Collective and single particle structure of ^{12}Be negative parity spectrum

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Progress in *Ab Initio* Techniques in Nuclear Physics
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Outline

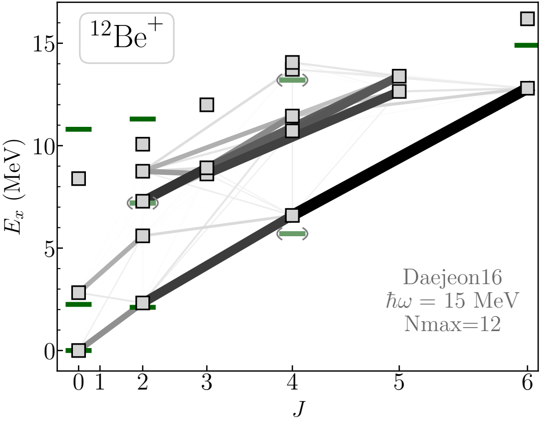
Goal: extract a more intuitive (approximate) picture for understanding the structure of ^{12}Be

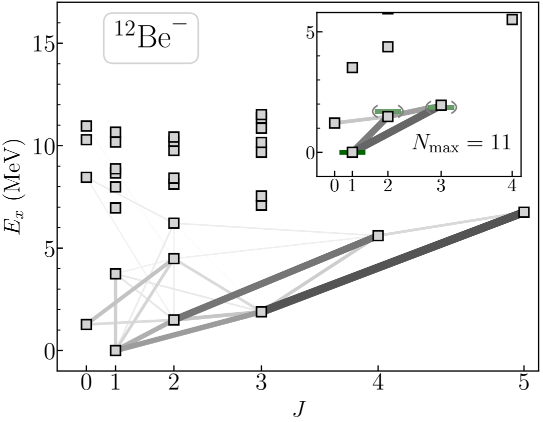
- Look for signatures of rotational dynamics (characteristic energies, enhanced transition strengths, etc.)

- Decompose wave functions by symmetries (SU(3), Sp(3, $\mathbb{R}$), SU(4), etc.)

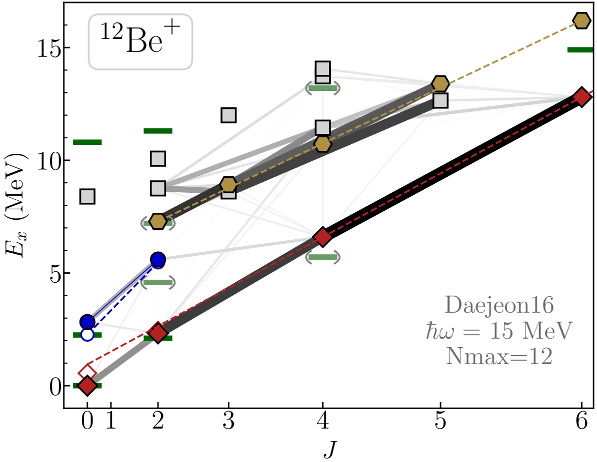
- Occupations of single particle orbitals (natural orbitals)

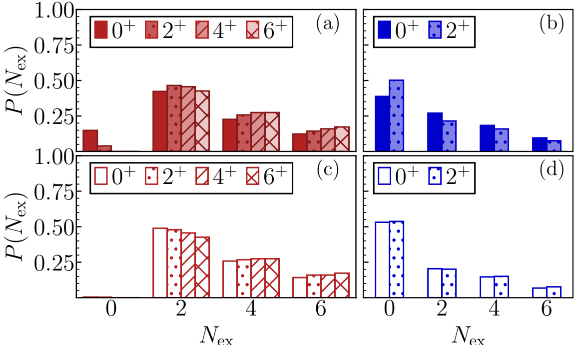
^{12}Be Spectrum





Intruder ground state band in ^{12}Be





Anna E. McCoy and Mark A. Caprio and Pieter Maris and Patrick J. Fasano, arXiv:2402.12606.

