN-only
- NCSMC prediction similar to FMD (AV18-like interaction)
- Consistent with current evaluation and capture data

![](_page_38_Figure_4.jpeg)

$$\begin{split} &\mathsf{NN-N3LO+3NInI}\\ &\hbar\Omega=20~\mathsf{MeV}\\ &\lambda_{\mathrm{SRG}}=2.0~\mathsf{fm}^{-1} \end{split}$$

#### Checking dependence on NN and 3N interactions

• Only comparing two interactions, both at  $N_{max} = 10$ 

$$NN-N3LO+3NInl$$

$$\hbar\Omega = 20 \text{ MeV}$$

$$\lambda_{SRG} = 2.0 \text{ fm}^{-1}$$

NN-N4LO+3NInIE7  $\hbar\Omega = 20 \text{ MeV}$  $\lambda_{
m SRG} = 2.0 \text{ fm}^{-1}$ 

![](_page_39_Figure_4.jpeg)

#### Checking dependence on NN and 3N interactions

- Only comparing two interactions, both at  $N_{max} = 10$
- Roughly 8% difference in  $S_{34}(E)$

$$NN-N3LO+3NInI$$

$$\hbar\Omega = 20 \text{ MeV}$$

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NN-N4LO+3NInIE7  $\hbar\Omega = 20 \text{ MeV}$  $\lambda_{
m SRG} = 2.0 \text{ fm}^{-1}$ 

![](_page_40_Figure_5.jpeg)

## Checking dependence on NN and 3N interactions

- Only comparing two interactions, both at  $N_{max} = 10$
- Roughly 8% difference in  $S_{34}(E)$
- Will analyze more interactions in a future work

![](_page_41_Figure_4.jpeg)

$$E_{c.m.}$$
 [MeV]

NN-N3LO+3NInI  $\hbar\Omega = 20 \text{ MeV}$   $\lambda_{SRG} = 2.0 \text{ fm}^{-1}$ 

NN-N4LO+3NInIE7  $\hbar\Omega = 20 \text{ MeV}$  $\lambda_{
m SRG} = 2.0 \text{ fm}^{-1}$ 

### SONIK <sup>3</sup>He+<sup>4</sup>He elastic scattering cross sections

 $\bullet$  Compare to elastic scattering results to further probe  $\psi_{\it sc}$ 

![](_page_42_Figure_2.jpeg)

Paneru et al., arXiv:2211.14641 (2022)

## SONIK <sup>3</sup>He+<sup>4</sup>He elastic scattering cross sections

- $\bullet$  Compare to elastic scattering results to further probe  $\psi_{\it sc}$
- Experiment done at TRIUMF in 2022  $\rightarrow$  lowest *E* measured to date

![](_page_43_Figure_3.jpeg)

Paneru et al., arXiv:2211.14641 (2022)

## SONIK <sup>3</sup>He+<sup>4</sup>He elastic scattering cross sections

- $\bullet$  Compare to elastic scattering results to further probe  $\psi_{\it sc}$
- Experiment done at TRIUMF in 2022  $\rightarrow$  lowest *E* measured to date

![](_page_44_Figure_3.jpeg)

• What is the source of discrepancy at large angles?

 $\begin{aligned} &\mathsf{NN-N3LO+3NInI}\\ &\hbar\Omega = 20 \; \mathsf{MeV}\\ &\lambda_{\mathrm{SRG}} = 2.0 \; \mathsf{fm}^{-1} \end{aligned}$ 

Paneru et al., arXiv:2211.14641 (2022)

#### Same large-angle discrepancy when comparing to 1964 Barnard et al.

![](_page_45_Figure_1.jpeg)

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• Rutherford obscures the fact that a constant shift accounts for the discrepancy

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$$\frac{d\sigma}{d\Omega_{\rm Ruth}} = \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 m v^2 \sin^2\left(\frac{\theta}{2}\right)}\right)^2$$

- Varied properties of the interaction
- Nothing in the NCMSC appears to reproduce the 15 mb shift

![](_page_48_Figure_5.jpeg)

• Rutherford obscures the fact that a constant shift accounts for the discrepancy

$$\frac{d\sigma}{d\Omega_{\rm Ruth}} = \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 m v^2 \sin^2\left(\frac{\theta}{2}\right)}\right)^2$$

- Varied properties of the interaction
- Nothing in the NCMSC appears to reproduce the 15 mb shift

How can we emulate a constant shift?

