

Coupling nuclear structure and relativistic
hydrodynamic calculations:
collectivity in small systems

Benjamin Bally

PAINT 2024 - TRIUMF - 01/03/2024



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 - ◇ Nuclear structure recognized as important to understand heavy-ion collisions
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 - Zhang *et al.*, PRL 128, 022301 (2022)
 - Jia *et al.*, PRL 131, 022301 (2023)

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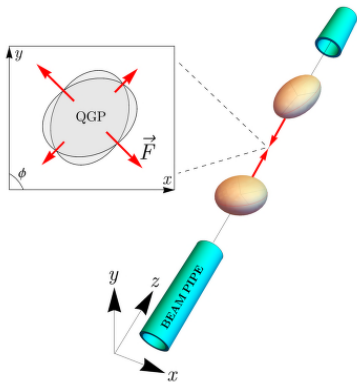
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- New proposal for a future LHC run (not before 2029)

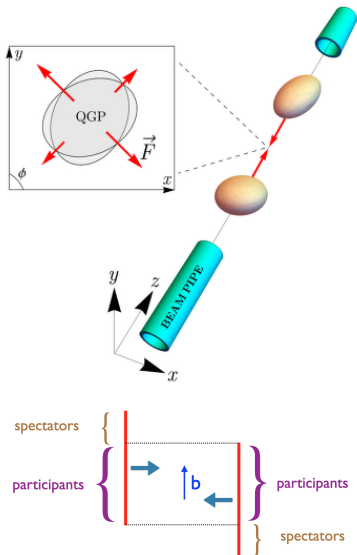
CERN-TH-2024-021

The unexpected uses of a bowling pin: exploiting ^{20}Ne isotopes for precision characterizations of collectivity in small systems

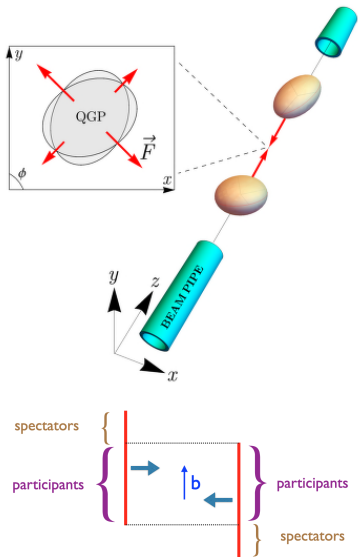
Giuliano Giacalone,^{1,*} Benjamin Bally,² Govert Nijss,³ Shihang Shen,⁴
Thomas Duguet,^{5,6} Jean-Paul Ebran,^{7,8} Serdar Elhatisari,^{9,10} Mikael Frosini,¹¹ Timo A. Lähde,^{12,13}
Dean Lee,¹⁴ Bing-Nan Lu,¹⁵ Yuan-Zhuo Ma,¹⁴ Ulf-G. Meißner,^{10,16,17} Jacquelyn Noronha-Hostler,¹⁸
Christopher Plumberg,¹⁹ Tomás R. Rodríguez,²⁰ Robert Roth,^{21,22} Wilke van der Schee,^{3,23,24} and Vittorio Somà⁵

- Large collaboration of theorists
- From low- and high-energy nuclear physics communities
 - ◇ Heavy-ion collisions
 - ◇ Nuclear structure (PGCM)
 - ◇ Nuclear structure (NLEFT)

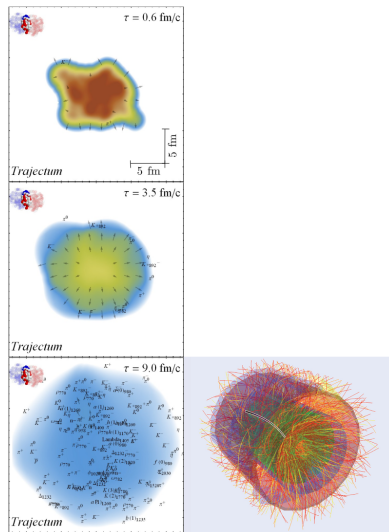




Ollitrault, EPJA 59, 236 (2023)