Elliott SU(3)

Labels (λ,μ) associated with deformation parameters β and γ

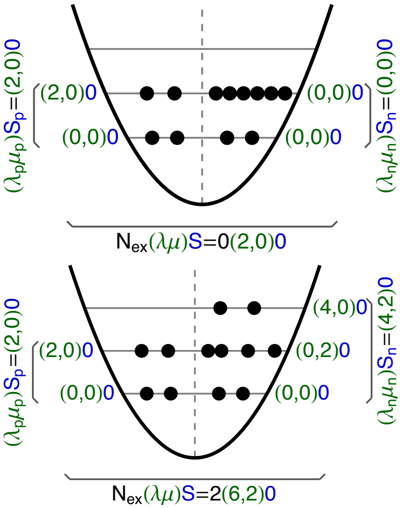
O. Castanos, J. P. Draayer, Y. Leschber, Z. Phys. A **329** *(1988) 3.*

Lowest energies correspond to most deformed state

D. J. Rowe, G. Thiamova, and J. L. Wood. Phys. Rev. Lett. **97** *(2006) 202501.*

$$\beta^2 \propto \langle Q\cdot Q\rangle / \langle r^2\rangle^2$$

$$H = \underbrace{H_0}_{\text{shell}} - \underbrace{\kappa Q\cdot Q}_{\text{correlations}} + L\cdot S$$



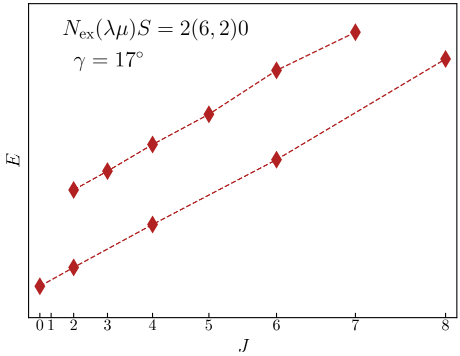
SU(3) symmetry of a configuration

- Each particle has SU(3) symmetry $(N,0)$, $N = 2n+\ell$
- Allowed spins dictated by antisymmetry constraints
- Final quantum numbers are $N_{ex}(\lambda\mu)S$.

Elliott's Rotational Model

SU(3) intrinsic state [with definite $(\lambda\mu)$] projects onto K_L bands with good L.

$$|(\lambda\mu)K_L L M_L\rangle, \qquad \cancel{L\times S \to J} \quad \to \quad K = K_K + K_S$$



Rotor Hamiltonian:
A. S. Davydov and G. F. Fillippov. Nucl. Phys. **8** (1958) 237.

SU(3) generators

Q_{2M}	*Algebraic quadrupole*
L_{1M}	*Orbital angular momentum*

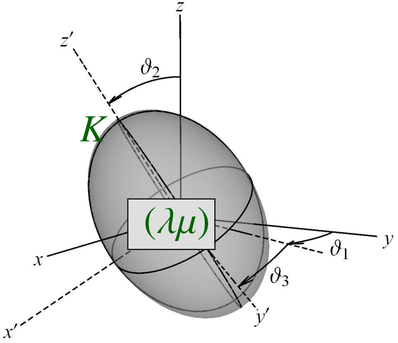
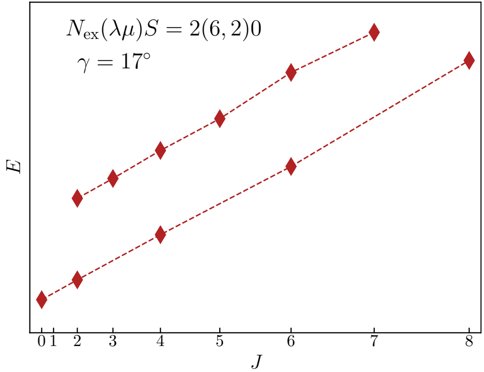
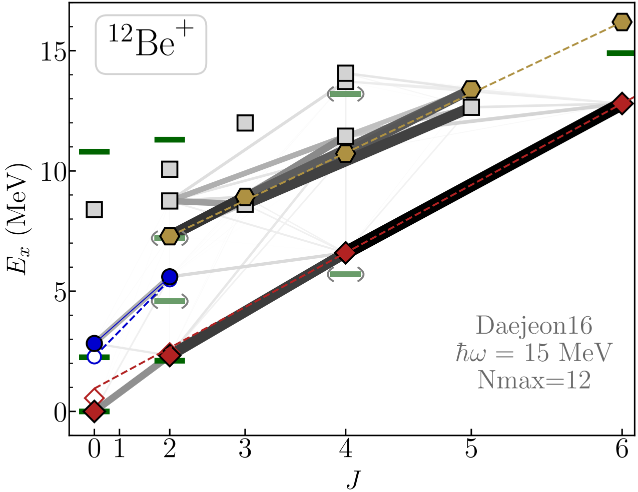


Figure: F. Iachello and M. A. Caprio, Understanding Quantum Phase Transitions. (2010) pp. 673–700.