![](_page_49_Figure_6.jpeg)

## The $1/2^+$ channel can produce this constant shift

![](_page_50_Figure_1.jpeg)

## The $1/2^+$ channel can produce this constant shift

![](_page_51_Figure_1.jpeg)

![](_page_51_Figure_2.jpeg)

## The $1/2^+$ channel can produce this constant shift

- More repulsion is needed in the  $1/2^+$  channel
- Already shown that changing *NN* and *3N* interactions does not fix
- We explicitly add repulsion to the 1/2<sup>+</sup> Hamiltonian kernel

$$V(r,r') = rac{V_0}{1+e^{(R-r_0)/a_0}} imes e^{(r-r')^2/a_0^2}$$

![](_page_52_Figure_5.jpeg)

$$\mathcal{H}_{RGM}(r,r') \rightarrow \left\langle lpha + {}^{3}He \left| \mathcal{A}^{\dagger}H\mathcal{A} \left| lpha + {}^{3}He \right\rangle + V(r,r') 
ight.$$

![](_page_53_Figure_1.jpeg)

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- Elastic and capture data inconsistent
- Cannot describe both simultaneously

![](_page_54_Figure_3.jpeg)

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- Elastic and capture data inconsistent
- Cannot describe both simultaneously

![](_page_55_Figure_3.jpeg)

![](_page_55_Figure_4.jpeg)

![](_page_56_Figure_1.jpeg)

- Cannot describe both simultaneously
- Considering all data provides new band

![](_page_56_Figure_4.jpeg)

![](_page_56_Figure_5.jpeg)

#### Data-Informed S-factor

• Consider spread of  $S_{34}(E)$  from different interactions as well as considering elastic data

![](_page_57_Figure_2.jpeg)

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- Consider spread of  $S_{34}(E)$  from different interactions as well as considering elastic data
- Discrepancy between elastic and capture data dominates the uncertainty

![](_page_58_Figure_3.jpeg)

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- Consider spread of  $S_{34}(E)$  from different interactions as well as considering elastic data
- Discrepancy between elastic and capture data dominates the uncertainty

![](_page_59_Figure_3.jpeg)

• For Solar Model calculations, I would provide the spread due to elastic vs. capture data inconsistency (right figure)

• We predict a  $1/2^+$  resonance roughly 2 MeV above  $p+^6$ Li threshold

![](_page_60_Figure_2.jpeg)

- We predict a  $1/2^+$  resonance roughly 2 MeV above  $p+^6$ Li threshold
  - Proton resonance due to close proximity to threshold

![](_page_61_Figure_3.jpeg)

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- Inclusion of  $p+^{6}$ Li channel will improve description resonance

![](_page_62_Figure_4.jpeg)

- We predict a  $1/2^+$  resonance roughly 2 MeV above  $p+^6$ Li threshold
  - Proton resonance due to close proximity to threshold
- Inclusion of  $p+^{6}$ Li channel will improve description resonance
- Could address discrepancy between data sets

![](_page_63_Figure_5.jpeg)

- Ab initio calculation of  ${}^{3}\text{He}(\alpha,\gamma){}^{7}\text{Be}$  capture reaction using the NCSMC
- Can provide both an *ab initio* prediction as well as a data-informed prediction
- The NCSMC allows the simultaneous analysis of elastic and capture data, revealing a discrepancy
- Future: Include  $p+^{6}$ Li channel
- Future: More robust uncertainty quantification

#### Thanks!

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