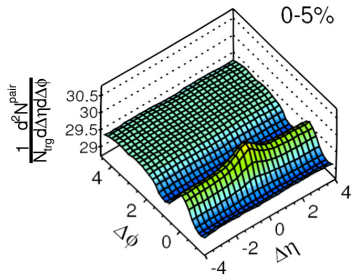


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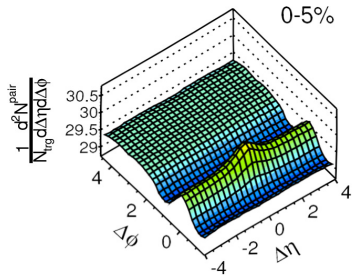


Giacalone *et al.*, PRL 131, 202302 (2023)

ALICE collaboration



credit: CMS collaboration

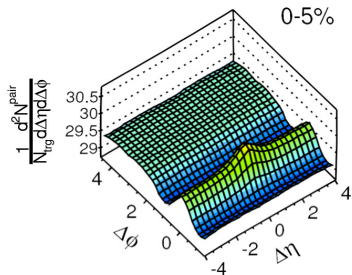


credit: CMS collaboration

- Probability distribution of particle emission

$$P(\phi, \eta) = P(\phi) = \frac{1}{2\pi} \sum_{n=-\infty}^{+\infty} V_n e^{-in\phi}$$

$V_2 \equiv$ elliptic flow, $V_3 \equiv$ triangular flow, ...



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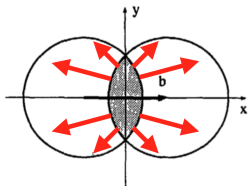
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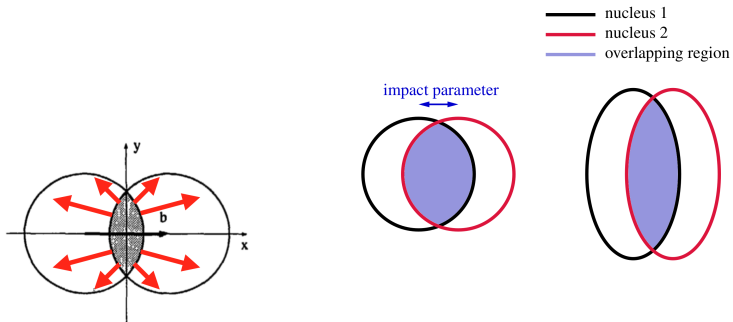
- Average of pair distribution

$$\left\langle \frac{dN_{\text{pair}}}{d\Delta\eta d\Delta\phi} \right\rangle = \langle P(\phi)P(\phi + \Delta\phi) \rangle = \frac{1}{2\pi} \left(1 + 2 \sum_{n=1}^{+\infty} \langle |V_n|^2 \rangle \cos(n\Delta\phi) \right)$$



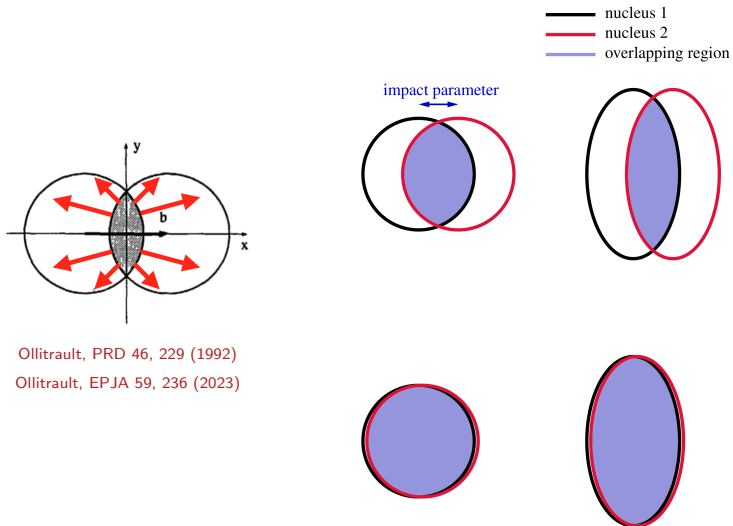
Ollitrault, PRD 46, 229 (1992)