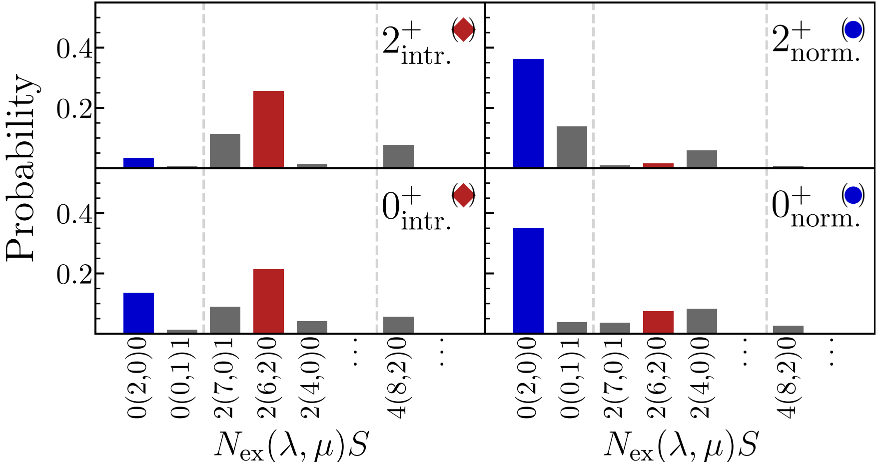
Elliott's rotational model: ^{12}Be





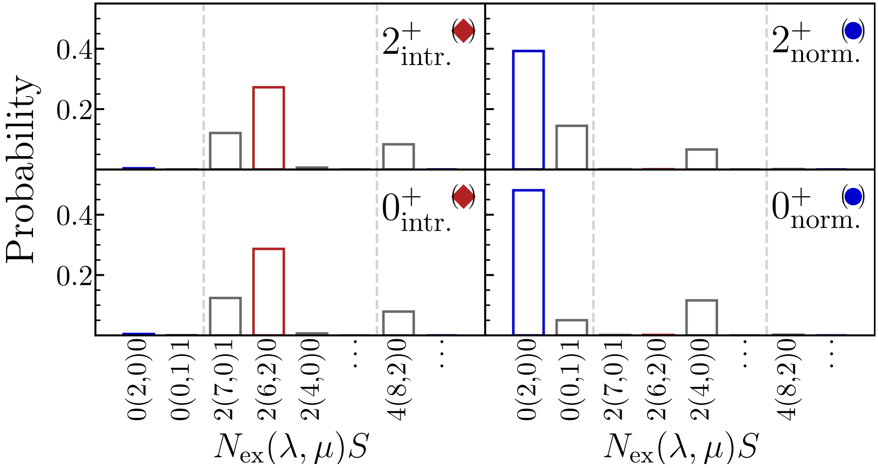
SU(3) decompositions

Mixed states

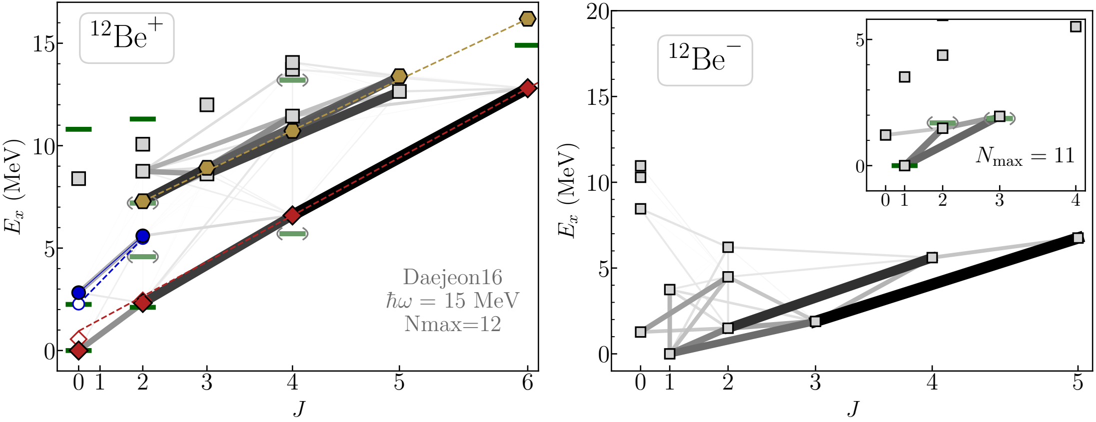


SU(3) decompositions

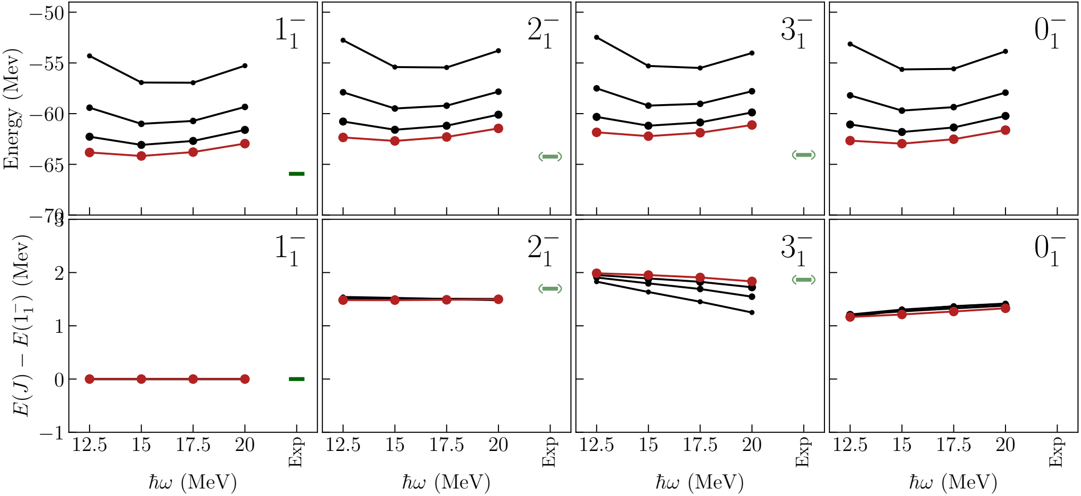
Pure states



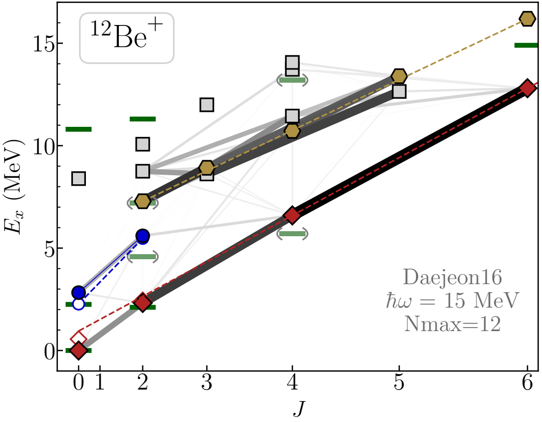
^{12}Be Spectrum

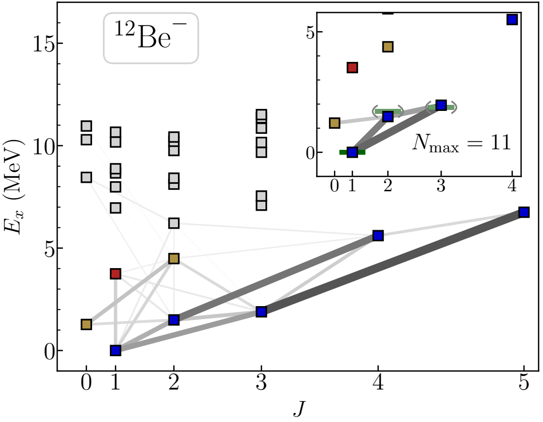


Energy convergence



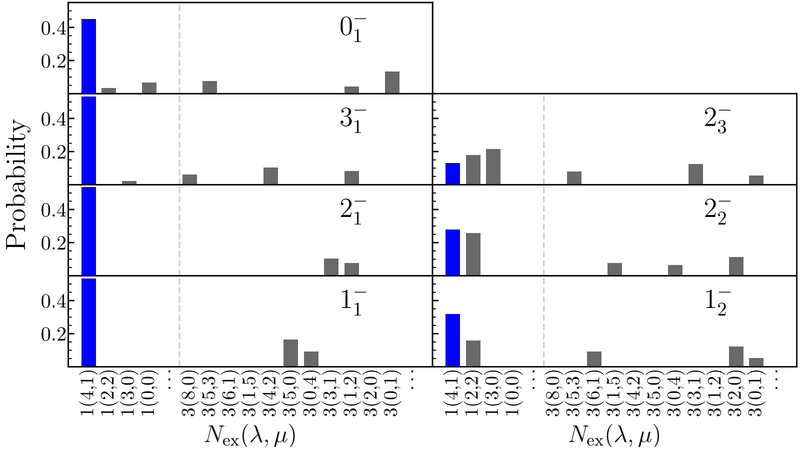
^{12}Be Spectrum





Elliott's model: $^{12}\mathrm{Be}^{-}$

"Most deformed SU(3): $N_{\mathrm{ex}}(\lambda,\mu)S = 1(4,1)0$ and $1(4,1)1$.