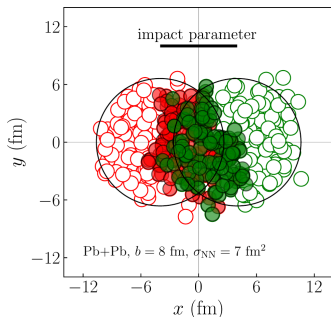
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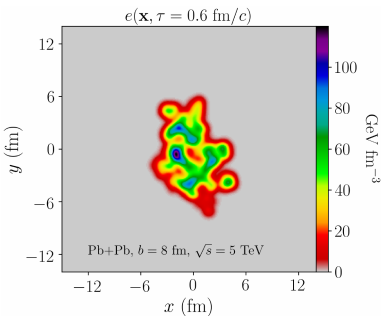
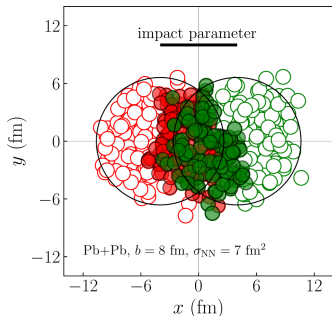
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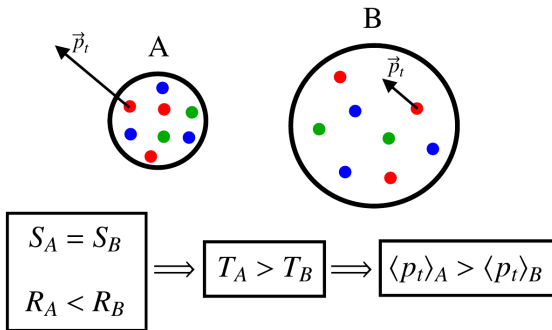
Courtesy of G. Giacalone

- Nucleons sampled using a model (often simply a Woods-Saxon density)



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- Nucleons sampled using a model (often simply a Woods-Saxon density)
- Participant nucleons translated into energy density using a prescription, e.g., T_RENTo model
- Run relativistic hydrodynamic simulations

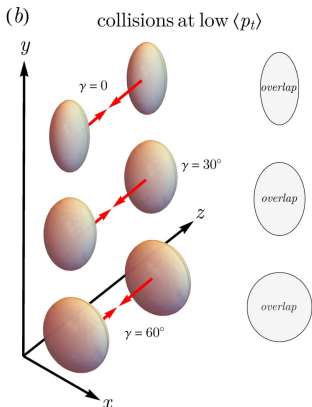


Courtesy of G. Giacalone

- $\langle p_t \rangle$ is the average transverse momentum of particles
- Looking at low $\langle p_t \rangle \Rightarrow$ looking at larger nuclear overlaps

- Pearson correlation coefficient

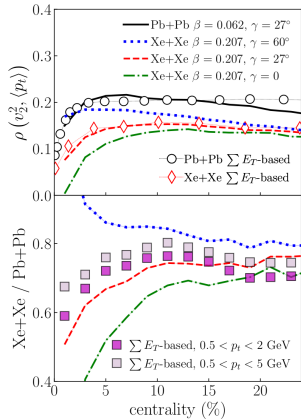
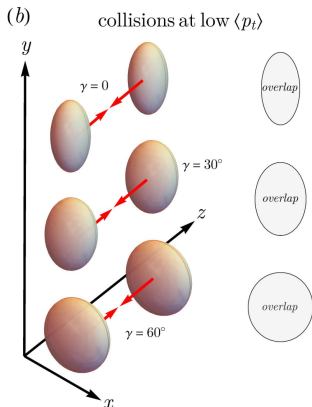
$$\rho(v_2^2, \langle p_t \rangle) = \frac{\langle \delta v_2^2 \delta \langle p_t \rangle \rangle}{\sqrt{\langle (\delta v_2^2)^2 \rangle \langle (\delta \langle p_t \rangle)^2 \rangle}} \quad \text{where } \delta o = o - \langle o \rangle$$



Bally *et al.*, PRL 128, 082301 (2022)