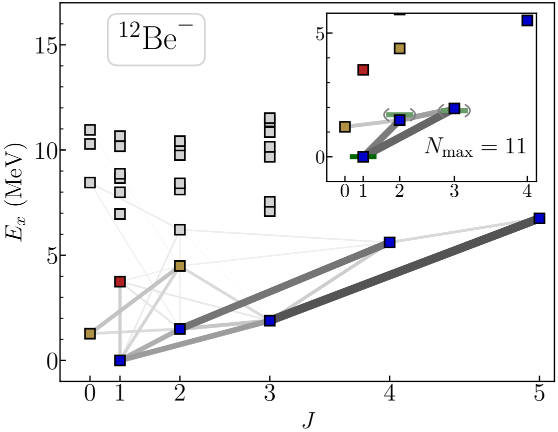
Elliott's model: $^{12}\text{Be}^{-}$

"Most deformed SU(3): $N_{\text{ex}}(\lambda,\mu)S = 1(4,1)0$ and $1(4,1)1$.

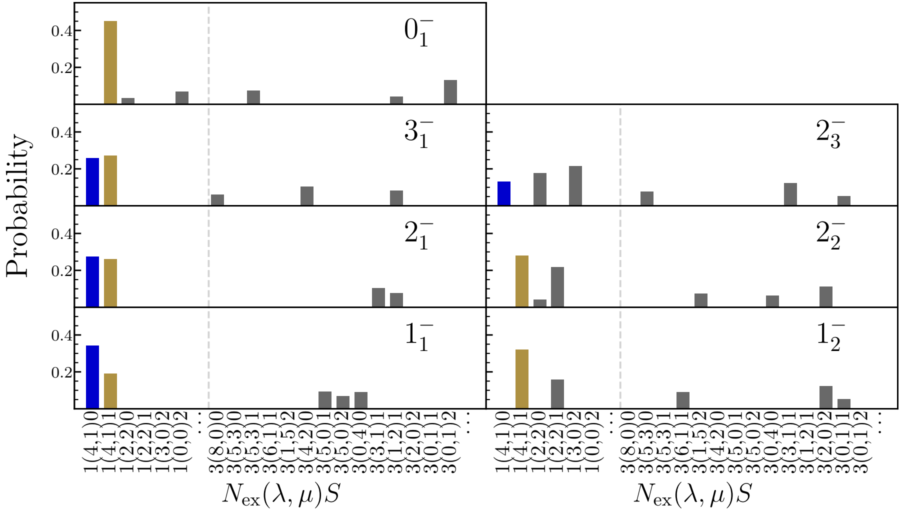
$\gamma \approx 25°$
L = 1,2,3,4,5

$\underline{S = 0}$
$K = 1 : J = 1,2,3,4,5$

$\underline{S = 1}$
$K = 0 : J = 0,2,4$
$K = 0 : J = 1,3,5$
$K = 1 : J = 1,2,3,4,5$
$K = 2 : J = 2,3,4,5,6$



SU(3) decompositions of $^{12}\mathrm{Be}^{-}$



Summary

What we know so far...

- Calculated energies for ^{12}Be in reasonable agreement with experiment, though many of the states assignments are tentative. We predict a previously unobserved 0_1^- at similar energy to the 2_1^-.

- Both positive and negative parity states in ^{12}Be exhibit signatures of collective behavior (enhanced $E2$ transition strengths).

- Negative parity states do not have an obvious rotational structure, but exhibit approximate SU(3) symmetry. *Need to look at* SU(4)

Let's look at the single particle structure

Natural Orbitals – Example: four-state, two-orbital system: $0s_{1/2}, 1s_{1/2}$

Eigenvector in harmonic oscillator basis:

$$|\Psi\rangle = \underbrace{\frac{1+\sqrt{3}}{4}|(0s_\uparrow)(0s_\downarrow)\rangle}_{N=0} + \underbrace{\frac{1-\sqrt{3}}{4}|(0s_\uparrow)(1s_\downarrow)\rangle - \frac{1-\sqrt{3}}{4}|(0s_\downarrow)(1s_\uparrow)\rangle}_{N=2} + \underbrace{\frac{1+\sqrt{3}}{4}|(1s_\uparrow)(1s_\downarrow)\rangle}_{N=4}$$

Density matrix $\langle\Psi|a_i^\dagger a_j|\Psi\rangle$:

$$\rho = \begin{pmatrix} 1/2 & 0 & -1/4 & 0 \\ 0 & 1/2 & 0 & -1/4 \\ -1/4 & 0 & 1/2 & 0 \\ 0 & -1/4 & 0 & 1/2 \end{pmatrix}$$

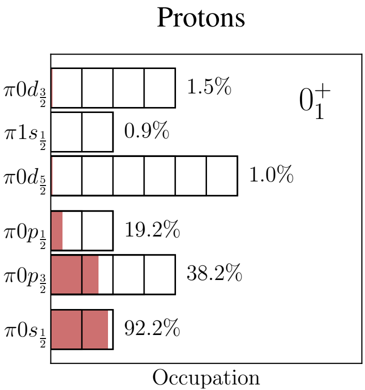
Natural orbitals (eigenvectors of ρ):

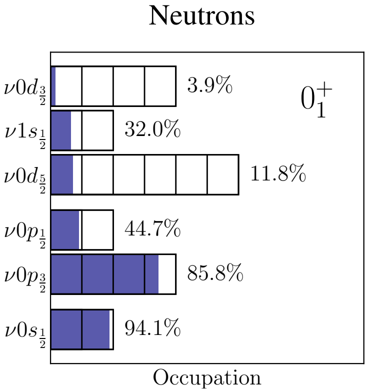
$$|0s'_{1/2}\rangle = \frac{1}{\sqrt{2}}|0s_{1/2}\rangle - \frac{1}{\sqrt{2}}|1s_{1/2}\rangle$$

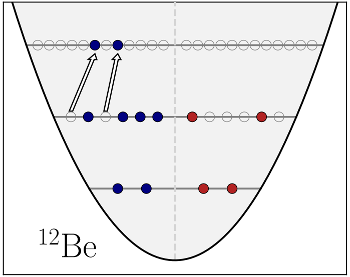
$$|1s'_{1/2}\rangle = \frac{1}{\sqrt{2}}|0s_{1/2}\rangle + \frac{1}{\sqrt{2}}|1s_{1/2}\rangle$$

Eigenvector in natural orbital basis:

Natural orbital occupations

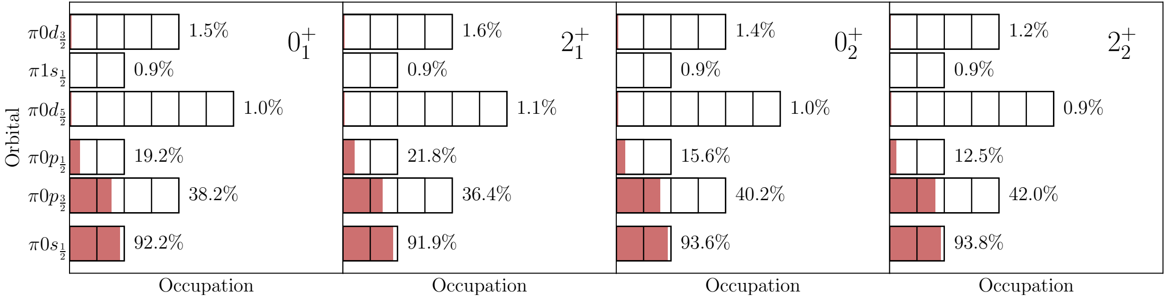






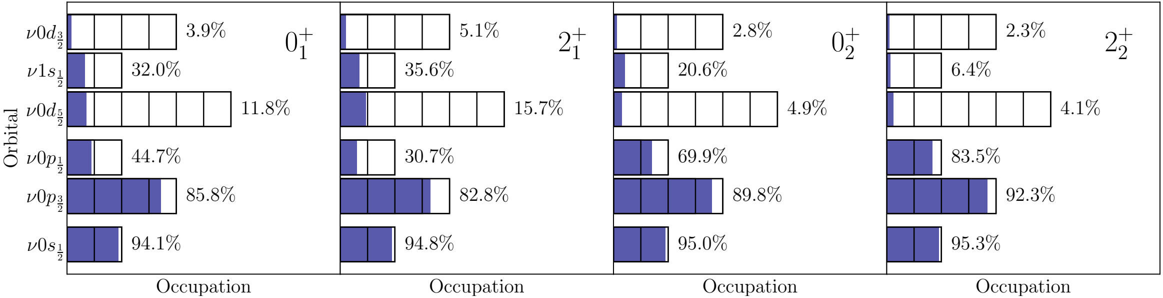
Natural orbital occupations

Protons



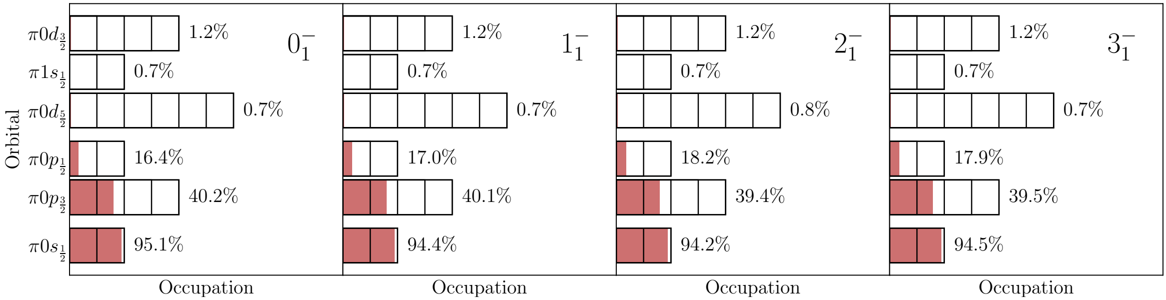
Natural orbital occupations

Neutrons



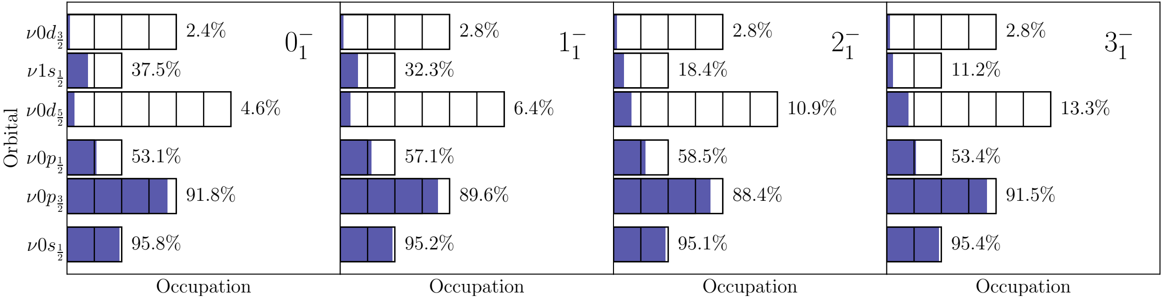
Natural orbital occupations

Protons



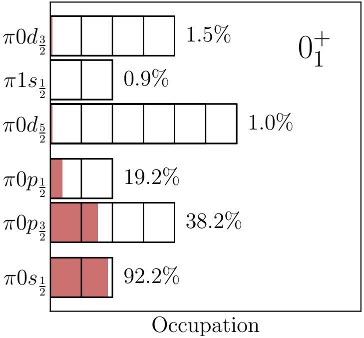
Natural orbital occupations

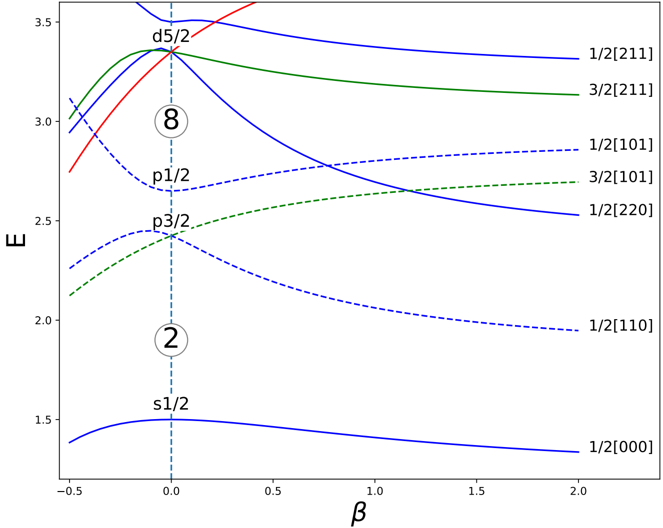
Neutrons



Nilsson model

$$H = H_0 - \hbar\omega r^2 \delta(\beta) Y_{20} + \alpha \mathbf{l} \cdot \mathbf{s}$$