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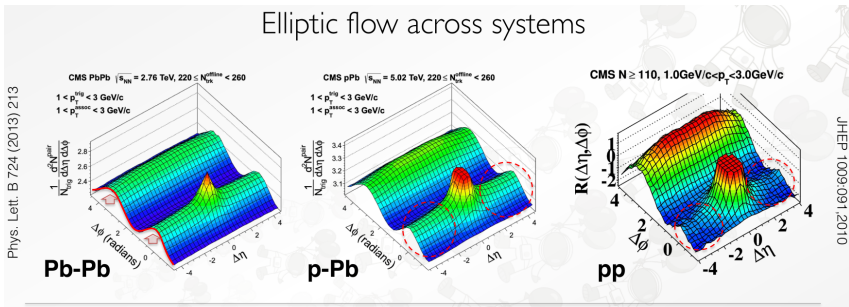
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- Collectivity appears in *small systems* (p+p, p+A, d+A, ...)
- Is it the same mechanism? Is hydrodynamic the correct description?
- How small can a QGP be?

- Experimental methods exist but have also limitations
 - ◇ $p + {}^{197}\text{Au}$ *versus* $d + {}^{197}\text{Au}$
 - ◇ Central ${}^{16}\text{O} + {}^{16}\text{O}$ *versus* peripheral ${}^{208}\text{Pb} + {}^{208}\text{Pb}$

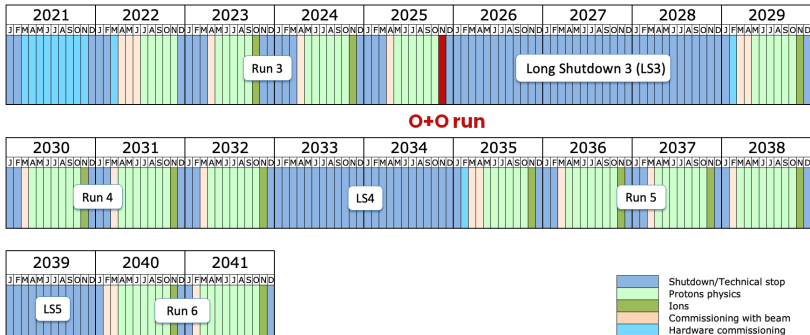
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 - ◇ Systems with similar A
 - ◇ Same centrality class
 - ◇ Overlap region well-defined

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⇒ **Test hydrodynamic description in a small systems**



Last update: April 2023

<https://lhc-commissioning.web.cern.ch/schedule/LHC-long-term.htm>

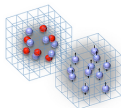
- ion+ion collisions scheduled 1 month/year until the end of operation

EFT \rightarrow Nuclear structure \rightarrow Relativistic hydrodynamic \rightarrow Hadron transport \rightarrow Nucl-th

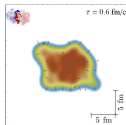
χ EFT
N3LO



PGCM



NLEFT



Trajectum



arXiv:2402.05995

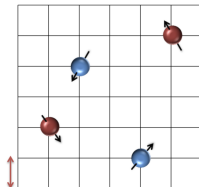
- TAURUS: <https://github.com/project-taurus>
- Trajectum: <https://sites.google.com/view/govertnijs/trajectum>
- SMASH: <https://github.com/smash-transport/smash>

- Nuclear Lattice Effective field Theory (NLEFT)

Lee, *Front. in Phys.* 8, 174 (2020)

Lähde and Meißner, *Lectures Notes in Phys.*, Springer (2019)

- Mesh with 8 sites and spacing $a = 1.315$ fm



- Minimal pionless EFT Hamiltonian with $SU(4)$ symmetry

- Pin-hole algorithm \rightarrow nucleon positions

Elhatisari *et al.*, *PRL* 119, 222505 (2017)

\Rightarrow see [Dean's presentation](#)

- Projected Generator Coordinate Method (PGCM)

$$|\Theta_{\epsilon}^{\sigma M}\rangle = \sum_{qK} f_{\epsilon}^{\sigma M}(q, K) P_{MK}^{\sigma} |\Phi(q)\rangle \quad \text{where } \sigma \equiv Z, N, J, \pi$$