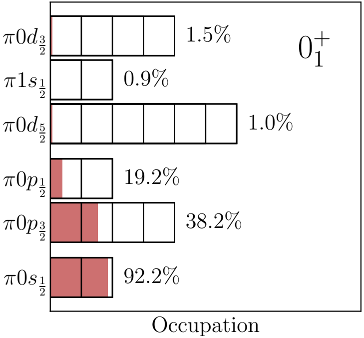


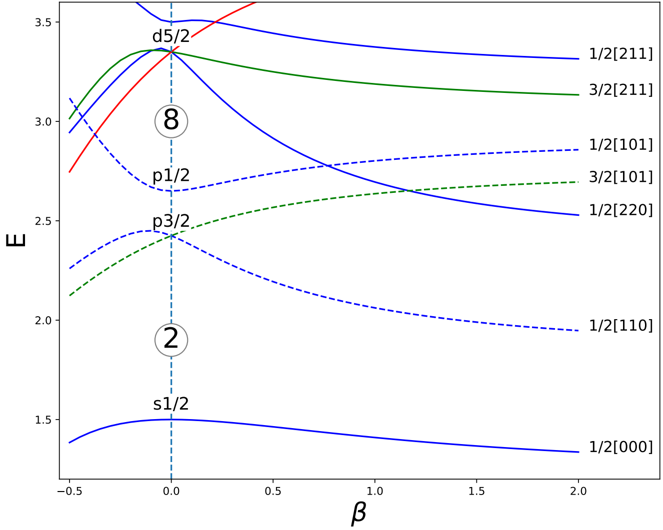


Nilsson model

At $\beta_p = 1.2$

$$|1/2[110]\rangle = 0.45\,|0p_{1/2}\rangle - 0.89\,|0p_{3/2}\rangle$$





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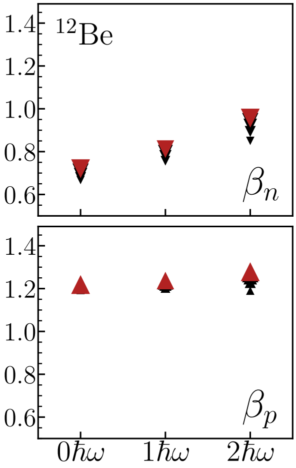

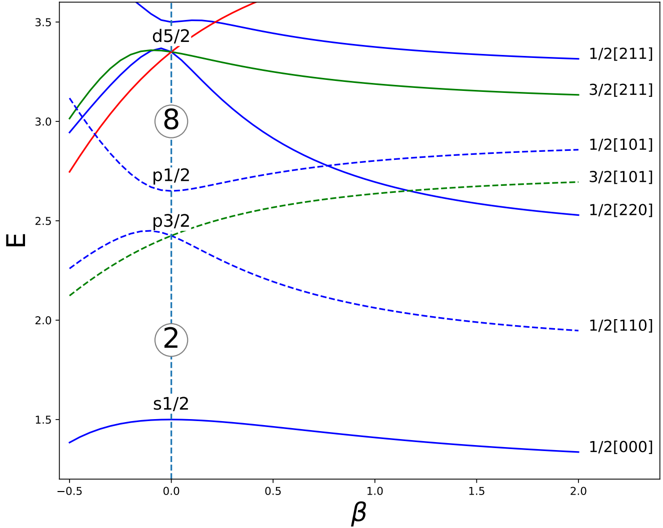

Acknowledgements

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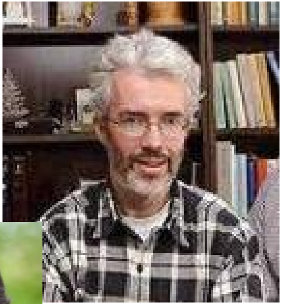
Mark Caprio *Univ. Notre Dame*
Patrick Fasano *ANL*
Pieter Maris *Iowa State Univ.*



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– Negative parity states do not have an obvious rotational structure, but exhibit approximate SU(3) symmetry. *Need to look at* SU(4)

– Occupations of single particle natural orbitals are qualitatively consistent with naive filling of Nilsson orbitals. *See Patrick Fasano's talk on Thursday!*