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- Full method: PGCM + Perturbation Theory

Frosini *et al.*, EPJA 58, 62 (2022)

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- Model space: $e_{\max} = 6$, $e_{3\max} = 18$, $\hbar\omega = 12$

Hamiltonian: Hütter N3LO

Hütter *et al.*, PLB 808, 135651 (2019)

Reference states $|\Phi\rangle$: real general Bogoliubov (VAPNP minimization)

Collective coordinates q : $\beta_{20}, \beta_{22}, \beta_{30}, \beta_{32}$

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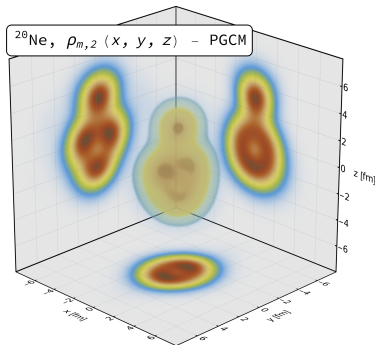
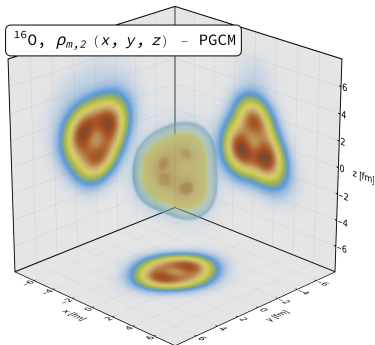
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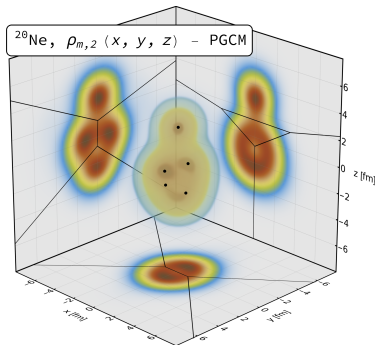
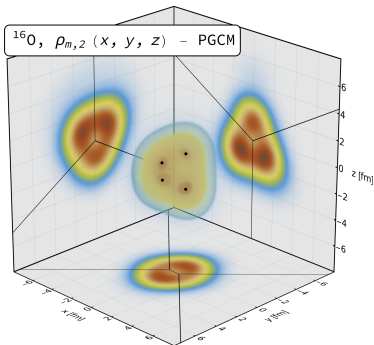
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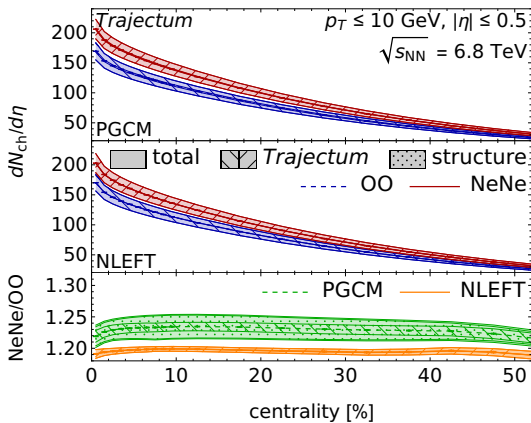
⇒ see **Andrea's presentation & my presentation of last year**



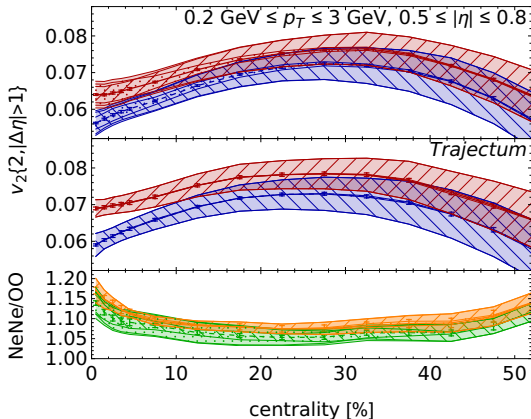
- Determine average deformation of PGCM ground state: \bar{q}
- One-body density: $\rho_m(x, y, z) = \sum_{st} \frac{\langle \Phi(\bar{q}) | a_{xyzst}^+ a_{xyzst} P^Z P^N | \Phi(\bar{q}) \rangle}{\langle \Phi(\bar{q}) | P^Z P^N | \Phi(\bar{q}) \rangle}$
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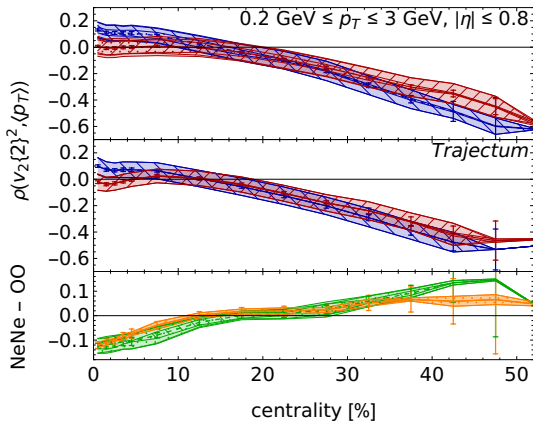
Results: elliptic flow $v_2\{2\}^2$



- In the 0-1% events:

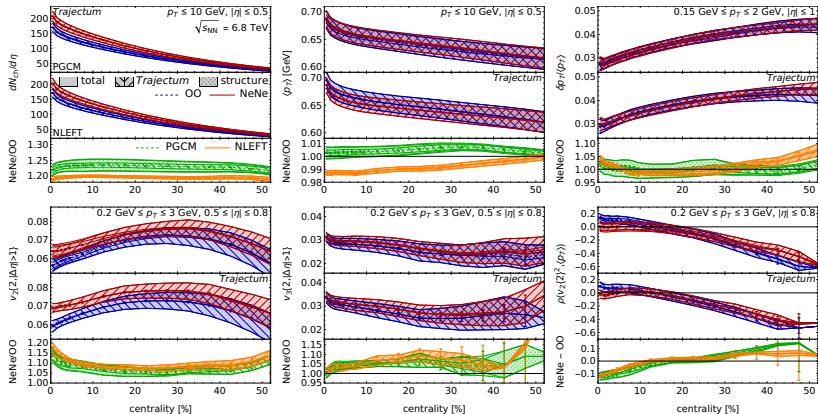
$$\frac{v_2\{2\}_{\text{NeNe}}}{v_2\{2\}_{\text{OO}}} = \begin{cases} 1.170(8)_{\text{stat.}} (30)_{\text{Traj.}}^{\text{str.}} (0)_{\text{syst.}}^{\text{str.}} & \text{(NLEFT)} \\ 1.139(6)_{\text{stat.}} (27)_{\text{Traj.}}^{\text{str.}} (28)_{\text{syst.}}^{\text{str.}} & \text{(PGCM)} \end{cases}$$

Results: Pearson coefficient $\rho(v_2\{2\}^2, \langle p_T \rangle)$



- Suppression of ρ due to the large deformation of ^{20}Ne

$$\rho_{\text{Ne+Ne}} - \rho_{\text{O+O}} \propto (\beta_{2,16\text{O}}^3 - \beta_{2,20\text{Ne}}^3)$$



- Ratios/differences between $^{20}\text{Ne} + ^{20}\text{Ne}$ and $^{16}\text{O} + ^{16}\text{O}$ reduce uncertainties
- NLEFT and PGCM in good agreement

- Interface between low- and high-energy nuclear physics
 - ◊ Combines several state-of-the-art frameworks and software
 - ◊ NLEFT and PGCM give consistent results
 - ◊ Many things to understand or improve
 - PGCM: less phenomenological sampling of nucleon positions
 - more methods and interactions

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- We make **predictions** for the future $^{16}\text{O} + ^{16}\text{O}$ run at LHC in 2025
- **Proposal: combine the $^{16}\text{O} + ^{16}\text{O}$ run with a $^{20}\text{Ne} + ^{20}\text{Ne}$ run**
 - ◊ Ratios/differences reduce considerably systematic uncertainties
 - ◊ Make use of the large ground-state deformation of ^{20}Ne

⇒ **Test hydrodynamic behavior in a small systems**